Problems in Geometry

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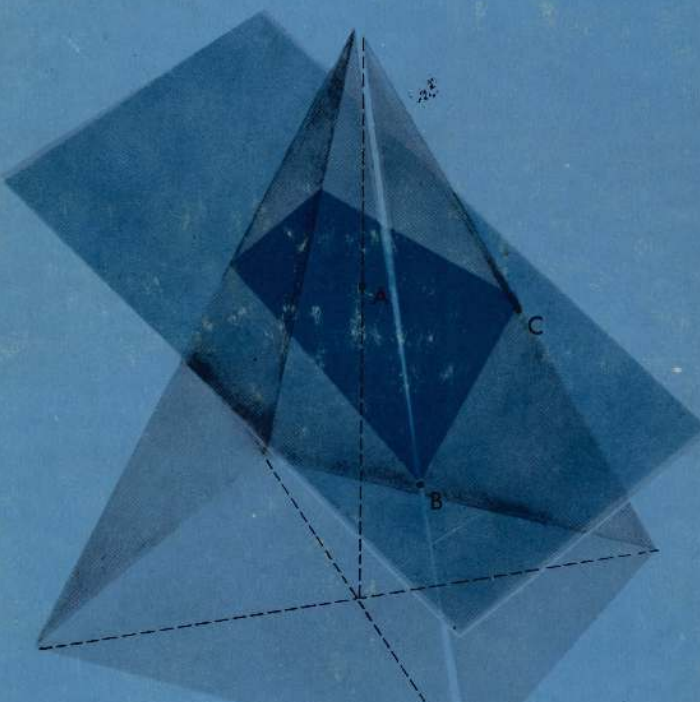

The book contains a collection of 1351 problems (with answers) in plane and solid geometry for technical schools and colleges. The problems are of varied content, involving calculations, proof, construction of diagrams, and determination of the spatial location of geometrical points.

It gives sufficient problems to meet the needs of students for practical work in geometry, and the requirements of the teacher for varied material for tests, etc.

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Problems in Geometry

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CHAPTER I

REVIEW PROBLEMS

1. The Ratio and Proportionality of Line Segments, Similarity of Triangles

1. Are the line segments AC and CD (AC and DB), into which the line segment AB is divided by the points C and D, commensurable, if:

(a) $AC : CD : DB = 3.5 : 4\frac{1}{6} : 3\frac{1}{3}$;

(b) $AC : CD : DB = \sqrt{2} : \frac{5}{2} : \frac{\sqrt{2}}{3}$?

2. Is it possible to construct a triangle from three line segments which are in the following ratios:

(a) 2 : 3 : 4; (b) 2 : 3 : 5; (c) 1 : 1 : 1;

(d) 2 : 5 : 80; (e) 2 : 75 : 75?

3. 1. Given on the axis Ox are the points A (6; 0) and B (18; 0). Find the coordinates of the point C which divides the line segment AB in the following ratios:

(a) $AC : CB = 1$; (b) $AC : CB = 1 : 2$;

(c) $AC : CB = 5 : 1$.

2. The point B divides the line segment in the ratio $m : n$. Find the lengths of the segments AB and BC if $AC = a$.

4. Given in the orthographic system of coordinates are two points: A (2; 4) and B (8; 12). Find the coordinates of the point M which divides the segment AB in the ratio:

(a) $AM : MB = 1$; (b) $AM : MB = 2 : 1$.

5. 1. Compute the scale if the true length $AB = 4$ km is represented in the drawing by a segment $AB = 8$ cm.

2. Compute the true length of the bridge which is represented on a map drawn to the scale 1 : 20,000 by a line segment 9.8 cm long.

6. Given a triangle ABC in which $AB = 20$ dm and $BC = 30$ dm. A bisector BD is drawn in the triangle (the point D lies on the side AC). A straight line DE is drawn through the point D and parallel to the side AB (the point E lies on the side BC), and another straight line EK is drawn through the point E and parallel to BD. Determine the side AC if $AD - KC = 1$ cm.

7. The sides of a triangle are 40 cm, 50 cm and 60 cm long. In what ratio is each bisector divided by the other ones as measured from the vertex?

8. The sides of an angle A are intersected by two parallel straight lines BD and CE, the points B and C lying on one side of this angle, and D and E on the other. Find AB if $AC + BC = 21$ m and $AE : AD = 5 : 3$.

9. Drawn from the point M are three rays. Line segments $MA = 18$ cm and $MB = 54$ cm are laid off on the first ray, segments $MC = 25$ cm and $MD = 75$ cm on the second one, and a segment MN of an arbitrary length on the third. A straight line is drawn through the point A and parallel to BN to intersect the segment MN at the point K. Then a straight line is drawn through the point K and parallel to ND. Will the latter line pass through the point C?

10. The bases of a trapezium are equal to m and n ($m > n$), and the altitude to h. Find: (1) the distance between the shorter base and the point at which the extended lateral sides intersect, (2) the ratio in which the diagonals are divided by the point of their intersection, (3) the distances between the point of intersection of the diagonals and the bases of the trapezium.

11. What must the diameter of an Earth's satellite be for an observer to see a total lunar eclipse at a distance of 1000 km from it?

12. The length of the shadow cast by a factory chimney is 38.5 m. At the same moment the shadow cast by a man 1.8 m in height is 2.1 m long. Find the height of the chimney.

13. Prove that two similar triangles inscribed in one and the same circle are equal to each other.

14. Inscribed in an angle are two mutually tangent circles whose radii are 5 cm and 13 cm. Determine the distances between their centres and the vertex of the angle.

15. A triangle ABC with an obtuse angle B is inscribed in a circle. The altitude AD of the triangle is tangent to the circle. Find the altitude if the side $BC = 12$ cm, and the segment $BD = 4$ cm.

16. Two circles whose radii are 8 cm and 3 cm are externally tangent. Determine the distance between the point of tangency of the circles and a line externally tangent to both of them.

17. A triangle ABC is inscribed in a circle. A straight line is drawn through the vertex B and parallel to the line tangent to the circle at the point A to intersect the side AC at the point D. Find the length of the segment AD if $AB = 6$ cm, $AC = 9$ cm.

18. A circle is inscribed in an isosceles triangle whose lateral side is 54 cm and the base is 36 cm. Determine the distances between the points at which the circle contacts the sides of the triangle.

19. Given a triangle ABC whose sides are: $AB = 15$ cm, $AC = 25$ cm, $BC = 30$ cm. Taken on the side AB is a point D through which a straight line DE (the point E is located on AC) is drawn so that the angle AED is equal to the angle ABC. The perimeter of the triangle ADE is equal to 28 cm. Find the lengths of the line segments BD and CE.

20. The bases of a trapezium are 7.2 cm and 12.8 cm -ong. Determine the length of the line segment which is parallel to the bases and divides the given trapezium into two similar trapeziums. Into what parts is one of the lateral sides (12.6 cm long) of the given trapezium divided by this segment?

21. Given in the triangle ABC: $AB = c$, $BC = a$, $AC = b$, and the angle BAC is twice as big as the angle ABC. A point D is taken on the extension of the side CA so that $AD = AB$. Find the length of the line segment BD.

22. In an acute triangle ABC the altitudes AD and CE are drawn. Find the length of the line segment DE if $AB = 15$ cm, $AC = 18$ cm and $BD = 10$ cm.

23. Prove that a straight line passing through the point of intersection of the extended lateral sides of a trapezium and also through the point of intersection of its diagonals divides both bases of the trapezium into equal parts.

24. Prove that if two circles are tangent externally, then the segment of the tangent line bounded by the points of tangency is the mean proportional to the diameters of the circles.

25. Inscribe a rectangle in a given triangle so that one of its sides lies on the base of the triangle, and the vertices of the opposite angles on the lateral sides of the triangle and that the sides of the rectangle are in the same ratio as 1 : 2.

26. Find the locus of the points which divide all the chords passing through the given point of a circle in the ratio of m to n.

2. Metric Relationships in a Right-Angled Triangle

27. 1. Compute the hypotenuse given the sides containing the right angle:

(a) 15 cm and 36 cm; (b) 6.8 and 2.6.

2. Compute one of the sides containing the right angle given the hypotenuse and the other side:

(a) 113 and 15; (b) 5 and 1.4; (c) 9 and 7.

3. Given two elements of a right-angled triangle compute the remaining four elements:

(a) $b = 6$, $b_c = 3.6$; (b) $a_c = 1$, $b_c = 9$; (c) $a = 68$, $h = 60$.

28. Prove that the ratio of the projections of the sides containing the right angle on the hypotenuse is equal to the ratio of the squares of these sides.

29. Prove that if in a triangle ABC the altitude CD is the mean proportional to the segments AD and BD of the base AB, then the angle C is a right one.

30. A perpendicular dropped from a point of a circle on its diameter divides the latter into segments whose difference is equal to 12 cm. Determine the diameter if the perpendicular is 8 cm long.

31. Given two line segments a and b. Construct a triangle with the sides a, b and $\sqrt{ab}$.

32. In a right-angled triangle the bisector of the right angle divides the hypotenuse in the ratio $m : n$. In what ratio is the hypotenuse divided by the altitude dropped from the vertex of the right angle?

33. In a right-angled triangle the perpendicular to the hypotenuse dropped from the midpoint of one of the sides containing the right angle divides the hypotenuse into two segments: 5 cm and 3 cm. Find these sides.

34. An altitude BD is drawn in a triangle ABC. Constructed on the sides AB and BC are right-angled triangles ABE and BCF whose angles BAE and BCF are right ones and $AE = DC$, $FC = AD$. Prove that the hypotenuses of these triangles are equal to each other.

35. The sides of a triangle are as $5 : 12 : 13$. Determine them if the difference between the line segments into which the bisector of the greater angle divides the opposite side is equal to 7 cm.

36. 1. One of the sides containing the right angle in a right-angled triangle is 6 cm longer than the other one; the hypotenuse is equal to 30 cm. Determine the bisector of the larger acute angle.

2. The sides containing the right angle are equal to 6 cm and 12 cm. Determine the bisector of the right angle.

37. A circle with the radius of 8 cm is inscribed in a right-angled triangle whose hypotenuse is equal to 40 cm. Determine the sides containing the right angle and the distance between the centres of the inscribed and circumscribed circles.

38. The base of an isosceles triangle is equal to 48 cm, and the lateral side to 40 cm. Find the distances between the centre of gravity and the vertices of this triangle.

39. The sides containing the right angle are: $AC = 30$ cm, $BC = 16$ cm. From C as centre with radius CB an arc is drawn to intersect the hypotenuse at point D. Determine the length of the line segment BD.

40. A quarter timber has the greatest bending strength if the perpendiculars dropped from two opposite vertices of the cross-section rectangle divide its diagonal into three equal parts. Determine the size of the cross-section for such a timber which can be made from a log 27 cm in diameter.

41. At what distance does the cosmonaut see the skyline if his spaceship is at an altitude of 300 km above the surface of the Earth, whose radius is equal to 6400 km?

42. A circle with the centre at the point M (3; 2) touches the bisector of the first quadrant. Determine: (1) the radius of the circle, (2) the distance between the centre of the circle and the origin of coordinates.

43. A rhombus is inscribed in a parallelogram with an acute angle of 45° so that the minor diagonal of the former serves as the altitude of the latter. The larger side of the parallelogram is equal to 24 cm, the side of the rhombus to 13 cm. Determine the diagonals of the rhombus and the shorter side of the parallelogram.

44. The base of an isosceles triangle is equal to 12 cm and the altitude to 9 cm. On the base as on a chord a circle is constructed which touches the lateral sides of the triangle. Find the radius of this circle.

45. The radius of a circle is equal to 50 cm; two parallel chords are equal to 28 cm and 80 cm. Determine the distance between them.

46. The radii of two circles are equal to 54 cm and 26 cm, and the distance between their centres to 1 m. Determine the lengths of their common tangent lines.

47. 1. From a point 4 cm distant from a circle a tangent line is drawn 6 cm long. Find the radius of the circle.

2. A chord 15 cm distant from the centre is 1.6 times the length of the radius. Determine the length of the chord.

48. The upper base BC of an isosceles trapezium $ABCD$ serves as a chord of a circle tangent to the median (midline) of the trapezium and is equal to 24 cm. Determine the lower base and the lateral side of the trapezium if the radius of the circle is equal to 15 cm and the angle at the lower base to 45°.

49. An isosceles trapezium with the lateral side of 50 cm, is circumscribed about a circle whose radius is equal to 24 cm. Determine the bases of the trapezium.

50. A circle is circumscribed about an isosceles trapezium. Find the distances between the centre of this circle and each base of the trapezium if the midline of the trapezium equal to its altitude is 7 cm long, and its bases are as 3 : 4.

51. A segment AE (1 cm long) is laid off on the side of a square $ABCD$. The point E is joined to the vertices B and C of the square. Find the altitude BF of the triangle BCE if the side of the square is equal to 4 cm.

52. Two sides of a triangle are equal to 34 cm and 56 cm; the median drawn to the third side is equal to 39 cm. Find the distance between the end of this median and the longer of the given sides.

53. In an obtuse isosceles triangle a perpendicular is dropped from the vertex of the obtuse angle to the lateral side to intersect the base of the triangle. Find the line segments into which the base is divided by the perpendicular if the base of the triangle is equal to 32 cm, and the altitude to 12 cm.

54. A rectangle whose base is twice as long as the altitude is inscribed in a segment with an arc of 120° and an altitude h. Determine the perimeter of the rectangle.

55. Determine the kind of the following triangles (as far as their angles are concerned) given their sides:

(1) 7, 24, 26; (2) 10, 15, 18; (3) 7, 5, 1; (4) 3, 4, 5.

56. 1. Given two sides of a triangle equal to 28 dm and 32 dm containing an angle of 120° determine its third side.

2. Determine the lateral sides of a triangle if their difference is equal to 14 cm, the base to 26 cm, and the angle opposite it to 60°.

57. In a triangle ABC the base $AC = 30$ cm, the side $BC = 51$ cm, and its projection on the base is equal to 46.2 cm. In what portions is the side AB divided by the bisector of the angle C?

58. Prove that if M is a point on the altitude BD of a triangle ABC, then $AB^2 - BC^2 = AM^2 - CM^2$.

59. The diagonals of a parallelogram are equal to 14 cm and 22 cm, its perimeter to 52 cm. Find the sides of the parallelogram.

60. Three chords intersect at one point inside a circle. The segments of the first chord are equal to 1 dm and 12 dm, the difference between the segments of the second

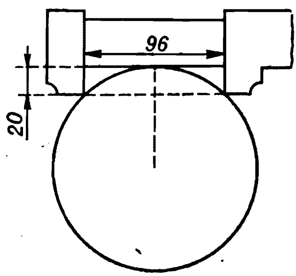

Fig. 1

one is equal to 4 dm, and the segments of the third chord are in the ratio of 4 to 3. Determine the length of each chord.

61. According to the established rules the radius of curvature of a gauge should not be less than 600 m. Are the following curvatures allowable:

(1) the chord is equal to 120 m and the sagitta to 4 m;

(2) the chord is equal to 160 m and the sagitta to 4 m?

62. Compute the radius of the log (Fig. 1) using the dimensions (in mm) obtained with the aid of a caliper.

3. Regular Polygons, the Length of the Circumference and the Arc

63. 1. What regular polygons of equal size can be used to manufacture parquet tiles?

2. Check to see whether it is possible to fit without a gap round a point on a plane: (a) regular triangles and regular hexagons; (b) regular hexagons and squares; (c) regular octagons and squares; (d) regular pentagons and regular decagons. What pairs (from those mentioned above) can be used for parqueting a floor?

64. Cut a regular hexagon into:

(1) three equal rhombuses; (2) six equal isosceles triangles.

65. A regular triangle is inscribed in a circle whose radius is equal to 12 cm. A chord is drawn through the midpoints of two arcs of the circle. Find the segments of the chord into which it is divided by the sides of the triangle.

66. Given the apothem of a hexagon inscribed in a circle $k_6 = 6$. Compute R, a_3, a_4, a_6, k_3, k_4.

67. Inscribed in a circle are a regular triangle, quadrilateral and hexagon whose sides are the sides of a triangle inscribed in another circle of radius $r = 6$ cm. Find the radius R of the first circle.

68. A common chord of two intersecting circles is equal to 20 cm. Find (accurate to 1 mm) the distance between the centres of the circles if this chord serves as the side of an inscribed square in one circle, and as the side of an inscribed regular hexagon in the other, and the centres of the circles are situated on different sides of the chord.

69. 1. Constructed on the diameter of a circle, as on the base, is an isosceles triangle whose lateral s de is equal to the side of a regular triangle inscr bed in this circle. Prove that the altitude of this triangle is equal to the side of a square inscribed in this circle.

2. Using only a pair of compasses, construct a circle and divide it into four equal parts.

70. A regular quadrilateral is inscribed in a circle and a regular triangle is circumscribed about it; the difference between the sides of these polygons is equal to 10 cm. Determine the circumference of the circle (accurate to 0.1 cm).

71. The length of the circumference of the outer circle of the cross section of a pipe is equal to 942 mm, wall thickness to 20 mm. Find the length of the circumference of the inner circle.

72. A pulley 0.3 m in diameter must be connected with another pulley through a belt transmi sion. The first pulley revolves at a speed of 1000 r.p.m. What diameter must the second pulley have to revolve at a speed of 200 r.p.m.?

73. Two artificial satellites are in circular orbits about the Earth at altitudes of h_1 and h_2 ($h_1 > h_2$), respectively. In some time the altitude of flight of each satellite decreased by 10 km as compared with the initial one. The length of which orbit is reduced to a greater extent?

74. A regular triangle ABC inscribed in a circle of radius R revolves about the point D which is the foot of the altitude BD of the triangle. Find the path traversed by the point B during a complete revolution of the triangle.

75. A square with the side $6\sqrt{2}$ cm is inscribed in a circle about which an isosceles trapezium is circumscribed. Find the length of the circumference of a circle constructed on the diagonal of this trapezium if the difference between the lengths of its bases is equal to 18 cm.

76. 1. A circle of radius 8 m is unbent to form an arc of radius 10 m. Find the central angle thus obtained.

2. A circle of radius 18 dm is unbent to form an arc subtending a central angle of 300°. Find the radius of the arc.

3. An arc of radius 12 cm subtending a central angle of 240° is bent to form a circle. Find the radius of the circle thus obtained.

4. An arc of radius 15 cm is bent to complete a circle of radius 6 cm. How many degrees did the arc contain?

5. Compute the length of 1° of the Earth meridian, taking the radius of the Earth to be equal to 6400 km.

6. Prove that in two circles central angles corresponding to arcs of an equal length are inversely proportional to the radii.

77. A regular triangle ABC with the side a moves without sliding along a straight line L, which is the extension of the side AC, rotating first about the vertex C, then B and so on. Determine the path traversed by the point A between its two successive positions on the line L.

78. On the altitude of a regular triangle as on the diameter a semi-circle is constructed. Find the length of the arc contained between the sides of the triangle if the radius of the circle inscribed in the triangle is equal to m cm.

4. Areas of Plane Figures

79. Determine the sides of a rectangle if they are in the ratio of 2 to 5, and its area is equal to 25.6 cm^2.

80. Determine the area of a rectangle whose diagonal is equal to 24 dm and the angle between the diagonals to 60°.

81. Marked off on the side BC of a rectangle $ABCD$ is a segment BE equal to the side AB. Compute the area of the rectangle if $AE = 32$ dm and $BE : EC = 5 : 3$.

82. The projection of the centre of a circle inscribed in a rhombus on its side divides the latter into the segments 2.25 m and 1.21 m long. Find the area of the rhombus.

83. Determine the area of a circle if it is less than the area of a square circumscribed about it by 3.44 cm^2.

84. The altitude BE of a parallelogram $ABCD$ divides the side AD into segments which are in the ratio of 1 to 3. Find the area of the parallelogram if its shorter side AB is equal to 14 cm, and the angle $ABD = 90°$.

85. The distance between the centre of symmetry of a parallelogram and its longer side is equal to 12 cm. The area of the parallelogram is equal to 720 cm^2, its perimeter being equal to 100 cm. Determine the diagonals of the parallelogram if the difference between them equals 24 cm.

86. 1. Determine the area of a rhombus whose side is equal to 20 dm and one of the diagonals to 24 dm.

2. The side of a rhombus is equal to 30 dm, the smaller diagonal to 36 dm. Determine the area of a circle inscribed in this rhombus.

87. The diagonals of a parallelogram are the axis of ordinates and the bisector of the first and third quadrants. Find the area of the parallelogram given the coordinates of its two vertices: (3; 3) and (0; —3).

88. The perimeter of an isosceles triangle is equal to 84 cm; the lateral side is to the base in the ratio of 5 to 4. Determine the area of the triangle.

89. The median of a right-angled triangle drawn to the hypotenuse is 6 cm long and is inclined to it at an angle of 60°. Find the area of this triangle.

90. A point M is taken inside an isosceles triangle whose side is a. Find the sum of the lengths of the perpendiculars dropped from this point on the sides of the triangle.

91. In an isosceles triangle ABC an altitude AD is drawn to its lateral side. The projection of the point D on the base AC of the triangle divides the base into the segments m and n. Find the area of the triangle.

92. Prove that the triangles formed by the diagonals of a trapezium and its lateral sides are equal.

93. The altitude of a regular triangle is equal to 6 dm. Determine the side of a square equal to the circle circumscribed about the triangle.

94. A square whose side is 4 cm long is turned around its centre by 45°. Compute the area of the regular polygon thus obtained.

95. Find the area of the common portion of two equilateral triangles one of which is obtained from the other by turning it round its centre by an angle of 60°. The side of the triangle is equal to 3 dm.

96. The area of a right-angled triangle amounts to 28.8 dm^2, and the sides containing the right angle are as 9 : 40. Determine the area of the circle circumscribed about this triangle.

97. In an isosceles trapezium the parallel sides are equal to 8 cm and 16 cm, and the diagonal bisects the angle at the base. Compute the area of the trapezium.

98. The perimeter of an isosceles trapezium is 62 m. The smaller base is equal to the lateral side, the larger base being 10 m longer. Find the area of the trapezium.

99. A plot fenced for a cattle-yard has the form of a right-angled trapezium. The difference between the bases of this trapezium is equal to 30 m, the smaller lateral side to 40 m. The area of the plot amounts to 1400 m^2. How much does the fence cost if 1 m of its length costs 80 kopecks?

100. A trapezium is inscribed in a circle of radius 2 dm. Compute the area of the trapezium if its acute angle is equal to 60° and one of its bases is equal to the lateral side.

101. Two parallelly running steel pipes of an air duct each 300 mm in diameter are replaced by one polyethylene tube. What diameter must this tube have to ensure the same capacity of the air duct?

102. The area of a circle whose radius is 18 dm is divided by a concentric circle into two equal parts. Determine the radius of this circle.

103. Find the cross-sectional area of a hexagonal nut (Fig. 2).

104. Find the area of a figure bounded by three semicircles shown in Fig. 3, if $AB = 4$ cm and $BD = 4\sqrt{3}$ cm.

105. 1. The length of the circumference of a circle is equal to 25.12 m. Determine the area of the inscribed regular triangle.

2. Determine the area of a circle inscribed in an equilateral triangle whose side is equal to 3.6 m.

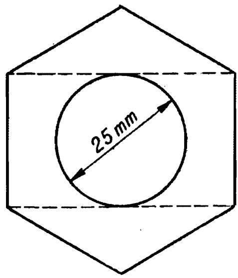

Fig. 2

106. Compute the area of a circle inscribed in an isosceles triangle whose base is equal to $8\sqrt{3}$ cm and the angle at the base to 30°.

107. Two circles 6 cm and 18 cm in diameter are externally tangent. Compute the area bounded by the circles and a line tangent to them externally.

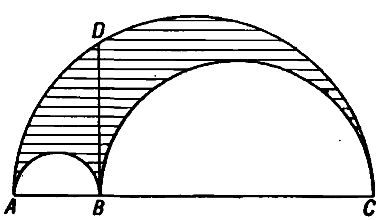

Fig. 3

108. On the hypotenuse of a right-angled isosceles triangle as on the diameter a semi-circle is constructed. Its end-points are connected by a circular arc drawn from the vertex of the right angle as centre, its radius being equal to the lateral side of the triangle. Prove that the sickle thus obtained is equal to the triangle.

109. A square with the side a is inscribed in a circle. On each side of the square as on the diameter a semicircle is constructed. Compute the sum of the areas of the sickles thus obtained.

110. The greatest possible circle is cut out of a semicircle. The same was done with each of the scraps thus obtained. What is the percentage of the waste?

111. The plan of a plot has the form of a square with the side 10.0 cm long. Knowing that the plan is drawn to the scale 1 : 10,000, find the area of the plot and the length of its boundary.

112. Figure 4 presents the plan of a plot drawn to the scale 1 : 1000. Compute the area of the plot given the

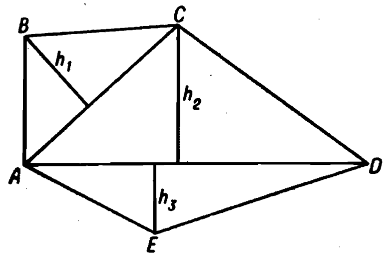

Fig. 4

following dimensions: $AC = 6$ cm, $AD = 7.6$ cm, $h_1 = 3$ cm, $h_2 = 4.8$ cm, $h_3 = 3.2$ cm.

113. 1. A straight line parallel to the base of a triangle divides its lateral side in the ratio 2 : 3 (as measured from the base). In what ratio is the area of the triangle divided by this line?

2. Given the sides of a triangle: 26 cm, 28 cm, 30 cm. A straight line is drawn parallel to the larger side so that the perimeter of the trapezium obtained is equal to 66 cm. Determine the area of the trapezium.

114. By what percentage will the area of a circle be increased if its radius is increased by 50 per cent?

115. 1. Construct a circle whose area is equal to: (a) the sum of the areas of two given circles; (b) the difference between their areas.

2. Construct a square whose area is n times greater than the area of the given square ($n = 2$; 4; 5).

CHAPTER II

SOLVING TRIANGLES

5. Solving Right-Angled Triangles

116. Find from the tables:

1. (a) $\sin 27°23'$; (b) $\cos 18°32'$; (c) $\cos \frac{\pi}{8}$; (d) $\tan 60°41'$; (e) $\cot 70°20'$; (f) $\sin 3°44'$; (g) $\cos 88°36'$; (h) $\tan 3°52'$.

2. (a) $\log \sin 22°8'$; (b) $\log \sin 80°23'$; (c) $\log \cos 87°50'$; (d) $\log \cos 63°15'$; (e) $\log \tan 37°51'$; (f) $\log \tan 85°12'$; (g) $\log \cot 77°28'$; (h) $\log \cot 15°40'$.

117. Using the tables, find the positive acute angle x if:
(1) $\sin x$ is equal to: 0.2079; 0.3827; 0.9858; 0.0579;
(2) $\cos x$ is equal to: 0.8643; 0.6490; 0.1846; 0.0847;
(3) $\tan x$ is equal to: 0.0148; 0.9774; 1.2576; 4.798;
(4) $\cot x$ is equal to: 0.8421; 1.2813; 2.0751; 0.0935.

118. Using the tables, find the positive acute angle x if:
(1) $\log \sin x$ is equal to: $\bar{1}.4044$; $\bar{1}.9314$; $\bar{1}.1716$; $\bar{2}.1082$;
(2) $\log \cos x$ is equal to: $\bar{1}.6418$; $\bar{1}.3982$; $\bar{1}.7810$; $\bar{2}.8475$;
(3) $\log \tan x$ is equal to: $\bar{2}.9625$; $\bar{1}.2570$; $\bar{1}.7793$; 0.7791;
(4) $\log \cot x$ is equal to: 1.5207; $\bar{2}.6952$; $\bar{1}.7839$; 0.8718.

119. Find with the aid of a slide-rule:
(1) $\sin 32°$, $\sin 32°40'$, $\sin 32°48'$, $\sin 71°15'$, $\sin 4°40'$;
(2) $\cos 30°$, $\cot 74°14'$, $\cos 81°12'$, $\cos 86°40'$;
(3) $\tan 2°30'$, $\tan 3°38'$, $\tan 43°15'$, $\tan 72°30'$;
(4) $\cot 2°$, $\cot 12°36'$, $\cot 42°54'$, $\cot 85°39'$.

120. Using a slide-rule, find the positive acute angle x if
(1) $\sin x$ is equal to: 0.53; 0.052; 0.0765; 0.694;
(2) $\cos x$ is equal to: 0.164; 0.068; 0.763; 0.857;

(3) $\tan x$ is equal to: 0.0512; 2.84; 0.863; 1.342;
(4) $\cot x$ is equal to: 0.824; 1.53; 0.065; 0.853.

121. Solve the following right-angled triangles with the aid of a slide-rule:
(1) $c \approx 8.53$, $A \approx 56°41'$; (2) $a \approx 360$ m, $B \approx 36°30'$;
(3) $c \approx 28.2$, $a \approx 16.4$; (4) $a \approx 284$ m, $b \approx 170$ m.

122. Solve the following right-angled triangles, using the tables of values of trigonometric functions:
(1) $c = 58.3$, $A = 65°14'$; (2) $a = 630$ m, $B = 36°30'$;
(3) $c = 82.2$, $a = 61.4$; (4) $a = 428$ m, $b = 710$ m.

123. Solve the following right-angled triangles, using the tables of logarithms of trigonometric functions:
(1) $c = 35.8$, $A = 56°24'$; (2) $a = 306$ m, $B = 63°32'$;
(3) $c = 22.8$, $b = 14.6$; (4) $a = 284$ m, $b = 170$ m.

In Problems 124 through 126 solve the isosceles triangles, introducing the following notation: $a = c =$ lateral sides, $b =$ base, $A = C =$ angles at the base, $B =$ angle at the vertex, $h =$ altitude, $h_1 =$ altitude drawn to a lateral side, $2p =$ perimeter, $S =$ area.

124. (1) $a \approx 590$, $A \approx 56°36'$;
(2) $a = 276$ m, $B = 123°$;
(3) $b = 25.6$, $A = 49°45'$;
(4) $b = 547.8$, $B = 40°42'$.

125. (1) $a = 87.5$, $b = 139.6$;
(2) $b = 92.6$, $h = 72.4$;
(3) $a = 200$ m, $h = 174$ m;
(4) $b = 820$, $h_1 = 666$.

126. (1) $b = 120.7$, $S = 1970$;
(2) $h = 98.4$ m, $S = 1880$ m^2;
(3) $2p = 406.5$, $A = 72°36'$;
(4) $S = 66$, $a = 16$.

127. The length of a line segment is equal to 52.0 cm and its projection on the axis to 36.4 cm. Find the angle between the line segment and the axis.

128. The summit of a mountain is connected with its foot by a suspension rope-way 4850 m long. Determine the height of the mountain if the average slope upgrade of the way is 27°.

129. A plane is seen at an angle of 35° at the moment it is flying above the observer at a distance of 5 km from him. At what altitude is it flying?

130. Figure 5 shows two wedges. The wedge B rests against the wedge A and can move in the vertical direction. How much will the wedge B rise if the wedge A is driven 0.75 m rightwards ($\alpha = 25°$).

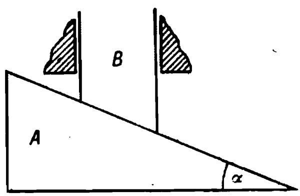

Fig. 5

131. The angle of slope of fine sand $\alpha \approx 31°$. What area is occupied by a heap of sand 1 m high?

132. The section of a ditch has the form of an isosceles trapezium whose lower base is equal to 80 cm, upper base to 160 cm, and altitude to 90 cm. Find the steepness of the walls of the ditch.

133. Figure 6 shows lamps installed along moving staircases in the Moscow metro. The side view of the support for the lamps has the form of a right-angled triangle whose vertical side is 10 cm and horizontal side is 17.3 cm long. Determine the angle of elevation of the staircase.

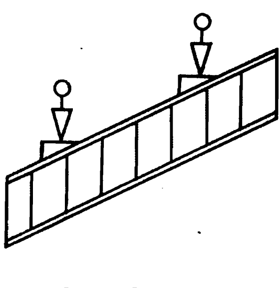

Fig. 6

134. A pendulum 70 cm long is swinging between two points 40 cm apart. Determine the arc of swinging.

135. Figure 7 presents a height as an element of a topographic map. The contour lines are drawn to connect the points lying at one and the same altitude, the horizontal cutting planes being passed through each 4 m. Determine the average steepness of slopes at various places of the height and in various directions if the map is drawn to the scale 1 : 10,000.

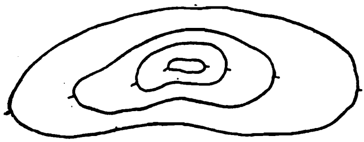

Fig. 7

136. From an observat on post situated at an altitude of 5.5 m above the river level the banks are seen at angles $\alpha_1 = 8°20'$ and $\alpha_2 = 3°40'$ (Fig. 8). Determine the width of the river at the place of observation. The angles of observation are contained in a plane perpendicular to the direction of the river.

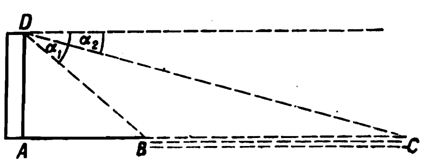

Fig. 8

137. Along railway lines there are posts with marks showing the gradient of the permanent way, e.g. $\left|\frac{0.007}{800}\right|$. The figures signify that the line rises or falls 0.007 metre per metre over a length of 800 metres. Calculate the gradient of the permanent way on this section in degrees, and the height of the ascent in metres.

138. A force $R = 42.0$ N is resolved into two mutually perpendicular forces, one of which is at an angle of 61°20′ to the given force R. Determine the value of each of the component forces.

139. A weight $P = 50$ N is suspended from a bracket (Fig. 9). Calculate the strain on arm a and the force compressing bracket c, if the angle $\alpha = 43°$.

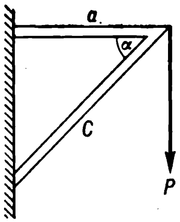

Fig. 9

140. 1. A barrel of petrol must be tilted 11°20′ to the horizontal. What force must be applied to tilt it if the weight of the petrol in the barrel is 1300 N? (Friction to be ignored.)

2. A motor car is travelling at a speed of 72 km/hr. To the driver it appears that the raindrops are falling at an angle of 40° to the perpendicular. At what speed is the rain striking the ground?

141. The speed of a motor boat in still water is 8.5 km/hr. The current of the river is 1.5 km/hr. The boatman must carry a load across the river and land it at a point on the other bank directly opposite. At what angle α to the point of landing must the boat be steered? And what will its speed be?

142. The corner of a room is represented in Fig. 10. It is taken that the plane of the floor is horizontal and the corner of the room vertical. Two points A and B are 0.5 m from the corner; the distance between them is 0.7 m. Determine the precise angle between the two walls of the room.

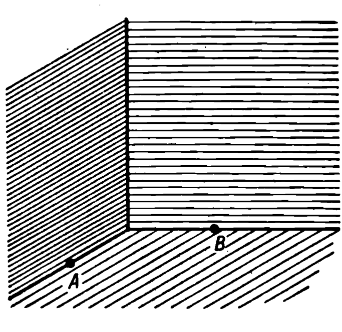

Fig. 10

143. How long will a transmission belt need to be, when the two pulleys are respectively 12 cm and 34 cm in diameter, and their centres are one metre apart?

144. Two points A and B lie on different sides of a motor road (Fig. 11). In order to get from A to B it is necessary to drive 3.5 km along a side road that joins the main road at an angle of 40°, then drive 2.5 km along the main road and turn right onto another side road, which makes an angle of 70° with the main road, and drive another 4 km. All sections of the roads traversed are straight. How much will the distance between A and B be shortened when a straight road is built between them?

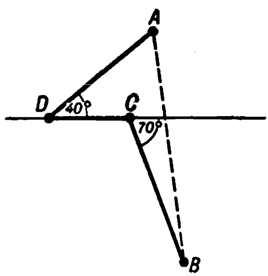

Fig. 11

145. The diagonals of a rhombus are equal to 2.3 dm and 3.6 dm. Determine the angles of the rhombus and its perimeter.

146. The base of an isosceles triangle is to its altitude as 3 : 4. Find the angles of the triangle.

147. The base of an isosceles triangle and the altitude dropped on a lateral side are equal to 18 cm and 13 cm, respectively. Determine the lateral side of the triangle.

148. Determine the radius of a circle and the length of the sagitta of the segment if a chord 9.0 cm long subtends an arc of 110°.

149. Find the central angle subtended by a circular arc of radius 14.40 dm if the chord is 22.14 dm long.

150. Find the radius of a circle if the angle between the tangent lines drawn from a point M is equal to 48°16′

and this point is 26 cm distant from the centre of the circle.

151. 1. A rhombus with an acute angle α is circumscribed about a circle of radius r. Find the area of the rhombus.

2. An isosceles trapezium with an acute angle α is circumscribed about a circle of radius r. Find the area of the trapezium.

152. 1. Compute the perimeter of a regular nonagon inscribed in a circle of radius $R = 10.5$ cm.

2. Determine the rad'us of a circle if the perimeter of an inscribed regular dodecagon is equal to 70 cm.

153. 1. Compute the perimeter of a regular fourteen-sided polygon circumscribed about a circle of radius $R = 90.3$ cm.

2. Determine the radius of a circle if the perimeter of a circumscribed regular eighteen-sided polygon is equal to 82.4 cm.

154. The diagonal d of a right-angled trapezium is perpendicular to the lateral side which orms an angle α with the base of the trapezium. Compute the perimeter if $d = 15$ cm and $\alpha = 43°$.

155. In an isosceles triangle the altitude is equal to 30 cm, and the altitude dropped on a lateral side to 20 cm. Determine the angles of the triangle.

156. In a right-angled triangle the bisector of the right angle divides the hypotenuse in the ratio of 2 to 3. Determine the angles of the triangle.

157. In an isosceles triangle with the base of 30 cm and the angle at the base of 63° a square is inscribed so that two of its vertices are found on the base, and the other two on the lateral sides of the triangle. Determine the area of the square.

158. Given in an isosceles triangle ABC: $AB = BC = a$ and $AC = b$. The bisectors of the angles A and C intersect at the point D. Determine the angle ADC.

159. In a square with the side a another square is inscribed so that the angle between the sides of these squares is equal to α. Determine the perimeter and area of the inscribed square.

160. Determine the angles of a right-angled triangle if the arc of radius equal to the smaller of the sides containing the right angle drawn from the vertex of the latter divides the hypotenuse in the ratio of 8 to 5.

161. Straight lines MA and MB are drawn tangent to a circle. The arc AB is equal to α ($\alpha < 180°$). The perimeter of the triangle AMB is equal to $2p$. Determine the distance AB between the points of tangency.

162. From the end-points of the arc ACB tangent lines are drawn which intersect at point M. Determine the perimeter of the figure $MACB$ if the radius of the arc is equal to R and its magnitude to α radians.

6. Solving Oblique Triangles

Law of Cosines

163. The sides of a triangle are equal to 27 cm and 34 cm, and the angle between them to $37°17'$. Compute the third side.

164. The sides of a triangle are equal to 15 cm, 18 cm and 22 cm. Compute the medium angle of the triangle.

165. Given an equilateral triangle ABC. The point D divides the side BC into the segments $BD = 4$ cm, and $CD = 2$ cm. Determine the segment AD.

166. The sides of a parallelogram are equal to 42.3 dm and 67.8 dm, and its angle to 56°. Find the diagonals of the parallelogram.

167. The sides of a parallelogram are equal to 32.5 cm and 38.3 cm, and one of its diagonals to 27.4 cm. Compute the angles of the parallelogram.

168. In a trapezium the lateral sides are equal to 72 cm and 93 cm, and one of its bases to 115 cm. The angles at the given base are equal to 68° and 42°. Compute the diagonals of the trapezium.

169. The chords AB and CD intersect at point M at an angle of 83°. Find the perimeter of the quadrangle

ADBC if $AB = 24$ cm, and the chord *CD* is divided by the point *M* into the segments equal to 8 cm and 12 cm.

170. In a triangle *ABC* the sides *AB* and *BC* are equal to 15 cm and 22 cm, respectively, and the angle between them to 73°28′. Find the segments of the side *AC* of the triangle into which it is divided by the bisector of the angle *B*.

171. Prove that for any triangle the following inequalities hold true

$$(1)\ a \geqslant 2\sqrt{bc}\sin\frac{A}{2};\quad (2)\ b \geqslant 2\sqrt{ac}\sin\frac{B}{2};$$

$$(3)\ c \geqslant 2\sqrt{ab}\sin\frac{C}{2}.$$

172. To determine the distance between the points *A* and *B*, where a bend of the river is situated (Fig. 12), a point *C* is chosen so that the distances *AC* and *BC* can be measured directly. Find *AB* if $AC = 820$ m, $BC = 650$ m, and the angle $ACB = 130°$.

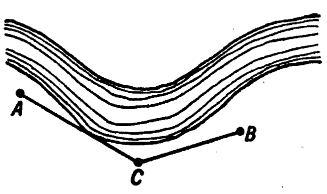

Fig. 12

173. At 7 o'clock in the morning a passenger plane took off from the town *A* and after 30 minutes' stay in the town *B* at 8.10 it turned by 35° to the right and at 9.00 landed in the town *C*. Determine the distance between the towns *A* and *C* if the average speed of the plane over each section of the flight was equal to 320 km/hr.

174. Applied at the point *M* is the force $P \approx 18.3$ N. One of its components $P_1 \approx 12.8$ N and the angle between the given force and its component P_1 $\alpha = 37°$. Compute the other component.

175. A material point is acted upon by the forces of 43 N and 55 N. Determine the angles between each of these forces and an equilibrium force of 70 N.

176. Resolve the force $P_1 \approx 240$ N into two forces $P_2 \approx 185$ N and $P_3 \approx 165$ N. At what angle to each other must the forces P_2 and P_3 act?

Law of Sines

177. 1. The perimeter of the triangle ABC is equal to 24 m, $\sin A : \sin B : \sin C = 3 : 4 : 5$. Find the sides and angles of the triangle.

2. Given in the triangle ABC: $a - c = 22$ dm, $\sin A : \sin B : \sin C = 63 : 25 : 52$. Find the sides and angles of this triangle.

178. The diagonal of a parallelogram divides its angle into two portions: 60° and 45°. Find the ratio of the sides of the parallelogram.

179. The hypotenuse of a right-angled triangle is equal to 15 cm, and one of the acute angles to 37°. Compute the bisector of the right angle.

180. Given in the triangle ABC: the angle $A = 63°18'$, $AC = 16$ cm, $BC = 19$ cm. Find the angle formed by the bisectors of the angles A and B.

181. The diagonal of an isosceles trapezium is 75 cm long and divides the obtuse angle into two unequal parts: 80° and 36°. Determine the sides of the trapezium.

182. From the end-points of a chord 18 cm long a tangent and a secant are drawn to form a triangle together with the chord. Determine the external portion of the secant if the angles of the triangle adjacent to the chord are equal to 136° and 27°.

183. To find the distance between the points A and B situated across the river an arbitrary point C is taken. Then all necessary measurements were carried out and the following results obtained: $AC = 140$ m, the angles $BAC \doteq 67°$, $BCA = 73°$. Using these data, find the distance AB.

184. To determine the height of a waste heap of a mine (Fig. 13) the basis $AB = 100$ m is chosen. The angles $\alpha_1 = 25°$ and $\alpha_2 = 17°$ were determined with the aid of a goniometer whose height is 1.4 m. Find the height of the waste heap, using the results of the measurements.

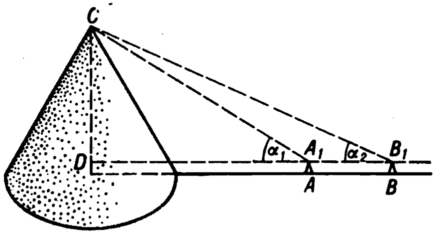

Fig. 13

185. It is required to determine the height of a tree growing on the slope of a hill. For this purpose the angles α and β, and the distance AB were measured: $\alpha = 27°$, $\beta = 12°$, $AB = 40$ m. Compute the height of the tree, using the results of the measurements (Fig. 14).

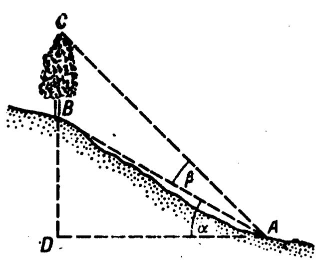

Fig. 14

186. A force P is resolved into two components one of which is $P_1 = 35.6$ N. The components form angles of 27°30′ and 39°45′ with the force P. Find this force.

Areas of Triangles, Parallelograms and Quadrilaterals

187. Compute the area of a triangle given two sides and an angle between them.

(1) $a = 42$ m, $b = 28$ m, $C = 82°36'$;
(2) $a = 28.3$ dm, $c = 73.4$ dm, $B = 112°44'$;
(3) $b = 254$ m, $c = 388$ m, $A = 39°21'$.

188. The area of a triangle is equal to 48 cm^2, two of its sides to 12 cm and 9 cm, respectively. Determine the angle formed by these sides.

189. Compute the area of a parallelogram given two of its sides and the angle contained by them:
(1) $AB = 18$ dm, $AD = 49$ dm, $A = 78°44'$;
(2) $AB = 2.3$ m, $AD = 11.5$ m, $A = 93°18'$;
(3) $AB = 234$ m, $BC = 48$ m, $A = 21°46'$.

190. The area of a parallelogram is equal to 14 dm^2, its sides to 3.8 dm and 4.6 dm. Determine the angles of the parallelogram.

191. Compute the area of a rhombus given its side and angle:
(1) $a = 43.6$ m, $\alpha = 74°28'$; (2) $a = 18$ cm, $\alpha = 120°8'$.

192. Compute the area of a quadrilateral given its diagonals and the angle between them:
(1) $d_1 = 24$ cm, $d_2 = 36$ cm, $\alpha = 48°34'$;
(2) $d_1 = 0.35$, $d_2 = 0.48$, $\alpha = 74°47'$.

193. Compute the area of a rectangle given its diagonals and the angle between the diagonals:
(1) $d = 9.3$ dm, $\alpha = 48°$;
(2) $d = 38$ cm, $\alpha = 85°15'$.

194. Compute the area of an isosceles trapezium given its diagonals and the angle between the diagonals:
(1) $d = 47$ cm, $\alpha = 54°30'$;
(2) $d = 0.6$ cm, $\alpha = 78°20'$.

195. A rectangle $ABCD$ is inscribed in a circle of radius R. Determine the area of this rectangle if the arc AB is equal to α.

196. Determine the area of a triangle given the radius of the circumscribed circle and two angles α and β.

197. Show that in any triangle

$$ab = 2Rh_c; \quad ac = 2Rh_b; \quad bc = 2Rh_a.$$

198. Prove that if in an isosceles trapezium the diagonals are mutually perpendicular, then the trapezium is equal to the isosceles triangle constructed on the diagonal of the trapezium as on a side containing the right angle.

Basic Cases of Solving Oblique Triangles

199. Given three sides:
(1) $a = 44$; $b = 58$; $c = 62$;
(2) $a = 29$; $b = 44$; $c = 59$;
(3) $a = 272.4$; $b = 1035$; $c = 1305$.

200. Given two sides and the included angle:
(1) $a = 420$; $b = 371$; $C = 67°19'$;
(2) $a = 22.9$ m; $c = 16.9$ m; $B = 39°52'$;
(3) $b = 38$; $c = 52$; $A = 122°34'$.

201. Given a side and two angles:
(1) $a = 730$; $B = 86°3'$; $C = 50°56'$;
(2) $b = 13.2$; $A = 21°48'$; $B = 123°42'$;
(3) $c = 97.5$ m; $A = 87°55'$; $C = 12°53'$.

Given two sides and the angle opposite one of them:

202. (1) $a = 28.9$; $b = 22.4$; $A = 81°22'$;
(2) $b = 354$; $c = 520$; $B = 43°55'$;
(3) $a = 402$ m; $c = 258$ m; $C = 38°16'$.

203. (1) $a = 0.38$; $b = 0.59$; $B = 64°11'$;
(2) $b = 45.5$; $c = 25.0$; $C = 33°19'$;
(3) $a = 1054$ m; $c = 1350$ m; $A = 48°46'$.

Particular Cases of Solving Oblique Triangles

Notation: a, b, c = the sides of a triangle; A, B, C = = angles opposite them; S = area; $2p$ = perimeter; R = radius of the circumscribed circle; r = radius of the inscribed circle; h_a, h_b, h_c = altitudes; l_a, l_b, l_c = = bisectors.

204. (1) $R = 8.9$; $A = 83°17'$; $B = 58°16'$;
(2) $S = 609.1$; $A = 45°28'$; $B = 54°23'$;
(3) $h_a = 53.7$; $B = 105°20'$; $C = 15°33'$.

205. (1) $l_a = 75.8$; $B = 98°30'$; $C = 14°20'$;
(2) $a + b = 488.8$; $A = 70°24'$; $B = 50°16'$;
(3) $a - b = 34$; $A = 108°$; $B = 28°$.

206. (1) $r = 5.4$; $A = 22°$; $B = 49°$;
(2) $S = 1460$ m^2; $a = 32.5$ m; $B = 114°50'$;
(3) $a = 72$; $b = 52$; $A = 2B$.

207. (1) $S = 25$; $ab = 58$; $\sin A = \cos B$;
(2) $a = 120.0$ m; $b = 29.6$ m; $h_c = 23.8$ m;
(3) $h_a = 8$; $h_b = 12$; $h_c = 18$.

Heron's Formula

208. Determine the area of a triangle given its sides:

(1) 13, 14, 15; (2) 12, 17, 25; (3) 14.5, 12.5, 3;

(4) 10, $17\frac{1}{3}$, $24\frac{2}{3}$; (5) $\sqrt{13}$, $\sqrt{10}$, $\sqrt{5}$.

209. Determine the sides of a triangle if:

(1) they are as 7 : 8 : 9, and the area of the triangle is equal to $48\sqrt{5}$ cm^2;

(2) they are as 17 : 10 : 9, and the area of the triangle is equal to 576 cm^2.

210. Determine the area of a quadrilateral given a diagonal equal to 34 cm and two sides 20 cm and 42 cm long lying on one side of the diagonal, the other two 16 cm and 30 cm long on its other side.

211. Determine the area of a parallelogram whose sides are equal to 15 cm and 112 cm, and one of the diagonals to 113 cm.

212. Determine the area of a parallelogram if one of its sides is equal to 102 cm, and the diagonals to 80 cm and 148 cm.

213. Determine the area of a trapezium whose bases are equal to 12 dm and 4 dm, and the lateral sides to 2.6 dm and 7.4 dm.

214. Determine the area of a trapezium whose bases are equal to 50 cm and 18 cm, and the diagonals to 40 cm and 36 cm.

215. The radii of two intersecting circles are equal to 68 cm and 156 cm, and the distance between their centres to 176 cm. Determine the length of the common chord.

216. The sides of a triangle are equal to 10 cm, 12 cm and 18 cm. Determine the area of the circle whose diameter is equal to the medium altitude of the triangle.

217. The sides of a triangle are equal to 26 cm, 28 cm and 30 cm. A semi-circle is inscribed in the triangle so that its diameter lies on the greater side of the triangle. Compute the area bounded by the sides of the triangle and the semi-circle.

Radii r and R of Inscribed and Circumscribed Circles and the Area S of a Triangle

218. Compute the area of an equilateral triangle if: (1) the radius of the circumscribed circle is equal to R; (2) the radius of the inscribed circle is equal to r.

219. Determine R and r for a triangle whose sides are: (1) 3, 4, 5; (2) 29, 8, 35; (3) 13, 14, 15.

220. The sides of a triangle are equal to 9 cm, 15 cm and 12 cm. Can we construct an isosceles triangle whose sides would be equal to the radii of the circles inscribed in and circumscribed about the given triangle?

221. Show that in any triangle $abc = 4pRr$.

222. Prove that if the lengths of the sides a, b, c of a triangle form an arithmetic progression, then the product Rr is equal to $\frac{1}{6}$ the product of the extreme terms of this progression.

223. Prove that in any triangle

$$(1)\ \frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}=\frac{1}{r}; \quad (2)\ h_a+h_b+h_c=$$

$$=\frac{ab+ac+bc}{2R}=\frac{S}{R}(\operatorname{cosec} A+\operatorname{cosec} B+\operatorname{cosec} C).$$

224. Determine the radii of the inscribed and circumscribed circles if the sides of the triangle are as 9 : 10 : 17, its area being equal to 144 cm^2.

225. The base of a triangle is equal to m, one of the lateral sides is to the radius of the circumscribed circle as 2 : 3. Determine the altitude dropped onto the third side of the triangle.

226. Find the area of a circle inscribed in a right-angled triangle one of whose sides containing the right angle is equal to 60 cm, and its projection on the hypotenuse to 36 cm.

227. One side of a triangle is equal to 25 cm. The ratio of its area to the radius of the incircle is equal to 35 cm, and the product of the area by the radius of the circumscribed circle to 2975 cm^3. Find the two other sides of the triangle.

228. The point of intersection of two mutually perpendicular chords divides one of them into the segments of 5 cm and 9 cm and cuts off a segment 12 cm long from the other one. Find the area of the circle.

229. The base of an isosceles triangle is to the lateral side as 6 : 5. The altitude drawn to the base is equal to 8 cm. Determine the radius of the incircle and that of the circumscribed circle.

230. In a circle of radius $R = 2$ dm a triangle is inscribed two angles of which are equal to 60° and 45°. Find the area of the triangle.

231. An acute triangle two sides of which are equal to 58 cm and 50 cm is inscribed in a circle of radius 36.25 cm. Find the third side and the area of the triangle.

232. Construct a right-angled triangle given the hypotenuse and the radius of the incircle.

233. Compute the area of a regular octagon and a regular decagon if: (1) $R = 10$ cm, (2) $r = 10$ cm.

234. Find the radii of the incircle and the circumscribed circle if the area of a regular icosagon is equal to 273.3 cm^2.

235. Find the area of a regular dodecagon inscribed in a circle about which a regular hexagon is circumscribed with the side of 8 cm.

236. A regular triangle and a regular hexagon are inscribed in one and the same circle of radius $R = 20$ cm so that the vertices of the triangle coincide with those of the hexagon. Compute the area bounded by the perimeters of the triangle and hexagon.

237. The area of a regular decagon is equal to 124.5 cm^2. Compute the area of the annulus bounded by the circles inscribed in and circumscribed about this decagon.

238. Given the perimeter P of a regular n-gon determine its area.

Miscellaneous Problems

239. In order to build a railway a tunnel had to be constructed between points A and B. To determine the length and direction of the tunnel in a given locality, a point C was chosen, from where the points A and B could be seen, and the following distances were defined: $AC = 370$ m, $BC = 442$ m and $\angle ACB = 108°40'$. Find out the direction and the length of the tunnel.

240. In a shaft, where a horizontal occurrence of a seam lies, from the same wall occurs two drifts AD and BC with corresponding lengths of 320 m and 380 m. The distance between the entrances of the drifts is 12 m. It is necessary for the ends of the drifts to be joined with a third drift. Calculate the direction and length of the third drift, if the measurements of the angles are as follows: $\alpha = 105°$, $\beta = 115°$ (Fig. 15).

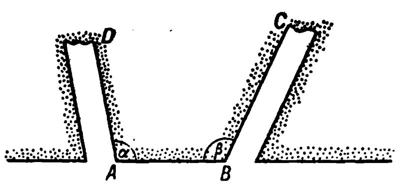

Fig. 15

241. Radio direction-finders, situated at points A and B 48 km apart, plotted the directions α_1 and β_1 for the enemy ship C_1. In 1 hour 10 minutes the observations were repeated to determine the direction of the angles α_2 and β_2. Determine the direction of movement and speed of the ship, if the following results were ascertained: $\alpha_1 = 78°30'$, $\beta_1 = 54°18'$, $\alpha_2 = 53°40'$, $\beta_2 = 98°36'$ (Fig. 16).

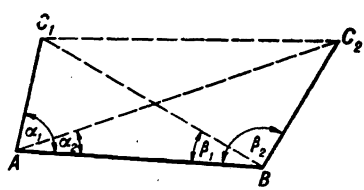

Fig. 16

242. In order to draw a line through a given point A, parallel to an inaccessible line on which two points C and D can be seen, a point B was chosen and the following measurements were noted: $AB = 100$ m, $\angle CAD = 25°$,

$\angle CAB = 110°$, $\angle ABC = 30°$, $\angle ABD = 80°$. Using the results of the measurements, give the direction Ax parallel to CD (Fig. 17).

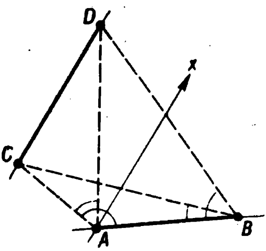

Fig. 17

243. Using the measurements of the previous problem, from the point A draw a perpendicular line to an inaccessible line on which points C and D can be seen.

244. To draw a straight line through a point M and extend it to a point C, which intersects the lines AA_1 and BB_1, when the point C cannot be seen from the point M, draw an arbitrary line AB through the point M and measure the angles AA_1B and B_1BA. Find the direction of the line MC, using the following data: $AB = 80$ m, $\angle CAB = 80°$, $\angle ABC = 63°$, $AM = 50$ m, $MB = 30$ m (Fig. 18).

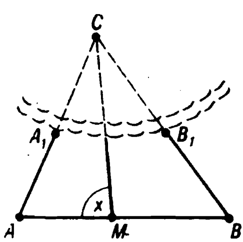

Fig. 18

245. Two forces $F_1 \approx 50$ N and $F_2 \approx 100$ N are acting on a point at an angle of 120°. Compute the resultant force and the angles which it makes with the forces F_1 and F_2.

246. 1. The force $R = 80$ N must be resolved into two forces F_1 and F_2. It is known that $F_1 = 60$ N and forms an angle of 50° with the resultant force. Find the force F_2 and its direction relative to the resultant force.

2. Resolve the force $R = 300$ N into two components forming angles of 35° and 45° with the resultant force.

247. The diagonals of a parallelogram intersect at an angle of 80°. Find the sides of the parallelogram if its greater diagonal is equal to 72 cm and inclined to the base at an angle of 30°.

248. In an isosceles trapezium the lateral side and diagonal are respectively equal to 15.5 dm and 20.3 dm and the angle at the base to 50°. Determine the area of the trapezium.

249. Determine the diagonals of a regular n-gon whose side is equal to a.

250. The diagonal of a trapezium inscribed in a circle of radius R forms angles α and 3α with its lateral sides. Determine the perimeter of the trapezium.

251. Circumscribed about, and inscribed in, a circle of radius R is a ring made up of n equal circles. Determine the radii of these circles at $n = 3, 4, 6$.

252. From the centre of an equilateral triangle with the side a a circle is circumscribed intersecting its sides so that the outside arcs amount to α radians. Determine the length of the portion of the circumference contained inside the triangle.

253. Drawn in a circle of radius R on both sides of the centre are two parallel chords subtending arcs α and β; the end-points of the chords are joined to each other. Determine the perimeter of the trapezium thus obtained.

254. Given in a triangle are the three sides: $AB = 52$ cm, $BC = 60$ cm, $AC = 56$ cm. Determine the portion of the area of this triangle found between the altitude and bisector drawn from the vertex B.

255. The perimeter of a rhombus is twice the perimeter of a square. Whose area is greater if the angle of the rhombus is equal to 14°30′?

256. Using the law of sines, prove that the bisector of an internal angle in a triangle divides the opposite to it side into portions proportional to the adjacent sides.

257. Through the end-points of a chord which divides the circle of radius $R = 18$ cm in the ratio of 7 to 12 tangent lines are drawn. Determine the area of the triangle formed by the chord and tangents.

258. Determine the area of a circle inscribed in a sector whose radius is equal to R, and the arc to α.

259. The sides of a parallelogram are equal to a and b $(a < b)$, and the angle between them to α. Determine the area of the quadrilateral formed by the bisectors of the angles of the parallelogram.

260. Compute the area of the hatched portion of a circle (Fig. 19) whose radius is equal to 20 cm if the arcs AB and CD are equal to 15° and 75° respectively.

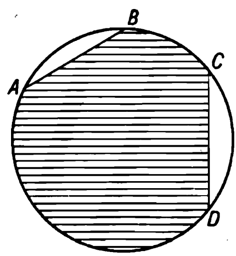

Fig. 19

261. Drawn through the centre of symmetry of a rhombus are its altitudes which serve as diagonals of a rectangle. Find the area of this rectangle if the side of the rhombus is equal to 10 cm and acute angle amounts to 60°.

262. Taken inside a circle of radius 8 cm is an arbitrary point M which is 2 cm distant from the centre of the circle. Drawn through this point are two mutually perpendicular chords and a diameter forming an angle of 45° with each chord. Compute the area of the quadrilateral whose vertices are the end-points of the chords.

263. Two sides of a triangle are equal to 24 cm and 12 cm, and the median of the third side to $\sqrt{279}$ cm. Find the area of the circumscribed circle.

264. Compute the area of a quadrilateral $ABCD$, in which $AB = AC = 17$ dm, $AD = 44$ dm, $CD = 39$ dm, and the angle between the side AB and diagonal AC is equal to 30°.

265. Two sides of a triangle are equal to 5 cm and 6 cm, and its area to 5.28 cm². Find the third side.

266. The parallel sides of a trapezium are equal to 10 cm and 20 cm, and non-parallel ones to $6\sqrt{2}$ cm and $2\sqrt{13}$ cm. Compute the diagonals of the trapezium.

267. In a parallelogram with an acute angle of 30° the diagonals are to each other as $\sqrt{2-\sqrt{3}} : \sqrt{2+\sqrt{3}}$. Find the ratio of the sides.

268. Compute the area of a triangle if:

(1) its sides are as 15 : 26 : 37, and the radius of the incircle is equal to 16 cm;

(2) the sides are as 29 : 25 : 6, and the radius of the circumscribed circle is equal to $36\frac{1}{4}$ cm.

269. Compute the area of a triangle, two sides of which are equal to 6 cm and 9 cm, and the bisector of the angle between them to $4\sqrt{3}$ cm.

CHAPTER III

STRAIGHT LINES AND PLANES IN SPACE

7. Basic Concepts and Axioms. Two Straight Lines in Space

270. 1. A circle and a plane have two common points. May we assert that the circle is contained in this plane?

2. Three points of a circle lie on a plane. May we assert that the circle is contained in the plane? Two planes have three common points not lying in a straight line. How are these planes situated?

271. How many planes can be drawn through: (a) one point; (b) two points; (c) three points not in a straight line; (d) four points each three of which are not in a straight line?

272. 1. A spherical surface and a straight line have two common points. Do other points of the line belong to this surface?

2. The same for: (a) a cylindrical surface, (b) a conical surface. Consider various positions of the two common points of the straight line and each of the surfaces.

273. 1. Prove that if a plane and a straight line not contained in this plane have a common point, then the latter is a unique point.

2. Prove that all straight lines intersecting a straight line a and passing through the point A not lying on the straight line a lie in one plane.

3. Prove that a straight line intersecting two parallel straight lines lies in the plane containing these lines.

274. 1. Straight lines a and b intersect at point M. Where do all straight lines lie which intersect each of the given lines and (a) do not pass through the point M, (b) pass through the point M?

2. Four rays are drawn from a point of space. How many planes can be drawn through these rays? Consider all possible cases.

3. Can two opposite sides of an oblique quadrilateral be parallel?

275. Given an equilateral triangle ABC and a point D not contained in its plane. The point D is joined to the centre O of the triangle and vertex B. Draw a plane through DB and DO. Will this plane pass through the midpoint of the side AC?

276. Given points A and B on one of the two intersecting planes α and β, and a point C on the other. Construct the lines of intersection of these planes and the plane γ passing through the points A, B, C.

277. Given a straight line AB not contained in the plane P. Construct the point of intersection of this line and the plane P.

278. Given points A, B and C outside the plane P. Construct the line of intersection of the planes ABC and P.

279. Given a triangular pyramid $SABC$ with the altitude SO. Construct an orthogonal projection of: (a) the point D of the edge SB; (b) the point E of the lateral face SAB; (c) the line segment MN connecting the points M and N of the lateral edges SA and SB, on the plane containing the base of the pyramid.

280. Construct a point E on a lateral face of a regular quadrangular pyramid given its projection E_1 on the plane containing the base of the pyramid.

281. Given points D and E on the lateral edges SA and SB of a triangular pyramid $SABC$. Construct the point of intersection of the straight line DE and the plane containing the base of the pyramid.

282. Given points D and E on the lateral edge AS and face SBC of a triangular pyramid $SABC$. Construct the trace of the straight line DE on the plane containing the base of the pyramid.

283. Given points E and F on the lateral edge AS and altitude SO of a quadrangular pyramid $SABCD$. Construct a second point of intersection of the straight line, passing through these points, and the surface of the pyramid.

284. 1. Construct the points of intersection of a straight line, passing through the point M located on one of the sides of the lower base of a cube and point N on its axis, and all the planes containing the faces of the cube.

2. Given a cube $ABCDA_1B_1C_1D_1$, a point M contained in its face DD_1C_1C, and a point N on the face BB_1C_1C. Construct the points of intersection of the straight line MN and the faces $ABCD$ and $A_1B_1C_1D_1$.

285. Construct the section of a parallelepiped by a plane passing through the three points given on the lateral edges. Consider two cases: (1) the cutting plane intersects all the lateral edges of the parallelepiped and

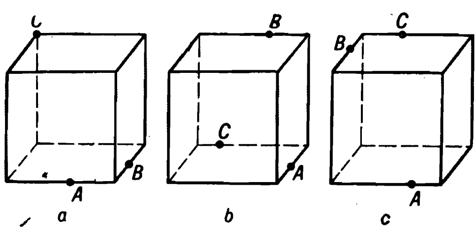

Fig. 20

(2) the cutting plane does not intersect the fourth edge.

286. Construct the section of a cube by a plane passing through the points A, B and C (Fig. 20).

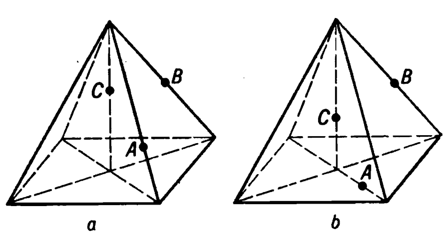

Fig. 21

287. Construct the section of a regular quadrangular pyramid by a plane passing through the points A, B and C (Fig. 21).

288. In a cube with the edge 8 cm long construct the section by a plane passing through the midpoints of three edges emanating from one vertex, and compute the area of the section.

289. 1. Through a point A lying on a given straight line a draw a line forming a given angle with the given line.

2. Through a point M situated outside a given line a draw a straight line forming a given angle with the given line.

290. At two points of a straight line two perpendiculars are erected to it. How can these perpendiculars be mutually situated?

291. The plane $ABCD$ of a trapezium intersects the plane α along a straight line a. Will a and AC on which a diagonal of the trapezium lies be skew lines?

292. Two straight lines are intersected by a third one. How many planes can be drawn through these lines?

293. 1. Given two intersecting straight lines. How can one of them be situated with respect to a straight line: (a) parallel to the other one; (b) intersecting the other one; (c) forming with the other one skew lines?

2. The same for two parallel lines.

3. The same for two skew lines.

294. Determine the angle between a diagonal of a cube and its edge which does not intersect this diagonal.

295. The base of a pyramid is a rectangle with the sides 12 cm and 16 cm long. The lateral edges are of the same length, and the altitude is equal to 24 cm. Determine the angle between a lateral edge of the pyramid and a diagonal of the base which does not intersect this edge.

8. Straight Lines Perpendicular and Inclined to a Plane

296. 1. A straight line a is perpendicular to the plane α containing a rhombus $ABCD$. What angles are formed by the line a and the sides of the rhombus?

2. A straight line a is perpendicular to two parallel lines lying in the plane α. How is the line a situated with respect to the plane α?

297. Through the side BC of a triangle ABC a plane α is drawn perpendicular to the side AB. A triangle BDC with a right angle B is constructed in the plane α. How is the side BD situated with respect to the plane ABC and the side BC with respect to the plane ABD?

298. 1. Through a given point in space draw a plane perpendicular to a given straight line.

2. At a given point in a plane erect a perpendicular to this plane.

299. Prove that if a straight line a lies in a plane perpendicular to a straight line b, then the latter lies in a plane perpendicular to a.

300. 1. Prove that if a lateral edge of a triangular pyramid is perpendicular to the opposite side of the base, then the vertex of the pyramid is projected on the altitude of the base.

2. Prove that the opposite edges of a regular triangular pyramid are mutually perpendicular.

301. Given in Fig. 22 is a rhombus $ABCD$: $MB = MD$. Prove that BD is perpendicular to the plane OMC.

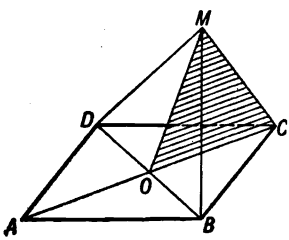

Fig. 22

302. Prove that the diagonal of a cube and the diagonal of a face, which do not pass through one and the same vertex, are mutually perpendicular.

303. At the point O of intersection of the diagonals of a square $ABCD$ a perpendicular OM is erected to its plane. Prove that MC is perpendicular to BD.

304. 1. At the point A lying on a given straight line a erect a perpendicular to this line.

2. From the point M not lying on a given straight line a drop a perpendicular to this line.

305. 1. Find the locus of points in space equidistant from a given point.

2. Find the locus of points in space equidistant from two given points.

3. Find the locus of points in space equidistant from three given points.

4. Find the locus of points in space equidistant from four given points. Consider all possible cases.

306. Find the locus of points in space equidistant from the vertices of: (a) a square, (b) a rectangle, (c) an isosceles trapezium. Is it possible to find in space points equidistant from all the vertices of any plane polygon?

307. Find the locus of points in space equidistant from the sides of: (a) a triangle, (b) a square, (c) a rhombus. Is it possible to find in space points equidistant from all the sides of any plane polygon?

308. 1. Find the locus of points in space equidistant from all the points of a circle.

2. Find the locus of points contained in a plane α which are situated at a given distance a from a given point M lying outside the plane α. Consider all the cases.

309. The edge of a cube is equal to a. Determine the distance between the diagonal of the cube and the diagonal of a face of the cube which does not intersect the former.

310. 1. Given two straight lines a and b. Construct a straight line intersecting the given lines and perpendicular to them. Consider all possible cases.

2. Through a given point draw a straight line perpendicular to two given skew lines.

311. Given outside the plane of a rhombus $ABCD$ is a point M situated from the sides AB and CD at distances MK and ML, respectively. Prove that the sum (or the difference) of the projections of the line segments MK and ML on the plane of the rhombus is equal to the altitude of the rhombus.

312. A point B is taken on a perpendicular AO (Fig. 23) to a plane α (the point O lies in the plane α). The points A and B are found at distances AC and BD, respectively, from the straight line a lying in the plane α and not passing through the point O. Find a mistake in Fig. 23 and correct it.

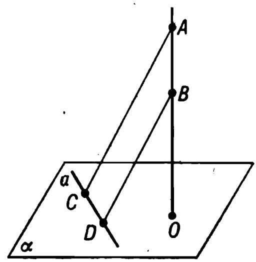

Fig. 23

313. Through a given point A contained in the plane α draw in this plane a straight line perpendicular to a given line a. Consider all cases of possible positions of the straight line a and the plane α.

314. From the vertex B of an isosceles triangle ABC ($AB = BC$) a perpendicular BM is erected to the plane of the triangle. Drop a perpendicular from the point M to the side AC, find the length of this perpendicular and the distance between this perpendicular and the vertex B if the angle ABC is equal to 120° and $AC = BM = 4$ dm.

315. At the vertex A of a right-angled triangle ABC (the angle C is a right one) a perpendicular AM is erected to the plane containing this triangle. Drop a perpendicular from the point M to the side BC. At what distance must the point M be situated from the plane of the triangle for the triangle AMC to be congruent with the triangle ABC?

316. The sides containing the right angle in a right-angled triangle are 9 cm and 16 cm long. At the midpoint of the hypotenuse a perpendicular 6 cm long is erected to the plane of the triangle. Find the distances between the end-point of the perpendicular and the sides containing the right angle, as also between that and the vertex of the right angle of the given triangle.

317. At the vertex of the right angle of a right-angled triangle a perpendicular 2 m long is erected to its plane. The end-point of the perpendicular is $2\sqrt{3}$ m distant from the hypotenuse, and 6 m from the mid-point of the hypotenuse. Determine the area of the triangle.

318. At the mid-point of the hypotenuse AB of a right-angled triangle ABC a perpendicular DM is erected to its plane. Drop perpendiculars ME and MF from the point M to the sides containing the right angle and find the perimeter of the triangle MEF if $MD = 12$ dm, $AC = 18$ dm and $BC = 32$ dm. Prove that the point M is equidistant from the vertices of the triangle ABC.

319. At the mid-point D of the side AC of a regular triangle ABC a perpendicular DM is erected to its plane. Drop a perpendicular ME from the point M to the side AB and find the radius of the circle circumscribed about the triangle DME if $DM = BD = 12$ cm.

320. Given in a triangle ABC: $AB = 13$ cm, $BC = 14$ cm and $AC = 15$ cm. At the vertex A a perpendicular AD 5 cm long is erected to the plane of the triangle. Find the distance between the point D and side BC.

321. A perpendicular AM is erected to the plane of a regular hexagon $ABCDEF$ at the point A and the point M is joined to the vertex C. Find the distances between the point M and each vertex of the hexagon if $AM = 3$ cm, and the area of the hexagon is equal to $81\sqrt{3}$ cm².

322. A perpendicular AM is erected to the plane of a parallelogram $ABCD$ at its vertex A. Drop a perpendicular ME from the point M to the side BC of the parallelogram. Consider two cases: (a) the angle A is acute; (b) the angle A is obtuse. Compute the area of the parallelogram if $AM = 2AD$ and the area of the triangle AME is equal to 24 cm².

323. Compute the distance between a point M and the plane of a regular triangle ABC if the side of the triangle is equal to a and $MA = MB = MC = b$.

324. A perpendicular OM 8 cm long is erected at the centre O of a regular triangle ABC. Find the distances between the point M and each vertex and side of the triangle if its area is equal to $27\sqrt{3}$ cm^2.

325. Compute the distance between a point M and the plane containing a triangle ABC if $AB = BC = 12$ cm; $MA = MB = MC = 10$ cm and the bisector of the angle B is 9 cm long.

326. The sides of a right-angled triangle are equal to 6 cm and 8 cm. Find the distance between the plane of the triangle and a point M such that is 13 cm distant from each vertex of the triangle.

327. The lateral side of an isosceles triangle is equal to 10 cm and its base to 12 cm. Find the distance between the plane of the triangle and a point M such that is 5 cm distant from each side of the triangle.

328. The sides containing the right angle in a right-angled triangle are equal to 12 cm and 9 cm. Find the distance between the plane of the triangle and a point M if the latter is 5 cm distant from each of the sides of the triangle.

329. In the triangles ABC and ABD the common side AB lies in the plane α. The projections of the sides AC and BD on this plane are perpendicular to AB; $AC = 8$ cm, $BD = 20$ cm, $AB = 15$ cm, the points C and D are, respectively, $4\sqrt{3}$ cm and 16 cm distant from the plane α. Compute the sides BC, AD and the distances between the perpendiculars CC_1 and DD_1 dropped onto the plane α.

330. The diagonals of a rhombus are equal to 30 cm and 40 cm. Find the distance between the plane of the rhombus and a point M if the latter is 20 cm distant from each of its sides.

331. At the vertex A of a square $ABCD$ whose side is equal to 3 dm a perpendicular AM is erected to the plane of the square. The line segment MB is 4 dm long. Compute the length of the line segment MC.

332. A point M is taken outside the plane of a parallelogram at equal distances from the vertices of acute angles and at equal distances from the vertices of obtuse angles. Prove that the line segment MO (where O is the point of intersection of the diagonals of the parallelogram) is perpendicular to the plane of the parallelogram. Find the distances between the point M and the sides of the parallelogram equal to m and n, respectively, if its area is equal to Q, and $MO = H$.

333. The forces $F_1 = 6\text{N}$ and $F_2 = 7\text{N}$ are applied to a point at an angle of 60° to each other. Applied to the same point is a third force $F_3 = 11\text{N}$ which is perpendicular to the former two. Find the resultant of the three given forces.

344. A plane is drawn through a lateral side of a trapezium whose bases are 12 cm and 16 cm long. The projection of the midline on this plane is equal to 8 cm. Determine the projections of the bases on this plane.

335. Given a rectangle $ABCD$. At some point O in its plane a perpendicular OK is erected to this plane. The point K is joined to the vertex B. At what position of the point O in the plane of the rectangle will the angle ABK be acute, right, obtuse?

9. Angles Formed by a Straight Line and a Plane

336. Given in a square $ABCD$ is a perpendicular CM to the plane of the square. Construct the angles of inclination of the straight lines MA and MB to the plane of the square and the angle at which the straight line MB is inclined to the plane MAC.

337. Given in a square $ABCD$: SO is a perpendicular to the plane of the square erected at the point O of intersection of its diagonals. Construct the angle of inclination of: (1) the straight line SA to the plane of the square; (2) the straight line SO to the plane SBC.

338. 1. The dimensions of a rectangular parallelepiped are 3 cm, 4 cm and 5 cm. Find the angles formed by its diagonal and the faces.

2. Determine the angle between the diagonal of a cube and its face.

339. Prove that if through the vertex of an angle ABC a straight line BD is drawn inclined to the plane of this angle and forming equal angles with the sides BA and BC of this angle, then its projection on the plane containing the angle is the bisector of the angle ABC.

340. Given in a triangle ABC: the angle $B = 60°$. From the vertex B a straight line is drawn inclined to the plane of the triangle and forming angles of 60° with the sides BA and BC. Find the angle between th s line and the plane containing the triangle.

341. In a regular quadrangular pyramid the altitude is equal to the side of the base. Find the angle of inclination of the lateral edge to the plane of the base and the angle between the lateral edge and the side of the base. Compare these angles.

342. In a regular triangular pyramid the lateral edge is equal to l and forms an angle of 30° with the plane containing the base. Find the side of the base.

343. From a point taken outside a plane two straight lines are drawn inclined to this plane whose lengths are 10 cm and 7 cm. The projections of these lines on the plane are as $6 : \sqrt{15}$. Determine the distance between the point and the plane.

344. 1. An inclined line is equal to a. How long is the projection of this line on a plane if the line is inclined to the plane at an angle of: (a) 30°; (b) 45°; (c) 60°?

2. A point is situated at a distance h from a plane. Find the length of an inclined line drawn from it at an angle of: (a) 30°, (b) 45°, (c) 60° to the plane.

345. From a point A in a plane α two straight lines AB and AC are drawn at an angle of 30° to the plane. The inclined lines are 4 dm and 6 dm long, and the angle between them is equal to 60°. Find the length of the projection on the plane α of the line segment BC connecting the end-points of the inclined lines and the angle between their projections.

346. From a point situated outside a plane draw two straight lines of an equal length inclined to this plane and forming an angle of 60° with each other. Find the

length of the inclined lines if their projections each 2 cm long form an angle of 90°.

347. At the vertex C of the acute angle of a rhombus $ABCD$ a perpendicular CE is erected to its plane and two inclined lines BE and DE are drawn. Find the angles of the rhombus if $BE = 8$ dm, $BD = 4$ dm and the lines BE and DE are inclined to the plane of the rhombus at an angle of 60°.

348. From a point M in a plane α two inclined lines $MA = 20$ cm and $MB = 30$ cm are drawn on one of its sides to form angles of 20° and 40° with the plane. Determine the distance between the end-points of the inclined lines if their projections lie on one straight line.

349. One side of an isosceles right-angled triangle 1 dm long is found in a plane α and the end-point of the other is $\frac{\sqrt{2}}{2}$ dm distant from the plane. Determine the angles at which the side and the hypotenuse are inclined to the plane α.

350. Drawn from a point A in a plane α are a straight line AB inclined at an angle of 45° to the plane and a straight line AC at an angle of 45° to the projection of the inclined line AB on the plane. Determine the angle ABC, the length of the line segment BC and the angle at which it is inclined to the plane α if $AB = AC = a$.

351. A line segment AB lies in a plane α. Drawn from its end-points are two intersecting lines $AD = BC = a$ each of which is inclined to the plane α at an angle of 30°. Determine the area of the quadrangle whose vertices are the points C and D and their projections on the plane α if the angle between the projections of the inclined lines is equal to 120° and $AB = \frac{a}{4}$.

352. A line segment 20 cm long intersects a plane. Its end-points are found at distances of 6 cm and 4 cm from the plane. Find the angle between the given line segment and the plane.

353. 1. From a point located at a distance of a from the plane two straight lines are drawn inclined to the plane at angles of 30° and 45° and at a right angle to each other. Determine the distance between the end-points of the inclined lines.

2. From a point located at a distance of a from a plane two inclined lines are drawn at an angle of 30° to the plane. Determine the angle between their projections if the distance between the end-points of the inclined lines is equal to $3a$.

354. A straight line AB lies in a plane α. Drawn from the points A and B, on one side of the plane and perpendicular to AB are two straight lines AD and BC inclined to the plane α at angles of 60° and 30°, respectively. Determine the distance between the end-points of the inclined lines if the distance between the end-points of their projections is equal to 12 cm, and $AD = 8\sqrt{3}$ cm, $BC = 14$ cm.

10. Parallelism of a Straight Line and a Plane

355. 1. A straight line is parallel to a plane. How is this line situated with respect to straight lines: (a) lying in the plane, (b) parallel to the plane, (c) intersecting the plane?

2. A straight line is perpendicular to a plane. How is this line situated with respect to straight lines: (a) lying in the plane, (b) parallel to the plane, (c) intersecting the plane?

356. 1. A straight line a is parallel to a plane α. How is the plane α situated with respect to straight lines: (a) parallel to the straight line a, (b) intersecting the straight line a, (c) crossing the straight line a?

2. What is the mutual position of two straight lines lying outside a plane and parallel to: (a) intersecting lines contained in the plane, (b) parallel lines contained in the plane?

357. Through the mid-points of the sides AB and AC of a triangle ABC a plane α is drawn which does not coincide with the plane ABC. What is the mutual position of the straight line BC and the plane α?

358. 1. How many straight lines parallel to a given plane can be drawn through a given point?

2. How many planes parallel to a given straight line can be drawn through a given point?

359. 1. How many straight lines parallel to a given line can be drawn in space?

2. Two straight lines are separately parallel to a third one. How many planes can be drawn through these lines so that at least two of them lie in each of these planes?

360. At a point A of a plane α a perpendicular AB is erected to this plane. The line segment BC is perpendicular to AB, $BC = 10$ cm. Find the projection of the line segment BC on the plane α.

361. Given two quadrangles $ABCD$ and $CDEF$ whose planes intersect. What must the quadrangle $ABCD$ be so that a plane passing through AB intersected the plane $CDEF$ along a straight line parallel to AB?

362. 1. Prove that in an oblique quadrangle (whose vertices do not lie in one plane) the line segments connecting the mid-points of adjacent sides form a parallelogram.

2. Given two straight lines a and b. Prove that the straight lines parallel to b and intersecting a lie in one plane.

363. 1. Prove that for two straight lines lying in intersecting planes to be parallel it is necessary and sufficient that they should be parallel to the line of intersection of the planes.

2. Given in planes α and β are points A and B. Through these points draw straight lines lying in the given planes and parallel to each other.

3. A point is given on each of the two intersecting planes. Draw a plane through these two points parallel to the line of intersection of the given planes.

364. 1. Through a given point in a given plane draw a straight line parallel to a given line which is parallel to the given plane.

2. Through a given point draw a straight line parallel to a given plane and intersecting a given straight line.

365. 1. Through two given points, not lying on a given straight line, draw a plane parallel to this line.

2. Given two straight lines. Through one of them draw a plane parallel to the other line.

3. Through a given point draw a plane parallel to two given straight lines which do not pass through this point.

366. 1. Through a given point outside a given plane draw a straight line parallel to this plane. How many such lines can be drawn?

2. Through a given point draw a plane parallel to a given straight line. How many such planes can be drawn?

3. Through a given point draw a straight line parallel to two intersecting planes.

367. Through the mid-point of a perpendicular to two parallel straight lines one can draw an infinite number of planes parallel to them. Prove that any one of these planes bisects the line segment connecting any two points of these lines.

368. 1. Find the locus of points in space equidistant from two parallel straight lines.

2. Find the locus of points of space equidistant from three parallel straight lines not lying in one plane.

369. At points A and B in a plane α perpendiculars $AC = 2.4$ cm and $BD = 12$ cm are erected to this plane. Through the end-points of the perpendiculars a straight line DE is drawn to intersect the plane α at point E. Find the length of the line segment DE if the perpendiculars are 28 cm apart.

370. Given a plane α and a triangle ABC. The side AB is parallel to the plane α and the extensions of the sides AC and BC intersect the plane α at the points D and E. Determine DE if $AB = 15$ cm and the points A and C are respectively 6 cm and 18 cm distant from the plane.

371. A perpendicular AC to a plane α intersects the latter at the point C. From a point B situated on the other side of the plane a straight line BD is drawn parallel to AC and intersecting the plane α at the point D. Find the length of the line segment AB if its projection on the plane α is equal to 32 cm and $AC = 15$ cm, $BD = 45$ cm.

372. A line segment BC 30 cm long is perpendicular to a straight line AB and a plane α containing the point C. The projection of an inclined line AD on the plane is equal to 10 cm and forms with the projection of AB on the same plane an angle of 120°. Determine the perimeter of the triangle ABD if AB is 20 cm long.

373. An isosceles triangle ABC, in which $AB = BC = 30$ cm and $AC = 24$ cm, is situated outside a plane α so that the side AC is parallel to the plane α and the vertex B is 18 cm farther from the plane than the base AC. Determine the kind of the triangle which is the projection of the given triangle on the plane α.

374. The bases of an isosceles trapezium are equal to 10 cm and 34 cm, and the altitude to 32 cm. Through the longer base a plane α is drawn at an angle of 60° to the altitude. Determine the projection of the lateral side of the trapezium on the plane α.

375. From the end-points of a line segment AB which is parallel to a plane α inclined lines AC and BD are drawn perpendicular to AB. The projections of these lines on the plane are respectively equal to 3 cm and 9 cm and lie on different sides of the projection of the line segment AB. Find the distance between the feet of the inclined lines if $AB = 16$ cm.

376. Straight lines AB and CD are situated on different sides of a plane α and are parallel to a straight line EF contained in this plane. Find the distance between AB and CD if they are, respectively, 17 cm and 25 cm distant from EF, and their projections are 15 cm distant from the same line.

377. An isosceles trapezium is situated outside a plane so that its bases are parallel to the plane. The projection of the trapezium on a plane α is also a trapezium circumscribed about a circle. Determine the angle between the lateral side of the given trapezium and a perpendicular to the plane α if the bases of the trapezium are equal to 2 dm and 6 dm, and the lateral side to 8 dm.

378. A rhombus, whose altitude is equal to $\frac{\sqrt{3}}{2}$ dm and the acute angle to 60°, lies with one of its sides on a plane α. The projection of the rhombus on this plane is a quadrangle one of whose angles is equal to 45°. Find the area of this quadrangle and the distance between the second side of the rhombus and the plane α.

379. Through the centre of the base of a regular quadrangular (triangular) pyramid draw a section parallel to a lateral edge of the pyramid. How many solutions does this problem have?

380. Determine the section figure cut from a triangular pyramid by a plane parallel to two skew edges if they are perpendicular to each other.

381. Given a regular quadrangular pyramid $SABCD$. Through a point M, which divides the side AB of the base in the ratio of 1 to 3, draw a plane parallel to the side AD of the base and the lateral edge SB. Compute the section area if $AD = 36$ cm and $SB = 30$ cm.

382. Draw the section of a regular triangular prism $ABCA_1B_1C_1$ by a plane passing through the vertex A of the base and mid-point M of the lateral edge BB_1 and parallel to the side BC of the base. Compute the section area if the side of the base of the prism is equal to a and the lateral edge to $2a$.

383. Cut a cube by a plane passing through the mid-points of two adjacent sides of the lower base and opposite vertex of the upper base. Compute the section area if the edge of the cube is equal to a.

384. In a regular quadrangular prism with the side of the base equal to 10 cm and lateral edge to 20 cm draw a section by a plane passing through its diagonal and parallel to one of the diagonals of the base. Determine the area of this section.

385. Each of the lateral edges of a regular quadrangular pyramid is equal to a. Draw a section through the mid-points of two adjacent sides of the base and the mid-point of the altitude. Find the area of the section.

11. Parallel Planes

386. 1. The diagonals of a rhombus $ABCD$ are perpendicular to a straight line a which is perpendicular to the plane of a triangle EFH. What is the mutual position of the planes containing the rhombus and triangle?

2. Prove that the section of a pyramid passing through the mid-points of its lateral edges is parallel to the base of the pyramid.

387. Prove that two planes parallel to a third plane are parallel to each other.

388. Prove that through two skew straight lines it is possible to draw two parallel planes. How many solutions does this problem have?

389. Find the locus of the points of space: (a) equidistant from two given parallel planes; (b) situated at a given distance from a given plane.

390. Prove that all the straight lines parallel to a given plane and passing through one and the same point are contained in one and the same plane which is parallel to the given one.

391. In a regular triangular pyramid draw a plane parallel to a lateral face and passing through: (a) the centre of the base; (b) the mid-point of the side of the base; (c) a vertex of the base; (d) the mid-point of the altitude.

392. In a regular quadrangular pyramid draw a plane parallel to a lateral face and passing through: (a) the centre of the base; (b) a side of the base; (c) the mid-point of the altitude.

393. Draw a plane parallel to a given one: (a) through a given point; (b) through a given straight line parallel to the given plane.

394. A line segment AB equal to 8 cm is perpendicular to planes α and β. A line segment CD (17 cm long) is situated so that its end-points are contained in these planes. Find the projections of the line segment CD on each of the planes.

395. The end-points of line segments AB and CD are contained in parallel planes α and β. Find the distance between these planes if $AB = 13$ cm, $CD = 20$ cm, and the sum of their projections on one of the planes is equal to 21 cm.

396. The plane of a triangle ABC with the sides 18 cm, 20 cm and 34 cm long is parallel to a plane α. A shining point S casts a shadow $A_1B_1C_1$ from the triangle ABC on the plane. Compute the area of the shadow if $SA : A_1A = 5 : 3$.

397. Two similar right-angled triangles ABC and $A_1B_1C_1$ are spaced so that their corresponding sides are parallel, and the line segment OO_1 which joins the centres of the circles circumscribed about them is perpendicular to the plane of the triangle ABC. Find the distance between the corresponding vertices of the triangles if the sides AC and BC containing the right angle are respectively equal to 10 cm and 24 cm, A_1C_1 is equal to 20 cm and OO_1 to 84 cm.

398. Two 120-degree angles are spaced so that their sides are correspondingly parallel, similarly directed and perpendicular to a line segment joining their vertices O and O_1. Marked from the vertices on the non-parallel sides are line segments $OA = 10$ cm and $O_1B = 20$ cm. Find the distance between their end-points A and B if $OO_1 = 18$ cm.

399. Two right-angled triangles ABC and $A_1B_1C_1$ with the sides $AB = A_1B_1 = 25$ cm and $AC = A_1C_1 = 20$ cm (Fig. 24) are situated in parallel planes so

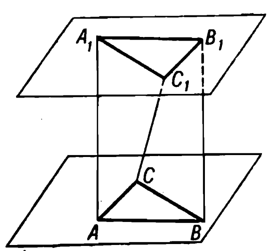

Fig. 24

that A_1 and B_1 are the projections of the vertices A and B on the plane $A_1B_1C_1$, whereas the projection of the vertex C on this plane does not coincide with the vertex C_1. Find the distance between the vertices C and C_1 of the right angles if the distance between the planes of the triangles is equal to 32 cm.

400. Two right angles are spaced so that their sides are correspondingly parallel and oppositely directed, and the vertex of one angle is an orthogonal projection of the vertex of the other on the plane of the former angle. Marked from the vertices on the non-parallel sides of the angles are line segments 7 cm and 24 cm long. Find the distance between the end-points of these line segments if the distance between the vertices of the angles is 60 cm.

401. A line segment AA_1 is perpendicular to planes α and β. Two equal and parallel line segments AC and A_1C_1 are drawn in these planes. On the line segment AC, as on the diagonal, a square $ABCD$ is constructed with the side equal to $30\sqrt{2}$ cm, and on the line segment A_1C_1, as on the greater diagonal, a rhombus $A_1B_1C_1D_1$ is constructed with the side $10\sqrt{10}$ cm long. Find the distance between the vertices D and D_1 of the square and rhombus if $AA_1 = 48$ cm.

402. The altitude of a regular quadrangular pyramid is equal to 16 dm, the side of the base to 24 dm. Compute the area of the section of this pyramid by a plane passing through the centre of the base and parallel to a lateral face of the pyramid.

403. Given a regular triangular prism $ABCA_1B_1C_1$ with the lateral edges AA_1, BB_1 and CC_1; O and O_1 are the centres of the bases of the prism. A point D divides the line segment OO_1 in the ratio of 5 to 1. Draw a plane through the point D and mid-points of the edges AB and A_1C_1 and construct the section of the given prism by this plane.

404. In a cube $ABCDA_1B_1C_1D_1$ draw a section through the vertex B, the mid-point of the edge AA_1 and the centre of the face CC_1D_1D, and find the perimeter of this section if the edge of the cube is equal to a.

405. The base of a regular prism is a hexagon with the side of 6 dm; the altitude of the prism is equal to 26 dm. Determine the area of the section drawn through two opposite sides of the upper and lower bases of the prism.

12. Dihedral Angles. Perpendicular Planes

406. From a point M situated on a face of a dihedral angle a perpendicular MN is drawn to the other face and from a point N a perpendicular NP is dropped to the edge of the dihedral angle. Prove that the line segment MP is perpendicular to the edge of the dihedral angle.

407. Prove that if from a point A taken inside a dihedral angle perpendiculars AB and AC are dropped to its faces, then the plane ABC is perpendicular to the edge of the dihedral angle.

408. Prove that if in a face of a right dihedral angle a perpendicular is erected to the edge, then it is perpendicular to the other face of the angle.

409. The end-points of a line segment AB are contained in the faces of a dihedral angle. The projection of this segment on one of the faces forms a right angle with the edge of the dihedral angle. Prove that the projection of AB on the other face is perpendicular to the edge of the dihedral angle.

410. Prove that the dihedral angles at the edges of the base of a regular pyramid are equal to one another.

411. Prove that the faces of a dihedral angle are perpendicular to the plane of its plane angle.

412. The altitude of a pyramid passes through the point of intersection of the diagonals of a rhombus serving as the base of the pyramid. Prove that the lateral faces are inclined to the base at equal angles.

413. 1. Find the locus of points of space equidistant from the faces of a dihedral angle.

2. Find the locus of points of space equidistant from two intersecting straight lines.

3. Find the locus of points of space equidistant from three pairwise intersecting straight lines.

414. Through the edge of a dihedral angle draw a plane so that it forms equal angles with the faces of the given dihedral angle.

415. At a point D of the side BC of a triangle ABC a perpendicular DE is erected to its plane. Planes are drawn through the point E and the sides of the triangle. Construct the plane angles of the dihedral angles formed with the edges AB, AC and BC.

416. Construct the edge and plane angle of the dihedral angle formed by opposite faces of a regular quadrangular pyramid.

417. 1. Through a given straight line draw a plane perpendicular to a given plane.

2. Through a given point draw a plane perpendicular to a given plane.

418. 1. Through a straight line in a plane draw a plane at a given angle to the given plane.

2. Through a straight line outside a plane which is parallel to this plane draw a plane at a given angle to the given plane.

419. Through a given straight line intersecting a given plane draw a plane at a given angle to this plane.

420. Two lateral faces of a pyramid are perpendicular to its base and form a dihedral angle of 40°. Find the angles of the parallelogram serving as the base of the pyramid.

421. On one face of a dihedral angle a point is taken at a distance a from the edge of the dihedral angle. Find the distance between this point and the other face if the dihedral angle is equal to: (1) 30°, (2) 45°, (3) 60°, (4) 90°.

422. The end-points of a line segment AB are found in the faces of a dihedral angle and the segment is perpendicular to one of them. Find the magnitude of the dihedral angle if the point A is twice as distant from the edge of the dihedral angle as the point B.

423. The pitch of a roof forms a dihedral angle $\alpha = 40°$ with the plane of the garret. The garret is 15 m wide. At what height from the plane of the garret is the ridge situated?

424. Two isosceles triangles ABC and ABD have a common base AB, and their planes form an angle of 60°. Determine the distance DO between the vertex D and the plane containing the triangle ABC if the altitude DK of the triangle ABD is equal to 12 cm.

425. An equilateral triangle with the side a is spaced so that one of its sides lies in a plane α and the opposite vertex is $\frac{3}{4}a$ distant from this plane. Determine the dihedral angle formed by the plane α and the plane containing the triangle.

426. An isosceles triangle ABC, in which $AB = BC = a\sqrt{2}$ and $AC = 2a$, is folded along the altitude BD so that the planes ABD and BDC form a right dihedral angle. Determine the angle between the side AB and its new position.

427. 1. In a right quadrangular pyramid the face angle at the vertex of the pyramid is equal to 60°. Find the angle of inclination of the lateral face to the base.

2. In a regular triangular pyramid the dihedral angle at the base is equal to α. Find the angle at which the lateral edge is inclined to the base of the pyramid.

3. In a regular triangular pyramid all the edges are of an equal length. Compute the angle between two adjacent faces.

428. In a regular triangular pyramid compute the dihedral angle at the base if the plane angle of the lateral face at the vertex of the pyramid is equal to 90°.

429. In a regular quadrangular pyramid compute the angle of inclination of the lateral face to the base if the side of the base and the lateral edge of the pyramid are equal to 10 cm and 13 cm, respectively.

430. In a regular quadrangular pyramid the side of the base is equal to 30 and the altitude to 20. Determine the dihedral angle at the lateral edge of the pyramid.

431. In a regular quadrangular pyramid the side of the base is equal to 12 cm and the apothem to 8 cm. Determine the dihedral angles between the lateral faces and between the lateral face and the base.

432. Given a triangle ABC with the sides $AB = 72$ cm, $AC = 58$ cm and $BC = 50$ cm. From the vertex C a perpendicular $CD = 90$ cm is erected to the plane containing the triangle. The point D is joined with straight lines to the vertices A and B. Determine the dihedral angle between the planes containing the triangles ABC and ABD.

433. A line segment AB 8 cm long touches with its end-points the faces of a right dihedral angle forming with them equal angles of 30°. Determine the distance between the projections of the end-points of the segment on the edge of the dihedral angle.

434. Taken on the faces of a dihedral angle of 60° are points A and B which are equidistant from the edge of the dihedral angle ($AC = BD = 27$ cm). Find the distance between the points A and B if $DC = 36$ cm.

435. Inside a right dihedral angle a point is taken which is equidistant from its faces and is situated at a distance of 8 cm from the edge of the dihedral angle. What are the distances between this point and the faces and between the projections of this point on the faces of the dihedral angle?

436. A line segment $AB = 20$ cm touches with its end-points two mutually perpendicular planes α and β. The projections of AB on these planes are equal to 16 cm and 15 cm, respectively. Determine the projection of AB on the line of intersection of the planes.

437. The side of a regular triangle equal to $16\sqrt{3}$ cm serves as the smaller base of an isosceles trapezium whose greater base is equal to 80 cm and the altitude to 18 cm. The planes containing the triangle and trapezium are mutually perpendicular. Find the distances between the vertex of the triangle not contained in the plane of the trapezium and the vertices of the trapezium.

438. The bases of two isosceles triangles serve as opposite sides of a rhombus. The planes of the triangles are perpendicular to the plane of the rhombus. Find the distance between the vertices of the triangles which do not coincide with the vertices of the rhombus if the lateral sides of the triangle are respectively equal to 10 cm and 17 cm, and the side of the rhombus to 16 cm (consider two cases).

439. A right-angled triangle is situated so that its hypotenuse lies on one of the faces of a dihedral angle and is parallel to its edge, and the vertex of the right angle on the other face. The hypotenuse of this triangle is equal to 25 cm and one of the sides containing the right angle to 20 cm. The hypotenuse and the vertex of the right angle are, respectively, $8\sqrt{3}$ cm and $4\sqrt{3}$ cm distant from the edge of the dihedral angle. Find the dihedral angle and the area of the projection of this triangle on the first face of this angle.

13. Areas of Projections of Plane Figures

440. The area of a plane polygon is equal to 120 dm^2. Find the area of the projection of this polygon on a plane forming an angle of 30° with the plane containing the polygon.

441. Given a triangle ABC with the sides $a = 25$ cm, $b = 29$ cm, $c = 36$ cm. Through the side AB a plane α is drawn at an angle of 45° to the plane containing the triangle ABC. Find the area of the projection of this triangle on the plane α.

442. Two triangles ABC and ABC_1 having a common base AB form a dihedral angle of 60°. The line segment CC_1 is perpendicular to the plane of the triangle ABC_1, whose angles A and B are equal to 30° and 60°, respectively, and the side AC_1 to 18 cm. Find the area of the triangle ABC.

443. Find the area of the projection of a semi-circle on a plane which forms with the latter a dihedral angle of 45° if the area of an isosceles triangle inscribed in the semi-circle amounts to 4 cm^2, its diameter serving as the base of the triangle.

444. A pipe whose inner and outer diameters are equal to d_1 and d_2 respectively is cut at an angle of 45° to the longitudinal axis. Determine the area of an elliptical annulus thus obtained.

445. A rhombus touches a plane α with its acute angle and is projected on this plane as a square whose side is equal to a. Find the area of the rhombus if its smaller diagonal is parallel to the plane α, and the distance between the vertex of the second acute angle and the plane α is equal to the diagonal of the square.

446. A right-angled triangle is spaced so that one of its sides containing the right angle is parallel to a plane α and the second side is projected on this plane half the true length. How many times is the area of the triangle greater than that of its projection on the plane α, and what is the angle of inclination of the plane containing the triangle to the plane α?

447. The base of an oblique prism is a rectangle with sides 10 cm and 8 cm. Two lateral faces have the shape of a rectangle; the lateral edges are inclined to the base at an angle of 60°. Find the area of a normal section which cuts all the lateral edges.

448. The base of a regular prism is a hexagon. The prism is cut by a plane passing through two opposite sides of the upper and lower bases. Find the area of the section if the cutting plane is inclined to the base at an angle of 40°32′ and the area of the base is equal to 152 dm^2.

449. Through opposite vertices of the lower and upper bases of a regular quadrangular prism a section is drawn parallel to a diagonal of the base. Determine the area of the section if its diagonal forms an angle of 75°10′ with the base, and the side of the base of the prism is equal to 16 cm.

450. In a triangular pyramid $SABC$ a section is drawn through the edge AC perpendicular to the edge SB cutting off a triangle with sides 5, 6, 9. Determine the area of the face ABC if it forms with the cutting plane an angle of 45°.

451. The area of the lateral surface of a regular pyramid is equal to 180 cm^2. The lateral face is inclined to the base at an angle of 53°8′. Determine the area of the base.

452. The side of the base of a regular hexagonal pyramid is equal to 5 cm. The lateral face is inclined to the base at an angle of 30°. Determine the area of the lateral and total surfaces of this pyramid.

453. The side of the smaller base of the frustum of a regular quadrangular pyramid is equal to 6 cm. The

lateral face is inclined to the base at an angle of 60° and has the area of 32 cm^2. Determine the side of the greater base.

454. How must an angle be situated with respect to a plane for its projection on this plane to have the shape of an angle: (a) equal to the given angle, (b) smaller than the given angle, (c) greater than the given angle, (d) equal to zero, (e) equal to 180°, (f) equal to 90°? Consider the three cases: the given angle is acute, right, obtuse.

14. Polyhedral Angles

455. 1. Is it possible to form a trihedral angle, using the following plane angles:
(a) 125°, 80°, 31°; (b) 110°, 80°, 50°;
(c) 72°, 56°, 38°; (d) 92°, 56°, 36°;
(e) 150°, 140°, 80°; (f) 140°, 130°, 90°?

2. Is it possible to form a convex polyhedral angle, using the following plane angles:
(a) 30°, 90°, 60°, 150°; (b) 50°, 60°, 110°, 140°;
(c) 42°, 62°, 72°, 82°, 92°;
(d) 23°, 38°, 85°, 92°, 98°, 100°?

456. In a trihedral angle two plane angles are equal to: (a) 85°45′ and 72°15′, (b) 85°45′ and 112°15′. Within what range does the third plane angle vary?

457. What plane angles has the trihedral angle of a: (1) regular triangular prism, (2) rectangular parallelepiped, (3) regular triangular pyramid?

458. In a trihedral angle two plane angles are equal to 45° each, and the dihedral angle between them to 90°. Find the third plane angle.

459. Determine the plane angles of each of the polyhedral angles of a regular quadrangular pyramid if its lateral edge is inclined to the base at an angle of 45°.

460. Determine the plane angles of each of the polyhedral angles of a regular quadrangular pyramid whose altitude is equal to the side of the base.

461. In a regular quadrangular pyramid the opposite lateral edges are mutually perpendicular. Find the plane angle at the vertex.

462. One of the plane angles of a trihedral angle is equal to 90°, the other two being equal to each other. A plane cutting off the edges three equal segments is perpendicular to the plane containing the right angle. Determine the other two plane angles.

463. In a trihedral angle two plane angles are equal to 60° each; the dihedral angle opposite the third plane angle is equal to 90°. Find the third plane angle of the trihedral angle.

464. In a trihedral angle two plane angles are equal to 45° each, the third to 60°. Find: (1) the angle between the plane of the third plane angle and opposite edge; (2) the dihedral angle between the equal plane angles.

465. In a trihedral angle all the plane angles are right ones. Laid off (from the vertex) on its edges are line segments each of which is equal to a. A plane is drawn through the end-points of the segments. Find the area of the section thus obtained.

466. In a trihedral angle each plane angle is equal to 60°. Laid off on one of the edges of the angle from the vertex M is a line segment MA 6 cm long, and through its end-point a plane is drawn perpendicular to the edge MA and cutting off segments MB and MC on the other two edges. Determine the area of the triangle MBC.

467. In a regular triangular pyramid all the plane angles at the vertex are right ones, and the lateral edge is equal to a. Find the distance between the centre of the base of the pyramid and the lateral face.

468. Inside a trihedral angle all of whose plane angles are right ones a line segment is drawn from the vertex whose end-point is 7 cm, 24 cm and 60 cm distant from the faces. Find the length of this segment.

CHAPTER IV

POLYHEDRONS AND ROUND SOLIDS

15. Prisms and Parallelepipeds

469. 1. How many edges, vertices and faces does a pentagonal (decagonal) prism have?

2. How many plane angles, dihedral angles and trihedral angles does a triangular (quadrangular, hexagonal) prism have?

3. What is the sum of the plane angles of a triangular (quadrangular, n-gonal) prism equal to?

470. 1. How many diagonals does a triangular (quadrangular, pentagonal, n-gonal) prism have?

2. How many diagonal sections can be drawn through a lateral edge of a triangular (quadrangular, n-gonal) prism?

3. How many diagonal sections can be drawn through all the lateral edges of a quadrangular (pentagonal, n-gonal) prism?

471. 1. What figures are yielded by diagonal sections of a right and oblique prisms?

2. Into how many parts is an n-gonal prism divided by diagonal sections passing through a lateral edge? What polygon is represented by each such part?

472. Find the point equidistant from all the vertices of a: (a) regular triangular prism, (b) right prism whose bases are right-angled triangles.

473. Prove that a section perpendicular to a lateral edge of the prism is perpendicular to each of its lateral faces.

474. Prove that in an oblique triangular prism the distance between a lateral edge and the opposite face is equal to the altitude of the triangle which is a normal section of a prism.

475. Prove that all the lateral edges of a prism are inclined to its base at an equal angle.

476. The base of an oblique parallelepiped is a rhombus; one of the lateral edges forms equal angles with the adjacent sides of the base. Prove that the vertex of the parallelepiped found on this edge is projected on the diagonal of the base.

477. Prove that if the diagonal planes of a parallelepiped are perpendicular to the base, then the parallelepiped is a right one.

478. The base of a parallelepiped is a rhombus. The plane of one of the diagonal sections is perpendicular to the base. Prove that the other diagonal section is a rectangle.

479. In a regular quadrangular prism $ABCDA_1B_1C_1D_1$ the diagonals AC_1 and BD_1 form an angle of 60°. Prove that the diagonal section of the prism is a square.

480. 1. Compute the acute angle between two diagonals of a cube.

2. Compute the acute angle between two diagonals of a rectangular parallelepiped whose dimensions are 2 dm, 3 dm and 6 dm.

481. The diagonal of a regular quadrangular prism is equal to d and inclined to the lateral face at an angle of 60°. Determine the side of the base of the prism.

482. The side of the base of a regular quadrangular prism is equal to 20 cm. Find the distance between the diagonal of the prism and a lateral side which does not intersect it.

483. Find the diagonal of a regular quadrangular prism in which: (a) the area of the base is equal to 450 cm², and the lateral edge to 40 cm; (b) the area of the base

is equal to 200 cm^2, and the area of the lateral face to $210\sqrt{2}$ cm^2.

484. The side of the base of a regular hexagonal prism is equal to a, and the altitude to $2a$. Compute its diagonals and the angles at which they are inclined to the base.

485. In a regular hexagonal prism each edge is equal to a. Find: (1) the diagonals of the prism; (2) the area of the section drawn through the greater diagonal of the base and the parallel side of the other base; (3) the distance between this section and the edges of the prism parallel to it.

486. Determine the diagonals of a rectangular parallelepiped given its dimensions: (1) 1, 2, 2; (2) 2, 3, 6; (3) 8, 9, 12; (4) 12, 16, 21.

487. The area of the diagonal section of a cube is equal to $16\sqrt{2}$ cm^2. Compute the edge of the cube, the diagonal of the base, the diagonal of the cube.

488. The dimensions of a rectangular parallelepiped are as 3 : 4 : 12, and its diagonal is equal to 26 cm. Find the dimensions of the parallelepiped.

489. In a right parallelepiped the sides of the base are equal to m and n and form an angle of 60°. The greater diagonal of the base is equal to the smaller diagonal of the parallelepiped. Find the diagonals of the parallelepiped.

490. In a regular triangular prism a section is drawn through a side of the lower base and opposite vertex of the upper base. (1) Prove that the section forms with the base a dihedral angle of 30° if the side of the base and the altitude of the section drawn to this side are equal to each other. (2) Compute the diagonal of the lateral face if the side of the base of the prism is equal to a and the section is inclined to the base at an angle of 60°.

491. In a regular quadrangular prism $ABCDA_1B_1C_1D_1$ the side of the base is one fourth the altitude of the prism. Points M and N are taken on the edges AA_1 and BB_1

so that $AM = \frac{1}{4} AA_1$ and $BN = \frac{1}{2} BB_1$. Find the angle DMN.

492. The base of a right prism is a rhombus with the side of 10 cm and altitude equal to 9.6 cm. The altitude of the prism is 12 cm high. Find the diagonals of the prism.

493. The base of a right prism is a parallelogram whose sides are equal to 13 cm and 15 cm and the altitude to 12 cm. The smaller diagonal of the prism is inclined to the base at an angle of 45°. Find the diagonals of the prism.

494. The base of an oblique prism is an equilateral triangle with the side a. The lateral edge is inclined to the plane containing the base at an angle of 60°. One of the vertices of the upper base is projected on the plane of the lower base in the centre of the circumscribed circle. Find the altitude of the prism and the area of each of the lateral faces.

495. In a right parallelepiped the sides of the base are equal to 15 cm and $2\sqrt{38}$ cm, and the sum of the diagonals of the base to 32 cm. The altitude of the parallelepiped is 12 cm high. Find the diagonals of the parallelepiped.

496. 1. Is it possible to cut a cube with a plane so as to obtain in section a triangle: (a) scalene, (b) isosceles, (c) equilateral?

2. Is it possible to cut a cube so as to obtain in section a triangle: (a) acute, (b) right-angled, (c) obtuse?

3. What quadrangles can be obtained by cutting a cube with a plane?

497. The edge of a cube is equal to 5 cm. Compute the perimeter and area of a section drawn through the end-points of three edges emanating from one vertex.

498. Given a cube $ABCDA_1B_1C_1D_1$ with an edge a. Constructed on the edge C_1D_1 is a line segment $C_1L = \frac{3}{4} a$, on the edge A_1B_1 a line segment $A_1M = \frac{a}{2}$ and on the edge BB_1 a line segment $B_1N = \frac{1}{4} a$. A plane

is drawn through the points L, M and N. Determine the perimeter of the section.

499. Given a cube $ABCDA_1B_1C_1D_1$ with the edge $a = 8$ cm. A plane is drawn through the mid-points of the edges CC_1, AB and AD. Compute the perimeter and the area of the section thus obtained.

500. Given a cube $ABCDA_1B_1C_1D_1$ with the edge a. A plane is drawn through the mid-points of the edges A_1B_1, B_1C_1 and AD. Determine the perimeter and the area of the section figure.

501. Given a cube $ABCDA_1B_1C_1D_1$. On the extensions of the edges line segments C_1P, BQ, DR are laid off, each of length a, and $CP = CR = CQ = 2AB$. Draw a plane through the points P, Q and R and compute the perimeter and the area of the section thus obtained.

502. Given a cube $ABCDA_1B_1C_1D_1$. Draw a plane through the mid-points of the edges A_1D_1 and BC and the vertex C_1. What polygon is obtained in section? Determine the perimeter and the area of this polygon if the edge of the cube is equal to a.

503. The area of the lateral face of a regular quadrangular prism is equal to Q. Determine the area of the diagonal section.

504. In a regular triangular prism a section is drawn through a side of the base equal to a and the mid-point of the opposite lateral edge. Find the area of the section if the area of the lateral face is equal to a^2.

505. In a right prism the sides of the base are equal to 4 cm, 13 cm and 15 cm, and the altitude to 10 cm. Determine the area of the section drawn: (a) through a lateral edge and the smaller altitude of the base, (b) through the smaller side of the lower base and opposite vertex of the upper base, (c) through the greater side of the lower base and the mid-point of the opposite edge.

506. The base of a right prism is a right-angled triangle whose hypotenuse is 25 cm long and one of the sides is equal to 20 cm. The altitude of the prism is 16 cm. A plane is drawn through the hypotenuse of one of the bases and the vertex of the right angle of the other base.

Determine the area of the section and the angle at which it is inclined to the base.

507. The base of a right prism is a rhombus with the side a and angle of 60°. The section of the prism passing through its greater diagonal is parallel to the diagonal of the base and inclined to the plane containing the base at an angle of 30°. Find the area of the section.

508. The base of a right prism is a trapezium whose bases are equal to 25 cm and 10 cm. One of the lateral sides of the trapezium is equal to 25 cm and the altitude to 24 cm. Determine the areas of the diagonal sections of this prism if the altitude of the prism is 20 cm high.

509. An isosceles right-angled triangle, whose lateral side is 10 cm long, serves as the base of a prism. The bisector of the right angle of the base is the projection of one of the lateral edges of the prism on the plane containing the base. Each lateral edge is inclined to the base at an angle of 45°. (1) Find the area of the section passing through a lateral edge and the mid-point of the hypotenuse of the base. (2) Determine the distance between the vertex of the right angle of the base and the opposite face. Compute the area of this face.

510. The altitude of a rectangular parallelepiped is equal to 16 cm and the sides of the bases to 10 cm and 12 cm. Compute the area of the section drawn through the smaller side of the lower base and the opposite side of the upper base.

511. 1. In a rectangular parallelepiped the sides of the base are equal to 9 cm and 40 cm, and the altitude to 10 cm. Find the area of the diagonal section.

2. In a rectangular parallelepiped the area of the diagonal section is to the area of the base as 25 to 24. The altitude and diagonal of the parallelepiped are equal to 10 cm and $10\sqrt{5}$ cm, respectively. Find the sides of the base.

512. In a right parallelepiped the sides of the base are equal to 8 cm and $\sqrt{66}$ cm, and the diagonals of the base are as 4 : 7. Determine the area of the diagonal sections of the parallelepiped if its smaller diagonal is equal to 10 cm.

513. The bases and two lateral faces of a parallelepiped are congruent rectangles with the sides a and b. The lateral edge is inclined to the base at an angle of 60°. Find the area of each section passing through two opposite edges.

514. The faces of a parallelepiped are congruent rhombuses with the side a and angle of 60°. Determine the area of its diagonal sections.

515. The base of an oblique parallelepiped is a rhombus with an acute angle of 60° and the side a. The lateral edge is equal to $3a$ and inclined to the base at an angle of 60°. Determine the areas of the diagonal sections if one of them is perpendicular to the base.

16. The Pyramid

516. 1. What is the sum of all plane angles of a pyramid equal to in: (a) triangular, (b) quadrangular, (c) pentagonal, (d) decagonal, (e) n-gonal pyramids?

2. How many angles has the base of a pyramid if the sum of all the plane angles of this pyramid is equal to 3600°?

517. Is it possible for a pyramid to have equal lateral edges if its base is a: (a) triangle, (b) rectangle, (c) rhombus, (d) isosceles trapezium, (e) non-isosceles trapezium, (f) regular polygon?

518. Is it possible for a pyramid to have equal dihedral angles at the base if its base is a: (a) triangle, (b) rectangle, (c) rhombus, (d) parallelogram, (e) isosceles trapezium, (f) non-isosceles trapezium, (g) regular polygon?

519. 1. If all the lateral edges of a pyramid are of equal length, then about its base a circle can be circumscribed whose centre will be a trace of the altitude of the pyramid on the base. Prove this.

2. Formulate and prove the converse.

520. 1. Prove that if the lateral faces of a pyramid are inclined to the plane containing the base at one and the same angle, then a circle can be inscribed in the base whose centre will be the trace of the altitude of the pyramid on the base.

2. Formulate and prove the converse.

521. Prove that the plane passing through the altitude of a regular triangular pyramid and the slant height of a lateral face is perpendicular to the plane containing this lateral face.

522. The base of a pyramid is a rhombus and the altitude of the pyramid passes through the point of intersection of the diagonals of the base. Prove that the lateral faces of the pyramid are equal.

523. The base of a pyramid is a right-angled triangle and the lateral edges are of equal length. Prove that one of them is perpendicular to the base.

524. Prove that if the base of a pyramid is an isosceles trapezium and all the lateral faces are equally inclined to the base, then the lateral side of the trapezium is equal to its median (midline).

525. 1. Given the side of the base a and the altitude h find the lateral edge and the slant height of a regular pyramid: (a) triangular, (b) quadrangular, (c) hexagonal.

2. Given the side of the base a and the lateral edge b determine the altitude and the slant height of a regular pyramid: (a) triangular, (b) quadrangular, (c) hexagonal.

526. 1. Given the side of the base a and the slant height m determine the altitude and the lateral edge of a regular pyramid: (a) triangular, (b) quadrangular, (c) hexagonal.

2. Given the lateral edge b and the slant height m find the side of the base and the altitude of a regular pyramid: (a) triangular, (b) quadrangular, (c) hexagonal.

527. The base of a pyramid is an isosceles triangle whose base is equal to 30 cm and the alternate angle to 120°. The altitude of the pyramid passes through the centre of the circle circumscribed about the base and is equal to 10 cm. Determine the lateral edges of the pyramid.

528. Each lateral edge of a pyramid is equal to 52 cm. The base of the pyramid is a triangle with the sides $2\sqrt{10}$ cm, 24 cm and $12\sqrt{10}$ cm. Find: (1) the altitude of the pyramid, (2) the angle between the lateral side and the base, (3) the dihedral angles at the base.

529. The base of a pyramid is a triangle ABC with the sides $AB = AC = 34$ cm and $BC = 32$ cm. The edge SA of the pyramid is equal to 30 cm and perpendicular to the base. Determine: (1) the edges SB and SC, (2) the angles of inclination of the lateral edges to the base, (3) the area of the face SBC and the angle at which it is inclined to the base.

530. The altitude of a regular triangular prism is equal to 6 cm, and the side of the base to 3 cm. The centre of the upper base and the vertices of the lower base serve as the vertices of a pyramid. Find the angles of inclination of the lateral edges and lateral faces of the pyramid to the base.

531. The base of a pyramid is a parallelogram whose sides are 3 cm and 7 cm. The altitude of the pyramid passes through the points of intersection of the diagonals of the base and is equal to 4 cm. The greater lateral edge of the pyramid is equal to 6 cm. Find the diagonals of the base.

532. 1. Compute the areas of the diagonal sections of a regular hexagonal pyramid if the lateral edge is equal to 10 cm and the altitude to 8 cm.

2. In the same pyramid find: (a) the angles at which the lateral edge and face are inclined to the base; (b) the dihedral angle between two adjacent lateral faces.

533. Construct a quadrangular pyramid in which all the eight edges are of the same length. Find the angle at the vertex of the diagonal section of this pyramid.

534. The base of a pyramid is a rectangle $ABCD$ with the sides of 18 cm and 24 cm. The lateral edge SB is perpendicular to the base and equal to 10.8 cm. Determine the areas of the diagonal sections of the pyramid.

535. In a regular quadrangular pyramid the area of the lateral face equals Q. The dihedral angle at the lateral edge amounts to 120°. Find the area of the diagonal section.

536. Find the areas of the diagonal sections of a quadrangular pyramid whose base is a parallelogram with the sides 23 cm and 11 cm long. The diagonals of the parallelogram are in the ratio of 2 to 3. The altitude of the pyramid passes through the point of intersection of the diagonals of the base. The smaller lateral edge of the pyramid is equal to 26 cm.

537. The altitude of a pyramid is divided into three equal parts and through the division points sections are drawn parallel to the base. Determine the areas of the sections if the area of the base is equal to 900 cm^2.

538. Through a point which divides the altitude of a pyramid in the ratio of 2 to 3 a section is drawn parallel to the base. The area of the section is 10 cm^2 smaller than the area of the base of the pyramid. Find the area of the section. (Consider two cases.)

539. In what ratio is the altitude of a pyramid divided by a section drawn parallel to the base if the area of the section is equal to: (1) half the area of the base; (2) $\frac{1}{4}$ the area of the base; (3) $\frac{4}{9}$ the area of the base; (4) $\frac{m}{n}$ the area of the base?

540. The area of the base of a pyramid is equal to 224 cm^2 and the area of a section parallel to it to 14 cm^2. The distance between them is equal to 27 cm. Find the altitude of the pyramid.

541. In a regular triangular pyramid with the side of the base a and the plane angle at the vertex of the right angle a section is drawn parallel to the base which cuts from each lateral edge a segment equal to m (as measured from the vertex). In what ratio is the altitude cut by the section?

542. The altitude of a regular triangular pyramid is 8 dm high and the side of the base is equal to 2 dm. Compute the area of a section drawn through the side of the base perpendicular to the opposite edge.

543. The altitude of a regular quadrangular pyramid is equal to 40 cm and the diagonal of the base to 60 cm. Draw a plane through the diagonal of the base of the pyramid perpendicular to a lateral edge. Compute the area of the section and the angle at which it is inclined to the plane containing the base.

544. In a regular quadrangular pyramid the side of the base is equal to 3 cm and the altitude to 2 cm. In what ratio does the section bisecting the dihedral angle at the base divide the area of the opposite face?

545. The base of a pyramid is a square. The three lateral edges are a, b and c. Find the fourth one.

17. The Truncated Pyramid

546. 1. How many diagonals can be drawn in a truncated: (a) quadrangular, (b) pentagonal, (c) n-gonal pyramid?

2. What figures are yielded by the diagonal sections of a regular truncated pyramid; irregular truncated pyramid?

3. How many diagonal sections can be drawn through one lateral edge in an n-gonal truncated pyramid, through all the edges?

547. In a regular triangular truncated pyramid the sides of the bases are equal to 5 dm and 11 dm, and the lateral edge to 4 dm. Find the altitude and the slant height of the pyramid.

548. In a regular triangular truncated pyramid the altitude and the slant height are equal to 15 cm and 17 cm, respectively. The altitude of the smaller base equals 6 cm. Find the sides of the bases and the lateral edge.

549. In a regular triangular pyramid the lateral edge is 12 cm long and forms an angle of 60° with the plane containing the larger base. The side of the smaller base is equal to $3\sqrt{3}$ cm. Find the side of the larger base, the altitude and the slant height of the pyramid.

550. In a regular triangular truncated pyramid the sides of the bases equal 18 cm and 12 cm, and the lateral faces are inclined to the plane of the greater base at an angle of 45°. Find the altitude and the lateral edge of this truncated pyramid and the altitude of the corresponding non-truncated pyramid.

551. The bases of a truncated pyramid are isosceles right-angled triangles whose hypotenuses are equal to 10 cm and 26 cm. The altitude of the pyramid passes through the centres of the circles circumscribed about the bases and is equal to 6 cm. Determine the lateral edges of the pyramid.

552. In a triangular truncated pyramid $ABCA_1B_1C_1$ the lateral face AA_1B_1B is an isosceles trapezium perpendicular to the bases. The sides of the lower base $AB = 50$ cm, $AC = 40$ cm and $BC = 30$ cm; the side of the upper base $A_1B_1 = 10$ cm, the altitude of this pyramid equals 15 cm. Find the angles of inclination of the lateral edges and lateral faces to the base.

553. In a regular quadrangular truncated pyramid the altitude is half the length of the lateral edge, and the sides of the bases are a and b. Find the altitude, the lateral edge and the slant height of the pyramid.

554. The lateral edge of a regular quadrangular pyramid is inclined to the base at an angle of $\arctan \frac{\sqrt{2}}{2}$. Prove that the opposite lateral faces of this truncated pyramid are mutually perpendicular.

555. In a regular quadrangular truncated pyramid the sides of the bases are equal to a and b. The lateral edge forms with the side of the greater base an angle of 60°. Determine the slant height, lateral edge, angle of inclination of the lateral edge to the greater base and altitude of the pyramid.

556. In a regular quadrangular truncated pyramid the diagonals are mutually perpendicular. The sides of the bases are equal to 20 cm and 10 cm. Determine the altitude, diagonals, lateral edge, slant height and diagonal of the lateral face of the pyramid.

557. In a regular quadrangular truncated pyramid the area of the smaller base is equal to 36 cm^2, and the area of the lateral face to 14 cm^2. The lateral face is inclined to the plane containing the greater base at an angle of 60°. Find the side of the greater base.

558. In a regular hexagonal truncated pyramid the side of the smaller base is equal to 14 cm. The slant height of the pyramid equal to 8 cm is inclined to the greater base at an angle of 30°. Determine the side of the greater base and the diagonals of the pyramid.

559. The sides of the bases of a regular triangular truncated pyramid are equal to 2 dm and 5 dm and the altitude to 1 dm. Through the vertex of the smaller base a section is drawn parallel to the opposite lateral face. Find the area of this section.

560. In a regular quadrangular truncated pyramid the diagonal of the lateral face is 8 cm long and perpendicular to the lateral side of the pyramid. The side of the greater base is to the lateral edge as 5 to 3. Determine the area of the diagonal section of the pyramid.

561. The diagonals of a regular quadrangular truncated pyramid are perpendicular to the lateral edges. The lateral edge of the pyramid is equal to 72 cm and its altitude to 56 cm. Find the sides of the bases of the pyramid and the area of its diagonal section.

562. The sides of the bases of a regular quadrangular truncated pyramid are equal to 18 dm and 12 dm and the lateral faces are inclined to the plane of the greater base at an angle of 60°. Find the diagonals of this pyramid and areas of its diagonal sections.

563. The slant height of a regular quadrangular truncated pyramid is equal to 20 dm and the sides of its bases to 12 dm and 6 dm. Determine the area of a section of this pyramid passing through opposite sides of the bases.

564. In a regular quadrangular truncated pyramid the sides of the bases are equal to 20 m and 10 m, the lateral edge is inclined to the greater base at an angle of 45°. Cut the pyramid by a plane passing through the side of the greater base perpendicular to the opposite lateral face and compute the area of the section.

565. In a regular hexagonal truncated pyramid the sides of the bases are equal to 12 cm and 18 cm and the lateral edge is inclined to the plane containing the greater

base at an angle of 45°. Determine the areas of the diagonal sections.

566. The bases of a truncated pyramid are rectangles. The sides of the greater base are equal to 30 cm and 40 cm, and those of the smaller base to 15 cm and 20 cm. One of the lateral edges is perpendicular to the planes containing the bases and equals 8 cm. Determine: (1) the lateral edges of the pyramid; (2) the angles of inclination of the lateral edges to the greater base; (3) the areas of the lateral faces; (4) the angles of inclination of the lateral faces to the greater base; (5) the diagonals; (6) the angles of inclination of the diagonals to the bases; (7) the areas of the diagonal sections; (8) the angles of inclination of the diagonal planes to the bases.

567. The areas of the bases of a truncated pyramid are equal to 16 cm^2 and 144 cm^2, and the altitude to 18 cm. Find the altitude of the corresponding non-truncated pyramid.

568. The areas of the bases of a truncated pyramid are Q_1 and Q_2. Find the area of the mid-section and the area of the section which is parallel to the bases and divides the altitude of the pyramid in the ratio of 1 to 2 (in the direction of the greater base).

18. Regular Polyhedrons

569. Two tetrahedrons with equal edges are brought together so that their contacting faces coincide. How many vertices, edges and faces has the polyhedron thus obtained? Will the obtained polyhedron be regular?

570. Prove that the straight lines passing through the midpoint of the altitude and the vertices of the base of a tetrahedron are mutually perpendicular.

571. The edge of a tetrahedron is equal to a. Determine the altitude, the slant height and the angle of inclination of the edge to the plane containing the face.

572. Prove that in a tetrahedron the sum of the distances between the centre of the base and the lateral faces is equal to the altitude of the tetrahedron.

573. A tetrahedron $ABCD$ is cut by a plane AEF so that the line segment EF is parallel to BD and passes through the centre of the face BCD. Find the area of the section if the edge of the tetrahedron is 9 dm long.

574. 1. Find the distance between the centres of two faces of a tetrahedron whose edge is a.

2. The distance between the centres of two faces of a tetrahedron is equal to 5 cm. Find the surface area of the tetrahedron.

575. Construct an octahedron with the edge a and determine: (1) the areas of the diagonal sections; (2) the distance between two opposite vertices; (3) the angles between adjacent faces; (4) the distance between the centres of two adjacent faces; (5) the distance between parallel faces.

576. 1. Through a point which divides the edge of a tetrahedron in the ratio of 1 to 4 a plane is drawn perpendicular to this edge. Find the perimeter of the obtained section if the edge of the tetrahedron is equal to a.

2. The same for an octahedron.

577. The edge of a tetrahedron is equal to a. Find the edge of an octahedron if their surface areas are equal.

578. The edge of an octahedron is equal to a. Find the area of the section passing through the diagonal of the octahedron perpendicular to its edge.

579. A cube is cut by planes passing through its centre and each side of the base. Into how many polyhedrons is the cube divided and what are they?

580. The edge of a cube is a. Find the ratio of the surface areas of a tetrahedron and an octahedron inscribed in it.

581. 1. How many planes of symmetry pass through one vertex in a: (a) tetrahedron, (b) octahedron, (c) hexahedron?

2. How many planes of symmetry has a: (a) tetrahedron, (b) octahedron, (c) hexahedron?

582. Join the centres of each pair of adjacent faces of an octahedron and draw planes through adjacent straight lines. Prove that the hexahedron thus obtained is a cube and compute its surface area if the edge of the octahedron is equal to a.

583. A cube is inscribed in an octahedron so that its vertices are found on the edges of the octahedron. Determine the diagonal of the cube if the edge of the octahedron is equal to a.

584. A regular quadrangular prism is inscribed in an octahedron so that its vertices lie on the edges of the octahedron. The lateral edge of the prism is twice the length of the side of its base. Determine the edges of the prism if the edge of the octahedron is equal to a.

19. The Right Circular Cylinder

585. 1. Find the locus of points of space situated at a given distance from a given straight line.

2. Find the locus of points of space situated at a given distance from a given cylindrical surface.

3. Find the locus of points of space situated at a given distance from two given parallel straight lines.

586. The diagonal of an axial section of a cylinder $d = 40$ cm and is inclined to the base at an angle $\alpha = 66°25'$. Determine the altitude of the cylinder and the radius of its base.

587. 1. The radius of the base of a cylinder is equal to 3 m, its altitude to 8 m. Find the diagonal of an axial section.

2. An axial section of a cylinder is a square whose area is S. Find the area of the base.

588. The altitude of a cylinder is equal to 16 dm, the radius of the base to 10 dm. The end-points of a given line segment which is 20 dm long lie on the circumferences of both base circles. Find the shortest distance between this segment and the axis of the cylinder.

589. The end-points of a line segment AB are situated on the circumferences of the base circles of a cylinder. The projection of the end-point of the segment on a parallel to it axial section divides the diameter of the base in the ratio of 1 : 5. Find the length of the segment

AB if the radius of the base and the altitude of the cylinder are equal to 18 cm and 32 cm, respectively.

590. A triangle ABC in which $AB = 15$ cm, $AC = 14$ cm and $BC = 13$ cm rotates about an axis which passes through the vertex C parallel to the side AB. Find the diagonal of the axial section of the cylinder generated by the side AB.

591. 1. What point is the centre of symmetry of a right circular cylinder?

2. How many axes of symmetry has a right circular cylinder? What are they?

3. What planes are the planes of symmetry of a right circular cylinder?

592. A plane is drawn through two elements of a cylinder. The area of the section constitutes 75 per cent of the area of an axial section of the cylinder. Find the distance between this section and the axis of rotation of the cylinder if the radius of its base is equal to 8 cm.

593. In a cylinder with the radius of the base of 40 cm and the altitude 30 cm a plane is drawn parallel to the axis and at a distance of 24 cm from it. Find the area of the section.

594. In a cylinder a plane is drawn parallel to its axis to cut arcs of 120° from the base circles. The perimeter of the section is equal to 60 cm, and its area to 225 cm^2. Determine the radius of the base of the cylinder and its altitude.

595. The altitude of a cylinder is 16 dm, the radius of the base, 10 dm. The cylinder is cut by a plane parallel to its axis so that the section figure is a square. Find the distance between the section and the axis of the cylinder.

596. Determine the cross-section and axial section of a pipe 3 m long whose outer and inner diameters are equal to 0.5 m and 0.4 m, respectively.

597. In a cylinder a section is drawn perpendicular to its base and dividing the base circle in the ratio of 1 to 5. Find the ratio of the areas of the given and axial sections of the cylinder.

598. A regular quadrangular prism is inscribed in a cylinder. The diagonal of the lateral face of the prism is equal to l and inclined to the base at an angle of 60°. Find the diagonal of an axial section of the cylinder.

599. The altitude of a cylinder $h = 4$ dm, the radius of the base $r = 14$ dm. A square is inscribed in this cylinder inclined to its axis so that all its vertices are found on circumferences of the base circles. Find the area of this square.

600. A plane is drawn through the diameter of the upper base and a chord of the lower base of a cylinder. The distances between this plane and the end-points of the diameter of the lower base which is perpendicular to the chord are as 1 : 2. The radius of the base of the cylinder is equal to 3 dm and its altitude to $\sqrt{3}$ dm. Determine the angle of inclination of the cutting plane to the plane containing the base of the cylinder.

601. In an equilateral cylinder (i.e. in a cylinder whose axial section is a square) a pyramid is inscribed whose base is an equilateral triangle and one of the lateral edges is an element of the cylinder. Compute the area of the greater face of the pyramid and the angle at which it is inclined to the base if the diagonal of the axial section of the cylinder equals $8\sqrt{2}$ cm.

602. A regular quadrangular prism whose edge is a is inscribed in a cylinder. Determine the area of the axial section of the cylinder.

603. A regular quadrangular pyramid is inscribed in a cylinder whose altitude $H = 5$ dm. Its lateral edge is inclined to the base at an angle $\alpha = 58°12'$. Determine the radius of the base of the cylinder.

604. A regular triangular prism is inscribed in a cylinder. A line segment joining the centre of the cylinder base to a point M of a lateral edge of the prism is equal to 16 cm and forms with the lateral edge an angle of 30°. The point M divides the lateral edge in the ratio of 1 to 4. Find the area of the axial section of the cylinder. (Two cases.)

605. The diagonal of an axial section of an equilateral cylinder is equal to d. Find the edges of a regular triangular prism inscribed in this cylinder.

606. A cylinder is inscribed in a right triangular prism; the sides of its base are 26 cm, 28 cm and 30 cm. Determine the area of the axial section of the cylinder and the area of its base if the altitude of the prism is equal to 40 cm.

607. A line segment is tangent to a cylinder and its end-points are contained in the planes of the bases of the cylinder. The end-points of the segment are 20 cm and 15 cm distant from the axis of the cylinder. Determine the length of this segment if the radius of the base is equal to 12 cm and the altitude to 25 cm.

608. The diameters AB and CD of the upper and lower bases of a cylinder are mutually perpendicular. A section of the cylinder passes through one diameter and the end-point of the other. The area of the section is to the area of the base of the cylinder as $1 : 2\sqrt{2}$. Prove that the diameter of the base of the cylinder is twice the length of its altitude.

609. A regular quadrangular pyramid is inscribed in a cylinder whose altitude equals 10 cm. The end-point of an element passing through a vertex of the base of the pyramid is 6 cm distant from its edge. Find the side of the base and the lateral edge of the pyramid.

20. The Right Circular Cone

610. Orally. 1. The generator of a cone is equal to 10 cm, and the altitude to 6 cm. Find the radius of the base.

2. The altitude of a cone is equal to H. The generator is inclined to the base at an angle of 30°. Find the radius of the base.

3. The radius of the base of a cone is R. The axial section is an equilateral triangle. Find its area.

611. 1. The radius of the base of a cone is R, the altitude is h. Find the generator.

2. The area of the base of a cone is Q, the generator is l. Find the altitude of a cone.
3. The generator of a cone l is inclined to its base at an angle of 60°. Find the radius of the base and the altitude of the cone.
4. The radius of the base of a cone is R, the area of the axial section is Q. Find the generator.

612. A right-angled triangle ABC with a right angle C revolves about the side BC. What figures are obtained by revolving: (1) the point A, (2) the side AC, (3) the hypotenuse AB, (4) the triangle ABC?

613. A trapezium $ABCD$ in which $AD \parallel BC$ $(AD > BC)$ and $AB \perp AD$ revolves about a straight line passing through the vertex of an acute angle parallel to the smaller lateral side. What figures are obtained by rotating: (1) the vertices of the trapezium, (2) the sides of the trapezium, (3) the trapezium itself?

614. The angle at the vertex of an axial section of a cone is a right one. The area of the section area is equal to 25 cm^2. Find the generator of the cone and the area of its base.

615. 1. The ratio of the area of the base of a cone and the area of the axial section is equal to $\pi\sqrt{3}$. Find the angle of inclination of the generator to the base.
2. Find the ratio of the area of the base of an equilateral cone (i.e. of a cone whose axial section is an equilateral triangle) to the area of its axial section.

616. The diameter of the base circle of a cone is equal to 54 cm and the generator to 45 cm. Determine the area of the axial section and the maximum angle between the elements.

617. Two mutually perpendicular elements of a cone are subtended by a chord of the base circle equal to a. The altitude of the cone equals $\frac{a}{2}$. (1) Determine the angle of inclination of the generator to the base and the angle at the vertex of the axial section of the cone. (2) Is it possible to draw in the base circle a chord longer than a?

618. The area of an axial section of a cone is equal to $40\sqrt{3}$ cm^2. The maximum angle between the elements is 120°. Determine the altitude of the cone, its generator and radius of the base circle.

619. An equilateral cone whose generator is l rolls along a plane α rotating about its vertex. Find the area of the base of the cone generated by: (1) the altitude of the given cone, (2) the element of the given cone permanently found at a maximum angle to the plane α.

620. The angle at the vertex in an axial section of a cone is equal to 120°, and the altitude to h. A second cone is constructed so that its vertex coincides with the vertex of the given cone and the generator is perpendicular to the generator of the given cone. Find the area of the axial section of the second cone if its altitude is also equal to h.

621. Through the vertex of a cone a plane is drawn at an angle of 60° to the altitude. The altitude of the cone is equal to 4 dm, the generator to 10 dm. Compute the area of the section.

622. Through the vertex of a cone a plane is drawn at an angle of 60° to the base. Compute the distance between the centre of the base of the cone and the cutting plane if the altitude of the cone is equal to 24 cm.

623. The radius of the base circle of a cone is R. The angle at the vertex of the axial section is equal to 120°. Find the area of a section drawn through two mutually perpendicular elements.

624. Through the vertex of a cone and at an angle of 30° to the base a plane is drawn cutting off a third of the base circle. The altitude of the cone equals 6 m. Determine the area of the section.

625. A chord divides the base circle of a cone in the ratio of 1 to 3. Through the vertex of the cone and this chord a section is drawn at an angle of 60° to the base. The altitude of the cone is equal to h. Find the ratio of the areas of the base and the section.

626. The maximum angle between the elements of a cone is equal to 126°54′. A plane is drawn through two elements the angle between which amounts to 60°. Find the angle between this plane and the base.

627. The radius of the base of a cone is equal to 16 cm. Find the area of a section parallel to the base which divides the altitude of the cone: (1) into two equal parts; (2) in the ratio of 1 to 3 (as measured from the vertex); (3) in the ratio of m to n.

628. The altitude of a cone is equal to 36 cm and the diameter of the base circle to 24 cm. The cone is cut by a plane parallel to the base so that the area of the section equals 64π cm^2. Determine the distance between the cutting plane and the base of the cone.

629. The altitude of a cone is equal to 15 cm. At a distance of 6 cm from the base a section parallel to the base is drawn whose area is 36π cm^2. Find the radius of the base circle.

630. Through the mid-point of an element $l = 10$ cm of an equilateral cone a plane is drawn perpendicular to this element. Determine the area of the section thus obtained if the projection of the section on the plane of the base of the cone is a circle.

631. In a cone with the altitude $h = 15$ cm a regular triangular pyramid is inscribed. Its faces are inclined to the base at an angle of $\alpha = 36°34'$. Determine the generator of the cone.

632. A regular tetrahedron can be inscribed in a given cone. Compute the maximum angle between the elements of such a cone.

633. A cube with the edge a is inscribed in an equilateral cone. Find the generator of the cone.

634. In an equilateral cone an equilateral cylinder is inscribed so that their axes coincide. Find the area of the axial section of the cone if that of the cylinder is equal to a^2.

635. Inscribed in an equilateral cone is a rectangular parallelepiped with a square base. The altitude of the parallelepiped is twice the length of the side of the base equal to a. Determine the area of the axial section of the cone.

21. The Truncated Cone

636. A rhombus $ABCD$ (AC being the greater diagonal) revolves about a straight line which is perpendicular to the diagonal AC and passing through the vertex C. What figures are obtained by revolving the vertices of the rhombus, the sides of the rhombus, the diagonals of the rhombus, the rhombus itself?

637. An isosceles trapezium $ABCD$ ($BC \parallel AD$ and $BC < AD$) revolves about a straight line perpendicular to the base AD and passing through the vertex A. What figures are obtained by revolving the vertices of the trapezium, the sides of the trapezium, the diagonals of the trapezium, the trapezium itself?

638. 1. The radii of the bases of a truncated cone are equal to 8 m and 5 m, the altitude to 4 m. Find the generator.

2. The radii of the bases of a truncated cone are R and r; the generator is inclined to the base at an angle of 45°. Find the altitude.

3. The altitude of a truncated cone is equal to H. Determine the generator if it is inclined to the base at an angle of 30°.

639. 1. The radii of the bases of a truncated cone are equal to 7 cm and 22 cm, and the generator to 25 cm. Find the altitude of the cone.

2. The generator l of a truncated cone is inclined to the plane containing the greater base at an angle of 45°. The radius of the smaller base is equal to r. Determine the altitude and the radius of the greater base of the cone.

640. The altitude of a truncated cone is equal to 12 cm, the generator to 13 cm. Find the radii of the bases of the cone if their ratio is 3 : 4.

641. The generator of a truncated cone equal to l is inclined to the base at an angle of 60° and perpendicular to the diagonal of the axial section. Find the radii of the bases and the altitude.

642. The radii of the bases of a truncated cone are R and r, the altitude is h. A circle can be inscribed in the axial section. Prove that $h = 2\sqrt{Rr}$.

643. The area of the circle inscribed in the axial section of a truncated cone is equal to Q. The generator is inclined to the base at an angle of 45°. Find the radii of the bases.

644. In a truncated cone the diagonals of the axial section are mutually perpendicular and divided at the point of intersection in the ratio of 7 to 24. The generator of the cone is equal to 50 cm. Determine the radii of the bases and the altitude of the cone.

645. A triangle whose sides are 25 cm, 29 cm and 36 cm long rotates about an axis passing through the vertex of the medium angle and perpendicular to the greater side. Determine the area of the axial section of the truncated cone generated by the medium side of the triangle.

646. The axial section of a truncated cone is a trapezium in which the diagonal is perpendicular to the lateral side. Determine the area of the diagonal section if the generator and the altitude of the truncated cone are equal to 15 dm and 12 dm, respectively.

647. Inscribed in a truncated cone whose radius of the smaller base circle is equal to 5 dm is another truncated cone. Their axes coincide, and the smaller base of the given cone serves as the greater base of the new one. The radius of the smaller base circle of the inscribed cone is equal to 2 dm. The generators of both cones are equal to each other and inclined to the base of the given cone at an angle of 60°. Determine the areas of the axial sections of both cones.

648. 1. The radii of the base circles of a truncated cone are R and r. Find the area of the mid-section parallel to the bases.

2. The areas of the base circles of a truncated cone are 16π cm^2 and 49π cm^2. Find the area of the section which is parallel to the bases and divides the altitude of the cone in the ratio of 1 to 2. (Consider two cases.)

649. The diagonals of the axial section of a truncated cone are mutually perpendicular, its altitude being equal to H. Find the area of a section of this truncated cone by a plane drawn through the mid-point of the altitude and parallel to the bases.

650. The areas of the bases of a truncated cone are equal to 36π cm^2 and 100π cm^2, the area of a section parallel to the bases to 64π cm^2. In what ratio is the altitude divided by this section?

651. The radii of the base circles of a truncated cone are equal to 16 cm and 12 cm, the altitude to 8 cm. Find the area of a section passing through the parallel chords of the base circles on one side of the altitude of the cone if each chord divides the base circle in the ratio of 1 to 3.

652. Through two elements of a truncated cone a plane is drawn cutting off the base circles arcs of 120°. Determine the area of the section and the angle at which it is inclined to the base if the radii of the base circles are equal to 10 cm and 2 cm, and the generator to 12 cm.

653. In a truncated cone AB and CD are diameters of the bases parallel to each other. The point M bisects the semicircle CD of the greater base circle. Find the area of the triangle ABM if the generator of the cone is equal to 13 cm, and the radii of the bases to 4 cm and 9 cm.

654. The radii of the base circles of a truncated cone are equal to 9 cm and 15 cm, the altitude to 8 cm. Through the diameter AB of the smaller base circle a section is drawn which is inclined to the base at the same angle as the generator of the cone and intersects the greater base circle at points C and D. Find the area of the trapezium $ABCD$.

655. The generator, altitude and radius of the greater base circle of a truncated cone are equal to 10 cm, 6 cm and 12 cm, respectively. Find the area of the projection of the lateral surface on the plane of the greater base.

656. The area of the projection of the lateral surface of a truncated cone on the plane of the greater base is equal to 189π dm^2. The generator and the altitude of

this cone are equal to 25 dm and 24 dm, respectively. Determine the area of the axial section of this cone.

657. A line segment 25 cm long joins two points situated on the base circles of a truncated cone. Determine the distance between this segment and the axis of the cone if the radii of the bases and the altitude of the truncated cone are equal to 14 cm, 13 cm and 20 cm, respectively.

CHAPTER V

AREAS OF POLYHEDRONS AND ROUND SOLIDS

22. Areas of Parallelepipeds and Prisms

658. 1. Determine the surface area of a cube whose edge is equal to: 10 cm, 2.5 cm, 5 cm.

2. Determine the surface area of a cube if its diagonal is equal to: 6 cm, 4 cm, 3 cm.

3. Determine the surface area of a cube given the area Q of its diagonal section.

4. Determine the edge of a cube if its surface area amounts to: 384 dm^2, 8.64 m^2.

659. The edge of a cube is equal to 8 cm. Draw the development of its surface area. Will a sheet of paper 40 cm $\times$ 30 cm size be sufficient for this purpose? Determine the lateral and total surface areas of the cube.

660. Determine the surface area of a rectangular parallelepiped if: (1) its dimensions are equal to 4 cm $\times$ $\times$ 6 cm $\times$ 8 cm; (2) the sides of the base are equal to 6 cm and 8 cm, and its diagonal to 26 cm; (3) its dimensions are as 1 : 3 : 5, and the area of the smaller face is equal to 27 dm^2.

661. The sides of the base of a rectangular parallelepiped are equal to 16 cm and 18 cm, and its diagonal to 34 cm. Determine the ratio of the total surface area to the lateral one.

662. The sides of the base of a rectangular parallelepiped are equal to 20 cm and 21 cm, and the diagonal sections are squares. Determine the area of the lateral surface of the parallelepiped.

663. The perimeter of the base of a rectangular parallelepiped is equal to 14 dm, its diagonal and altitude being equal to 13 dm and 12 dm, respectively. Determine the total surface area of the parallelepiped.

664. The areas of the faces of a rectangular parallelepiped are as 3 : 5 : 15, and the total surface area of the parallelepiped is equal to 184 dm^2. Find its dimensions.

665. 1. How many containers of the rectangular shape can be manufactured from an iron sheet 140 cm $\times$ 70 cm size if the dimensions of the container must be 35 cm $\times$ $\times$ 20 cm $\times$ 10 cm.

2. How much linen (80 cm wide) is it required to pack a box whose size is 2.4 m $\times$ 1.6 m $\times$ 1.5 m? Add 2 per cent of the found amount for seams.

666. How much mortar is it required to plaster a building 42.5 m long, 12.5 m wide and 6.4 m high? The building has 30 windows of size 1.4 m $\times$ 2.2 m. Mortar consumption is 20 kilograms per square metre.

667. A box without a cover is made up from eight boards. Each board is 6 m long and 0.4 m wide. The box is 2.5 m long, 1.4 m wide and 1.2 m high. The overlap in fastening the boards amounted to 4 per cent of the total surface area of the box. What percentage of the material remains unused?

668. In a regular quadrangular prism the diagonal is inclined to the base at an angle of 30°, and the lateral surface area of the prism is equal to $48\sqrt{6}$ m^2. Determine the side of its base.

669. In a regular triangular prism the side of the base is equal to a. The diagonal of a lateral face is inclined to the plane of another lateral face at an angle of 30°. Find the lateral surface area of the prism.

670. The altitude of a regular triangular prism is equal to $14\sqrt{3}$ cm, and the ratio of the areas of the base and the lateral face is 2 : 7. Determine the side of the base of the prism.

671. The diagonal of a regular quadrangular prism is inclined to the lateral face at an angle of 30°. The side of the base is equal to a. Determine the area of the lateral surface of the prism.

672. 1. In a regular hexagonal prism the side of the base is equal to the lateral edge. Determine the area of the lateral surface of the prism if the area of the smaller diagonal section is equal to $25\sqrt{3}$ cm^2.

2. The area of the greater diagonal section of a regular hexagonal prism is equal to Q. Find the area of the lateral surface of the prism.

673. 1. The lateral surface area of a regular hexagonal prism is equal to 48 dm^2. Find the areas of the diagonal sections.

2. The side of the base of a regular hexagonal prism is equal to a. The greater diagonal of the prism is inclined to the base at an angle of 60°. Find the total surface area of the prism.

674. Through a side of the lower base of a regular triangular prism a plane is drawn which intersects the opposite lateral edge at its mid-point and is inclined to the base at an angle of 30°. Find the lateral surface area of this prism if the side of its base is equal to 10 cm.

675. In a regular triangular prism the base is a right-angled triangle whose sides containing the right angle are equal to 10 cm and 24 cm. The diagonal of the greater lateral face is inclined to the base at an angle of 60°. Determine the area of the lateral surface of the prism.

676. The base of a right parallelepiped is a rhombus whose greater diagonal is four times longer than the radius R of the incircle. The smaller diagonal of the parallelepiped is inclined to the base at an angle of 60°. Determine the area of the lateral surface of the parallelepiped.

677. The base of a right prism is a rhombus whose side is equal to a and the acute angle to 60°. The section drawn through the longer diagonal of one base and the vertex of an obtuse angle of the other base represents a right-angled triangle. Find the total surface area of the prism.

678. The base of a right parallelepiped is a rhombus with an acute angle of 60°. The area of the larger diagonal section is equal to Q. Find the area of the lateral surface of the parallelepiped.

679. The base of a right parallelepiped is a rhombus; the areas of the diagonal sections are equal to 60 cm^2 and 80 cm^2. Determine the area of the lateral surface of the parallelepiped.

680. In a right triangular prism $ABCA_1B_1C_1$ the area of the face AA_1B_1B is equal to 26 dm^2. Through the edge AA_1 a plane is drawn perpendicular to the face BB_1C_1C. Compute the area of the lateral surface of the prism if the area of the section is equal to 24 dm^2 and the angle C of the base amounts to 53°8′.

681. The base of a right prism is an isosceles trapezium whose bases are equal to 22 cm and 12 cm. A plane is drawn through the greater base of the trapezium, which is the lower base of the prism, and the opposite side of the upper base. Find the surface area of the prism if the area of the section is equal to 408 cm^2 and it is inclined to the base at an angle of 60°.

682. The base of a right parallelepiped is a parallelogram with the sides 4 dm and 9 dm long and angle of 60°. The greater diagonal of the base is equal to the smaller diagonal of the parallelepiped. Determine the total surface area of the parallelepiped.

683. The base of a right prism is an isosceles trapezium in which a circle of radius 8 cm can be inscribed. The lateral side of this trapezium is equal to 20 cm. Find the altitude of the prism if the area of its lateral surface is equal to 160 cm^2.

684. The sides of the base of a right triangular prism are to one another as 3 : 25 : 26. The lateral edge is equal to 10 cm, and the total surface area amounts to 288 cm^2. Determine the area of the lateral surface of the prism.

685. A wooden beam of a square cross section (60 cm × × 60 cm) is 3 m 25 cm long. At a distance of 1 m 57 cm from the end it is cut into two equal parts. Determine the surface area of the cut-away half.

686. 1. In an oblique triangular prism the distances between the lateral edges are equal to 3 cm, 4 cm and 5 cm, and the lateral edge to 6 cm. Find the area of the lateral surface of the prism.

2. In an oblique quadrangular prism the lateral edge is equal to 3 dm, and the area of the lateral surface to 66 dm^2. Find the distances between the neighbouring edges of the prism if they are to one another as 1 : 2 : 3 : 5.

687. The base of an oblique prism is a regular triangle with the side a. The length of the lateral edge is b. One of the lateral edges forms with the adjacent sides of the base angles of 60°. Find the area of the lateral surface of the prism.

688. In an oblique triangular prism two lateral faces are perpendicular to each other, their areas being equal to 60 cm^2 and 80 cm^2. Find the area of the lateral surface of the prism if its lateral edge is 10 cm long.

689. The base of a prism is a regular triangle with the side equal to $6\sqrt{3}$ dm. A vertex of the upper base is projected at the mid-point of a side of the lower base. The altitude of the prism is 12 dm high. Find the area of the lateral surface of the prism.

690. Through the mid-point of the altitude of a regular tetrahedron a section is drawn parallel to the base. Another section is drawn through a side of the first one parallel to the opposite edge. Determine the area of the lateral surface of the oblique prism thus obtained if the edge of the tetrahedron is equal to a.

· 691. The base of a prism is a regular triangle whose side is a. One of the vertices of the upper base is projected in the centre of the lower base. The lateral edges of the prism are inclined to the base at an angle of 60°. Find the total surface area of the prism.

692. The base of a prism is a square with the side a. One of the lateral faces is a square, another is a rhombus with an angle of 60°. Determine the total surface area of the prism.

693. The base of an oblique parallelepiped is a rhombus whose side is equal to 15 cm and acute angle to 60°.

The greater diagonal section of the parallelepiped is perpendicular to the base. Determine the total surface area of the parallelepiped if the lateral edge is 10 cm long and inclined to the base at an angle of 60°.

694. The base of a parallelepiped is a square with the side *a*. The altitude of the parallelepiped is also equal to *a*. The projection of one of the lateral edges on the plane containing the lower base coincides with half its diagonal. Find the area of the lateral surface of the parallelepiped.

695. In an oblique hexagonal prism a perpendicular section yields a regular hexagon. The cutting plane forms with the base of the prism a dihedral angle of 60°. Find the area of the lateral surface of the prism if the area of its base is equal to $72\sqrt{3}$ cm, and the lateral edge to 10 cm.

23. Areas of Pyramids

696. Given the side of the base *a* and altitude *h* determine the total area of a regular pyramid: (1) triangular, (2) quadrangular, (3) hexagonal.

697. Determine the area of the lateral surface of a regular (a) triangular, (b) quadrangular, (c) hexagonal pyramid if the side of the base is equal to *a* and the angle of inclination of the lateral face to the base is equal to 60°.

698. The area of the lateral surface of a regular pyramid is equal to *Q*, the side of the base to *a*. Find the lateral edge and the altitude if the pyramid is: (a) triangular, (b) quadrangular, (c) hexagonal.

699. The area of the base is to the area of the lateral surface as 1 : 2; the side of the base is equal to *a*. Find the altitude of the pyramid: (a) triangular, (b) quadrangular, (c) hexagonal.

700. The diagonal section of a regular quadrangular pyramid is equal to the base whose side is *a*. Find the area of the lateral surface of the pyramid.

701. 1. Determine the surface area of a regular tetrahedron with the edge a.

2. Determine the surface area of a regular octahedron if its edge is equal to 5 cm.

702. The side of the base of a regular quadrangular pyramid is equal to 5 dm; the dihedral angles at the lateral edges to 120°. Find the area of the lateral surface of the pyramid.

703. The altitude of a regular hexagonal pyramid is equal to 8 cm, and the side of the base to $4\sqrt{3}$ cm. Determine the total surface area of the pyramid.

704. 1. Determine the total surface area of a regular triangular pyramid if the side of its base is equal to a, and the dihedral angle at the base to 60°.

2. The area of the lateral surface of a regular pyramid amounts to 36 cm^2 and the area of the base to 9 cm^2. Determine the dihedral angle at the base.

705. How many iron sheets are required to cover a roof which has the shape of a regular quadrangular pyramid with the side of the base equal to 4.2 m and the pitch 6.5 m long? The size of the metal sheets is 140 cm $\times$ $\times$ 70 cm. Add 10 per cent for joints and waste.

706. The area of the section of a regular quadrangular pyramid passing through a slant height and the altitude is equal to 9 dm^2. The lateral face of the pyramid is inclined to the base at an angle of 60°. Determine the area of the lateral surface of the pyramid.

707. Determine the area of the lateral surface of a regular hexagonal pyramid whose lateral edge is equal to 15 cm, and the diameter of the circle inscribed in the base to $18\sqrt{3}$ cm.

708. In a regular quadrangular pyramid the dihedral angles at the lateral edges are equal to 120°. Prove that the diagonal section of this pyramid is equal to its lateral face.

709. In a regular triangular pyramid the side of the base is equal to 20 cm, and the opposite lateral edge is 15 cm distant from it. Determine the area of the lateral surface of the pyramid.

710. In a regular quadrangular pyramid a cube is inscribed so that four of its vertices lie on the lateral edges of the pyramid. The altitude of the pyramid is twice the length of the edge of the cube. Find the ratio of the areas of the lateral surface of the pyramid and the total surface of the cube.

711. The side of the base of a pyramid is a. One of the lateral edges is perpendicular to the base and equal to its side. Determine the area of the lateral surface of the pyramid if its base is a: (a) regular triangle, (b) square, (c) regular hexagon.

712. The base of a pyramid is a rectangle whose area amounts to 100 dm^2. Two lateral faces are perpendicular to the base, and two others are inclined to it at angles of 30° and 60°. Find the total surface area of the pyramid.

713. The base of a pyramid is a rectangle whose diagonal is equal to m, and the angle between the diagonals to 60°. The angle of inclination of the lateral edge to the base is also equal to 60°. Determine the area of the lateral surface of the pyramid.

714. The base of a pyramid is a rhombus with the side a and acute angle of 60°. The dihedral angles at the base are also equal to 60°. Determine the total surface area of the pyramid.

715. The base of a pyramid is a rhombus with the side a and acute angle of 60°. Two adjacent lateral faces containing an angle of 60° are perpendicular to the base and two others are inclined to it at an angle of 45°. Determine the area of the lateral surface of the pyramid.

716. One of the lateral faces and the base of a pyramid are equilateral triangles forming a right dihedral angle. The area of the lateral surface of the pyramid is equal to $\sqrt{3}(1+\sqrt{5})$ m^2. Determine the side of the base.

717. The base of a pyramid is a right-angled triangle whose sides are 15 cm and 20 cm; each lateral face is inclined to the base at an angle of 60°. Find the total surface area of the pyramid.

718. The base of a pyramid is an isosceles trapezium whose parallel sides are equal to 10 dm and 20 dm. The lateral faces of the pyramid are inclined to the base at equal angles. The altitude of the pyramid is 10 dm. Find the total surface area of the pyramid.

719. The base of a pyramid is a rhombus with an acute angle of 60°. The altitude of the pyramid passes through the vertex of the obtuse angle of the rhombus and is equal to H. Two faces form with the base angles of 45°. Find the total surface area of the pyramid.

720. The sides of the base of a triangular pyramid are equal to 26 cm, 28 cm and 30 cm. The lateral faces are inclined to the base at an angle of 60°. Find the total surface area of the pyramid.

721. In a cube with the edge a the centre of one face is joined to the vertices of the opposite face. Determine the total surface area of the pyramid thus obtained.

722. The centre of the upper base of a cube with the edge a is connected with the mid-points of the sides of the lower base which are also joined to one another in a consecutive order. Compute the total surface area of the pyramid thus constructed.

723. A regular triangular pyramid and a regular prism have equal bases, altitudes and areas of lateral surfaces. The side of their base is equal to a. Determine the altitude.

724. The plane angles at the vertex of a pyramid are equal to 60°, 60° and 90°. Each of the lateral edges is equal to a. Determine the total surface area of the triangular pyramid.

725. The base of a pyramid is a rhombus. The altitude of the pyramid passes through the vertex of an acute angle of the rhombus. Determine the angles of inclination of the lateral faces to the base if the area of the lateral surface of the pyramid is $\sqrt{3}$ times the area of its base.

24. Areas of Truncated Pyramids

726. The sides of the bases of a regular truncated pyramid are equal to 10 cm and 6 cm, and slant height to 15 cm. Compute the lateral and total surface areas of the pyramid: (a) triangular, (b) quadrangular, (c) hexagonal.

727. The sides of the bases of a regular hexagonal truncated pyramid are equal to 18 cm and 12 cm, and its altitude to 13 cm. Find the area of the lateral surface of the pyramid.

728. The sides of the bases of a regular truncated pyramid are a and b; its altitude is $\frac{a+b}{2}$. Find the area of the lateral surface of the pyramid: (a) triangular, (b) quadrangular, (c) hexagonal.

729. The sides of the bases of a truncated pyramid are equal to 16 cm and 12 cm. Through the mid-point of the altitude a section is drawn parallel to the base. In what ratio is the area of the lateral surface divided by this section?

730. The sides of the bases of a regular quadrangular truncated pyramid are to each other as 1 : 5; the altitude of the pyramid is equal to 21 cm, and the area of the diagonal section to 630 cm^2. Find the area of the lateral surface of the pyramid.

731. The sides of the bases of a regular triangular truncated pyramid are equal to 6 dm and 4 dm. The lateral faces are inclined to the greater base at an angle of 60°. Determine the total surface area of this truncated pyramid.

732. In a regular quadrangular truncated pyramid the sides of the bases are equal to 20 cm and 10 cm, the lateral edge is inclined to the greater base at an angle of 45°. Compute the total surface area of this truncated pyramid.

733. In a regular triangular truncated pyramid the sides of the bases are equal to 18 cm and 12 cm, the lateral edges are inclined to the base at an angle of 45°. Find the lateral surface area of the truncated pyramid.

734. The areas of the bases of a regular hexagonal truncated pyramid are equal to 60 cm^2 and 40 cm^2, and the lateral faces are inclined to the greater base at an angle of 66°25′. Find the area of the lateral surface of the truncated pyramid.

735. In a truncated pyramid the similar sides of the bases are to each other as 7 : 3. In what ratio is the area of its lateral surface divided by a section which is parallel to the base and divides the lateral edge in the ratio of 1 to 2?

736. The area of the smaller base of a regular quadrangular truncated pyramid is equal to a^2, and that of the lateral surface to Q. The angle of inclination of the lateral face to the greater base is equal to 60°. Find the side of the greater base of the pyramid.

737. The areas of the greater base, lateral face and smaller base of a regular quadrangular pyramid are as 25 : 16 : 9. Find the total surface area of this truncated pyramid if the area of its diagonal section equals $16\sqrt{30}$ cm².

738. The diagonals of a regular quadrangular truncated pyramid and of the larger base form an angle of 60° and are equal to 8 dm and 5 dm, respectively. Determine the total surface area of the pyramid.

739. The lateral edge of a regular triangular truncated pyramid is equal to 20 cm and inclined to the larger base at an angle of 30°. The side of the smaller base of the pyramid is equal to 12 cm. Find the area of the lateral surface of the pyramid.

740. In a regular triangular truncated pyramid the area of the lateral surface is equal to $12\sqrt{601}$ cm², and the altitude and lateral edge to 12 cm and 13 cm, respectively. Find the sides of the bases.

741. In a regular hexagonal truncated pyramid the altitude is equal to the side of the smaller base and amounts to 6 dm. The lateral edge is inclined to the greater base at an angle of 45°. Find the area of the lateral surface of the pyramid.

742. Inside a regular quadrangular truncated pyramid a non-truncated pyramid is constructed whose base is the greater base of the given truncated pyramid, and the vertex is the centre of the smaller base. The sides of the bases are a and b. The areas of the lateral surfaces

are equal to each other. Determine the altitude of the pyramids.

743. The bases of a truncated pyramid are right-angled isosceles triangles whose hypotenuses are equal to $4\sqrt{2}$ dm and $6\sqrt{2}$ dm. The lateral edge joining the vertices of the right angles of the bases, is perpendicular to the base and equal to 4 dm. Find the area of the lateral surface of the pyramid.

744. The bases of a truncated pyramid are rhombuses with the respective sides a and b and angle of 60°. The altitude of the pyramid passes through the points of intersection of the diagonals of the rhombuses and equals to $\frac{1}{4}(a-b)$. Find the area of the lateral surface of the pyramid.

745. In a triangular truncated pyramid $ABCA_1B_1C_1$ the lateral face AA_1C_1C is an isosceles trapezium perpendicular to the bases. Compute the area of the total surface of the pyramid if $AC = 80$ cm, $AB = BC = 50$ cm, $A_1C_1 = 40$ cm and the altitude $H = 16$ cm.

746. In a triangular truncated pyramid $ABCA_1B_1C_1$ the bases are right-angled triangles. The lateral edge AA_1 is perpendicular to the bases and equals 7 cm. The sides AC and BC containing the right angle are respectively equal to 48 cm and 14 cm, the hypotenuse $A_1C_1 = 25$ cm. Compute the area of the total surface of the truncated pyramid.

747. The sides of the bases of a regular quadrangular truncated pyramid are equal to 25 cm and 15 cm. A section passing through a side of the lower base and an opposite side of the upper base is perpendicular to the lateral face. Determine the area of this section if the area of the total surface of the truncated pyramid amounts to 2050 cm^2.

25. Areas of Cylinders

748. How much varnish is it required to paint the external surface of a cylindrical tube 2 m long, whose outer diameter is equal to 50 cm, if the varnish consumption is 200 grams per square metre of the surface?

749. Determine the total surface area of a grindstone whose diameter is equal to 50 cm and thickness to 15 cm if it has a square hole at the centre 8 cm $\times$ 8 cm in size.

750. How much varnish is it required to paint 100 cylindrical buckets both from outside and inside if the diameter of the bottom is equal to 24 cm, the height of the bucket to 32 cm and the varnish consumption to 160 grams per square metre?

751. Determine the total surface area of a cylindrical tube 1.48 m long if its outer diameter is equal to 0.94 m and wall thickness to 0.06 m.

752. Two fire tubes 0.4 m in diameter each pass through a steam boiler. How much iron is it required to make such a boiler which is 4 m long and has an outer diameter of 1.4 m?

753. 1. Find the area of the lateral surface of an equilateral cylinder whose generator is equal to l.

2. How many times is the area of the lateral surface of an equilateral cylinder greater than the area of its base?

754. 1. The area of an axial section of a cylinder is equal to Q. Find the area of its lateral surface.

2. The diagonal of an axial section of a cylinder is equal to l and inclined to the base at an angle of 60°. Find the area of the lateral surface of the cylinder.

755. 1. (Orally.) How will the area of the lateral surface of a cylinder change if: (a) its altitude is increased five times and the diameter of the base circle remains unchanged; (b) the diameter of the base circle is decreased half the length without changing the altitude; (c) the altitude is increased three times and the diameter of the base circle four times; (d) the altitude is increased six times and the diameter of the base circle is decreased to one third?

2. Two cylindrical supports are to be painted. Which of the supports will take more paint if one of them is half the length and twice the thickness of the other?

756. The area of the lateral surface of a cylinder is equal to the area of a circle whose diameter is an element

of the cylinder. Find the relationship between the altitude and the radius of the base circle of the cylinder.

757. The area of the lateral surface of the cylinder is equal to the area of the circle circumscribed about the axial section. Find the ratio of the altitude of the cylinder and the radius of the base circle.

758. The edge of a cube is equal to the radius of a cylinder, and the surface area of the cube is equal to the area of the lateral surface of the cylinder. Determine the altitude of the cylinder.

759. Find the ratio of the areas of the lateral surfaces of an equilateral cylinder and (a) a rectangular parallelepiped (with a square base) inscribed in it; (b) a cube circumscribed about it.

760. A rectangle with the sides equal to a and b rotates first about the side a, and then about b. Find: (a) the ratio of the areas of the lateral surfaces of the solids of revolution thus obtained; (b) the ratio of the total surface areas of these solids.

761. The radius of the base circle of a cylinder is equal to R, the altitude to H. Two axial sections form a dihedral angle of 30°. Find the portion of the surface area subtended by this dihedral angle.

762. The area of a section of a cylinder which is drawn parallel to its axis and cuts off the base circle an arc of 120° is equal to Q. Determine the area of the lateral surface of the cylinder.

763. The development of the lateral surface of a cylinder is a rectangle whose diagonal is equal to d and inclined to the base of the rectangle at an angle of 30°. Find the area of the total surface of the cylinder.

764. A cylinder is cut by a plane which is parallel to its axis and cuts off the base circle an arc of 120°. Determine the area of the total surface of the greater portion of the cylinder if $R = 10$ cm and $H = 15$ cm.

765. The total surface area of an equilateral cylinder is equal to 9.6 m^2. Determine the area of its lateral surface.

766. Determine the area of the total surface of a cylinder if the area of its base is equal to 490π cm^3, and the area of the axial section to 400 cm^2.

767. A line segment joining the end-points of perpendicular diameters of the base circles of a cylinder is equal to 10 cm and is 4 cm distant from the axis of the cylinder. Find the area of the total surface of the cylinder.

768. The area of the section of a cylinder drawn through the diagonal of the axial section and perpendicular to the latter is equal to 24π cm^2. Determine the area of the total surface of the cylinder if the cutting plane is inclined to the base at an angle of 60°.

769. Through a point on the circle of the upper base of a cylinder a section is drawn at an angle of 45° to the base. Find the curved areas of the portions thus obtained if the radius of the base circle is equal to 10 cm, and the altitude to 50 cm.

770. Points M and N divide the axis of a cylinder in the ratio of 1 : 2 : 3. Through these points planes are drawn which are not parallel to the cylinder bases and do not intersect them. The area of the lateral surface of the cylinder is equal to Q. In what portions is the area of the lateral surface divided by the cutting planes?

771. At an angle of 60° to the base of a cylinder a plane is drawn which does not intersect its base. Find the area of this section if the lateral area of the cylinder is equal to Q, and the altitude to H.

772. Find the area of a section of a cylinder which is inclined to the base at an angle of 60° and cuts from one of the base circles an arc of 90° if the area of the lateral surface of the cylinder is equal to Q, and the altitude to H. (Consider two cases.)

26. Areas of Cones

773. 1. The radius of the base circle of a cone is equal to R, and the altitude to H. Find the area of the lateral surface of the cone.

2. The altitude of a cone is equal to H, the generator to l. Find the area of the total surface of the cone.

774. Determine the area of the total surface of a solid obtained by revolving a right-angled triangle about the greater side if its hypotenuse is equal to 12 cm and one of the sides containing the right angle to $4\sqrt{5}$ cm.

775. 1. How will the lateral surface area of a cone change if: (a) the radius of the base circle is increased two times; (b) the generator is reduced to one third?
2. How will the lateral surface area of a cone change if the radius of the base circle is reduced to one third and the generator is increased four times?

776. Prove that the area of the lateral surface of a cone is equal to that of a cylinder whose base is the base of the cone, and the altitude is equal to half the length of the generator of the cone.

777. 1. The area of the lateral surface of a cone is twice the area of its base. Prove that the cone is an equilateral one.
2. The area of the lateral surface of an equilateral cone is equal to Q. Find the altitude of the cone.

778. The area of the base of a cone $S \approx 28.26$ dm^2. The altitude of the cone is equal to 3 dm. Find the angle of inclination of the generator to the base and the area of the lateral surface of the cone.

779. 1. A cone and a cylinder have equal bases, altitudes and areas of the lateral surfaces. Find the maximum angle between the elements of the cone.
2. An equilateral cylinder and a cone have equal areas of the lateral surfaces. Find the ratio of their total surface areas.

780. A cone and a cylinder have a common base, and the vertex of the cone is found at the centre of the other base of the cylinder. The area of the lateral surface of the cylinder is equal to that of the cone. Find the angles between the elements of the cone and those of the cylinder.

781. An oil container has the form of a cylinder with a cone at the top. The radius of the base circle of the container is equal to 6 m, the altitude of the cylinder to 5 m, and the generator of the cone to 7.5 m. Determine the surface area of the container.

782. How much will the painting of a conical spire of a tower cost if the length of the circumference of its base circle is equal to 18.84 m, and the angle between the elements of the axial section to $23°4'$. The painting of 1 m^2 costs 15 kopecks.

783. The angle at the vertex of an axial section of a cone is a right one. The perimeter of the axial section is equal to p. Find the area of the total surface of the cone.

784. 1. A right-angled triangle rotates about the hypotenuse. Find the surface area of the solid of revolution if the sides containing the right angle are equal to 10 cm and 24 cm.

2. A triangle with the sides 26 cm, 28 cm and 30 cm long rotates about the medium side. Compute the surface area of the solid of revolution thus obtained.

785. A right-angled triangle with the hypotenuse equal to $2a$ and acute angle of $30°$ rotates about an axis passing through the vertex of the right angle and parallel to the hypotenuse. Find the surface area of the solid thus obtained.

786. A regular hexagon with the side a rotates about the greater diagonal. Determine the surface area of the solid of revolution thus generated.

787. Through the vertex of a cone and at an angle of $30°$ to the base a plane is drawn cutting off the base circle an arc of $60°$. Find the area of the lateral surface of the cone if the distance between the cutting plane and the centre of the base circle of the cone is equal to a.

788. The section of a cone by a plane passing through its vertex and cutting off one fourth from the base circle is an equilateral triangle whose area is Q. Find the total surface area of the cone.

789. 1. The angle of the development of the lateral surface of a cone is equal to $120°$, its generator to 30 cm. Compute the diameter of the base circle of the cone.

2. The radius of the base circle of a cone is equal to R, and its generator to l. Find the angle of the development of the lateral surface of the cone.

790. Draw the development of the surface of a cone if the radius of the base circle is equal to 4 cm, and the generator is three times its length. Determine the area of the total surface of the cone, the area of its axial section and the maximum angle between the elements.

791. 1. Given a circular sector of 90° whose radius is equal to 18 cm. Find the total surface area of a cone whose lateral surface is made up of this sector.

2. The lateral surface of a cone is made up of a quarter of a circle. Determine the total surface area of this cone if the lateral surface area is equal to Q.

792. 1. The altitude of a cone is equal to 8 cm, the radius of the base circle to 6 cm. The lateral surface of the cone is developed on a plane. Find the angle of the obtained sector.

2. Compute the angle of the development of the lateral surface of a cone: (a) if the maximum angle between the elements is a right one; (b) if the generator is inclined to the base at an angle of 60°.

793. 1. A conic surface is made up of a semicircle. Find the angle between the elements of the axial section of the cone.

2. The lateral surface area of a cone equals 20 cm^2 and is developed into a sector with an angle of 72°. Determine the total surface area of the cone.

794. The generator is inclined to the base at an angle of 45°. Inscribed in the cone is an equilateral cylinder whose altitude is a. Find the area of the lateral surface of the cone.

795. An equilateral cylinder is inscribed in an equilateral cone. Find the ratio of the areas of the lateral surfaces of the cone and cylinder.

796. A regular pyramid with the side of the base a is inscribed in an equilateral cone. Find the area of the lateral surface of the cone if the pyramid is: (a) triangular, (b) quadrangular, (c) hexagonal.

797. Find the ratio of the radius of the base circle to the generator of the cone if its lateral surface is the mean proportional to the area of the base circle and the total surface area of the cone.

798. The area of the lateral surface is the mean proportional to the area of the base circle of a cone and the area of its total surface if: (a) the area of the lateral surface of the cone is equal to the area of a circle whose radius is equal to the altitude of the cone, (b) the area of the total surface is equal to the area of a circle whose radius is the generator of the cone. Prove this.

799. The radius of the base circle is equal to R, and the altitude of the cone to H. A cylinder with the maximum area of the lateral surface is inscribed in the cone. Find the area of the lateral surface of the cylinder.

27. Areas of Truncated Cones

800. 1. The radii of the base circles of a truncated cone are equal to 4 cm and 20 cm, the altitude to 30 cm. Find the area of the lateral surface of the truncated cone.

2. The radius of the smaller base circle of a truncated cone is equal to 8 cm, the altitude to 6 cm. The generator is inclined to the base at an angle of 45°. Find the area of the lateral surface of the truncated cone.

801. The area of the lateral surface of a truncated cone is equal to 128π cm^2, the generator to 8 cm. Find the radii of the base circles if they are to each other as 2 to 5.

802. A vent pipe has to be fitted with a hood having the shape of a truncated cone whose altitude is equal to 60 cm, and the diameters of the base circles to 2 m and 40 cm. How much sheet iron (in square metres) is it required to construct the hood if 5 per cent of the material is provided for the seams?

803. How much varnish is it required for the outside and inside painting of 500 buckets having the form of a truncated cone if the diameters of its base circles are equal to 25 cm and 30 cm, the altitude to 40 cm, and the varnish consumption to 160 grams per square metre?

804. Tha generator of a truncated cone is equal to l and inclined to the base at an angle of 60°. Determine the area of the total surface of this truncated cone if the ratio of the areas of its base circles is equal to 9.

805. The generator l of a truncated cone is perpendicular to the diagonal of the axial section passing through it. The generator is inclined to the base at an angle of 60°. Determine the area of the lateral surface of the cone.

806. The generator of a truncated cone is inclined to the lower base at an angle of 60° and perpendicular to a straight line joining its upper end-point to the lower end-point of the opposite element. Determine the area of the total surface of the truncated cone if the radius of its greater base circle is equal to R cm.

807. In a truncated cone the radii of the base circles are equal to 8 cm and 20 cm, and the altitude to 16 cm. Find the radius of the base circle of a cylinder having the same altitude whose total surface area is equal to that of the truncated cone.

808. The radii of the base circles of a truncated cone are equal to 15 cm and 30 cm, and the altitude to 20 cm. Find the dimensions of an equilateral cone whose total surface area is equal to the lateral surface area of the given truncated cone.

809. The areas of the base circles of a truncated cone are equal to 100π cm^2 and 256π cm^2, and the area of the axial section to 208 cm^2. Find the area of the lateral surface of this truncated cone.

810. The radius of the smaller base circle of a truncated cone is equal to 6 cm, the area of the lateral surface to 90π cm^2, the difference of the radii of the base circles to half the length of the generator. Find the radius of the greater base circle.

811. In a truncated cone the radii of the base circles are equal to 6 cm and 10 cm, and the generator to 5 cm. Removed from this cone is another truncated cone whose greater base circle is the smaller base circle of the given cone, and the generator is equal to 5 cm. Determine the area of the total surface of the obtained solid with a through hole.

812. A circle can be inscribed in the axial section of a truncated cone. Prove that the area of the lateral surface of the cone is equal to the area of a circle whose radius is equal to the generator of the cone.

813. A section of a truncated cone by a plane passing through two elements has an acute angle of 60°, cuts off from the base circles arcs equal to 90°, and has an area of 150 cm^2. Find the area of the lateral surface of this truncated cone.

814. An equilateral triangle with the side a rotates about an axis parallel to one of its sides and at a distance a from it. Find the surface area of the solid thus obtained.

815. A regular hexagon with the side a rotates about one of its sides. Find the surface area of the solid thus generated.

816. A rhombus with the side a and acute angle of 60° revolves about an axis passing through the vertex of the acute angle of the rhombus perpendicular to its diagonal. Find the surface area of the solid of revolution.

817. In an isosceles trapezium the bases are equal to 15 cm and 9 cm, and the lateral side to 5 cm. The trapezium revolves about an axis passing through the end-point of the greater base and perpendicular to it. Compute the total surface area of the solid thus generated.

818. The radii of the base circles of a truncated cone are equal to 12 cm and 6 cm, and the area of its greater base circle is the mean proportional between the areas of the lateral surface and the smaller base. Find the area of the axial section of this cone.

CHAPTER VI

VOLUMES OF POLYHEDRONS AND ROUND SOLIDS

28. Volumes of Parallelepipeds

819. Orally. 1. Compute the volume of a cube if its edge is equal to: 10 cm, 2.5 dm, 0.6 cm.

2. Compute the volume of a cube if its surface area is equal to: 54 cm^2, 150 dm^2, 300 cm^2.

3. Compute the volume of a cube if its diagonal is equal to: 8 cm, 27 mm, d.

4. Compute the volume of a cube if the area of its diagonal section is equal to: 10 cm^2, 42 m^2, S dm^2.

820. A hollow iron cube has an edge $l = 20$ cm, the walls being 5 cm thick. One face of the cube is provided with a square hole as big as 16 cm^2. Find the mass of the cube if the density of iron is $7.86 \cdot 10^3$ kg/m^3.

821. How will the volume of a cube change if: (1) its edge is increased five times, (2) its diagonal is reduced to one third, (3) its surface area is increased twice?

822. 1. The edge of a cube is increased by 1 cm resulting in an increase of its volume by 37 cm^3. Find the original volume of the cube.

2. The volume of a cube is numerically equal to the area of its face multiplied by 4. Find the volume of the cube.

823. The surface area (in square units) and volume of a cube (in cubic units) are expressed by one and the same number. Find the edge of such a cube.

824. 1. Three lead cubes with edges of 6 cm, 8 cm and 10 cm are melted into one cube. What is the length of the edge of the obtained cube?

2. A lead cube whose edge is 20 cm long is melted into three cubes whose edges are in the same proportion as 3 : 4 : 5. Find the volumes of the cubes thus obtained.

825. 1. The overall dimensions of a rectangular parallelepiped are 1 cm, 5 cm and 10 cm. Find its volume.

2. The edges of a rectangular parallelepiped are in the same proportion as 2 : 3 : 6, the diagonal being equal to 14 cm. Find the volume of the parallelepiped.

826. The diagonal of the base of a rectangular parallelepiped is equal to 6 dm and forms an angle of 30° with the larger side of the base. Find the altitude of the parallelepiped if its volume is equal to 54 dm^3.

827. 1. The areas of the three faces of a rectangular parallelepiped are equal to 20 cm^2, 28 cm^2 and 35 cm^2. Find the volume of the parallelepiped.

2. Given the areas of the faces S_1, S_2 and S_3, find the volume of a rectangular parallelepiped.

3. The areas of the three faces of a rectangular parallelepiped are in the proportion 3 : 6 : 10. The volume of the parallelepiped is equal to 150 cm^3. Find the dimensions of the parallelepiped.

828. How many trips must a 3-ton dumper carry out to transport 10,000 bricks to a construction site, if the dimensions of a brick are: 2.5 dm × 1.2 dm × 0.65 dm, its density being $1.8 \cdot 10^3 \frac{kg}{m^3}$?

829. 1. A stack of deals (pine boards) 8 m × 8 m × 4 m in size is loaded on a barge. What is the weight of this load (in tons) if the density of pine is $0.5 \cdot 10^3 \frac{kg}{m^3}$?

2. A timber beam having the form of a rectangular parallelepiped (5 dm × 30 dm × 2 dm) is lowered into water and a load is placed on it such that the beam turns out to be submerged 0.9 its volume. Determine the mass of the load if the density of wood is equal to $0.84 \cdot 10^3 \frac{kg}{m^3}$.

830. A raft is made up of 16 timbers of a rectangular cross section each having the size 3.6 m $\times$ 0.2 m $\times$ $\times$ 0.25 m. What weight can be transported by this raft if for the sake of security it is loaded only 80 per cent of the maximum load? The density of wood is $0.7 \cdot 10^3 \frac{kg}{m^3}$.

831. A ditch 1.6 m deep is to be dug along the contour shown in Fig. 25. How long will it take two navvies to complete this job if each of them is able to dig out 0.62 cubic metre per hour?

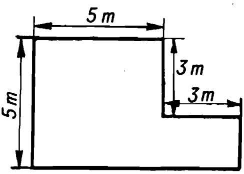

Fig. 25

832. A tank having the form of a rectangular parallelepiped with the base 3.2 m $\times$ 1.2 m contains 900,000 kg of water. How much galvanized iron (in m^2) was spent to make the tank, provided the seams took 5 per cent of the material?

833. A tank having the form of a rectangular parallelepiped with a square base and brim-filled with water is placed on an edge so that the bottom of the tank forms an angle of 30° with the horizontal plane. How much water (in per cent) ran out of the tank if its altitude is twice the length of the side of the base?

834. 1. Find the surface area of a cube which is equal to a rectangular parallelepiped whose dimensions are 4 dm $\times$ 6 dm $\times$ 9 dm.

2. A cube and a rectangular parallelepiped have equal surface areas. The dimensions of the parallelepiped are as 1 : 6 : 6, its volume being equal to 562.5 dm^3. Find the volume of the cube.

835. Given a cube with the edge 12 cm long and a right parallelepiped equal to the cube. The acute angle of the base of the parallelepiped is equal to 30°, and its edges are as 12 : 9 : 4. Determine the area of the total surface of the parallelepiped.

836. Prove geometrically the following formulas:
(1) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$,
(2) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

837. The diagonal of the base of a rectangular parallelepiped is equal to 16 cm. The greater side of the base subtends an arc of the circumscribed circle containing 120°. The lateral surface of the parallelepiped is equal to 24 cm^2. Find the volume of the parallelepiped.

838. The altitude of a rectangular parallelepiped with a square base is equal to 40 cm, and its total surface area to 2208 cm^2. Determine the volume of the parallelepiped.

839. In a rectangular parallelepiped $ABCDA_1B_1C_1D_1$ the sides of the base are equal to 8 cm and 6 cm, and the area of the section ACD_1 to 48 cm^2. Determine the volume of the parallelepiped.

840. Determine the volume of a right parallelepiped whose edges are equal to 4 m each, and the angle of the base to 60°.

841. The base of a right parallelepiped is a rhombus with the side a and an angle of 60°. The area of the lateral surface of the parallelepiped is equal to $8a^2$. Find the volume of the parallelepiped.

842. The base of a right parallelepiped is a parallelogram whose sides are equal to a and $4a$, and the acute angle $\alpha = 60°$. The greater diagonal of the parallelepiped is equal to $5a$. Determine its volume.

843. The base of a right parallelepiped is a parallelogram whose greater side is equal to 25 cm, and the smaller diagonal serves as the altitude of the parallelogram and is equal to 15 cm. The smaller diagonal of the parallelepiped is inclined to the base at an angle of 45°. Find the volume of the parallelepiped.

844. The base of a right parallelepiped is a rhombus with the side a. The section of the parallelepiped passing through two opposite sides of the bases has an area equal to Q and is inclined to the base at an angle of 45°. Find the volume of the parallelepiped.

845. A section is drawn through the side of the lower base of a right parallelepiped equal to 12 cm and the

opposite side of the upper base at an angle of 60° to the base. The area of the section is equal to 192 cm². Find the volume of the parallelepiped.

846. The diagonal of the base of a right parallelepiped is equal to d. Through this diagonal and the end-point of the lateral edge which does not intersect this diagonal a section is drawn at an angle of 30° to the base. The area of the section is equal to Q. Find the volume of the parallelepiped.

847. A rectangular parallelepiped whose dimensions are 10 dm, 6 dm and 8 dm is deformed so that two of its lateral faces turned out to be parallelograms with an acute angle of 30°. How is its volume changed?

848. An oblique parallelepiped has a rectangular base with the sides equal to 7 cm and 24 cm. One of the diagonal sections is a parallelogram whose plane is perpendicular to the base of the parallelepiped and the area is equal to 250 cm². Find the volume of the parallelepiped.

849. The base of a parallelepiped is a rhombus with the side a and an acute angle of 60°. The vertex of the obtuse angle of the upper base is projected in the centre of the lower base. The lateral edge is inclined to the base at an angle of 60°. Find the volume of the parallelepiped.

850. In an oblique parallelepiped two lateral faces have areas of S_1 and S_2 and form an angle of 150°. Find the volume of the parallelepiped if its lateral edge is equal to l and inclined to the base at an angle of 60°.

29. Volumes of Prisms

851. The side of the base of a regular prism is equal to a, the lateral edge to b. Find the volume if the prism is: (a) triangular; (b) quadrangular; (c) hexagonal.

852. A steel pipe whose cross section is a regular hexagon with the side equal to 5 cm has a longitudinal hole of 3 cm × 3 cm. Determine the weight of one linear metre of the pipe. The density of steel is $7.86 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}$.

853. A field having the shape of a triangle with the sides of 222 m, 156 m and 90 m is to be fertilized. To this end it has to be covered with a peat layer 0.6 cm thick. How much peat (in tons) is it required for this purpose? The density of peat is equal to $0.4 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}$.

854. Determine the capacity of a shed whose dimensions are: $a = 25$ m, $b = 10$ m, $c = 4$ m (Fig. 26).

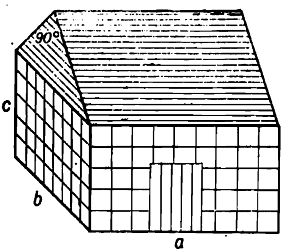

Fig. 26

855. In a regular triangular prism the perpendicular dropped from a vertex of the base to the opposite side of the other base is equal to a and inclined to the base at an angle of 60°. Determine the volume of the prism.

856. The volume of a regular triangular prism is equal to 90 cm³, and the radius of the incircle to 3 cm. Find the altitude of the prism.

857. In a regular triangular prism a plane is drawn through a side of the lower base and the opposite vertex of the upper base at an angle of 45° to the base. The area of the section is equal to Q. Determine the volume of the prism.

858. The greatest diagonal of a regular hexagonal prism is equal to l and inclined to the lateral face of the prism at an angle of 30°. Find the volume of the prism.

859. The lateral face of a regular hexagonal prism is a square whose diagonal is equal to d. Find the edge of a cube which is equal to the given prism.

860. When digging a foundation pit having the shape of a regular octagonal prism with the side of the base 6 m long the earth excavated amounted to 600 tons.

Determine the depth of the pit if the density of earth is equal to $1.8 \cdot 10^3 \frac{kg}{m^3}$.

861. Through two opposite vertices of the base of a regular hexagonal prism a plane is drawn which cuts equal segments from four lateral edges of the prism. The section area is equal to $27\sqrt{3}$ dm², and the cutting plane is inclined to the base at an angle of 45°. Find the volume of the prism.

862. Compute the mass of a metal hollow bar whose cross section and dimensions (in millimetres) are given in Fig. 27. The bar is 1.5 m long, the density of the metal being equal to $7.8 \cdot 10^3 \frac{kg}{m^3}$.

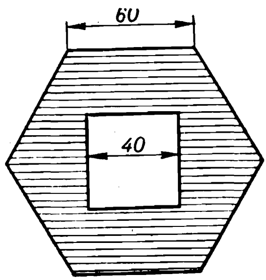

Fig. 27

863. The cross section of an irrigation canal has the shape of an isosceles trapezium whose bases are equal to 3 m and 2.5 m. The depth of the canal is equal to 1 m, the depth of the water flow in the canal to 0.7 m, the rate of flow to $2 \frac{km}{hr}$. Find the amount of water passing through the cross section of the canal during 24 hours.

864. A lead bar, whose mass is equal to 18 kg, has the shape of a right prism 30 cm high. The base of the prism is an isosceles trapezium, the parallel sides of which are equal to 3.5 cm and 11.5 cm and the lateral side to 8.5 cm. Find out whether the bar is solid or it has blow holes (in the latter case determine their volume). The density of lead is equal to $11.3 \cdot 10^3 \frac{kg}{m^3}$.

865. The cross section of a concrete slab has the shape of a right-angled trapezium, whose parallel sides are equal to 15 cm and 10 cm, and the altitude to 25 cm. The slab is 3 m long. How many slabs can a 5-ton lorry carry at a time? The density of concrete is equal to $2.6 \cdot 10^3 \frac{kg}{m^3}$.

866. The base of a right prism is a right-angled triangle whose sides are equal to 20 cm and 21 cm. The greater lateral face of the prism is a square. Determine the volume of the prism.

867. The base of a right prism is a triangle whose sides are 6 cm, 25 cm and 29 cm long. Through the mid-points of two greater sides of the base a section is drawn parallel to the lateral face. The area of the section is equal to 24 cm^2. Find the volume of the prism.

868. The base of a right prism is a trapezium $ABCD$. The diagonal B_1D of the prism forms with the diagonal BD of the base an angle of 45°. Find the volume of the prism if: (a) the parallel sides $AD = 25$ cm and $BC = 16$ cm and non-parallel sides $AB = 8$ cm and $CD = 10$ cm; (b) parallel sides $AD = 34$ cm and $BC = 22$ cm, and non-parallel sides $AB = 17$ cm and $CD = 25$ cm.

869. In an equilateral cylinder whose altitude is equal to 4 dm a triangular prism is inscribed. The sides of the base of the prism divide the base circle of the cylinder in the proportion 3 : 4 : 5. Find the volume of the prism.

870. The base of a right prism is a trapezium. The areas of its lateral faces are as 1 : 1 : 1 : 2. The volume of the prism is equal to $12\sqrt{3}$ dm^3, and its altitude to the greater side of the base. Find the sides of the base.

871. Given a cube with the edge of 15 cm and an equal triangular prism in which the sides of the base and the altitude are as 87 : 75 : 18 : 50. Determine the area of the total surface of the prism.

872. Through one edge of a cube a section is drawn dividing another edge in the ratio of 3 to 5. In what ratio is the volume of the cube divided by this section?

873. The areas of the faces of a right prism are equal to 300 cm², 240 cm², 180 cm², 96 cm², 96 cm². Find its volume.

874. The base of a right prism is a right-angled triangle whose sides are equal to 32 cm and 24 cm. The radius of the sphere circumscribed about this prism is equal to 25 cm. Determine the volume of the prism.

875. Given a right prism $ABCDA_1B_1C_1D_1$ whose base is an isosceles trapezium ($AB = CD$) in which a circle of radius r can be inscribed. The area of the face CDD_1C_1 is equal to Q cm². Determine the lateral surface area and the volume of the prism.

876. Compute the surface area and volume of a right prism whose base is a regular triangle inscribed in a circle of radius $r = 4$ dm, and the altitude is equal to the side of a regular hexagon circumscribed about the same circle.

877. Prove that the volume of an oblique prism is equal to the product of the area of the section perpendicular to the lateral edges by the length of the lateral edge.

878. The distances between the lateral edges of an oblique prism are equal to 6 dm, 25 dm and 29 dm, and the lateral edge to 10 dm. Find the volume of the prism.

879. The sides of the base of an oblique prism are equal to 18 cm, 20 cm and 34 cm. The lateral edge 12 cm long is inclined to the base at an angle of 30°. Find the volume of the prism.

880. The base of a prism is a regular triangle with the side a. Two lateral faces of the prism are rhombuses with an angle of 60°. Find the volume of the prism.

881. The base of an oblique prism is a regular hexagon $ABCDEF$ whose side is a. The lateral edges are inclined to the base at an angle of 60°. The edge AA_1 is projected on the plane of the base as the segment AM where M is the mid-point of the diagonal BF of the base. Find the volume of the prism.

882. The base of an oblique prism is a trapezium whose bases are 6 dm and 10 dm long. One of the lateral sides of the trapezium is perpendicular to the base, the other being inclined to it at an angle of 45°. The diagonal of the lateral face of the prism corresponding to the smaller lateral side of the base is perpendicular to the base of the prism and forms with the lateral edge an angle of 30°. Find the volume of the prism.

883. In an oblique triangular prism the area of one of the lateral faces is equal to Q and the distance between this face and the opposite edge is equal to d. Find the volume of the prism.

884. In an oblique hexagonal prism two opposite lateral faces are perpendicular to the base and represent rhombuses whose diagonals are equal to 6 cm and 8 cm; the base of the prism is a regular hexagon. Find the volume of the prism.

885. The base of an oblique prism is a quadrangle $ABCD$ whose diagonals are mutually perpendicular. The diagonal section AA_1C_1C is perpendicular to the base; $BD = 32$ dm, the section area $AA_1C_1C = 1000$ dm^2. Determine the volume of the prism.

886. In a prism $ABCDA_1B_1C_1D_1$ the diagonals of the base AC and BD are perpendicular to each other, $BD = 6$ dm. The diagonals of the prism $BD_1 = DB_1 = 10$ dm. The section BDD_1B_1 is inclined to the base at an angle of 60°. The diagonal A_1C of the prism is equal to $2\sqrt{21}$ dm. Determine the volume of the prism.

30. Volumes of Pyramids

887. Fing the volume of a regular triangular, quadrangular and hexagonal pyramids if: (1) the side of the base is equal to a, the lateral edge to b; (2) the altitude is equal to h, the lateral edge to b; (3) the side of the base is equal to a, the slant height to l.

888. 1. In a regular pyramid the side of the base is equal to a, the lateral face is inclined to the base at an angle of 60°. Compute the volume of the pyramid: (a) triangular, (b) quadrangular, (c) hexagonal.

2. In a regular pyramid the side of the base is equal to a, the lateral edge is inclined to the base at an angle

of 30°. Compute the volume of the pyramid: (a) triangular, (b) quadrangular, (c) hexagonal.

889. Orally. 1. How will the volume of a pyramid change if the altitude and the side of the base are increased two times, five times, n times?

2. The altitude of a pyramid is reduced to one fourth its length, and the sides of the base are doubled. How will the volume of the pyramid change?

890. In a regular triangular pyramid the lateral edge is twice as long as the altitude. The side of the base is equal to a. Find the volume of the pyramid.

891. The side of the base of a regular triangular pyramid is equal to 2 dm, and a perpendicular dropped from a vertex of the base to the opposite face bisects the slant height. Determine the volume of the pyramid.

892. The centre of the upper base of a regular prism and the mid-points of the sides of the lower base are the vertices of a pyramid. By how many times is the volume of the prism greater than that of the pyramid? Consider a triangular, quadrangular and hexagonal prisms.

893. Drawn from the vertex A of a cube are diagonals of the faces containing the point A. The end-points B, C and D of the diagonals are joined to one another. Find the volume of the tetrahedron $ABCD$ if the edge of the cube is equal to a.

894. Find the volume of a regular quadrangular pyramid if: (1) the section of the pyramid passing through the altitude and lateral edge is an equilateral triangle with the side a; (2) the lateral face is a regular triangle with the side a.

895. The side of the base of a regular quadrangular pyramid is equal to a; dihedral angles at the lateral edges contain 120° each. Find the volume of the pyramid.

896. Given a regular quadrangular pyramid. The area of the section passing through a diagonal of the base and perpendicular to a lateral edge is equal to S. The plane of the section is inclined to the base at an angle of 60°. Find the volume of the pyramid.

897. In a regular quadrangular pyramid a cube is inscribed so that four of its vertices are found on the lateral edges of the pyramid and bisect them. Find the volume of the pyramid if the volume of the cube is equal to 27 dm^3.

898. 1. A sphere is inscribed in a regular quadrangular pyramid whose altitude is equal to 96 cm. Determine the volume of the pyramid if the radius of the sphere is 21 cm long.

2. A sphere is circumscribed about a regular quadrangular pyramid whose altitude is 32 cm high. Determine the volume of the pyramid if the radius of the sphere is equal to 25 cm.

899. The lateral face of a regular hexagonal pyramid is inclined to the base at an angle of 60° and has the area of Q cm^2. Find the volume of the pyramid.

900. Find the volume of a regular hexagonal pyramid in which the section passing through the smaller diagonal of the base and the vertex of the pyramid is inclined to the base at an angle of 60° and has an area of Q cm^2.

901. The volume of a regular hexagonal pyramid is equal to $45\sqrt{3}$ dm^3 and the area of the section passing through the greater diagonal of the base and the altitude of the pyramid to 15 dm^2. Find the lateral edge of the pyramid.

902. The side of the base of a regular triangular pyramid is equal to a, and the altitude drawn from a vertex of the base to the opposite lateral face to b. Determine the volume of the pyramid.

903. A stack of straw has the shape of a rectangular parallelepiped with a pyramid mounted on the upper base. The stack is 6 m long, 4 m wide and 5 m high (less the height of the pyramid). The smaller lateral faces of the pyramid are inclined to the base at an angle of 60°. Find the weight of the straw stack, if the density of straw is equal to 80 $\frac{kg}{m^3}$.

904. The highest of the Egyptian pyramids—the pyramid of Cheops is 144 m high, and the side of its square base is 230 m long. The internal passages and rooms of the pyramid constitute 30 per cent of its volume. How many trips would be required for ten 6-ton lorries to carry the total amount of stone used for its construction? The density of stone is equal to $2.5 \cdot 10^3 \frac{kg}{m^3}$.

905. Given in Fig. 28 are the horizontal and vertical projections of the upper portion of a milestone with the dimensions in centimetres. Determine its weight if 1 m^3 of the material weighs 25.48 kN.

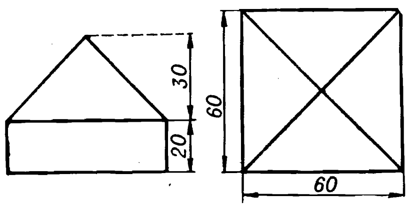

Fig. 28

906. Prove that the volume of a triangular pyramid is equal to one sixth the volume of a parallelepiped constructed on any three edges of the pyramid emanating from one vertex.

907. 1. The base of a pyramid is a right-angled triangle one of whose sides is equal to a and the adjacent angle to 30°. Find the volume of the pyramid if the lateral edges are inclined to the base at an angle of 60°.

2. The base of a pyramid is a triangle whose sides are equal to 12 cm, 20 cm and 28 cm. Each of the lateral edges is inclined to the base at an angle of 45°. Compute the volume of the pyramid.

908. 1. The base of a pyramid is a right-angled triangle whose sides are equal to 24 cm and 32 cm. Each of the lateral edges is inclined to the base at an angle of 60°. Compute the volume of the pyramid.

2. The base of a pyramid is an isosceles triangle whose base is equal to 12 cm and altitude to 18 cm. Each of the lateral edges is 26 cm long. Determine the volume of the pyramid.

909. A wooden triangular pyramid is sawn up into two pieces. The cutting plane intersects all the three lateral edges, cutting away one fourth from one edge (as measured from the vertex), one third from the other edge (as measured from the same vertex), and a half from the third edge. What portion of the total weight of the pyramid does the smaller piece constitute? (The weight of the sawdust should be neglected.)

910. 1. Two faces of a triangular pyramid are isosceles triangles whose hypotenuses are equal to c and form an angle of 45°. Determine the volume of the pyramid.

2. Two mutually perpendicular faces of a pyramid are equilateral triangles whose side is equal to 12 cm. Find the volume of the pyramid.

911. The base of a pyramid is an isosceles triangle whose equal sides are 78 cm long (each), the third side equalling 60 cm. Each dihedral angle at the base contains 45°. Find the volume of the pyramid.

912. The base of a pyramid is a rectangle whose area is equal to 1 m^2. Two lateral faces are perpendicular to the base and two others are inclined to it at angles of 30° and 60°. Find the volume of the pyramid.

913. The base of a pyramid is a rectangle. One of the lateral faces of the pyramid is perpendicular to the base, the rest of them being inclined to the base at an angle of 30°. The altitude of the pyramid is equal to 5 dm. Compute the total surface area and the volume of the pyramid.

914. Given in a pyramid $SABC$ the sides of the base: $AB = 13$ cm, $BC = 14$ cm and $AC = 15$ cm. A plane is drawn through the vertex A and the altitude of the pyramid. The pyramid is such that this plane is perpendicular to the side BC of the base. In what ratio is the volume of the pyramid divided by this plane?

915. The base of the pyramid is a rectangle. The face containing the longer side of the base is perpendicular to the base, represents an equilateral triangle and forms with the greater lateral face an angle of 60°. Find the volume of the pyramid if the area of the greater lateral face is equal to Q.

916. Determine the capacity of the plastic container whose shape and dimensions are shown in Fig. 29.

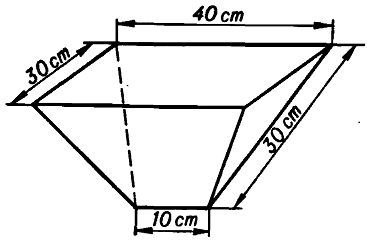

Fig. 29

917. A uniform body has the shape of a rectangular parallelepiped. A piece is cut away from it so that the cutting plane passes through the mid-points of three edges emanating from one vertex. What is the percentage of the cut-away material?

918. Prove that in a regular pyramid the sum of the distances between any point of the base and the lateral faces is a constant.

919. The plane angles at the vertex of a triangular pyramid are right ones, the lateral edges are equal to a, b and c. A point M taken on the base is x, y and z units distant from the faces opposite the edges a, b and c, respectively. Prove that

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$$

920. The base of a pyramid is a rhombus whose side is equal to 16 cm and acute angle to 60°. Dihedral angles at the base of the pyramid contain 45° each. Compute the volume of the pyramid.

921. Determine the volume of a pyramid whose altitude is equal to 10 cm, each of the dihedral angles at the base to 40°32′ and the area of the lateral surface to 660 cm^2.

922. The base of a pyramid is a rhombus with the side equal to 6 cm and acute angle to 53°8′. One of the lateral edges is perpendicular to the base and equal to the greater diagonal of the rhombus. Determine the volume of the pyramid.

923. Planes parallel to the base of a pyramid trisect its altitude. In what proportion is the volume of the pyramid divided by these planes?

924. Planes parallel to the base divide a pyramid into three equal portions. In what ratio is the altitude of the pyramid divided by these planes?

925. In a regular quadrangular pyramid the side of the base is equal to a and the lateral face is inclined to the base at an angle of 60°. A plane is drawn through a side of the base and perpendicular to the opposite lateral face. Find the volumes of the portions into which the pyramid is divided by the cutting plane.

926. Through the centre of the base of a regular triangular pyramid a section is drawn parallel to a lateral face. Find the ratio of the volumes of the obtained portions.

31. Volumes of Truncated Pyramids

927. Determine the volume of a regular truncated pyramid: (a) triangular, (b) quadrangular, (c) hexagonal, in each of which the sides of the bases are equal to 4 dm and 2 dm, and the altitude to 5 dm.

928. The sides of the bases of a regular truncated pyramid are equal to 12 dm and 8 dm, the lateral faces are inclined to the greater base at an angle of 60°. Compute the volume of the pyramid: (a) triangular, (b) quadrangular, (c) hexagonal.

929. The sides of the bases of a regular truncated pyramid are equal to 8 cm and 4 cm, the lateral edges are inclined to the greater base at an angle of 45°. Compute the volume of the pyramid: (a) triangular, (b) quadrangular, (c) hexagonal.

930. The sides of the bases of a regular truncated pyramid are equal to 6 m and 4 m, the acute angle of the lateral face to 60°. Compute the volume of the pyramid: (a) triangular, (b) quadrangular, (c) hexagonal.

931. The lateral edge of a regular truncated pyramid is equal to $\sqrt{48}$ cm, the sides of the bases to 10 cm and

4 cm. Find the volume of the pyramid: (a) triangular, (b) quadrangular, (c) hexagonal.

932. The sides of the bases of a regular triangular truncated pyramid are equal to 9 cm and 3 cm; the slant height equals the sum of the radii of the incircles. Find the volume of the pyramid.

933. In a regular triangular truncated pyramid the lateral edge is equal to the radius R of the circle circumscribed about the greater base and inclined to this base at an angle of 60°. Determine the volume of the pyramid.

934. In a regular triangular truncated pyramid the areas of the bases are equal to $4\sqrt{3}$ dm^2 and $\frac{9\sqrt{3}}{4}$ dm^2, and the area of the lateral surface to 31.5 dm^2. Find the volume of the pyramid.

935. The slant height of a regular quadrangular truncated pyramid is equal to 10 dm, and the sides of its bases to 24 dm and 12 dm. Determine the volume of the truncated pyramid.

936. Determine the volume of a regular quadrangular truncated pyramid if its diagonal equal to 40 cm is perpendicular to the lateral edge equal to 30 cm.

937. The volume of a regular quadrangular truncated pyramid is equal to 109 cm^3, its altitude to 3 cm, and diagonals of the bases are to each other as 5 : 7. Find the sides of the bases of the pyramid.

938. In a regular quadrangular truncated pyramid the area of the smaller base is equal to 16 cm^2, and the area of the lateral face to 64 cm^2. Find the volume of the pyramid if its lateral face is inclined to the base at an angle of 60°.

939. In a triangular truncated pyramid the altitude is equal to 6 m, the sides of one base to 29 m, 52 m and 27 m, and the perimeter of the other base to 72 m. Determine the volume of the truncated pyramid.

940. In a regular hexagonal truncated pyramid the radii of the circles inscribed in the bases are equal to $2.5\sqrt{2}$ dm and $3.5\sqrt{2}$ dm; the distance between the opposite sides of the bases is equal to 9 dm. Find the volume of the pyramid.

941. The volume of a regular hexagonal truncated pyramid is equal to 2808 dm^3, and the sides of the bases and the altitude are as $2 : 5 : 6\sqrt{3}$. Find the volume of the corresponding non-truncated pyramid.

942. The ratio of the areas of the bases of a truncated pyramid is 9 : 4, its volume is equal to 152 dm^3, and the altitude to 6 dm. Determine the volume of the corresponding non-truncated pyramid.

943. In a truncated pyramid the areas of the bases are equal to S_1 and S_2, and the altitude to h. Determine the volume of the corresponding non-truncated pyramid.

944. In a regular triangular truncated pyramid the side of the greater base is equal to 50 cm. The area of a section drawn through a side of the greater base and the opposite vertex of the smaller base amounts to $500\sqrt{3}$ cm^2. Find the volume of the pyramid if the cutting plane is perpendicular to the lateral edge.

945. In a regular quadrangular truncated pyramid the side of the greater base is equal to 9 dm, and the volume to $325.5\sqrt{3}$ dm^3. The distance between the centre of the smaller base and the side of the greater base is equal to this side. Determine the area of the lateral surface of the pyramid.

946. The altitude of a truncated pyramid is equal to 3 m, and its volume to 95 m^3. The ratio of the perimeters of the bases is 2 : 3. Determine the areas of the bases.

947. In a regular quadrangular truncated pyramid the slant height and sides of the bases are as 5 : 9 : 3, and the volume is equal to 1248 m^3. Determine the total surface area of the pyramid.

948. In a truncated pyramid the area of one base exceeds that of the other base by 24 cm^2. The altitude of the truncated pyramid is equal to 18 cm and its volume to 336 cm^3. Determine the ratio of the sides of the bases.

949. Determine the volume of a regular triangular truncated pyramid in which the sides of the bases are equal to 8 m and 4 m, and the area of the total surface is twice the sum of the areas of the bases.

950. In a regular quadrangular truncated pyramid the dihedral angle at the base is equal to 60°. The section of the pyramid passing through the slant height and the centres of the bases is a trapezium in which a circle of radius r can be inscribed. Find the volume of the pyramid.

951. Given a truncated pyramid $ABCDA_1B_1C_1D_1$ whose bases are squares with sides 30 cm and 12 cm long. The lateral face AA_1B_1B is perpendicular to the bases, the angle $AA_1B_1 = 120°$, and the diagonal AB_1 is equal to 28 cm. A plane AB_1C_1D is drawn. Find the volumes of the portions into which the pyramid is divided by this plane.

952. The bases of a triangular truncated pyramid are isosceles triangles with the sides equal to 14 cm, 25 cm, 25 cm and 2.8 cm, 5 cm, 5 cm. The lateral face containing the bases of the isosceles triangles is an isosceles trapezium whose plane forms a right dihedral angle with the base of the pyramid, and the opposite lateral edge is inclined to the base at an angle of 45°. Determine the volume of the pyramid.

953. A right quadrangular truncated pyramid is cut by planes drawn through the end-points of the diagonals of the upper base and perpendicular to them. Determine the volume of the remaining portion of the truncated pyramid if its altitude is equal to h, and the sides of the bases to a and b.

954. Out of a truncated pyramid a prism is cut whose base is equal to the smaller base of the pyramid and the altitude to the altitude of the pyramid. What is the volume (in per cent) of the waste if the corresponding sides of the bases are as 2 : 3?

955. In a triangular truncated pyramid through a side of the smaller base a plane is drawn parallel to a lateral edge. The corresponding sides of the bases are as 5 : 2. In what ratio is the volume of the truncated pyramid divided by this plane?

956. The corresponding sides of the bases of a truncated pyramid are as $m : n$. Find the ratio in which the mid-section divides its volume.

32. Volumes of Cylinders

957. 1. What is the ratio of the volumes of a cylinder and its model made to the graphic scale 1 : 2, 1 : 3, 1 : n?

2. What is the ratio of the volumes of two cylinders having equal altitudes, equal diameters of the base circles?

958. How will the volume of a cylinder change if: (a) its diameter is increased twice, and the altitude three times, (b) the diameter is reduced to half its length, and the altitude is increased four times?

959. Prove that an equilateral cylinder is equal to a prism in which the base is a square whose side is equal to the radius of the base circle of the cylinder and the altitude to the length of the circumference of the base circle.

960. 1. Given two cylinders with equal altitudes. The volume of the first cylinder is equal to 0.25 m^3, and the diameter of its base circle to 0.75 m. The diameter of the second cylinder is equal to 1.5 m. Compute the volume of the second cylinder.

2. Two cylinders have equal bases. The volume of the first cylinder is equal to 7.5 dm^3, and its altitude to 21 cm. The altitude of the second cylinder is equal to 7 cm. Compute the volume of the second cylinder.

961. It is required to make a cylindrical oiler containing 600 g of machine oil. The height of the oiler is to be equal to 10 cm. Determine the diameter of its bottom if the density of oil is equal to 900 $\frac{kg}{m^3}$.

962. What is the weight of the iron wire intended for two-wire communication between points A and B if $AB = 2$ km, the diameter of the wire is equal to 0.8 mm, and the density of iron to $7.8 \cdot 10^3 \frac{kg}{m^3}$?

963. How many barrels of the cylindrical form (1.5 m long and 0.8 m in diameter) are required to transport 16.5 tons of kerosene contained in a tank-car? The density of kerosene is equal to $0.8 \cdot 10^3 \frac{kg}{m^3}$.

964. How much silage will go in a silo of the cylindrical form if the inner diameter of the silo is equal to 3.70 m, and the height to 6.98 m? The average density of silage is equal to $670 \frac{kg}{m^3}$.

965. Wire 4 mm in diameter is drawn from a steel pig having the form of a rectangular parallelepiped whose dimensions are: 100 cm $\times$ 30 cm $\times$ 30 cm. Determine the length of the wire obtained from the given pig

966. It is required to lead-coat a cable 50 mm in diameter. How much lead is it required for this purpose if the cable is 5 km long and the lead coat 3 mm thick? The density of lead is equal to $11.4 \cdot 10^3 \frac{kg}{m^3}$.

967. Determine the volume of earth to be excavated in digging a foundation ditch for a water tower of the cylindrical form. The outer diameter of the tower equals 8 m, the ditch is to be 2 m deep and 1 m wide.

968. A construction site requires a water supply of 10 cubic metres per hour. Assuming the rate of flow in the water pipe to be equal to $1.5 \frac{m}{sec}$, determine the diameter of the pipe to be used for this purpose.

969. To reduce the weight of an intermediate floor the latter is made from prefabricated hollow concrete slabs. Assuming that the slab is made of a uniform material, determine the percentage by which the weight of a slab having the dimensions 586 cm $\times$ 119 cm $\times$ 22 cm can be reduced by making cylindrical hollows so that the minimum thickness of the concrete layer equals 3.5 cm, and the axes of the cylindrical hollows are parallel to the slab edge equal to 586 cm.

970. The radius of the base circle of a cylinder is equal to 10 cm. The area of the section which is parallel to the

axis of the cylinder and 6 cm distant from it is equal to 80 cm^2. Compute the volume of the cylinder.

971. The length of a line segment connecting the endpoints of mutually perpendicular diameters of the bases of a cylinder is equal to 11 cm; the radius of the base circle of the cylinder to 6 cm. Compute the volume of the cylinder.

972. The diagonal of an axial section of a cylinder is equal to l and inclined to the base at an angle of 30°. Find the volume of the cylinder.

973. The area of an axial section of an equilateral cylinder is equal to S. Find the volume of the cylinder.

974. 1. The volume of an equilateral cylinder is equal to 128π dm^3. Compute the surface area of the cylinder.

2. The volume of an equilateral cylinder is equal to V. Find the area of the lateral surface of the cylinder.

975. 1. A cylinder of the maximum possible size is made up out of a cube. How much material (in per cent) is removed in this operation?

2. A cube of the maximum possible size is made up out of a cylinder whose height is $H = R\sqrt{2}$. What is the percentage of the material removed?

976. A regular prism is inscribed in a cylinder and then a cylinder in this prism. Find the ratio of the volumes of these cylinders. Consider the cases when the prism is: (a) triangular, (b) quadrangular, (c) hexagonal.

977. The volume of a cylinder is equal to V. Find the volume of: (1) a circumscribed regular quadrangular prism, (2) an inscribed regular triangular prism.

978. Through a diagonal of an axial section of a cylinder a plane is drawn perpendicular to it. The area of the obtained section is equal to Q. Find the volume of the cylinder if the generator is to the radius of the base circle as 2 to $\sqrt{3}$.

979. The area of a section of a cylinder drawn through a diagonal of an axial section and perpendicular to this section is equal to 18π. Determine the volume of the cylinder if this section is inclined to the base at an angle of 60°.

980. 1. Determine the volume of an equilateral cylinder if it is numerically equal to the total surface area of the cylinder.

2. Determine the volume and the total surface area of an equilateral cylinder if its volume is numerically equal to its lateral surface area.

981. 1. The lateral surface of a cylinder is developed in a square with the diagonal d. Find the volume of the cylinder.

2. The radius of the base circle of a cylinder is R, and in the development of its lateral surface the generator forms with the diagonal an angle of 60°. Find the volume of the cylinder.

982. A metal sheet 20 cm × 10 cm is bent to make a tube. Find the volume of the tube. (Consider two cases.)

983. Through a chord of the base circle of a cylinder which divides its circumference in the ratio of 1 to 2 a plane is drawn parallel to the generator. In what ratio is the volume of the cylinder divided by the plane?

984. In a cone with the generator $l = 25$ cm and altitude $H = 20$ cm a cylinder is inscribed whose surface area amounts to 306π cm². Find the volume of this cylinder.

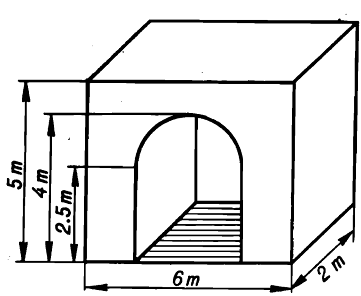

Fig. 30

985. How many bricks are required to construct an arch with a cylindrical vault whose dimensions are given in Fig. 30 if 1 m³ of masonry takes 400 bricks?

33. Volumes of Cones

986. 1. What is the ratio of the volumes of two cones which have equal altitudes, equal diameters of the base circles?

2. What is the ratio of the volumes of a cone and its model made to the scale 1 : 2, 1 : 3, 1 : n?

3. In what ratio will the volume of a cone change if its altitude and radius of the base circle are reduced to $\frac{1}{4}$?

987. 1. The diameter of the base circle is increased three times. How must its altitude be changed so that its volume remained unchanged?

2. The altitude of a cone is reduced to half its length. How must the diameter of its base circle be changed so that its volume remained unchanged?

988. 1. How will the volume of a cone change if its generator and altitude are increased three times?

2. The radius of the base circle of a cone is increased three times, and its altitude is reduced to one third. How is the volume of the cone changed?

3. The altitude of a cone is increased twelve times. How must the diameter of the base circle be changed so that the volume of the cone is increased three times?

989. 1. Prove that the volume of the cone is equal to one third the product of the lateral surface area by the distance between the centre of the base and generator.

2. Prove that the volume of the cone is equal to one sixth the product of the length of the circumference of the base circle by the area of the axial section.

990. A cylindrical and conical vessels have equal bases and altitudes. By how many times does the capacity of the cylindrical vessel exceed that of the conical one?

991. Sorted grain is gathered in a conical heap 0.7 m high. What is the mass of the grain if the generator of the cone has a natural slope (it is inclined to the horizontal plane at an angle of 45°). The density of grain in the heap is equal to 700 $\frac{kg}{m^3}$.

992. A building organization has to transport a conical heap of sand. How many 3-ton dumpers are required to transport the sand if the measurements gave the following results: the length of the circumference of the base circle is equal to 35.2 m, the generator to 9.5 m? Each dumper has to carry out five trips. The density of sand equals $1.9 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}$.

993. How many carts are needed to transport a haystack having the form of a cylinder with a conical top if the diameter of the base circle is equal to 6 m, and the height of the haystack to 5 m, the height of the conical portion being 1.5 times the height of the cylindrical portion? The density of hay is equal to $30 \frac{\text{kg}}{\text{m}^3}$. The cart carries about 600 kg at a time.

994. Determine the volume of a cone whose generator is equal to l, and the length of the circumference of the base circle to C.

995. 1. The area of the base circle of a cone is 16π dm^2, and the area of the lateral surface, 20π dm^2. Determine the volume of the cone.

2. The area of the lateral surface of a cone is equal to 15π dm^2 and that of the total surface to 24π dm^2. Determine the volume of the cone.

996. The area of the axial section of a cone is equal to 120 cm^2, and the generator to 17 cm. Find the volume of the cone.

997. Find the relationship between the altitude and the radius of the base circle of a cone in which: 1) the lateral surface area and the volume, (2) the total surface area and the volume, (3) the base circle area and the volume are numerically equal.

998. Poured in a conical vessel whose altitude is equal to 16 cm and the diameter of the base circle to 24 cm is an amount of liquid equal to one eighth its volume. Determine the level of the liquid in the vessel.

999. The altitude of a cone is divided into three equal parts and through the points of division planes are drawn parallel to the base. The volume of the cone is equal to V. Find the volume of its mid-portion.

1000. Through a point dividing the altitude of a cone in the ratio of 1 to 2 a plane is drawn parallel to the base. In what ratio is the volume of the cone divided by the cutting plane?

1001. A vessel has the form of a cylinder with a conical bottom. The height of the cylindrical portion is four times the height of the conical portion. Half the volume of the vessel is occupied by a liquid. What portion of the volume of the cylindrical part of the vessel remains empty?

1002. A conical vessel is made up of a metal sheet which has the form of a circular sector with a central angle of 120° and radius R. Find the capacity of the vessel.

1003. The volume of a cone is equal to $\frac{32}{3}\pi\sqrt{5}$ m^3, and the radius of the base circle to 4 m. Determine the central angle of the development of the lateral surface of the cone.

1004. Find the volume of a solid generated by revolving: (1) an equilateral triangle about its side a; (2) an isosceles triangle with the base 10 dm and lateral side 13 dm long about a lateral side; (3) a triangle with the sides 7 cm, 8 cm and 9 cm about the longer side; (4) a triangle with the sides 15 cm, 18 cm and 27 cm about the smaller side.

1005. Prove that the volume of a solid obtained by revolving a triangle about its side is equal to one third the product of the area of the triangle by the length of the circle circumscribed by its vertex.

1006. Prove that a solid generated by revolving a rhombus about its side is equal to a cylinder whose generator is the side of the rhombus, and radius is its altitude.

1007. An isosceles trapezium whose bases and lateral side are respectively equal to 5 cm, 21 cm and 17 cm rotates about the smaller base. Determine the volume of the solid of revolution.

1008. A trapezium whose bases are equal to 5 cm and 12 cm, and lateral sides to 24 cm and 25 cm revolves about a straight line passing through the vertex of the smaller angle and perpendicular to the bases. Compute the volume of the solid obtained.

1009. Compute the volume of a solid generated by revolving a triangle about a straight line passing through a vertex of the triangle and parallel to the longer side if its sides are given: (1) 29 cm, 25 cm, 6 cm; (2) a dm, a dm, a dm.

1010. Cones are inscribed in, and circumscribed about a regular triangular pyramid. Find the ratio of the volumes of the cones.

1011. In a regular hexagonal pyramid the lateral edge is twice the length of the base. The apothem of the base is equal to 6 cm. Find the volumes of the cones inscribed in, and circumscribed about, the pyramid.

1012. A cube is inscribed in an equilateral cone so that one of its faces lies in the plane of the base of the cone, and four of its vertices are found on the lateral surface of the cone. Find the ratio of the volumes of the cone and cube.

1013. 1. Prove that if a triangle ABC revolves about the side $BC = a$, then the volume of the solid thus generated $V = \frac{4}{3}\pi\frac{Q^2}{a}$, where Q is the area of the triangle.

2. Prove that the volumes of the solids generated by revolving a triangle consecutively about each of its sides are inversely proportional to the sides.

1014. A cone is inscribed in a cube so that the base of the cone is inscribed in one of the faces of the cube and the vertex of the cone is the centre of the opposite face. Find the ratio of the volumes of the cube and cone.

1015. A cone in which $R = 18$ cm and $l = 30$ cm is provided with a cylindrical hole so that their axes coincide. The diameter of the hole is equal to 6 cm. Find the volume and surface area of the solid obtained.

34. Volumes of Truncated Cones

1016. The radii of the bases of a truncated cone are equal to 1 dm and 3 dm, and the generator to 2.9 dm. Find the volume of the cone.

1017. What is the capacity (in litres) of the vessel having the form of a truncated cone the radii of whose base circles are equal to 35 cm and 20 cm, and the generator is inclined to the greater base at an angle of 60°?

1018. A vessel has the form of a truncated cone in which the lengths of the circumferences of the base circles are equal to 96 cm and 66 cm and the altitude to 27 cm. Compute the capacity of this vessel (in litres, accurate to one decimal place).

1019. In a truncated cone the altitude is equal to the radius of the smaller base; the radius of the greater base is equal to 12 cm, and the generator is inclined to the base at an angle of 45°. Find the volume of the cone.

1020. The radii of the base circles of a truncated cone are equal to 3 cm and 10 cm, and the volume to 1112π cm^3. Find the altitude and generator of the cone.

1021. The altitude of a truncated cone is equal to 8.4 dm, and the radius of one of the base circles to 0.7 dm. Find the area of the lateral surface of the cone if its volume is equal to 16.492π dm^3.

1022. The generator l of a truncated cone is equal to the diameter of the smaller base circle and inclined to the greater base at an angle of 60°. Find the volume of the cone.

1023. The volume of a truncated cone is equal to 416π cm^3. The radii of the base circles and the generator are as 5 : 2 : 5. Find the surface area of this truncated cone.

1024. In a truncated cone the diagonals of the axial section are perpendicular to each other and the generator is inclined to the greater base at an angle of 60° and equals l. Find the volume of this truncated cone.

1025. In a truncated cone the line segment l, which connects the centre of the greater base with the end-point of the diameter of the smaller base, is perpendicular to the generator and inclined to the greater base at an angle of 30°. Find the volume of this cone.

1026. The radii of the base circles of a truncated cone are equal to 6 cm and 8 cm, and the volume of the regular quadrangular pyramid inscribed in it is equal to 1480 cm^3. Find the volume of the cone.

1027. The radii of the base circles of a truncated cone are equal to 4 cm and 10 cm. Planes drawn parallel to the bases divide the altitude of the cone into three equal parts. In what proportion is the volume of the truncated cone divided by these planes?

1028. The ratio of the areas of the bases of a truncated cone is 1 : 2, and the radius of the circle inscribed in the axial section is equal to R. Find the volume of the truncated cone.

1029. The lateral sides of the axial section of a truncated cone are tangent to the inscribed circle at points the distance between which is equal to a. The angle of inclination of the generator to the base is equal to 45°. Find the volume of the cone.

1030. Find the relationship between the radii of the bases of a truncated cone if the conical surface, whose vertex lies at the centre of the lower base and whose base is the upper base of the truncated cone, divides the volume of the given cone in the ratio of 4 to 15.

1031. A cone is inscribed in a truncated cone so that its vertex coincides with the centre of the upper base of the truncated cone, and its base with the lower base of the given cone. Find the relationship between the radii of the base circles of the truncated cone if the volume of the inscribed cone constitutes half the volume of the truncated cone.

1032. Given in a truncated cone: the radii of the base circles R and r, and the altitude H. Out of it two cones are cut away whose bases coincide with the bases of the given cone, and the generator of one cone serves as the extension of the generator of the other. Determine the volume of the remaining portion.

1033. A tetrahedron is inscribed in a truncated cone so that one of its faces is inscribed in the smaller base circle of the cone, and the opposite vertex is found at the centre of the greater base of the cone. The edge of the tetrahedron has the length a and is equal to the generator of the cone. Find the volume of the truncated cone.

1034. 1. Given the radii of the base circles R and r; determine the ratio of the volumes of the truncated cone and the corresponding non-truncated cone.

2. In what ratio is the volume of a truncated cone divided by the mid-section?

1035. A truncated cone, whose radii of the base circles are equal to R and r, and the area of the axial section is the mean proportional between the areas of the bases, is provided with a coaxial cylindrical hole of radius $\frac{r}{\sqrt{3}}$. Find the volume of the remaining portion.

1036. Find the dimensions of an equilateral cylinder equal to a truncated cone in which the areas of the base circles are equal to π cm^2 and 16π cm^2, and the area of the axial section to $61\frac{1}{4}$ cm^2.

1037. A regular triangle with the side a revolves about an axis passing through one of its vertices and perpendicular to its side. Find the volume of the solid thus generated.

1038. A rhombus with the side a and acute angle of 60° revolves about an axis drawn through the vertex of this angle and perpendicular to the side. Find the surface area and volume of the solid obtained.

1039. A square whose side is equal to a revolves about an axis passing outside the square through one of its vertices and perpendicular to its diagonal. Find the surface area and volume of the solid obtained.

1040. An isosceles trapezium in which a diagonal is perpendicular to the lateral side 6 dm long and the acute angle at the base is equal to 60° revolves about a lateral side. Find the volume of the solid of revolution.

1041. A regular hexagon with the side a rotates about one of its sides. Find the surface area and volume of the obtained solid.

1042. A regular hexagon with the side a revolves about an axis parallel to one of its sides and contained in its plane. The axis of revolution is a units distant from the centre of the hexagon. Find the volume of the solid thus generated.

CHAPTER VII

THE SPHERE

35. Spheres

1043. 1. Find the locus of the centres of spheres passing through two given points.

2. Find the locus of the centres of spheres passing through three given points which do not lie in one straight line.

3. Through what four points can a sphere be drawn? Consider the cases when the points are contained and not contained in one plane.

1044. 1. Find the locus of the centres of spheres tangent to a given straight line at a given point.

2. Find the locus of the centres of spheres tangent to a given plane at a given point.

3. Find the locus of the centres of spheres tangent to a given sphere at a given point.

1045. 1. Find the locus of the centres of spheres tangent to two given straight lines contained in one plane. Consider the cases of parallel and intersecting lines.

2. Find the locus of the centres of spheres tangent to three given straight lines contained in one plane. Consider the following cases: the lines are parallel; two lines are parallel and the third one intersects them; the lines intersect pairwise; the lines intersect at one point.

3. Find the locus of the centres of spheres tangent to two given planes. Consider the cases of parallel and non-parallel planes.

4. Find the locus of the centres of spheres tangent to three given planes. Consider the following cases: the planes are parallel; two planes are parallel, and the third one intersects them; the planes intersect pairwise; the planes pass through one straight line; the planes have only one common point.

1046. Find the locus of the centres of spheres of a given radius tangent to: (a) a straight line; (b) a plane; (c) a right circular cylindrical surface; (d) a sphere.

1047. 1. The sides of what quadrangles can a sphere be tangent to?

2. The vertices of what quadrangles can a sphere pass through?

1048. Several planes are drawn through a point M situated inside a sphere. Prove that the least section of the sphere is one whose centre is the point M.

1049. The areas of the great and small circles are equal to 225π cm^2 and 144π cm^2, respectively. Find the distance between the small circle and the centre of the sphere.

1050. 1. The latitude of Moscow is 55°45′. Compute the radius of the parallel on which Moscow lies assuming that the Earth is a sphere whose radius is equal to 6370 km.

2. Compute the length of the circumference of the Polar circle if the radius of the Earth is approximately equal to 6400 km.

1051. Two parallel planes divide the diameter of a sphere in the proportion 1 : 2 : 3. In what proportion is the area of the sphere divided by these planes?

1052. The diameter of a sphere is divided by seven points into eight equal parts. Through the first and fifth points planes are drawn perpendicular to this diameter. By how many times does the area of one section exceed that of the other?

1053. The angle between two radii of a sphere is equal to 60° and the distance between the end-points of the radii to 15 cm. Find the shortest distance between the end-points of the radii as measured along an arc on the surface of the sphere.

1054. Two points A and B situated on a sphere of radius 54 cm are joined to the centre of the sphere. The shortest distance between the points A and B as measured along an arc on the sphere is equal to 132 cm. Compute the angle AOB, where O is the centre of the sphere, and the distance between the points A and B as measured along a straight line.

1055. A plane is drawn through the end-point of the radius R of a sphere and at an angle of $30°$ to it. Find the area of the circle yielded by this cutting plane.

1056. Through a point of a spherical surface a plane is drawn at an angle of $45°$ to a plane tangent at this point. Find the area of the section if the radius of the sphere is equal to R.

1057. A tangent plane and a cutting plane are drawn through a point of a spherical surface. Find the dihedral angle formed by these planes if the area of the section is equal to one fourth the area of the great circle.

1058. The radius of a sphere is equal to 13 dm. At what distance from the centre of the sphere must a cutting plane be drawn so that it passes through points of the spherical surface the rectilinear distances between which are equal to 6 dm, 8 dm and 10 dm?

1059. The radius of a sphere is equal to 12.5 cm. A cutting plane is drawn at a distance of 9 cm from the tangent plane. Find the radius of the section.

1060. A conical surface contacts a sphere along a circle whose radius is equal to 12 cm. The radius of the sphere is equal to 13 cm. Find the distance between the vertex of the cone and the centre of the sphere.

1061. An equilateral cone has a base equal to 16 cm. A sphere is constructed on its altitude as on the diameter. Determine the length of the line of intersection of the sphere and cone.

1062. 1. Two equal spheres of radius R are arranged so that the centre of one of them is situated on the surface of the other. Find the length of the line along which they intersect.

2. The radii of two spheres are equal to 15 dm and 20 dm, and the distance between their centres to 25 dm. Find the length of the circle along which their surfaces intersect.

1063. The base and altitude of a hemisphere serve as the base and altitude of the cone inscribed in it. A plane is drawn parallel to the base which bisects the altitude. Prove that the area of the annulus contained between the lateral surface of the cone and the surface of the hemisphere is equal to half the area of the base of the cone.

1064. A hollow sphere is cut by two planes one of which passes through its centre and the other touches its inner surface. Prove that the sections contained between the inner and outer surfaces of the hollow sphere are equal to each other.

1065. Three equal circles lie on a sphere and have a common point pairwise. The radius of the sphere is equal to R. Find the radius of the circles if their common points lie on the great circle.

36. Areas of Spheres and Their Parts

1066. How much material is it required for manufacturing the shell of a balloon 10 m in diameter if the seams constitute 5 per cent of the spherical surface area?

1067. 1. The radius of one sphere is equal to 0.5 m, the radius of the other to 2 m. Find the ratio of the surfaces of the spheres.

2. The surface of one sphere is n times the surface of the other. Determine the ratio of their diameters at $n = 4, 5, 9$.

1068. Prove that if the diameters of three spheres form a right-angled triangle, then the surface of the greater sphere is equal to the sum of the remaining two.

1069. Prove that the total surface of an equilateral cone is equal to the surface of a sphere whose diameter is the altitude of the cone.

1070. Prove that the lateral surface of an equilateral cone whose base is the great circle of the sphere is equal to half the surface of the sphere.

1071. Prove that the total surface of a cylinder whose generator is equal to the radius of the base circle is equal to the surface of the sphere in which the base circle of the cylinder serves as the great circle.

1072. Find the radius of the base circle of a cylinder whose altitude is equal to 0.6 m if the total surface of the cylinder is equal to the surface of a sphere whose radius is equal to 12 dm.

1073. A plane is drawn tangent to a sphere. A point M is taken on this plane at a distance of 8 cm from the surface of the sphere and 16 cm distant from the point of tangency. Determine the area of the sphere.

1074. On different sides of the centre of a sphere two parallel sections are drawn; their areas are equal to 36π dm^2 and 64π dm^2, and the distance between them to 8 dm. Determine the surface of the sphere.

1075. Prove that if an equilateral cone and a hemisphere have a common base, then the lateral surface of the cone is equal to $\frac{2}{3}$ the surface of the hemisphere, and the line of their intersection to half the circumference of the base circle.

1076. The diameter of the base circle of a spherical segment is equal to 10 cm. The arc of the axial section contains 120°. Determine the surface of this segment.

1077. The radii of the base circles of a spherical segment are equal to 20 cm and 24 cm, and the radius of the sphere to 25 cm. Determine the surface area of the spherical zone (Consider two cases.)

1078. Determine the area of the spherical surface of a spherical segment given its altitude equal to 30 cm and the radius of the base circle to 40 cm.

1079. A shining point is found at a distance from a sphere equal to its radius. What portion of the surface of the sphere is illuminated by the point?

1080. An empty cone is placed on a sphere of radius 26 cm. What portion of the surface of the sphere is covered by the cone if the radius of its base circle is equal to 10 cm?

1081. 1. Determine the total surface of a solid generated by revolving a circular segment whose arc is equal to 90° about its altitude if the base of the segment is equal to b.

2. Solve the same problem for the case when the arc of the segment contains 120°, and its area is equal to Q.

1082. A circular sector with an angle of 90° and area of 28 cm^2 rotates about the medium radius. Find the surface of the solid thus generated.

1083. At what distance from the centre of a sphere whose radius is equal to 113 cm a cutting plane should be drawn so that the ratio of the spherical surface of the smaller segment to the lateral surface of the cone which has a common base with the segment and whose vertex is found at the centre of the sphere is equal to 1.75?

1084. The surface of a sphere is divided by a cutting plane in the ratio of 1 to 4. Prove that the surface of the spherical sector corresponding to this section is divided by the section into two equal parts. (The spherical and conical surfaces of the sector are equal to each other.)

1085. Prove that if a semi-circle divided into three equal parts rotates about its diameter, then the sum of the areas of the segment surfaces is equal to the area of the spherical zone.

1086. A cutting plane divides the surface of a sphere into two segment surfaces equal to 16 cm^2 and 48 cm^2. Find the area of the section.

1087. Prove that the lateral surface of a cone inscribed in a spherical segment is the mean proportional between the areas of the base and the lateral surface of the segment.

1088. The area of the lateral surface of a spherical segment is equal to the sum of the areas of its bases one of which is the great circle. Find the altitude of the spherical segment if the radius of the sphere is equal to R.

1089. Prove that the area of a spherical segment

$$S = \pi \sqrt{4r_1^2h^2 + (r_2^2 - r_1^2 + h^2)^2},$$

where r_1 and r_2 are the radii of the base circles of the segment and h is its altitude.

37. Volumes of Spheres and Their Parts

1090. Prove that a cylinder whose altitude constitutes $\frac{3}{4}$ the radius of the base circle is equal to a sphere of the radius equal to the radius of the base circle of the cylinder.

1091. 1. The area of the surface of a sphere is equal to 100π cm^2. Find its volume.

2. Find the area of the surface of a sphere if its volume is equal to V.

1092. How will the volume of a sphere change if: (1) its radius is increased twice, by 200 per cent? (2) its diameter is reduced by 75 per cent, to one third?

1093. 1. The diameter of the Mars is 0.53 the diameter of the Earth. What are the surface and volume of the Mars as compared with those of the Earth?

2. The diameter of the Jupiter is 11 times greater than that of the Earth. By how many times does the Jupiter exceed the Mars in surface and volume?

1094. The average depth of all the oceans is equal to 4 km. Knowing that the oceans cover 70 per cent of the Earth surface, find the approximate volume of the water contained in all the oceans.

1095. Prove that the volume of the walls of a hollow sphere is equal to the volume of a truncated cone the radii of the base circles of which are equal to the radii of the spherical surfaces, its altitude being four times greater than the thickness of the walls of the sphere.

1096. Will a hollow iron ball float on the water surface if its outer diameter is equal to 28 cm, and the wall thickness to 0.5 cm?

1097. A hollow iron ball, whose external radius is equal to 15.4 cm, is floating in water half-submerged. Compute the thickness of the walls of this ball if the density of iron is equal to $7\cdot8\cdot10^3\ \frac{\text{kg}}{\text{m}^3}$.

1098. The radii of three balls are as 1 : 2 : 3. (1) Prove that the volume of the greater ball is three times the sum of the volumes of the two smaller ones. (2) Compute the volume of each ball if the volume of the greater ball exceeds the sum of the volumes of the smaller balls by 192π cm^3.

1099. (1) Find the surface area of a sphere, whose volume and surface area are expressed by equal numbers. (2) The same condition for a hemisphere.

1100. An equilateral cylinder and a sphere have equal volumes. Find the ratio of their surface areas.

1101. Find the diameter of a sphere which is equal to a cone the radius of the base circle of which is equal to 6 cm, and the altitude to 24 cm.

1102. Three lead balls 3 cm, 4 cm and 5 cm in diameters are melted to yield one ball. Determine the ratio (in per cent) of its surface area to the surface area of each of the given balls.

1103. A maximum possible ball is made out of (a) an equilateral cylinder, (b) an equilateral cone, and (c) a cube. How much material (in per cent) is removed as waste in each case?

1104. A cylindrical vessel whose diameter is equal to 12 cm, and altitude to 72 cm is filled with water to half its height. What will the increase in the water level be if a ball 10 cm in diameter is placed in the vessel?

1105. A cylindrical pipe 28 cm long ends in a hemisphere. Compute the capacity of the pipe if its diameter is equal to 3.6 cm.

1106. A vessel has the shape of an overturned cone whose axial section is an equilateral triangle with the side 20 cm long. The vessel is filled with water so that its surface touches a ball of radius 4 cm plunged into the water. Find the level of the water in the vessel after the ball is removed.

1107. The diameter of the base of a spherical segment is equal to 16 cm, and the arc of the axial section contains 60°. Determine the volume of the segment.

1108. Compute the volume of a spherical sector if the radius of the circle of the corresponding segment is equal to 12 cm, and the radius of the sphere to 15 cm.

1109. The radii of parallel sections of a sphere are equal to 20 cm and 24 cm, and the radius of the sphere to 25 cm. Determine the volume of the portion of the sphere contained between these sections. (Consider two cases.)

1110. Determine the volume of a spherical segment given its altitude equal to 4 cm and the radius of the base circle equal to 8 cm.

1111. How many iron rivets of the cylindrical shape can be manufactured from one kilogram of metal if the head of the rivet represents a spherical segment whose altitude is equal to 6 mm and the radius of the sphere to 18 mm, the length of the rivet to 20 mm, and the diameter of the cylindrical portion to 10 mm? The density of iron is $7.8 \cdot 10^3 \ \frac{\text{kg}}{\text{m}^3}$.

1112. Using the formula $V = \pi h^2 \left(R - \frac{h}{3}\right)$, deduce the following formula for computing the volume of a spherical segment:

$$V = \frac{1}{6} \pi h (3r^2 + h^2),$$

where r is the radius of the base circle of the segment.

1113. The volume of a spherical sector is divided into two equal portions by the base of a segment (the spherical segment is equal to the cone). Find the ratio of the altitudes of the segment and cone.

1114. A ball whose radius is equal to 30 cm is provided with a cylindrical hole bored along its diameter. Compute the volume of the remaining portion if the radius of the cylindrical hole is equal to 18 cm.

1115. Two equal spheres are situated so that the surface of one passes through the centre of the other. What portion of the volume of the sphere does the common part of the two spheres constitute?

1116. A circular sector with an angle of 120° and radius R rotates about the medium radius. Find the volume of the solid of revolution.

1117. A circular sector with an angle of 90° and area amounting to 157 cm^2 revolves about a straight line passing through the centre of the corresponding circle perpendicular to the medium radius of this sector. Find the volume of the solid of revolution thus generated.

1118. A line segment AB 16 cm long is divided by a point C into two parts which are to each other as 1 to 3. Constructed on each of the segments AB, AC and BC as on the diameters are semicircles lying on one side of the segment AB. The figure bounded by the three semicircles revolves about the axis AB. Find the volume of the solid of revolution.

1119. Prove that the volume of a solid generated by revolving a segment of a circle with the chord a about the diameter parallel to this chord is independent of the radius of the circle.

1120. Constructed on the base AC of an isosceles triangle ABC whose altitude BO is equal to the base AC are a semicircle and a tangent line DE parallel to AC. Prove that the solids generated by revolving the trapezium $ADEC$ and the semicircle about the axis AB are equal to each other.

1121. The surface area of a spherical segment is equal to S. Find the volume of the segment if the radius of the sphere is equal to R.

1122. Determine what part of the volume of the sphere is constituted by the volume of a spherical sector whose spherical and conic surfaces are equal to each other.

1123. The radius of a sphere is equal to 5 cm. At what distance from the centre should a section be drawn so that the volume and surface of the smaller segment are expressed by equal numbers?

38. Inscribed and Circumscribed Spheres

1124. 1. Is it possible to circumscribe a spherical surface about an oblique prism?

2. At what condition is it possible to circumscribe a sphere about a right prism?

3. At what condition will the centre of a sphere circumscribed about a right triangular prism be situated on one of the faces of the prism?

1125. 1. Is it possible to inscribe a sphere in a cube, in a rectangular parallelepiped?

2. At what condition is it possible to inscribe a sphere in a right or oblique parallelepiped?

3. At what condition is it possible to inscribe a sphere in a right triangular prism?

1126. 1. About what pyramid is it possible to circumscribe a sphere? How are the centre and the radius of the sphere found?

2. Prove that if a pyramid is a right one, then a sphere can be circumscribed about and inscribed in it.

1127. 1. Is it possible to circumscribe a spherical surface about any right circular cylinder?

2. At what condition a sphere can be inscribed in a right circular cylinder?

3. Prove that a sphere can be inscribed in and circumscribed about any right circular cone.

1128. 1. The edge of a cube is equal to a. Find the radii of the inscribed and circumscribed spheres.

2. The radius of a sphere is equal to R. Find the edges of the inscribed and circumscribed cubes.

1129. A sphere is tangent to all the edges of a cube. The radius of the sphere is equal to R. Find the area of the portion of the sphere contained inside the cube.

1130. The radius of a sphere is equal to 18 dm. A regular quadrangular prism whose altitude is equal to 28 dm is inscribed in the sphere. Find the surface area of this prism.

1131. A sphere of radius R can be inscribed in a right parallelepiped the acute angle of whose base is equal to 45°. Find the volume of the parallelepiped.

1132. A sphere of radius 2 m is circumscribed about a regular triangular prism. The side of its base is equal to 3 m. Find the volume of the prism.

1133. A regular prism is circumscribed about a sphere whose radius is R. Find the surface area and volume of the prism. Consider a: (1) triangular, (2) quadrangular and (3) hexagonal prism.

1134. A regular hexagonal prism is circumscribed about a sphere. Determine the volume of the prism if its altitude is equal to h.

1135. A regular triangular prism is inscribed in a sphere whose radius is equal to 14 cm. The diagonal of its lateral face is 26 cm long. Find the lateral surface of this prism.

1136. A regular prism is circumscribed about a sphere, and another sphere is circumscribed about this prism. Find the ratio of the surface areas of the spheres if the prism is: (a) triangular, (b) quadrangular, (c) hexagonal.

1137. The side of the base of a regular n-gonal pyramid is equal to a, the altitude is also equal to a. Find the radii of the inscribed and circumscribed spheres if: (1) $n = 3$, (2) $n = 4$, (3) $n = 6$.

1138. (1) In a regular triangular pyramid the altitude is equal to h and the lateral edge to b. Find the radii of the inscribed and circumscribed spheres. (2) The same condition for a regular quadrangular pyramid. (3) The same condition for a regular hexagonal pyramid.

1139. Given the edge a of a regular octahedron determine the radii of the inscribed and circumscribed spheres.

1140. In a regular pyramid the altitude is equal to h, and the radius of the circle circumscribed about the base to r. At what ratio of h and r the centre of the circumscribed sphere lies: (1) inside the pyramid, (2) on its base, (3) outside the pyramid.

1141. In a given pyramid each of the lateral edges is equal to 18 cm, and the altitude to 10 cm. Determine the radius of the circumscribed sphere.

1142. In a regular quadrangular pyramid the side of the base is equal to a, and the plane angle at the vertex to 60°. Determine the surface area of the sphere inscribed in the pyramid.

1143. The side of the base of a regular triangular pyramid is equal to a, and the lateral edges are mutually perpendicular. Find the radius of the circumscribed sphere.

1144. The base of a regular pyramid is a triangle whose side is equal to 15 dm. One of the lateral edges is equal to 10 dm and perpendicular to the base. Find the radius of the circumscribed sphere.

1145. A sphere touches all the sides of the base of a regular triangular pyramid and all its lateral faces. Find the volume of the portion of the sphere contained inside the pyramid if the altitude of the pyramid is equal to 3 dm, and the dihedral angle at the base to 60°.

1146. Prove that the volume of a circumscribed pyramid is equal to one third the product of the surface area of the pyramid by the radius of the sphere.

1147. A sphere is inscribed in a regular quadrangular pyramid whose altitude is equal to 24 dm and the side of the base to 14 dm. Determine the volume of the sphere.

1148. The side of the base of a regular quadrangular pyramid is equal to a, and the dihedral angle at the base to 60°. Find the surface area of the insphere.

1149. In a regular triangular pyramid each of the lateral edges is equal to b and inclined to the base at an angle of 30°. Find the surface area of the circumscribed sphere.

1150. The slant height of a regular quadrangular truncated pyramid circumscribed about a sphere is equal to a. Find the area of the lateral surface of the pyramid.

1151. A regular quadrangular truncated pyramid in which the dihedral angle at the base is equal to 60° is circumscribed about a sphere of radius R. Determine the total surface area of the pyramid.

1152. In a regular quadrangular truncated pyramid the sides of the base are equal to 6 m and 8 m, and the altitude to 14 m. Find the radius of the circumscribed sphere.

1153. 1. A sphere is circumscribed about an equilateral cylinder. Find the ratios of their volumes and surface areas.

2. A cylinder is circumscribed about a sphere. Find the ratios of their surface areas and volumes.

1154. Inscribed in a sphere of radius R is a cylinder, the diagonal of the axial section of which is inclined to the base at an angle of 30°. Find the volume of the cylinder.

1155. The diameter of the circle of a cylinder inscribed in a sphere divides the great circle in the ratio of 1 to 2. Find the surface area and volume of the cylinder if the radius of the sphere is equal to R.

1156. Find the ratio of the volumes of an equilateral cone and a sphere inscribed in (circumscribed about) it.

1157. The generator of an equilateral cone is equal to l. Determine the surface areas and volumes of the inscribed and circumscribed spheres.

1158. An equilateral cone is inscribed in a sphere of radius 6 m. Find the total surface area and volume of the cone.

1159. The altitude of a cone is equal to h, the generator to l. Find the radii of the inscribed and circumscribed spheres.

1160. The altitude of a cone is equal to half the diameter of the sphere circumscribed about it. How many times is the volume of the sphere greater than that of the cone?

1161. If a cone is circumscribed about a sphere and the altitude of the cone is twice the diameter of the sphere, then the volume and the total surface area of the cone is twice the volume and the surface area of the sphere. Check this.

1162. The altitude of a cone is equal to 40 cm, and the generator to 50 cm. Find the radius of the inscribed hemisphere whose base lies on the base of the cone.

1163. A sphere is inscribed in an equilateral cone whose volume is equal to V. The plane passing through the circle of tangency divides the sphere into two segments. Find the volume of each segment.

1164. A sphere is inscribed in a truncated cone, the radii of the base circles of which are equal to 9 cm and 25 cm. Determine the surface area and volume of the sphere.

1165. The radii of the base circles of a truncated cone are equal to 9 m and 12 m, the altitude to 21 m. Find the radius of the circumscribed sphere.

1166. Determine the total surface area and volume of a truncated cone circumscribed about a sphere if the generator is equal to 26 cm, and the radius of the sphere to 12 cm.

1167. The surface area of a sphere is equal to S. Circumscribed about the sphere is a truncated cone whose generator is inclined to the base at an angle of 60°. Find the lateral surface of the truncated cone.

1168. Prove that if it is possible to inscribe a sphere in a truncated cone whose generator is inclined to the base at an angle of 45°, then the lateral surface of the cone is twice the surface area of the sphere.

1169. Inscribed in a spherical sector are two mutually tangent spheres whose radii are equal to 2 dm and 6 dm. Find the radius of the sphere.

1170. The total surface area of a spherical segment is three times greater than the surface area of the sphere inscribed in it. Determine the altitude of the segment if the radius of its spherical surface is equal to R.

1171. An equilateral cone is inscribed in a spherical sector with an angle in the axial section equal to 90°. The vertex of the cone is found on the spherical surface of the sector, and the base of the cone rests against the conical surface of the sector. Find the ratio of the volumes of the cone and sector.

CHAPTER VIII

APPLYING TRIGONOMETRY TO SOLVING GEOMETRIC PROBLEMS

39. Polyhedrons

1172. Find the sides of the base of a rectangular parallelepiped if its altitude is equal to H, and α and β are the angles at which the diagonal of the parallelepiped and the diagonal of the lateral face are inclined to the base.

1173. The base of a rectangular parallelepiped is a square with the side a. The diagonal of the parallelepiped is inclined to the base at an angle of α. Find the area of the section passing through the diagonal of the parallelepiped and mid-points of two opposite lateral edges.

1174. The base of a right prism is a rhombus with an acute angle α. At what angle to the base must a cutting plane be drawn to obtain in section a square with the vertices lying on the lateral edges?

1175. The angles formed by the diagonal of a rectangular parallelepiped with its edges emanating from the same vertex are equal to α, β and γ. Prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. Compute the angle γ if $\alpha = 41°10'$ and $\beta = 59°20'$.

1176. The diagonal of a rectangular parallelepiped forms with the faces angles α, β and γ. Prove that: (1) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$, (2) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 2$.

1177. In a regular triangular prism the perpendicular dropped from a vertex of the base to the opposite side of the other base is equal to d and inclined to the base at an angle α. Find the side of the base of the prism.

1178. Drawn through a side of the base and the midpoint of the opposite lateral edge of a regular triangular prism is a section whose area is equal to Q and the angle at the vertex to α. Find the altitude of the prism.

1179. Through a side of the base of a regular triangular prism a plane is drawn to obtain a triangular section. The perimeter of the triangle is twice as long as the perimeter of the base of the prism. Find the angle between the cutting plane and the base of the prism.

1180. The base of a right triangular prism is an isosceles triangle with an angle of 120°. A plane is drawn through the base of this triangle and the opposite vertex of the upper base. The section of the prism is turned out to be a right-angled triangle. Find the angle of inclination of the cutting plane to the base.

1181. The base of a right prism is an isosceles trapezium with an acute angle α and lateral side a equal to the smaller base. Find the area of the diagonal section of the prism if the diagonal of the prism is inclined to the base at an angle of $\frac{\alpha}{2}$.

1182. The base of an inclined prism is an isosceles triangle whose altitude is H. Each of the lateral edges of the prism is also equal to H, one of them forming an angle α with each of the adjacent sides of the base. Find the altitude of the prism.

1183. In a regular pyramid the plane angle at the vertex is equal to α. Compute the angle of inclination of the lateral face to the base of the pyramid at $n = 4$ and $\alpha = 72°34'$.

1184. In a regular pyramid the lateral edge is inclined to the base at an angle α. Find the dihedral angle at the base of the pyramid.

1185. In a regular pyramid the dihedral angle at the base is equal to α. Compute the angle of inclination of the lateral edge to the base of the pyramid at $n = 6$ and $\alpha = 63°26'$.

1186. Determine the angle between two altitudes drawn from two vertices of a regular tetrahedron to the opposite faces.

1187. In a regular quadrangular pyramid the lateral edge is inclined to the base at an angle α. Determine the dihedral angle at the lateral edge.

1188. The areas of the lower and upper bases and lateral surface of a regular quadrangular truncated pyramid are as $m : n : p$. Find the angle between the lateral face and the lower base.

1189. In a regular quadrangular pyramid the angle between the opposite lateral edges is equal to α. Determine the plane angle at the vertex at $\alpha = 50°28'$.

1190. From a point situated at a distance h from a plane two straight lines are drawn at an angle α to the plane, their projections forming an angle β. Determine the distance between the feet of the inclined lines.

1191. A rectangle $ABCD$ with the sides $AB = a$ and $BC = b$ $(a < b)$ is projected on a plane passing through the side AB. Find the angle of inclination of the plane of projection to the plane of the rectangle if the projection obtained is a square.

1192. In a trihedral angle two plane angles are equal to each other, the third one being equal to α. Determine each of the two equal plane angles if the dihedral angle between them is a right one.

1193. Determine the radius of the circle inscribed in the base of a regular quadrangular pyramid whose edge forms an angle α with the altitude, and the volume is equal to V.

1194. In a regular triangular pyramid the lateral edge is equal to a and forms an angle α with the altitude. A section is drawn through a side of the base perpendicular to the opposite lateral edge. Find the area of the section.

1195. In a regular triangular pyramid the side of the base is equal to a and the lateral edge is inclined to the base at an angle α. Through the centre of the base a plane is drawn parallel to two non-intersecting edges of the pyramid. Compute the area of the section at $a = 8.4$ dm and $\alpha = 62°17'$.

1196. In a regular quadrangular pyramid the side of the base is equal to a, and the angle between the altitude and lateral edge to α. Through a point, dividing the side of the base in the ratio of 1 to 3, a plane is drawn perpendicular to the base of the pyramid and parallel to the side of the base. Find the area of the section.

1197. In a regular quadrangular pyramid the lateral edge is equal to b and inclined to the base at an angle α. Find the area of the section passing through the diagonal of the base of the pyramid parallel to the lateral edge.

1198. The altitude of a regular quadrangular pyramid is equal to H. Drawn through a diagonal of the base of the pyramid and the mid-point of the opposite edge is a section which forms an angle α with the diagonal plane passing through the same diagonal of the base. Find the area of the section at $H = 12.4$ cm and $\alpha = 51°43'$.

1199. Through a side of the base of a regular quadrangular pyramid a section is drawn perpendicular to the opposite lateral face. Determine the area of the section if the side of the base of the pyramid is equal to a and the section is inclined to the base at an angle α.

1200. The base of a pyramid is a rhombus and its vertex is projected in the point of intersection of the diagonals of the base. Construct the section of the pyramid by a plane passing through the smaller diagonal of the rhombus parallel to the lateral edge of the pyramid. Find the area of the section if the side of the rhombus is equal to a, its acute angle to α, and the greater lateral edge of the pyramid is inclined to the base at an angle β.

1201. The diagonal of a regular quadrangular truncated pyramid is twice the length of the diagonal of the smaller base equal to d and is inclined to the greater base at an angle α. Find the area of the diagonal section.

1202. The base of the pyramid is an isosceles triangle with the lateral side b and the angle at the base α. The lateral edge is inclined to the base at an angle β. Find the altitude of the pyramid.

1203. The base of a right prism is an isosceles triangle with the base a and the angle at the vertex α. Through a given side of the base a section is drawn which forms a dihedral angle α with the base of the prism. Determine the radius of the sphere inscribed in the pyramid thus obtained.

1204. In a regular triangular truncated pyramid the dihedral angle formed by the greater base and lateral face is equal to φ, and the sides of the bases are equal to a and b. Find the altitude of the pyramid.

40. Round Solids

1205. The diagonal of a rectangle equal to d is inclined to the base at an angle α. The rectangle is bent to form a cylinder. Find the radius of the base circle of the cylinder. (Consider two cases.)

1206. In an equilateral cylinder a point on the upper base circle is joined to one of the points of the lower base circle. The angle between the radii drawn to these points is equal to 120°. Determine the angle between the line segment connecting these points and the axis of the cylinder.

1207. In an equilateral cylinder the radius of the base of which is equal to R cm a point of the upper base circle is joined to a point of the lower base circle. The straight line passing through these points is d cm distant from the axis of the cylinder. Determine the angles of inclination of this straight line to the bases of the cylinder if $R = 15$ and $d = 12$.

1208. In a cylinder a section is drawn parallel to the axis which cuts off the lower base circle an arc equal to α. The line segment joining the centre of the upper base circle to the mid-point of the chord subtending the arc α is equal to m and inclined to the base at an angle α. Find the area of the section.

1209. Constructed in a cylinder is an isosceles triangle ABC ($AB = BC$), and AC is the diameter of the lower base circle, and B is a point lying on the upper base circle; the angle ABC is equal to α and the altitude of the cylinder to h. Find the radius of the base circle of the cylinder.

1210. The section of a cylinder by a plane parallel to its altitude is a square and is situated at a distance d from its axis. The cutting plane cuts an arc α off the base circle. Find the area of the section if $d = 9.5$ cm and $\alpha = 152°30'$.

1211. Drawn in the base circle of a cylinder is a chord AB subtending an arc of 90°. The end-points of the chord are joined to the centre O of the other base circle. The area of the triangle AOB thus obtained is equal to Q and its plane is inclined to the base of the cylinder at an angle α. Find the radius of the base circle and altitude of the cylinder.

1212. A tangent line is drawn to a cylinder at an angle α to its elements. Determine the distance between the centre of the lower base circle and this line if its distance from the point of tangency is equal to d and the radius of the base circle to R.

1213. The altitude of a cone is equal to H, and the angle between the altitude and generator to α. Find the area of the section drawn through two elements, the angle between which is equal to β.

1214. The generator of an equilateral cone is equal to L. Find the area of the section drawn through two elements if the angle of inclination of the cutting plane to the base is equal to α.

1215. The area of the lateral surface of a cone is four times the area of the base circle. Find the angle at which the generator is inclined to the base.

1216. The maximum angle between the elements of a cone is equal to α. Find the ratio of the total surface area of the cone to its lateral surface area.

1217. Two elements of a cone and a chord of the base circle form an isosceles triangle with the angle at the vertex α and area Q. The plane of the triangle is inclined to the base at an angle β. Find the altitude of the cone.

1218. The radius of the base circle of a cone is equal to R, and the generator is inclined to the base at an angle α. In this cone a plane is drawn through its vertex and at an angle β to the base. Determine the area of the section.

1219. Through the vertex of a cone a plane is drawn which cuts an arc α from the base circle. The angle at the vertex of the section thus obtained is equal to β. Find the angle of inclination of the section to the base if $\alpha = 120°$ and $\beta = 90°$.

1220. The angle at the vertex of the axial section of a cone is equal to α. Find the central angle of the development of its lateral surface.

1221. The central angle of the development of the lateral surface of a cone is equal to α. Find the angle at the vertex of the axial section of the cone.

1222. Tangent to the lateral surface of a cone is a straight line forming an angle α with the element of a cone passing through the point of tangency. The angle of inclination of the generator to the base is equal to β. The point of tangency is located at a distance d from the plane of the base. Find the segment of the tangent line as measured from the point of tangency to the point of intersection with the base of the cone.

1223. In a truncated cone, the radii of the base circles of which are equal to R and r, a plane is drawn at an angle β to the base. This plane cuts an arc α from each base circle. Determine the area of the section.

1224. In a truncated cone, the radii of the base circles of which are R and r, a plane is drawn at an angle α to the base, the area of the section which does not intersect the axis of the cone is equal to Q. Determine the length of each of the arcs cut off by this plane from the base circles.

1225. Through two elements of a truncated cone containing an angle α a plane is drawn which intersects the base circles of the cone along the chords equal to a and b ($a > b$). Determine the area of the section.

1226. The altitude of a truncated cone is equal to h; the generator is inclined to the lower base at an angle α and forms an angle β with the straight line passing through its upper end-point and the lower end-point of the opposite element. Determine the area of the axial section of this cone.

1227. Inscribed in a sphere with a surface area S is a cylinder, the diagonal of the axial section of which is inclined to the base at an angle α. Find the area of the axial section of the cylinder.

1228. An equilateral cylinder is inscribed in a cone. Find the altitude of the cylinder if the altitude of the cone is h and the angle at the vertex of the axial section is equal to 2α.

1229. The radius of the base circle of a cone is equal to R, and the generator is inclined to the base at an angle α. A sphere is inscribed in the cone. Find the distance between the vertex of the cone and the plane of the circle along which the spherical surface contacts the lateral surface of the cone.

1230. Circumscribed about a cone is a sphere, the area of the great circle of which is equal to Q, and a sphere is inscribed in a cone. Find the distance between the centres of the spherical surfaces if the generator of the cone is inclined to the base at an angle α.

1231. Circumscribed about a sphere is a truncated cone, whose generator is inclined to the greater base at an angle α. The length of the circumference of tangency is equal to C. Find the surface area of the sphere.

1232. In a truncated cone the radii of the base circles are equal to R and r, and the generator is inclined to the greater base at an angle α. Find the radius of the sphere circumscribed about the truncated cone.

1233. Given a spherical segment. Drawn through a point dividing the altitude of the segment in the ratio of 1 to 4 and perpendicular to it is a section, whose area is equal to one fourth the area of the base of the segment. Determine the arc of the axial section of the segment.

1234. The angle of the axial section of a spherical sector is equal to α. In what ratio is the area of the conical surface of the sector divided by the plane drawn through the mid-point of the medium radius and perpendicular to it. Analyse the formula of the ratio. Compute it at $\alpha = 45°, 60°, 90°, 120°$.

41. Areas and Volumes of Prisms

1235. The base of a rectangular parallelepiped is a square with the side a. The diagonal of the parallelepiped forms an angle α with the lateral face. Find the volume and lateral surface of the parallelepiped.

1236. The diagonal of a rectangular parallelepiped is inclined to the base at an angle α. The sides of the bases are equal to a and b. Determine the volume of the parallelepiped.

1237. In a rectangular parallelepiped the diagonal is equal to d and inclined to the base at an angle α. One of the sides of the base forms with the diagonal of the base an angle β. Find the volume of the parallelepiped.

1238. The diagonal of a rectangular parallelepiped is equal to d and forms an angle α with the base and an angle β with the smaller lateral face. Determine the volume of the parallelepiped.

1239. The area of the diagonal section of a rectangular parallelepiped is equal to Q. The diagonal of the base equal to d forms an angle α with the side of the base. Determine the surface area and volume of the parallelepiped.

1240. The side of the base of a regular quadrangular prism is equal to a. From one vertex of the base diagonals are drawn in two adjacent lateral faces. The angle between the diagonals is equal to α. Compute the area of the lateral surface of the prism at $a = 25.3$ cm and $\alpha = 80°16'$.

1241. The base of a rectangular parallelepiped is a rhombus with an acute angle α and smaller diagonal d. The greater diagonal of the parallelepiped is inclined to the base at an angle $\frac{\alpha}{2}$. Find the total surface area and volume of the parallelepiped.

1242. In a right parallelepiped the acute angle of the base is equal to α, one of the sides of the base to a; the section drawn through the other side of this base and the opposite side of the other base has an area Q and is inclined to the base at an angle β. Determine the volume of the parallelepiped and compute it at $a = 12.3$ cm, $Q = 203.8$ cm^2, $\alpha = 48°25'$, $\beta = 63°26'$.

1243. Find the volume of a right quadrangular prism whose diagonal is equal to d and inclined to the base at an angle α, and the acute angle between the diagonals of the rectangular base is equal to β.

1244. The diagonal of the lateral face of a regular quadrangular prism is equal to d and forms an angle α with the diagonal of the prism. Determine the area of the lateral surface of the prism.

1245. In a regular quadrangular prism the diagonal is equal to d and forms an angle α with the lateral edge. Determine the volume of the prism.

1246. In a rectangular parallelepiped the lateral edge is equal to H and forms an angle α with the diagonal of the parallelepiped. The angle between the diagonal and the side of the base of the parallelepiped is equal to β. Determine the volume of the parallelepiped.

1247. The area of the diagonal section of a rectangular parallelepiped amounts to Q, and the diagonal of the base is equal to d and forms an angle α with the side of the base. Determine the volume of the parallelepiped and compute it at $Q = 244.6$ cm^2, $d = 30.2$ cm and $\alpha = 63°26'$.

1248. In a regular triangular prism the diagonal of the lateral face is equal to d and forms an angle α with the plane of the base. Determine the volume of the prism.

1249. In a regular triangular prism $ABCA_1B_1C_1$ the side of the base is equal to a. Through the vertex B_1 and mid-points of the edges AB and AC straight lines are drawn, the angle between which is equal to α. Determine the volume of the prism.

1250. In a regular hexagonal prism the diagonal joining the opposite vertices of two bases is equal to d and forms an angle α with the base of the prism. Determine the volume and area of the lateral surface of the prism and compute it at $d = 0.38$ m and $\alpha = 73°16'$.

1251. In a regular triangular prism the diagonal of the lateral face is equal to d and forms an angle α with the diagonal of the other lateral face. Determine the area of the lateral surface of the prism and compute it at $d = 28.5$ dm and $\alpha = 50°22'$.

1252. The base of a right parallelepiped is a rhombus with the side a and an acute angle α. The greater diagonal is inclined to the base at an angle $\frac{\alpha}{2}$. Determine the volume of the parallelepiped and compute it at $a = 0.83$ m and $\alpha = 72°20'$.

1253. In a regular triangular prism a plane is drawn through a side of the lower base and the opposite vertex of the upper base. The angle between this plane and the base of the prism is equal to α, and the area of the section to Q. Determine the volume of the prism.

1254. Drawn through one of the vertices in a regular triangular prism is a section, which divides the opposite lateral face into two congruent rectangles. The area of the section is equal to Q, and it is inclined to the base of the prism at an angle α. Find the area of the lateral surface and volume of the prism.

1255. In a regular triangular prism the diagonals of the lateral face intersect at an angle α and each of them is equal to d. Determine the area of the lateral surface and volume of the prism.

1256. The altitude of a right prism is equal to h; its base is a right-angled trapezium (with an acute angle α) circumscribed about a circle of radius r. Find the volume of the prism.

1257. The base of a right prism is a right-angled triangle with an acute angle α. The diagonal of the greater lateral face is equal to d and inclined to the base at an angle β. Determine the volume of the prism and the area of the lateral surface and compute it at $d = 8.3$ dm, $\alpha = 81°19'$ and $\beta = 58°53'$.

1258. The base of a right prism is a right-angled triangle with an acute angle α. The area of the greater lateral face is equal to Q. Determine the area of the lateral surface of this prism.

1259. The base of a right prism is an isosceles triangle whose perimeter is equal to $2p$ and the angle between the equal sides to α. The perpendicular dropped from the vertex of this angle to the opposite side of the other base is inclined to the base at an angle β. Determine the volume of the prism.

1260. A right prism whose base is a right-angled triangle with the hypotenuse c and an acute angle α is circumscribed about a sphere. Find the volume of the prism.

1261. The base of an oblique parallelepiped is a rhombus with the side a and an acute angle α. One of the vertices of the upper base is projected in the point of intersection of the diagonals of the lower base. Determine the volume of the parallelepiped if the lateral faces are inclined to the base at an angle β. Compute it at $a = 83$ cm, $\alpha = 78°16'$, $\beta = 64°49'$.

1262. Determine the volume of a parallelepiped, in which each of the edges is equal to a, and each of the plane angles at one of the vertices is equal to α ($\alpha < 90°$). Compute it at $\alpha = 69°20'$ and $a = 30.3$ cm.

1263. The area of the base of an oblique prism is equal to Q, and the lateral edge equal to b is inclined to the base at an angle α. Find the volume of the prism.

1264. The base of an oblique prism is an equilateral triangle. One of the vertices of the prism is projected in the centre of the base. Determine the volume of the prism if each of its lateral edges equal to b is inclined to the base at an angle α.

1265. Two lateral faces of an oblique triangular prism are congruent rhombuses with the side b and an acute angle α. The angle of the base of the prism formed by equal sides is also equal to α. Find the volume of the prism.

1266. The base of a right prism is an isosceles triangle with the base a and opposite angle α. The area of the lateral surface of the prism is equal to the area of the base. Find the volume of the prism.

1267. The base of a right prism is an isosceles trapezium with an acute angle α and the smaller base a equal to the lateral side. The diagonal of the prism is inclined to the base at an angle $\frac{\alpha}{2}$. Determine the volume of the prism.

1268. In an oblique parallelepiped one of the diagonal sections is perpendicular to the base, the smaller diagonal is equal to the lateral edge of the parallelepiped, and the acute angle of the section is equal to α. The base of the parallelepiped is a rectangle with the smaller side a and acute angle between the diagonals 2α. Find the volume of the parallelepiped.

1269. Inscribed in a sphere of radius R is a regular quadrangular prism whose diagonal is inclined to the base at an angle α. Determine the area of the lateral surface and the volume of the prism.

42. Areas and Volumes of Pyramids

1270. In a regular triangular pyramid the lateral edge is equal to b and forms an angle α with the side of the base. Determine the area of the lateral surface of the pyramid.

1271. In a regular triangular pyramid the lateral edge is equal to b and inclined to the base at an angle α. Determine the volume of the pyramid.

1272. In a regular triangular pyramid the slant height is equal to m, and the lateral face is inclined to the base at an angle α. Determine the volume of the pyramid.

1273. In a regular triangular pyramid the radius of the circle circumscribed about its base is equal to R. The slant height is inclined to the base at an angle α. Determine the total surface area of the pyramid.

1274. In a regular triangular pyramid the lateral edge is inclined to the altitude at an angle α. The radius of the incircle is equal to r. Determine the volume of the pyramid.

1275. In a regular quadrangular pyramid the lateral edge is equal to b, and the plane angle at the vertex to α. Determine the total surface area of the pyramid and compute it at $b = 0.72$ m and $\alpha = 48°52'$.

1276. In a regular quadrangular pyramid the slant height is equal to m. The lateral edge is inclined to the plane at an angle α. Determine the total surface area of the pyramid.

1277. The altitude of a regular quadrangular pyramid is equal to H, and the plane angle at the vertex to α. Determine the volume of the pyramid.

1278. The lateral edge of a regular quadrangular pyramid is equal to l, and the dihedral angle at the base to α. Find the volume and total surface area of the pyramid.

1279. The base of a pyramid is a triangle whose perimeter is equal to P, and the dihedral angle at the base to α. Determine the volume of the pyramid.

1280. In a regular n-gonal pyramid the slant height is equal to m, and the dihedral angle at the base to α. Find the total surface area and volume of the pyramid.

1281. The area of the total surface of a regular quadrangular pyramid is equal to Q, and the dihedral angle at the base to φ. Find the volume of the pyramid.

1282. The volume of a regular quadrangular pyramid is equal to V. Find the side of the base of the pyramid if the angle between the lateral edge and the altitude is equal to α.

1283. A sphere of radius R is inscribed in a regular quadrangular pyramid. The dihedral angle at the base of the pyramid is equal to α. Find the area of the lateral surface and the volume of the pyramid.

1284. In a regular triangular pyramid a perpendicular equal to p is dropped from the foot of the altitude to a lateral face. The plane angle at the vertex is equal to α. Find the volume of the pyramid.

1285. The base of a pyramid is a right-angled triangle. The lateral edge of the pyramid passing through the vertex of the right angle of this triangle is equal to l and perpendicular to the base. Two other lateral edges of the pyramid are inclined to the base at angles α and β. Determine the volume of the pyramid.

1286. The base of a pyramid is a right-angled triangle in which one of the acute angles is equal to α. Each of the lateral edges of the pyramid is equal to b and inclined to the base at an angle β. Find the volume of the pyramid.

1287. The base of a pyramid is a right-angled triangle ABC, in which the side $BC = a$ and the angle A is equal to α. The lateral faces of the pyramid passing through the sides AC and BC, containing the right angle are perpendicular to the base, the third lateral face being inclined to the base at an angle β. Find the volume of the pyramid.

1288. The base of a pyramid is a right-angled triangle, in which one of the acute angles is equal to α. Inscribed in the pyramid is a cone, the radius of the base circle of which is equal to R, and the generator is inclined to the base at an angle β. Find the volume of the pyramid.

1289. The base of a pyramid is a right-angled triangle. Each lateral edge is equal to l and inclined to the base at an angle α. The lateral face of the pyramid passing through one of the sides containing the right angle forms a dihedral angle β with the base. Find the volume of the pyramid.

1290. The base of a pyramid is an isosceles triangle with the lateral side b and plane angle α at the vertex. Each of the lateral edges is also equal to b. Find the volume of the pyramid.

1291. The base of a pyramid is an isosceles triangle, in which the angle at the vertex is equal to β, and the base to b. The lateral faces are inclined to the base at an angle α. Find the total surface area of the pyramid.

1292. The base of a pyramid is a triangle with angles α and β; each of the lateral edges is equal to b and forms an angle γ with the altitude. Determine the volume of the pyramid.

1293. In a triangular pyramid two of the lateral faces are isosceles right-angled triangles, whose hypotenuses are equal to c (each) and form an angle α. Determine the volume of the pyramid.

1294. The base of a pyramid is a rhombus with the side a. Two of the lateral faces of the pyramid are perpendicular to the base and form an obtuse angle β; two others are inclined to the base at an angle α. Find the area of the lateral surface of the pyramid.

1295. The base of the pyramid is a rhombus. Two lateral faces of the pyramid forming an obtuse angle α are perpendicular to the base, and two others are inclined to the base at an angle β. The distance between the vertex of the obtuse angle of the base of the pyramid and the plane of the inclined face is equal to d. Find the area of the lateral surface of the pyramid.

1296. From a regular quadrangular pyramid with the side of the base a and the plane angle at the vertex α a triangular pyramid is cut off by a plane passing through a diagonal of the base of the given quadrangular pyramid and parallel to its lateral edge. Determine the volume of the cut-off pyramid. Compute it at $a = 7.81$ dm and $\alpha = 63°17'$. •

1297. The base of a pyramid is a rhombus whose greater diagonal is equal to d, and the acute angle to α. Each lateral face is inclined to the base at an angle β. Find the total surface area of the pyramid.

1298. The base of a pyramid is a parallelogram whose diagonals intersect at an angle α. The altitude of the pyramid passes through the point of intersection of the diagonals of the base and is equal to H. The lateral edges are equal to b and c. Find the volume of the pyramid.

1299. The base of a pyramid is a square. Two opposite lateral faces of the pyramid are isosceles triangles whose planes form dihedral angles α and β with the base. The projection of the vertex of the pyramid on the plane of the base is found outside the base of the pyramid at a distance m from the nearest side. Find the volume of the pyramid.

1300. The base of a pyramid is a square. Two lateral faces of the pyramid are perpendicular to its base, and two others form an angle α with the base. A cube is inscribed in the pyramid so that four of its vertices lie on the lateral edges of the pyramid, and four others on the base of the pyramid. Find the area of the lateral surface of the pyramid if the edge of the cube is equal to a.

1301. In a regular quadrangular pyramid the altitude is equal to H and forms an angle φ with the lateral face. Drawn through one of the sides of the base is a cutting plane perpendicular to the opposite face. Find the volume of the pyramid cut by this plane from the given pyramid.

1302. The lateral edges of a regular triangular truncated pyramid are inclined to the plane of the greater base at an angle α, and the sides of the bases are equal to a and b $(a > b)$. Find the volume of this truncated pyramid.

1303. In a regular quadrangular truncated pyramid the side of the greater base and the slant height are equal to a (each). The section of the pyramid by a plane passing through a side of the greater base perpendicular to the lateral face forms a dihedral angle α with the plane of the greater base. Determine the area of the lateral surface of the pyramid and compute it at $a = 82.5$ cm, $\alpha = 18°28'$

1304. The lateral edge of a regular quadrangular truncated pyramid is inclined to the side of the greater base at an angle φ. Find the volume of the pyramid if the sides of its bases are equal to a and b.

1305. Inscribed in a regular triangular truncated pyramid is a sphere of radius R. Find the area of the lateral surface of the truncated pyramid if the dihedral angle at the greater base is equal to α.

1306. The area of the lateral surface of a regular quadrangular truncated pyramid is equal to Q. Find the volume of the pyramid if the areas of its bases are as 1 : 2, and the lateral face is inclined to the greater base at an angle α.

1307. Inscribed in a sphere is a pyramid, whose base is a right-angled triangle with the hypotenuse c and an acute angle α. The lateral edges are equal to one another and are inclined to the base at an angle β. Find the volume of the pyramid.

43. Areas and Volumes of Round Solids

1308. 1. The diagonal of the axial section of a cylinder is equal to d, and forms an angle α with the base. Determine the volume and area of the lateral surface of the cylinder and compute it at $d = 10.3$ dm, $\alpha = 71°43'$.

2. The area of the axial section of a cylinder is equal to Q, the angle between this section and the axis of the cylinder to α. Determine the volume of the cylinder and compute it at $Q = 478.6\ \text{cm}^2$, $\alpha = 62°13'$.

1309. The section of a cylinder by a plane parallel to the axis of the cylinder represents a square. Determine the surface area and volume of the cylinder if the section cuts an arc α from the base circle and is drawn at a distance d from the axis of the cylinder.

1310. The base of a right prism is a right-angled triangle with an acute angle α and altitude h dropped to the hypotenuse. Determine the volume of the cylinder circumscribed about this prism if its generator is also equal to h. Compute it at $h = 0.7$ m, $\alpha = 38°53'$.

1311. A rectangular parallelepiped is inscribed in a cylinder. Find the area of the lateral surface of the cylinder if it is known that the smaller side of the base is equal to a, the acute dihedral angle between the diagonal planes to α, and the diagonal of the parallelepiped forms an angle β with its greater lateral face.

1312. A pyramid is inscribed in a cylinder so that its base is inscribed in the lower base of the cylinder, and the vertex is found at the centre of the upper base of the cylinder. The base of the pyramid is an isosceles triangle with an angle β at the vertex. The lateral edges of the pyramid are inclined to the base at an angle α. Find the volume of the pyramid if the radius of the base of the cylinder is equal to R.

1313. Inscribed in a regular quadrangular pyramid is a cylinder, in which the radius of the base circle is equal to the generator. Determine the volume of the cylinder if the side of the base of the pyramid is equal to a, and the dihedral angle at the base to α. Compute it at $a = 28.3$ cm, $\alpha = 15°$.

1314. Determine the volume and surface area of a cone given: (1) the generator l and maximum angle α between two elements; (2) the area Q of the axial section and angle α between the generator and the base.

1315. The angle at the vertex of the axial section of a cone is equal to α, and its perimeter to P. Find the total surface area of the cone.

1316. Determine the total surface area of a cone if the area of its base circle is equal to πQ cm^2, and the angle between the generator and altitude to α. Compute it at $Q = 428.6$ cm^2, $\alpha = 36°56'$.

1317. Find the angle at the vertex of the axial section of a cone if the area of the lateral surface of the cone is equal to Q, and the total surface area to S.

1318. The axial section of a cone is an isosceles triangle, in which the angle at the base is equal to α. The radius of the circle inscribed in this triangle is equal to r. Determine the volume of a cone and compute it at $r = 9.2$ dm, $\alpha = 70°14'$.

1319. The difference between the generator and altitude of a cone is equal to d, and the angle at the vertex of the axial section of the cone to 2α. Determine the volume of the cone and compute it at $d = 8.4$ cm, $\alpha = 83°28'$.

1320. Inscribed in the base of a cone is a square, whose side is equal to a. The plane passing through the vertex of the cone and one of the sides of this square cuts the cone forming a triangle in the section with the angle α

at its vertex. Determine the volume of the cone and compute it at $a = 0.39$ m and $\alpha = 72°32'$.

1321. Find the volume of a cone if its total surface area is equal to Q, and the angle at the vertex in the axial section is equal to α.

1322. Through the vertex of a cone and at an angle α to the base a plane is drawn cutting from the base circle an arc β. Find the volume of the cone if this plane is d cm distant from the centre of the base circle.

1323. A circular sector ABC is bent to form a cone. Find the volume of the cone if the chord AC is equal to a and the angle ABC to α.

1324. A right-angled triangle with the hypotenuse c and an acute angle α rotates about the hypotenuse. Find the surface area and volume of the solid of revolution thus obtained.

1325. An isosceles triangle ABC, in which $AB = BC = b$ and $\angle C = \angle A = \alpha$, rotates about a lateral side. Find the surface area and volume of the solid of revolution.

1326. In an isosceles trapezium $ABCD$ with an acute angle β $AB = CD$ and O is the mid-point of the greater base AD. Find the volume of the solid generated by the rotation of this trapezium about the greater base if $OC = m$ and the angle BOC is equal to α.

1327. Constructed on a common base are two cones, one inside the other, so that the distance between their vertices is equal to d. Find the volume of the solid bounded by conical surfaces of the cones if the angle at the vertex of the axial section of the greater cone is equal to α, and the smaller one to β.

1328. The base of a right prism is an isosceles trapezium with an acute angle α. A cone is inscribed in the prism so that its base is inscribed in the base of the prism, and the vertex is found in the plane of the other base. The radius of the base circle is equal to R, and the angle at the vertex of the axial section of the cone is equal to β. Find the total surface area of the prism.

1329. In a regular quadrangular pyramid the side of the base is equal to m, and the plane angle at the vertex to α. Find the volume of the cone circumscribed about the pyramid.

1330. In a regular triangular pyramid the vertex of the base is found at a distance d from the opposite lateral face. Find the total surface area of the cone inscribed in the given pyramid if the dihedral angle at the base of the pyramid is equal to α.

1331. Circumscribed about a cone is a pyramid, whose base is a right-angled triangle. The radius of the base circle of the cone is equal to r, the acute angle of the triangle to α, and the generator of the cone is inclined to the base at an angle β. Find the area of the lateral surface of the pyramid.

1332. The radii of the base circles of a truncated cone are as 1 : 2, and the generator l is inclined to the greater base at an angle α. Find the volume of the cone.

1333. In a truncated cone the diagonals of the axial section are mutually perpendicular, and the generator is inclined to the greater base at an angle α and equals l. Find the surface area of the truncated cone.

1334. The generator of a truncated cone equal to l is inclined to the lower base at an angle α and perpendicular to the diagonal of the axial section. Determine the area of the lateral surface of the cone.

1335. The generator of a truncated cone is equal to l and inclined to the base at an angle α; the ratio of the areas of the base circles is equal to 9. Find the volume of the truncated cone.

1336. Find the total surface area and volume of a truncated cone if its generator l is inclined to the greater base at an angle α, and the diagonals of the axial section form also the angle α.

1337. In a triangle ABC the angle A is an obtuse one, and the angle C is equal to α; the sides AC and BC are equal to a and $2a$, respectively. The triangle revolves about an axis passing through the vertex C and perpendicular to AC. Find the volume of the solid of revolution (the axis of revolution is contained in the plane of the triangle ABC).

1338. 1. An isosceles triangle with the angle at the base α and lateral side b revolves about an axis passing through the end-point of the base perpendicular to it. Find the volume of the solid of revolution.

2. Solve the same problem for the case, when the axis of revolution passes through the end-point of the base parallel to the lateral side.

1339. 1. A rhombus with the side a and an acute angle α revolves about an axis passing through the vertex of the obtuse angle perpendicular to the smaller diagonal. Find the volume and surface area of the solid of revolution.

2. Solve the same problem for the case when the axis of revolution passes through the vertex of the acute angle perpendicular to the side of the rhombus.

1340. A sector of a circle, whose radius is equal to R and the angle at the vertex to α, revolves about a diameter, which does not intersect the arc of the sector and forms an angle β with the nearest radius. Find the volume of the spherical sector obtained and the surface area of the corresponding spherical zone.

1341. A segment of a circle of radius R, having an arc equal to α, revolves about a diameter which does not intersect the arc of the segment and forms an angle β with the radius drawn to the nearest end-point of the arc. Find the surface area of the solid generated by revolving this segment. Compute it at $R = 5.7$ cm, $\alpha = 92°32'$, $\beta = 16°58'$.

1342. In a regular triangular pyramid the side of the base is equal to a and the lateral edge is inclined to the base at an angle α. Find the volume of the circumscribed sphere.

1343. The side of the base of a regular quadrangular pyramid is equal to a, and the dihedral angle at the base to α. Find the surface area of the insphere.

1344. A sphere is circumscribed about a regular quadrangular pyramid. Find the surface area of the sphere if the side of the base of the pyramid is equal to a and the plane angle at the vertex of the pyramid to α.

1345. Inscribed in a sphere of radius R is a cylinder, in which the diagonal of the axial section is inclined to the base at an angle α. Determine the volume of the cylinder and compute it at $R = 0.92$ m, $\alpha = 78°12'$.

1346. Circumscribed about a sphere of radius r is a cone with an angle α at the vertex of the axial section. Find the volume of the cone.

1347. A cone is inscribed in a sphere of radius R. Find the volume of the cone if the angle at the vertex of the axial section of the cone is equal to α.

1348. The altitude of a cone is equal to H, and the generator is inclined to the base at an angle α. Find the surface area of the insphere. Compute it at $H = 24.2$ cm, $\alpha = 32°14'$.

1349. Inscribed in a sphere, the area of the great circle of which is equal to Q is a cone, whose generator is inclined to the base at an angle α. Find the volume of the sphere inscribed in this cone.

1350. Inscribed in a sphere is a cone with an angle α at the vertex of the axial section. Find the total surface area of the cone if the surface area of the sphere is equal to Q. Compute it at $Q = 304.6$ cm^2, $\alpha = 122°18'$.

1351. A truncated cone is circumscribed about a sphere. Find the area of the lateral surface and the volume of this cone if the radius of the sphere is equal to r, and the generator is inclined to the base at an angle α.

1352. A truncated cone is inscribed in a sphere of radius R, and an axial section is drawn. The bases of the obtained trapezium subtend the arcs α and β ($\alpha > \beta$) of the great circle obtained in the section by this plane. Find the area of the lateral surface of the cone. (The arc β is not superimposed on the arc α.)

ANSWERS

1. (b) AC and CD are incommensurable; AC and DB are commensurable. 3. 1. (a) (12; 0), (b) (10; 0), (c) (16; 0). 2. $\frac{am}{m+n}$ and $\frac{an}{m+n}$. 4. (a) (5; 8), (b) $\left(6;\ 9\frac{1}{3}\right)$. 5. 1. 1 : 50,000, 2. 1 km 960 m. 6. 25 dm. 7. 2 : 1, 3 : 2, 11 : 4. 8. 9 m. 10. (1) $\frac{nh}{m-n}$, (3) $\frac{mh}{m+n}$ and $\frac{nh}{m+n}$. 11. Not less than 9 km. 12. 33 m. 14. 11.25 cm and 29.25 cm. 15. 8 cm. 16. $4\frac{4}{11}$ cm. 17. 4 cm. 18. 24 cm; $12\sqrt{3}$ cm and $12\sqrt{3}$ cm. 19. 5 cm and 19 cm. 20. 9.6 cm; 7.2 cm and 5.4 cm. 21. $\frac{ac}{b}$. 22. 12 cm. 25. *Hint.* It is required to prove that the greater side of the rectangle is the fourth proportional quantity for the base of the triangle, altitude and sum of the altitude and half the base. 27. 3. (a) $c = 10$, $a = 8$, $a_c = 6.4$, $h = 4.8$; (b) $h = 3$, $c = 10$, $a = \sqrt{10}$, $b = \sqrt{90}$; (c) $a_c = 32$, $c = 144.5$, $b = 127.5$, $b_c = 112.5$. 30. 20 cm. 31. The construction is possible at $a > b$, if $b + \sqrt{ab} > a$. 32. $\frac{m^2}{n^2}$. 33. 4 cm and $4\sqrt{3}$ cm. 35. 17 cm, $15\frac{9}{13}$ cm and $6\frac{7}{13}$ cm. 36. 1. ≈ 20.1 cm. 2. ≈ 5.7 cm. 37. 32 cm and 24 cm; $\sqrt{80}$ cm. 38. $21\frac{1}{3}$ cm and $\frac{8\sqrt{97}}{3}$ cm. 39. $15\frac{1}{17}$ cm. 40. $9\sqrt{6}$ cm and $9\sqrt{3}$ cm. 41. $\approx$ 1982 km. 42. (1) $\frac{\sqrt{2}}{2}$, (2) $\sqrt{13}$. 43. 24 cm and 10 cm; $10\sqrt{2}$ cm. 44. $2\sqrt{13}$ cm. 45. 78 cm or 18 cm. 46. 96 cm

and 60 cm. 47. 1. 2.5 cm. 2. 40 cm. 48. 48 cm and $12\sqrt{2}$ cm or 120 cm and $48\sqrt{2}$ cm. 49. 64 cm and 36 cm. 50. 3 cm and 4 cm. 51. 3.2 cm. 52. 15 cm. 53. 25 cm and 7 cm. 54. $3h(\sqrt{7}-1)$. 56. 1. 52 dm. 2. 16 cm and 30 cm. 57. 10 cm and 17 cm. 59. 14 cm and 12 cm. 60. 13 dm, 8 dm and 7 dm. 61. (1) Inadmissible, (2) permissible. 62. 67.6 mm. 63. 2. (a) Possible, using two different ways, (b) impossible, (c) possible, (d) possible. Triangles and hexagons; octagons and squares. 65. $4\sqrt{3}$ cm. 66. $4\sqrt{3}$, 12, $4\sqrt{6}$, $4\sqrt{3}$, $2\sqrt{3}$, $2\sqrt{6}$. 67. $4\sqrt{3}$ cm. 68. $\approx$15.8 cm. 70. $\approx$30.6 cm. 71. 816.4 mm. 72. 1.5 m. 74. $3\pi R$. 75. $\pi\sqrt{369}$ cm. 76. 1. 288°. 2. 21.6 dm. 3. 8 cm. 4. 144°. 5. $\approx$111.6 km. 77. $\frac{4\pi a}{3}$. 78. $\frac{\pi m}{2}$. 79. 3.2 cm and 8 cm. 80. $144\sqrt{3}$ dm^2. 81. 819.2 dm^2. 82. $\approx$11.42 m^2. 83. $\approx$12.56 cm^2. 84. 339.5 cm^2. 85. 22 cm and 46 cm. 86. 1. 384 dm^2. 2. $\approx$651.1 dm^2. 87. 18 (unit)^2. 88. $\approx$329.9 cm^2. 89. $18\sqrt{3}$ cm^2. 90. $\frac{a\sqrt{3}}{2}$. 91. $\frac{(m+n)^2\sqrt{m}}{4\sqrt{n}}$. 93. 7.1 cm. 94. $32(\sqrt{2}-1)$ dm^2. 95. $\frac{3\sqrt{3}}{2}$ dm^2. 96. $\approx$211.1 dm^2. 97. $48\sqrt{3}$ cm^2. 98. 216 m^2. 99. 128 roubles. 100. $3\sqrt{3}$ dm^2. 101. $\approx$424 mm. 102. $\approx$12.7 dm. 103. $\approx$11.33 cm^2. 104. 12π cm^2. 105. 1. $\approx$20.8 m^2, 2. $\approx$3.39 m^2. 106. $48\pi(7-4\sqrt{3})$ cm^2. 107. $\approx$10.5 cm^2. 109. a^2. 110. 25 per cent. 111. 1 km^2, 4 km. 112. $39.4a$. 113. 1. 16:9. 2. $298\frac{2}{3}$ cm^2. 114. By 125 per cent. 116. 1. (a) 0.4599; (c) 0.9239; (f) 0.0651; (h) 0.0676; 2. (a) $\bar{1}.5760$; (c) $\bar{2}.5776$; (f) 1.0759; (h) 0.5522. 117. (1) 12°, 27°30′, 80°20′, 3°19′; (2) 30°12′, 49°32′, 79°22′, 85°8′; (3) 51′, 44°21′, 51°30′, 78°13′; (4) 49°54′, 37°58′, 25°44′, 84°40′. 118. (1) 14°47′, 58°38′, 8°32′, 44′; (2) 64°, 75°31′, 57°51′, 85°58′; (3) 5°14′, 86°50′, 31°2′, 87°33′; (4) 71°40′, 7′, 58°49′, 7°39′. 119. (1) 0.53, 0.54, 0.54, 0.95, 0.08; (2) 0.87, 0.272, 0.153, 0.058; (3) 0.044, 0.063, 0.94, 3.17; (4) 28.6, 4.47, 1.075, 0.076. 120. (1) 32°, 3°, 4°20′, 44°; (2) 80°35′, 86°5′, 40°15′, 31°10′; (3) 2°50′, 70°35′, 40°50′, 53°30′; (4) 50°30′, 33°10′, 86°20′, 49°53′.

121.

	a	b	c	A	B	S
1	7.15	4.70	(8.53)	56°41′	33°19′	16.8
2	(360)	266	445	53°30′	(36°30′)	47,800
3	(16.4)	23	(28.2)	35°25′	54°35′	190
4	(284)	(170)	330	59°10′	30°50′	24,100

122.

	a	b	c	A	B	S
1	52.9	24.4	(58.3)	(65°14′)	24°46′	645.38
2	(630 m)	466 m	784 m	53°30′	(36°30′)	146,790 m²
3	(61.4)	54.7	(82.2)	48°20′	41°40′	1679.29
4	(428 m)	(710 m)	829 m	31°5′	58°55′	151,940 m²

123.

1	29.8	19.8	(35.8)	(56°24′)	33°36′	295.02
2	(306 m)	615 m	687 m	26°28′	(63°32′)	94,095 m²
3	17.5	(14.6)	(22.8)	50°12′	39°48′	127.75
4	(284 m)	(170 m)	331 m	59°6′	30°54′	24,140 m²

124.

	$a=c$	b	$A=C$	B	h	h_1	$2p$	S
1	(590 m)	650 m	(56°36′)	66°48′	—	—	—	1601 a
2	(276 m)	485 m	28°30′	(123°)	—	—	—	31,950 m²
3	19.8	(25.6)	(49°45′)	80°30′	—	—	—	193.5
4	487.5	(547.8)	69°39′	40°42′	—	—	—	202,200

125.

1	(87.5)	(139.6)	37°05′	105°50′	—	—	—	3 683
2	85.9	(92.6)	57°24′	65°12′	(72.4)	—	—	3 352
3	(200 m)	197 m	60°27′	59°06′	(174 m)	—	—	17,139 m²
4	703	(820)	54°18′	71°24′	—	(666)	—	234,099

126.

1	68.6	(120.7)	28°22′	123°16′	(32.6)	—	—	(1970)
2	100.2 m	38.2 m	79°01′	21°58′	(98.4 m)	—	—	(1880 m²)
3	156.4	93.7	(72°36′)	34°48′	—	—	(406.5)	6982
4	(16)	8.6	74°29′	31°02′	—	—	—	(66)

127. 45°34′. 128. ≈2200 m. 129. ≈3.5 km. 130. ≈0.35 m. 131. ≈8.7 m². 132. 66°02′. 133. 30°02′. 134. ≈40.6 cm. 136. ≈48 m. 137. ≈5.6 m; ≈24′. 138. ≈20.1 N and 36.9 N. 139. ≈54 N and

73 N. 140. 1. ≈ 260 N. 2. $\approx 23.8 \frac{\text{m}}{\text{sec}}$. 141. $\approx 10°10'$ and 8.4 $\frac{\text{km}}{\text{hr}}$. 142. The mistake is 1°10'. 143. ≈ 274 cm. 144. ≈ 3.87 km. 145. 65°9', 114°51', 8.5 m. 146. 69°27', 41°06'. 147. ≈ 13 cm. 148. ≈ 5.5 cm, 2.3 cm. 149. 100°28'. 150. ≈ 10.6 cm. 151. (1) $\frac{4r^2}{\sin \alpha}$. (2) $\frac{4r^2}{\sin \alpha}$. 152. 1. ≈ 64.6 cm. 2. ≈ 11.3 cm. 153. 1. ≈ 576.8 cm. 2. ≈ 13 cm. 154. $d\left[\sqrt{2}\cos(45°-\alpha)+\cot\frac{\alpha}{2}\right] \approx 59.3$ cm. 155. 70°32', 38°56'. 156. 56°19' and 33°41'. 157. ≈ 219 m². 158. $2 \operatorname{arc\,cot} \frac{\sqrt{4a^2-b^2}}{b+2a}$. 159. $\frac{2a\sqrt{2}}{\cos(45°-\alpha)}$, $\frac{a^2}{2\cos^2(45°-\alpha)}$. 160. 56°19', 33°41'. 161. $\frac{P\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{4}}$. 162. $R\left(\alpha+2\tan\frac{\alpha}{2}\right)$. 163. ≈ 20.6 cm. 164. 54°19'. 165. ≈ 5.3 cm. 166. ≈ 56.4 dm and 98 dm. 167. 44°36', 135°24'. 168. ≈ 110 cm and 78 cm. 169. ≈ 64.9 cm and 65.6 cm. 170. ≈ 9.3 cm and 13.6 cm. 172. ≈ 1334 m. 173. ≈ 458 km. 174. ≈ 11.2 N. 175. 128°14' and 140°01'. 176. 93°37'. 177. 1. 10 m, 8 m and 6 m. 90°, 53°8' and 36°52'. 2. 126 dm, 50 dm, 104 dm, 104°15', 22°37', 53°8'. 178. ≈ 1.225. 179. ≈ 17.3 cm. 180. 56°2'. 181. ≈ 78.6 cm, 51.3 cm, 39.9 cm. 182. ≈ 18.3 cm. 183. ≈ 208.3 m. 184. ≈ 90 m. 185. ≈ 10.7 m. 186. ≈ 51.3 N or 25.7 N. 187. (1) 583.2 m²; (2) 958.1 dm², (3) 31,250 m². 188. 62°44' or 117°16'. 189. (1) 865.2 dm²; (2) 26.4 m²; (3) 4165 m². 190. 53°12' and 126°48'. 191. (1) 1831 m²; (2) 280.2 cm². 192. (1) 323.9 cm²; (2) $S = 0.08106$ (unit)². 193. (1) 32.15 dm²; (2) 719.5 cm². 194. (1) 899.3 cm²; (2) $S = 0.18$ cm². 195. $2R^2 \sin\alpha$. 196. $2R^2 \sin\alpha \times \sin\beta \sin(\alpha+\beta)$.

199.

	a	b	c	A	B	C	S
1	(44)	(58)	(62)	42°51'	63°43'	73°26'	1223
2	(29)	(44)	(59)	28°12'	45°47'	106°1'	613.2
3	(272.4)	(1035)	(1305)	1°44'	6°34'	171°42'	2034

200.

1	(420)	(371)	440	61°39'	51°2'	(67°19')	71,900
2	(22.9)	14.7	(16.9)	92°36'	(39°52')	47°31'	124.1
3	79	(38)	(52)	122°34'	23°51'	33°36'	832.8

201.

1	(730)	1068	831	43°1′	(86°3′)	(50°56′)	302,700
2	5.9	(13.2)	9.0	21°48′	(123°42′)	34°30′	22.03
3	437 m	429.5 m	(97.5 m)	(87°55′)	79°12′	(12°53′)	20,920 m^2

202.

1	(28.9)	(22.4)	21.9	(81°22′)	50°1′	48°37′	242.9
2	—	(354)	(520)	—	(43°55′)	—	—
3	(402 m)	383 m	(258 m)	74°48′	66°56′	(38°16′)	47,710 m^2
		248 m		105°12′	36°32′		30,870 m^2

203.

1	(0.38)	(0.59)	0.65	35°26′	(64°11′)	80°23′	0.1106
2	38.0	(45.5)	(25.0)	56°41′	90°	(33°19′)	475
3	(1054 m)	1171 m	(1350 m)	(48°46′)	56°44′	74°30′	594,800 m^2
		608 m			25°44′	105°30′	308,800 m^2

204.

1	17.7	15.1	11.1	(83°17′)	(58°16′)	38°27′	83.2
2	32.9	37.5	45.5	(45°28′)	(54°23′)	80°9′	(609.1)
3	178.2	200.3	55.7	59°7′	(105°20′)	(15°33′)	4787

205.

1	211.7	227.3	56.9	67°10′	(98°30′)	(14°20′)	5,958
2	269.3	219.8	245.8	(70°24′)	(50°16′)	(59°20′)	25,450
3	67.1	33.1	49.0	(108°)	(28°)	44°	772.7

206.

1	15.7	31.6	39.6	(22°)	(49°)	109°	234.7
2	(32.5 m)	116.5 m	99.0 m	14°41′	(114°50′)	50°29′	(1460)
3	(72)	(52)	48	92°22′	46°11′	41°27′	1240

207.

1	14.6	4.0	13.1	105°14′	15°13′	59°33′	(25)
2	(120 m)	(29.6 m)	135.2 m 100.0 m	53°30′ 126°30′	11°26′	115°4′ 42°4′	1608.88 m² 1190 m²
3	33.9	22.6	15.1	127°10′	32°6′	20°44′	135.6

207. (3) *Hint.* First find the ratio of the sides of the triangle and then its angles. 208. (1) 84; (2) 90; (3) 15; (4) $69\frac{1}{3}$; (5) $3\frac{1}{2}$. 209. (1) 14 cm; 16 cm; 18 cm; (2) 36 cm; 40 cm; 68 cm. 210. 576 cm². 211. 16.8 dm². 212. 4896 cm². 213. 19.2 dm². 214. 576 cm². 215. 120 cm. 216. ≈69.8 cm². 217. ≈93 cm². 219. (1) 1 and 2.5; (2) $2\frac{1}{3}$ and $24\frac{1}{6}$; (3) 4 and $8\frac{1}{8}$. 220. Possible. 224. 4 cm and $21\frac{1}{4}$ cm. 225. $h=\frac{m}{3}$. 226. ≈1256 cm². 227. 17 cm and 28 cm. 228. ≈207 cm². 229. 3 cm and 6.25 cm. 230. $(3+\sqrt{3})$ dm². 231. 1440 cm². 233. (1) $200\sqrt{2}$ cm² and ≈294 cm², (2) ≈331 cm² and ≈325 cm². 234. ≈9.4 cm and ≈9.3 cm. 235. 144 cm². 236. $300\sqrt{3}$ cm². 237. 4.04π cm². 238. $\frac{p^2}{4n}\cot\frac{180°}{n}$. 239. ≈39°18′ with respect to the direction of AC. $AB\approx 661$ m. 240. $\angle BCD=57°6'$, ≈258 m. 241. $\angle AC_1C_2=129°18'$; $\approx 50\ \frac{\text{km}}{\text{hr}}$. 242. $\angle BAX=78°51'$. 243. $\angle CAY=58°51'$. 244. $\angle AMC=75°50'$. 245. ≈86.6 N; 90° and 30°. 246. (1) ≈61.9 N; 47°56′. (2) ≈174.7 N and 215.4 N. 247. ≈46.3 cm and 39.4 cm. 248. ≈195.5 dm². 249. $d_k=$
$$=\frac{a\cdot\sin\left[\frac{180°}{n}(k+1)\right]}{\sin\frac{180°}{n}},\ k=1,\ 2,\ \ldots,\ \frac{n-2}{2}\ \text{if } n \text{ is even,}$$
$k=1, 2, \ldots, \frac{n-3}{2}$ if n is odd. 250. $4R(\sin 2\alpha\cos\alpha+\cos 2\alpha)$. 251. $R(2\sqrt{3}+3)$, $R(\sqrt{2}+1)$, R. 252. $\frac{a\sqrt{3}(2\pi-3\alpha)}{6\cos\frac{\alpha}{2}}$. 253. $2R\left(\sin\frac{\alpha}{2}+\sin\frac{\beta}{2}+2\cos\frac{\alpha+\beta}{2}\right)$. 254. 144 cm². 255. Of the rhombus. 257. ≈619.4 cm². 258. $\frac{\pi R^2\sin^2\frac{\alpha}{2}}{4\cos^4\left(45°-\frac{\alpha}{4}\right)}$ 259.

$\frac{1}{2}(a-b)^2 \times \sin\alpha$. 260. $100(3\pi+\sqrt{6})$ cm^2. 261. $\approx$35.2 cm^2. 262. 124 cm^2. 263. 153.6 cm^2. 264. 402.25 dm^2. 265. 2.2 cm and $\approx$10.8 cm. 266. $\approx$15.2 cm and 17.1 cm. 267. 1. 268. (1) 2496 cm^2; (2) 240 cm^2. 269. 14.1 cm^2. 270. 1. No. 271. (d) Generally speaking, none. 274. 1. (b) In various planes passing through the point M. 2. One or four, or six. 3. Cannot. 288. $8\sqrt{3}$ cm^2. 292. One or none. 293. 1. (a) Skew, or intersecting, (b), (c) skew, or intersecting, or parallel. 2. (a) Parallel, or coincides, (b), (c) skew, or intersecting. 3. (a) Skew, or intersecting, (b) either skew, or intersecting, or parallel. 294. $\approx$54°44′. 295. 83°49′. 296. 2. Either perpendicular, or inclined, or parallel, or contained in the plane α. 298. 1. and 2. Single solution. 304. 1. Many solutions. 2. Single solution. 309. $\frac{a\sqrt{6}}{6}$. 310. 2. Single solution. 314. $\approx$4.2 dm and 1.1 dm. 315. $AM = BC$. 316. 10 cm, 7.5 cm, $\approx$11.7 cm. 317. 16 m^2. 318. $\approx$53.4 dm. 319. $3\sqrt{5}$ cm. 320. 13 cm. 321. 3 cm, $\approx$7.9 cm, $\approx$13.1 cm, 15 cm. 322. 24 cm^2. 323. $\frac{1}{3}\sqrt{9b^2-3a^2}$. 324. 10 cm and $\approx$8.5 cm. 325. 6 cm. 326. 12 cm. 327. 4 cm. 328. 4 cm. 329. 17 cm, 25 cm, 17 cm. 330. 16 cm. 331. 5 cm. 332. $\frac{\sqrt{4m^2H^2+Q^2}}{2m}$ and $\frac{\sqrt{4n^2H^2+Q^2}}{2n}$. 333. $\approx$15.8 N. 334. $9\frac{1}{7}$ cm and $6\frac{6}{7}$ cm. 338. 1. 25°6′, 34°27′, 45°. 2. 35°16′. 340. 54°44′. 341. $\approx$54°44′ and 65°54′. 342. $1.5l$. 343. $\approx$3.5 cm. 344. 1. $\frac{a\sqrt{3}}{2}$, $\frac{a\sqrt{2}}{2}$, $\frac{a}{2}$. 2. $2h$, $h\sqrt{2}$, $\frac{2h\sqrt{3}}{3}$. 345. $3\sqrt{3}$ cm, 70°32′. 346. $\approx$2.8 cm. 347. 60° and 120°. 348. $\approx$43.6 cm. 349. 45° and 30°. 350. a, 45°. 351. $\frac{5}{8}a^2$. 352. 30°. 353. (1) $a\sqrt{6}$. (2) 120°. 354. 13 cm. 355. 1. (a) Either parallel, or skew; (b) either parallel, or skew, or intersecting; (c) either skew, or intersecting. 2. (c) Either intersecting, or skew, or parallel. 356. 1. (a) Parallel to the straight line, or passes through it; (b) either intersects the straight line, or is parallel to it; (c) either parallel to the straight line, or intersects it, or passes through it. 2. (a) Either intersect, or are skew; (b) parallel. 361. Parallelogram, rhombus, trapezium with the bases AB and CD. 362. 2. The problem has no solution if b is parallel to a. 364. 2. No solution if the given straight line is parallel to the given plane and the distances between the point and plane, and between the straight line and plane are different. 365. 1. Many solutions if the given points lie on a straight line parallel to the given one. 2. No solution if the given lines intersect. 3. One solution, or many solutions. 369. 37 cm. 370. 22.5 cm. 371. 68 cm. 372. $\approx$91.6 cm. 374. 20 cm. 375. 20 cm. 376. 28 cm or $\approx$41.0 cm. 377. 30°. 378. 0.5 dm^2, $\approx$0.7 dm. 379. Many solutions. 381. 189 cm^2. 382. $\frac{1}{4}a^2\sqrt{7}$. 383. $\frac{7}{24}a^2\sqrt{17}$. 384. $\approx$173.2 cm^2. 385. $\frac{5}{16}a^2\sqrt{2}$.

394. 15 cm and 15 cm. 395. 12 cm. 396. 368.64 cm^2. 397. 85 cm. 398. 32 cm. 399. 40 cm. 400. 65 cm. 401. 52 cm. 402. 180 dm^2. 404. $\frac{a}{2}(2\sqrt{5}+\sqrt{17})$. 405. 252 dm^2. 413. 2. Two planes. 3. Four straight lines perpendicular to the plane containing the given lines. 418. 1. Two planes. 2. Two planes. 419. Let α be the given angle, and β the angle between the given straight line and the given plane. Then, if $\alpha < \beta$—no solution, if $\alpha = \beta$—one solution, if $\beta < \alpha < 90°$—two solutions, if $\alpha = 90°$—one solution. 420. 40° and 140°. 421. $\frac{a}{2}$, $\frac{a\sqrt{2}}{2}$, $\frac{a\sqrt{3}}{2}$, a. 423. $\approx$6.3 m. 424. $\approx$10.4 cm. 425. 60°. 426. 60°. 427. 1. $\approx$54°44′. 2. arc tan $\left(\frac{1}{2}\tan\alpha\right)$. 3. $\approx$70°32′. 428. $\approx$54°44′. 429. $\approx$65°22′. 430. $\approx$111°6′. 431. $\approx$124°14′, 41°24′. 432. $\approx$66°2′. 433. $4\sqrt{2}$ cm. 434. 45 cm. 435. $4\sqrt{2}$ cm and 8 cm. 436. 9 cm. 437. $16\sqrt{3}$ cm and 50 cm. 438. $\approx$18.4 cm or 26.4 cm. 439. 60°, $75\sqrt{3}$ cm^2. 440. $\approx$104 dm^2. 441. $\approx$255 cm^2. 442. $108\sqrt{3}$ cm^2. 443. $\pi\sqrt{2}$ cm^2. 444. $\frac{\pi(d_2^2-d_1^2)\sqrt{2}}{4}$. 445. $a^2\sqrt{2}$. 446. Two times, 60°. 447. $40\sqrt{3}$ cm^2. 448. 200 cm^2. 449. 10 dm^2. 450. 20 $(\text{unit})^2$. 451. 144 cm^2. 452. 75 cm^2 and $\approx$140 cm^2. 453. 10 cm. 455. 1. (b), (c) possible; (a), (d), (e), (f) impossible. 2. (a), (c) possible; (b), (d) impossible. 456. (a) $13°30' < x < 158°$; (b) $26°30' < x < 162°$. 457. (3) The plane angles of the trihedral angles at the vertices of the base: one—60°, the two others are equal to each other, each being more than 30°, but less than 90°. The plane angles of the trihedral angle at the vertex are equal to one another. Each of them is more than 0°, but less than 120°. 458. 60°. 459. At the vertex 60° each, at the base 90°, 60°, 60°. 460. At the vertex 48°12′ each, at the base 90°, 65°54′, 65°54′. 461. 60°. 462. 60° each. 463. 75°31′. 464. (1) $\approx$35°16′; (2) 90°. 465. $a^2\frac{\sqrt{3}}{2}$. 466. $\approx$62.4 cm^2. 467. $\frac{1}{3}a$. 468. 65 cm. 469. 2. $n = 3$.18, 9.6, $n = 4$, 24, 12, 8, $n = 6$, 36, 18, 12. 3. 1440°; 2160°, 720° $(n-1)$. 470. 1. 0, 4, 10, $n(n-3)$. 2. 0, 1, $n-3$. 3. 2, 5, $\frac{n(n-3)}{2}$. 471. 2. $n-2$. 480. 1.70°32′. 2. $\approx$33°12′, $\approx$50°12′, $\approx$62°. 481. $\frac{d\sqrt{3}}{2}$. 482. $\approx$14.1 cm. 483. (a) 50 cm, (b) 29 cm. 484. $a\sqrt{7}$; $2a\sqrt{2}$; 49°6′, 45°. 485. (1) $2a$, $a\sqrt{5}$, (2) $\frac{3\sqrt{7}a^2}{4}$, (3) $a\sqrt{\frac{3}{7}}$, $2a\sqrt{\frac{3}{7}}$. 487. 4 cm, $4\sqrt{2}$ cm, $4\sqrt{3}$ cm. 488. 6 cm, 8 cm, 24 cm. 489. $\sqrt{m^2+n^2+mn}$ and $\sqrt{m^2+n^2+3mn}$. 490. (2) $\frac{1}{2}a\sqrt{13}$. 491. 120°. 492. 20 cm and $12\sqrt{2}$ cm. 493.

$\approx$22 cm and $\approx$28 cm. 494. a, $\frac{2a^2\sqrt{3}}{3}$, $\frac{a^2}{2}\sqrt{\frac{13}{3}}$, $\frac{a^2}{2}\sqrt{\frac{13}{3}}$. 495. 13 cm and $\approx$29.6 cm. 496. 1. (a) Yes, (b) yes, (c) yes. 2. (a) Yes, (b) no, (c) no. 3. Square, rectangle, parallelogram, rhombus, trapezium. 497. $15\sqrt{2}$ cm, $\frac{25\sqrt{3}}{2}$ cm^2. 498. $\frac{a}{8}$ $(2\sqrt{17}+5\sqrt{5}+\sqrt{65})$. 499. $p=4(\sqrt{2}+2\sqrt{10})$ cm, $S=\frac{56\sqrt{11}}{3}$ cm. 500. $3a\sqrt{2}$, $\frac{3}{4}a^2\sqrt{3}$. 501. $3a\sqrt{2}$, $\frac{a^2\sqrt{3}}{2}$. 502. $2a\sqrt{5}$, $\frac{1}{2}a^2\sqrt{6}$. 503. $Q\sqrt{2}$. 504. $\frac{a^2}{2}$. 505. (a) 32 cm^2, (b) $\approx$31.2 cm^2, (c) $\approx$ 45.5 cm^2. 506. 250 cm^2; 53°8′. 507. a^2. 508. 600 cm^2 and $\approx$588 cm^2. 509. (1) 50 cm^2. (2) 5 cm, $\approx$ 141.4 cm^2. 510. 200 cm^2. 511. 1. 410 cm^2. 2. 12 cm and 16 cm. 512. 48 cm^2 and 84 cm^2. 513. ab, $ab\sqrt{3}$, $\frac{a}{2}\sqrt{3a^2+4b^2}$. 514. a^2 and $a^2\sqrt{2}$. 515. $\frac{9a^2}{2}$ and $3a^2$. 516. 1. (a) 720°, (b) 1080°, (c) 1440°, (d) 3240°, (e) 360° $(n-1)$. 2. 11. 517. (a) Yes, (b) yes, (c) no, (d) yes, (e) no, (f) yes. 518. (a) Yes, (b) no, (c) yes, (d) no, (e) both yes and no (f) both yes and no, (g) yes. 527. 20 cm. 528. (1) 48 cm, (2) 67°23′; (3) 82°30′; 71°34′; 82°30′. 529. (1) $\approx$45.4 cm and $\approx$45.4 cm, (2) 41°26′, (3) $\approx$678.8 cm^2; 45°. 530. 73°54′; 81°47′. 531. 6 cm and $\approx$8.9 cm. 532. (1) 48 cm^2; $\approx$44.4 cm^2 and $\approx$44.4 cm^2. 2. (a) 53°8′ and 57°, (b) 130°24′. 533. 90°. 534. 162 cm^2 and 270 cm^2. 535. Q. 536. 360 cm^2 and 240 cm^2. 537. 400 cm^2 and 100 cm^2. 538. $1\frac{19}{21}$ cm^2 or $5\frac{5}{8}$ cm^2. 539. (1) $\sqrt{2}-1$, (2) 1, (3) 1:2, (4) $(\sqrt{n}-\sqrt{m}):\sqrt{m}$. 540. 36 cm. 541. $\frac{a-m\sqrt{2}}{m\sqrt{2}}$. 542. $1\frac{5}{7}$ dm^2. 543. 720 cm, 36°52′. 544. 96: 25. 545. $\sqrt{a^2+c^2-b^2}$. 547. 2 dm, $\sqrt{7}$ dm. 548. $4\sqrt{3}$ cm, $20\sqrt{3}$ cm, $\approx$21.9 cm. 549. $9\sqrt{3}$ cm, $6\sqrt{3}$ cm, $\sqrt{117}$ cm. 550. $\sqrt{3}$ cm, $\sqrt{15}$ cm, $3\sqrt{3}$ cm. 551. 10 cm, 10 cm, 10 cm. 552. $\alpha=\beta=\gamma=$ 36°52′, 90°, 51°20′, 43°9′. 553. $\frac{a-b}{\sqrt{6}}$, $\sqrt{\frac{2}{3}}(a-b)$, $\frac{(a-b)\sqrt{15}}{6}$. 555. $(a-b)\frac{\sqrt{3}}{2}$, $a-b$, 45°, $\frac{\sqrt{2}}{2}(a-b)$. 556. $15\sqrt{2}$ cm, 30 cm, $10\sqrt{5}$ cm, $5\sqrt{19}$ cm, $10\sqrt{7}$ cm. 557. 8 cm. 558. 22 cm, $\approx$36.2 cm, $\approx$31.7 cm. 559. $\approx$198 cm^2. 560. $\approx$28.7 cm^2. 561. 81 cm, 17 cm, $2744\sqrt{2}$ cm^2. 562. $\approx$21.8 dm, $\approx$110.2 dm^2. 563. $\approx$195.5 dm^2. 564. $\approx$183.7 m^2. 565. 180 cm^2 and $\approx$174.3 cm^2. 566. (1) 17 cm, $\approx$21.5 cm, $\approx$26.2 cm, (2) 28°4′, 21°48′, 17°45′, (3) 180 cm^2, 240 cm^2, 510 cm^2, $\approx$484 cm^2, (4) 90°, 90°, 28°4′, 21°48′, (5) $\approx$26.2 cm, $\approx$50.6 cm, $\approx$43.5 cm, $\approx$36.9 cm, (6) $\approx$17°45′,

$\approx 9°5'$, $\approx 10°36'$, $\approx 12°31'$, (7) 300 cm^2 and ≈ 540.9 cm^2, (8) 90° and $\approx 33°42'$. 567. 9 cm. 568. $\frac{(\sqrt{Q_1}+\sqrt{Q_2})^2}{4}$ and $\frac{(\sqrt{Q_1}+2\sqrt{Q_2})^2}{9}$. 569. 5, 9.6. No. 571. $\frac{a\sqrt{6}}{3}$, $\frac{a\sqrt{3}}{2}$, 54°44'. 573. ≈ 22.1 dm^2. 574. 1. $\frac{a}{3}$. 2. $225\sqrt{3}$ cm^2. 575. (1) a^2, (2) $a\sqrt{2}$, (3) 109°28', (4) $\frac{a\sqrt{2}}{3}$, (5) $\approx 0.8a$. 576. 1. $\frac{2a}{5}(\sqrt{3}+1)$. 2. $\frac{4a}{5}(\sqrt{3}+1)$. 577. $\frac{a\sqrt{2}}{2}$. 578. $\frac{a^2\sqrt{2}}{2}$. 579. Two regular quadrangular pyramids and 24 equal triangular pyramids. 580. 2:1. 581. 1. (a) three, (b) four (c) three. 2. (a) 6, (b) 9, (c) 9. 582. $4\frac{a^2}{3}$. 583. $a\sqrt{6}(\sqrt{2}-1)$. 584. $a(\sqrt{2}-1)$ and $2a(\sqrt{2}-1)$. 586. 36.7 cm, 8 cm. 587. 1. 10 m. 2. $\frac{1}{4}\pi S$. 588. 8 dm. 589. 40 cm. 590. ≈ 26.9 cm. 592. ≈ 5.3 cm. 593. 1920 cm^2. 594. $5\sqrt{3}$ cm, 15 cm. 595. 6 dm. 596. ≈ 7.1 dm^2, 0.3 m^2. 597. 1:2. 598. $\frac{l\sqrt{5}}{2}$. 599. 400 dm^2. 600. 60° 601. $20\sqrt{3}$ cm^2, 53°8'. 602. $a^2\sqrt{2}$. 603. ≈ 4.4 dm^2. 604. $160\sqrt{3}$ cm^2 or $640\sqrt{3}$ cm^2. 605. $\frac{d\sqrt{2}}{2}$, $\frac{d\sqrt{6}}{4}$. 606. 640 cm^2, 64π cm^2. 607. ≈ 35.4 cm. 609. $7.5\sqrt{2}$ cm, 12.5 cm. 614. $5\sqrt{2}$ cm, 25π cm^2. 615. 1. 30°. 616. 972 cm^2, 73°44'. 617. (1) 45°, 90°, (2) no. 618. $2\sqrt{10}$ cm, $4\sqrt{10}$ cm, $2\sqrt{30}$ cm. 619. (1) $\frac{9}{16}\pi l^2$, (2) $\frac{\pi l^2}{4}$. 620. $\frac{h^2\sqrt{3}}{3}$. 621. 48 dm^2. 622. 6 cm. 623. $\frac{2R^2}{3}$. 624. 216 m^2. 625. π. 626. 31°4'. 627. (1) 64π cm^2, (2) 16π cm^2, (3) $\frac{\pi n^2R^2}{(m+n)^2}$ or $\frac{\pi m^2R^2}{(m+n)^2}$. 628. 8 cm. 629. 10 cm. 630. $\frac{75\pi\sqrt{3}}{8}$ cm^2. 631. ≈ 43.1 cm. 632. 70°32'. 633. $a\frac{(2+\sqrt{6})}{\sqrt{3}}$. 634. $\frac{a^2\sqrt{3}(7+4\sqrt{3})}{12}$. 635. $\frac{a^2(11+4\sqrt{6})}{2\sqrt{3}}$. 640. 20 cm, 15 cm. 641. $R=l$, $r=\frac{l}{2}$, $H=\frac{l\sqrt{3}}{2}$. 643. $R=\sqrt{\frac{Q}{\pi}}(\sqrt{2}+1)$, $r=\sqrt{\frac{Q}{\pi}}(\sqrt{2}-1)$. 644. $R=24\sqrt{2}$ cm, $r=7\sqrt{2}$ cm, $H=31\sqrt{2}$ cm. 645. 1020 cm^2. 646. 192 dm^2. 647. $39\sqrt{3}$ dm^2 and $21\sqrt{3}$ dm^2. 648. 1. $\frac{\pi}{4}(R+r)^2$. 2. 25π cm^2 or 36π cm^2. 649. $\frac{1}{4}\pi H^2$. 650. Into two equal parts.

651. 168 cm^2. 652. 72 $\sqrt{2}$ cm^2, 65°54′. 653. 60 cm^2. 654. 30 ($\sqrt{21}+$ $+3$) cm^2. 655. 128π cm^2. 656. 648 dm^2. 657. 11.2 cm. 658. (3) $3Q\sqrt{2}$. 659. Sufficient. 256 cm^2, 384 cm^2. 660. (3) 414 dm^2. 661. 23:17. 662. 2378 cm^2. 663. 192 dm^2. 664. 2 dm, 6 dm, 10 dm. 665. 1. 3. 2. $\approx$25.1 m. 666. $\approx$23 tons. 667. $\approx$30.4 per cent. 668. 6 cm. 669. $3a^2\sqrt{2}$. 670. 16 cm. 671. $4a^2\sqrt{2}$. 672. 1. 150 cm^2. 2. $3Q$. 673. 1. 16 dm^2 and 8 $\sqrt{3}$ dm^2. 2. $15a^2\sqrt{3}$. 674. 3 dm^2. 675. 1560 $\sqrt{3}$ cm^2. 676. $\frac{64\sqrt{3}R^2}{3}$. 677. $a^2(2\sqrt{2}+\sqrt{3})$. 678. $\frac{4Q}{\sqrt{3}}$. 679. 200 cm^2. 680. 84 dm^2. 681. $\approx$ 1655 cm^2. 682. $\approx$ 283.1 dm^2. 683. 2 cm. 684. 270 cm^2. 685. 4.626 m^2. 686. 1. 72 cm^2. 2. 2 dm, 4 dm, 6 dm, 10 dm. 687. $ab(\sqrt{3}+1)$. 688. 240 cm^2. 689. $\approx$422.3 dm^2. 690. $\frac{a^2}{4}(1+\sqrt{3})$. 691. $\frac{a^2\sqrt{3}}{3}\left(\sqrt{13}+\frac{7}{2}\right)$. 692. $a^2(4+\sqrt{3})$. 693. $\approx$931 cm^2. 694. $2a^2\sqrt{5}$. 695. $\approx$294 cm^2. 699. (a) $\frac{a}{2}$, (b) $\frac{a\sqrt{3}}{2}$, (c) $\frac{3a}{2}$. 700. $3a^2$. 701. 1. $a^2\sqrt{3}$. 2. $\approx$86.6 cm^2. 702. 25 $\sqrt{2}$ dm^2. 703. 192 $\sqrt{3}$ cm^2. 704. 1. $\frac{3a^2\sqrt{3}}{4}$. 2. 75°31′. 705. 62. 706. $\approx$41.57 dm^2. 707. 648 cm^2. 709. $\approx$624.5 cm^2. 710. $\frac{2\sqrt{5}}{3}$. 711 (a) $a^2\left(1+\frac{\sqrt{7}}{4}\right)$, (b) $a^2(1+\sqrt{2})$, (c) $a^2\left(3+\frac{\sqrt{7}}{2}\right)$. 712. 100 $(\sqrt{3}+2)$ dm^2. 713. $\frac{m^2}{8}(\sqrt{15}+\sqrt{39})$. 714. $\frac{3a^2\sqrt{3}}{2}$. 715. $\frac{a^2\sqrt{3}}{2}(1+\sqrt{2})$. 716. 2 m. 717. 450 cm^2. 718. $\approx$579.5 dm^2. 719. $\frac{2H^3}{3}\sqrt{3}(2+\sqrt{2})$. 720. 1008 cm^2. 721. $a^2(\sqrt{5}+1)$. 722. $2a^2$. 723. $\frac{a}{6}$. 724. $\frac{a^2}{2}(2+\sqrt{3})$. 725. 90° and 30°. 727. 1260 cm^2. 728. (a) $\frac{a+b}{2}\sqrt{3(a^2+ab+b^2)}$, (b) $(a+b)\sqrt{2(a^2+b^2)}$, (c) $3(a+b)\sqrt{a^2-ab+b^2}$. 729. 13:15. 730. $\approx$2148 cm^2. 731. $\approx$3984 cm^2. 732. $\approx$1019.6 cm^2. 733. 45 $\sqrt{15}$ cm^2. 734. 50 cm^2. 735. 11:34 or 26:19. 736. $\sqrt{a^2+\frac{1}{2}Q}$. 737. 392 cm^2. 738. $\approx$95.4 dm^2. 739. $\approx$1072 cm^2. 740. $(8+2.5\sqrt{3})$ cm and $(8-2.5\sqrt{3})$ cm. 741. $162\sqrt{7}$ dm^2. 742. $\frac{1}{2}\sqrt{\frac{(2a^2-b^2)b}{2a+b}}$. 743. 70 dm^2. 744. a^2-b^2. 745. 39.6 dm^2. 746. 1197 cm^2. 747. 400 cm^2. 748. $\approx$0.628 kg. 749. $\approx$6635 cm^2. 750. $\approx$13.5 kg. 751. 2.6576π m^2. 752. $\approx$31 m^2. 753. 1. πl^2. 2. Four times. 754. 1. πQ.

2. $\frac{\pi l^2 \sqrt{3}}{4}$. 756. $H = 8R$. 757. $\frac{H}{R} = 4 \pm 2\sqrt{3}$. 758. $\frac{3R}{\pi}$. 759. (a) $\frac{\pi\sqrt{2}}{4}$, (b) $\frac{\pi}{4}$. 760. (a) 1, (b) $\frac{b}{a}$. 761. $\frac{\pi RH}{6}$. 762. $\frac{2\pi Q\sqrt{3}}{3}$. 763. $\frac{d^2\sqrt{3}\,(\sqrt{3}+2\pi)}{8\pi}$. 764. $\frac{200\,(5\pi+3\sqrt{3})}{3}$. 765. 6.4 m. 766. 1380π cm². 767. $16\pi\sqrt{2}\times(2\sqrt{2}+3)$ cm². 768. $24\pi(1+2\sqrt{3})$ cm². 769. 200π cm² and 800π cm². 770. $\frac{Q}{6}, \frac{Q}{3}, \frac{Q}{2}$. 771. $\frac{Q^2}{2\pi H^2}$. 772. $\frac{(\pi-2)\,Q^2}{8\pi^2 H^2}$ or $\frac{(3\pi+2)\,Q^2}{8\pi^2 H^2}$. 773. 1. $\pi R\sqrt{R^2+H^2}$ 2. $\pi\sqrt{l^2-H^2}\times$ $\times(l+\sqrt{l^2-H^2})$. 774. ≈ 502.6 cm². 775. 2. Will be increased $1\frac{1}{2}$ times. 777. 2. $\sqrt{\frac{3Q}{2\pi}}$. 778. 45°; $9\pi\sqrt{2}$ cm². 779. 1. 120°. 2. 1. 780. 60°. 781. ≈ 443 m². 782. 21 roubles 30 kopecks. 783. $\frac{\pi P^2}{4}(\sqrt{2}-1)$. 784. 1. $\approx 313.8\pi$ cm². 2. 1344π. 785. $\frac{1}{2}\pi a^2\sqrt{3}\times$ $\times(5+\sqrt{3})$. 786. $2\pi a^2\sqrt{3}$. 787. $\frac{8}{3}\pi a^2\sqrt{5}$. 788. $\frac{2}{3}\pi Q(\sqrt{3}+\sqrt{6})$. 789. 1. 20 cm. 2. $\frac{360°\,R}{l}$. 790. ≈ 201 cm², ≈ 45.3 cm², $\approx 38°56'$. 791. 1. ≈ 318.1 cm². 2. $1.25Q$. 792. 1. 216°. 2. (a) $\approx 254°36'$. (b) 180°. 793. 1. 60°. 2. 24 cm². 794. $\frac{9}{4}\pi a^2\sqrt{2}$. 795. $\frac{1}{6}(7+4\sqrt{3})$. 796. (a) $\frac{2}{3}\pi a^2$, (b) πa^2, (c) $2\pi a^2$. 797. $\frac{1}{2}(\sqrt{5}-1)$. 799. $0.5\pi RH$. 800. 1. 816π cm². 2. $132\pi\sqrt{2}$ cm². 801. $11\frac{3}{7}$ cm and $4\frac{4}{7}$ cm. 802. ≈ 4 m². 803. ≈ 87.2 kg. 804. $\frac{13}{8}\pi l^2$. 805. $\frac{3}{2}\pi l^2$. 806. $\frac{11}{4}\pi R^2$ cm². 807. 16 cm. 808. $R \approx 19.4$ cm. 809. 260π cm². 810. 9 cm. 811. ≈ 678.6 cm². 813. ≈ 769.5 cm². 814. $\pi a^2(6+\sqrt{3})$. 815. $6\pi a^2\sqrt{3}$. 816. $4\pi a^2\sqrt{3}$. 817. ≈ 1603 cm². 818. ≈ 565.8 cm². 820. 54.39 kg. 821. (3) Increased by ≈ 2.8 times. 822. 1. 27 cm³. 2. 64 (unit)³. 823. 6. 824. 1. 12 cm. 2. 1 dm³, $2370\frac{10}{27}$ cm³ and $4629\frac{17}{27}$ cm³. 825. 2. 288 cm³. 826. ≈ 3.5 dm. 827. 1. 140 cm³. 2. $\sqrt{S_1S_2S_3}$. 3. 3 cm, 5 cm, 10 cm. 828. 12. 829. 1. 128 tons. 2. 18 kg. 830. ≈ 6772 N. 831. 40 hrs. 832. ≈ 29.8 m². 833. ≈ 14.4 per cent. 834. 1. 216 dm³. 2. 1 m³. 835. Three cases: 1392 cm²; 1344 cm²; 1104 cm². 837. $48(3-\sqrt{3})$ cm³. 838. 5760 cm³. 839. ≈ 399 cm³. 840. ≈ 55.43 m³. 841. $a^3\sqrt{3}$. 842. $4a^3\sqrt{3}$. 843. 4.5 dm³. 844. $\frac{Q^2}{2a}$. 845. $768\sqrt{3}$ cm³.

846. $\frac{Q^2\sqrt{3}}{d}$. 847. Reduced by 240 dm^3. 848. 1680 cm^3. 849. $\frac{3}{4}a^3$. 850. $\frac{S_1S_2}{2l}$. 852. $\approx$431 N. 854. 1625 m^3. 855. $\frac{1}{8}a^3$. 856. $\approx$1.9 cm. 857. $\frac{1}{2}Q\sqrt[4]{6a^2}$. 858. $\frac{1}{8}l^3\sqrt{2}$. 859. $\frac{1}{2}d\sqrt[6]{54}$. 860. $\approx$1.9 m. 861. $81\sqrt[4]{72}$ dm^3. 862. $\approx$90.7 kg. 863. 89,880 m^3. 864. 94.6 cm^3. 865. 20. 866. 6090 cm^3. 867. 480 cm^3. 868. (a) $\approx$3700 cm^3, (b) $\approx$1873 cm^3. 869. $4(3+\sqrt{3})$dm^3. 870. 2 dm, 2 dm, 2 dm, 4 dm. 871. 2520 cm^2. 872. $\frac{5}{11}$. 873. 1440 cm^3. 874. 11.52 dm^3. 875. $4Q$ cm^2 and $2Qr$ cm^3. 876. $\approx$137.6 dm^2 and 96 dm^3. 878. 600 dm^3. 879. 864 cm^3. 880. $\frac{a^3\sqrt{2}}{4}$. 881. $\frac{9a^3}{4}$. 882. 222 dm^3. 883. $\frac{1}{2}Qd$. 884. $\approx$ 311.8 cm^3. 885. 16 m^3. 886. $\approx$ 207.8 dm^3. 887. (1) $\frac{1}{12}a^2\sqrt{3b^2-a^2}$, $\frac{1}{6}a^2\sqrt{4b^2-2a^2}$, $\frac{1}{2}a^2\sqrt{3(b^2-a^2)}$; (2) $\frac{1}{4}(b^2-h^2)h\sqrt{3}$, $\frac{2}{3}(b^2-h^2)h$, $\frac{1}{2}(b^2-h^2)h\sqrt{3}$; (3) $\frac{1}{24}a^2\sqrt{12l^2-a^2}$, $\frac{1}{6}a^2\sqrt{4l^2-a^2}$, $\frac{1}{4}a^2\sqrt{3(4l^2-3a^2)}$. 888. 1. (a) $\frac{1}{24}a^3\sqrt{3}$, (b) $\frac{1}{6}a^3\sqrt{3}$, (c) $\frac{3}{4}a^3\sqrt{3}$. 2. (a) $\frac{1}{36}a^3\sqrt{3}$, (b) $\frac{1}{18}a^3\times\sqrt{6}$, (c) $\frac{1}{2}a^3$. 891. $\approx$ 0.75 dm^3. 892. 12, 6, 4. 893. $\frac{1}{3}a^3$. 894. (1) $\frac{1}{12}a^3\sqrt{3}$; (2) $\frac{1}{6}a^3\sqrt{2}$. 895. $\frac{1}{6}a^3$. 896. $\frac{4}{9}S\sqrt{6S}$. 897. 72 dm^3. 898. 1. $\approx$100.35 dm^3. 2. 12.29 dm^3. 899. $Q\sqrt{\frac{3Q\sqrt{3}}{2}}$. 900. $Q\sqrt{\frac{Q\sqrt{3}}{2}}$. 901. 6.5 dm. 902. $\frac{a^3b}{12\sqrt{3a^2-4b^2}}$. 903. $\approx$ 126.7 kN. 904. 74,060. 905. 2753.8 N. 907. 1. $\frac{a^3\sqrt{3}}{18}$. 2. 560 cm^3. 908. 1. $2560\sqrt{3}$ cm^3. 2. 864 cm^2. 909. 24. 910. 1. $\frac{1}{24}c^3$. 2. 216 cm^3. 911. 14.4 dm^3. 912. $\frac{1}{3}$ m^3. 913. $\approx$ 366.5 dm^2, 250 dm^3. 914. 5:9. 915. $Q\sqrt{\frac{Q}{2\sqrt{3}}}$. 916. $\approx$9.51. 917. $\approx$2.1 per cent. 920. 512 cm^3. 921. $V\approx$ 1671 cm^3. 922. $\approx$103 cm^3. 923. 19:7:1. 924. $(\sqrt[3]{3}-\sqrt[3]{2}):(\sqrt[3]{2}-1):1$. 925. $\frac{a^3\sqrt{3}}{16}$ and $\frac{5a^3\sqrt{3}}{48}$. 926. 8:19. 927. (a) 20.2 dm^3, (b) 46.7 dm^3, (c) 121.2 dm^3. 928. (a) $\approx$87.8 dm^3, (b) 351.0 dm^3, (c) 1579.6 dm^3.

929. (a) $\approx$ 37.3 cm^3, (b) $\approx$ 105.6 cm^3, (c) $\approx$ 388.0 cm^3. 930. (a) 17.9 m^3. (b) $\approx$ 35.8 m^3. (c) 0; $H_{pyr.} \neq 0$. 931. (a) 78 $\sqrt{3}$ cm^3, (b) 52 $\sqrt{30}$ cm^3, (c) 468 cm^3. 932. $\frac{117\sqrt{3}}{4}$ cm^3. 933. $\frac{21R^3}{32}$. 934. $\frac{37}{24}\sqrt{107}$ dm^3. 935. 2688 dm^3. 936. 13,584 cm^3. 937. 7 cm and 5 cm. 938. $\frac{832\sqrt{3}}{3}$ cm^3. 939. $180(5+\sqrt{6})$ m^3. 940. $\approx$ 188.8 dm^3. 941. 3 m^3. 942. 216 dm^3. 943. $\frac{hS_1\sqrt{S_1}}{3(\sqrt{S_1}-\sqrt{S_2})}$. 944. 12,537 cm^3. 945. 265.6 dm^2. 946. 45 m^2 and 20 m^2. 947. 840 m^2. 948. 2. 949. $\approx$ 24.9 m^3. 950. $\frac{104r^3}{9}$. 951. 1080 $\sqrt{3}$ cm^3 and 3600 $\sqrt{3}$ cm^3. 952. $\approx$ 1333 cm^3. 953. $\frac{1}{3}h(5ab-a^2-b^2)$. 954. $\approx$368 per cent. 955. 4:9. 956. $\frac{7m^2+4mn+n^2}{7n^2+4mn+m^2}$. 960. 1. 1 m^3. 2. 2.5 dm^3. 961. $\approx$ 4.6 cm. 962. $\approx$15.7 kg. 963. 28. 964. $\approx$50.3 tons. 965. $\approx$7163 m. 966. $\approx$28.6 tons. 967. $\approx$ 44 m^3. 968. $\approx$ 49 mm. 969. $\approx$ 40.5 per cent. 970. 500π cm^3. 971. 252 cm^3. 972. $\frac{3\pi l^3}{32}$. 973. $\frac{\pi S\sqrt{S}}{4}$. 974. 1. 96π dm^2. 2. $\sqrt[3]{16\pi V^2}$. 975. 1. $\approx$ 21.5 per cent. 2. $\approx$ 36.3 per cent. 976. (a) 4, (b) 2, (c) $1\frac{1}{3}$. 977. (1) $\frac{4V}{\pi}$, (2) $\frac{3\sqrt{3}V}{4\pi}$. 978. $Q\sqrt{\frac{Q\sqrt{3}}{2\pi}}$. 979. $54\pi\sqrt{3}$. 980. 1. 54π. 2. 16π and 24π. 981. 1. $\frac{d^3\sqrt{2}}{16\pi}$. 2. $\frac{2\pi^2R^3\sqrt{3}}{3}$. 982. $\frac{1000}{\pi}$ or $\frac{500}{\pi}$. 983. $\frac{8\pi+3\sqrt{3}}{4\pi-3\sqrt{3}}$. 984. 648π. 985. 15,134. 987. 2. Increase $\sqrt{2}$ times. 988. 2. Increased three times. 3. Reduce by half. 991. $\approx$ 251.3 kg. 992. 32. 993. 5. 994. $\frac{C^2\sqrt{4\pi^2l^2-C^2}}{24\pi^2}$. 995. 1. 16π dm^3. 2. 12π dm^3. 996. 320π cm^3 or 600π cm^3. 997. (1) $H=\frac{3R}{\sqrt{R^2-9}}$, $R>3$, (2) $H=\frac{6R^2}{R^2-9}$, $R>3$, (3) $H=3$, R—any. 998. 8 cm 999. $\approx$0.26 V. 1000. 1:26 or 8:19. 1001. $\frac{13}{24}$. 1002. $\frac{2\pi R^3\sqrt{2}}{81}$. 1003. 240°. 1004. (1) $\frac{\pi a^3}{4}$, (2) $\frac{4800\pi}{13}$ dm^3, (3) $\frac{320\pi}{3}$ cm^3, (4) 1440π cm^3. 1007. 3525π cm^3. 1008. 3064π cm^3. 1009. (1) $\frac{9600\pi}{29}$ cm^3, (2) $\frac{\pi a^3}{2}$ dm^3.

1010. 4. 1011. 144π cm^3 and 192π cm^3. 1012. $\frac{\pi(26-15\sqrt{3})}{72}$. 1014. $\frac{12}{\pi}$. 1015. 2400π cm^3, 960π cm^2. 1016. ≈ 28.6 dm^3. 1017. ≈ 63.3 l. 1018. ≈ 14.3 l. 1019. ≈ 1583 cm^3. 1020. 24 cm, 25 cm. 1021. 22.95π. 1022. $\frac{7}{24}\pi\sqrt{3}l^3$. 1023. ≈ 804.3 cm^2. 1024. $\frac{5\pi l^3\sqrt{3}}{48}$. 1025. $\frac{37\pi l^3}{72}$. 1026. ≈ 2325 cm^3. 1027. 19 : 37 : 61. 1028. $\frac{(2+3\sqrt{2})\pi R^3}{3}$. 1029. $\frac{7\pi a^3\sqrt{2}}{6}$. 1030. $R = 1.5\,r$. 1031. $R=\frac{r(\sqrt{5}+1)}{2}$. 1032. $\frac{2}{3}\pi RrH$. 1033. $\frac{7\pi a^3\sqrt{6}}{27}$. 1034. 1. $1-\left(\frac{r}{R}\right)^3$. 2. $\frac{7R^2+4Rr+r^2}{R^2+4Rr+7r^2}$. 1035. $\frac{1}{3}\pi^2R^2r$. 1036. $H = 7$ cm, $R = 3.5$ cm. 1037. $\frac{\pi a^3\sqrt{3}}{4}$. 1038. $6\pi a^2$, $\frac{3\sqrt{3}\pi a^3}{4}$. 1039. $4\pi a^2\sqrt{2}$. 1040. ≈ 1217.5 dm^3. 1041. $6\pi a^2\sqrt{3}$, $4.5\pi a^3$. 1042. $3\pi a^3\sqrt{3}$. 1049. 9 cm. 1050. 1. 3585 km. 2. $\approx$ 15,920 km. 1051. 5 : 9. 1052. $2\frac{1}{7}$. 1053. ≈ 15.7 cm. 1054. 2.44 radians, ≈ 101.5 cm. 1055. $\frac{3\pi R^2}{4}$. 1056. $\frac{\pi R^2}{2}$. 1057. 30°. 1058. 12 dm. 1059. 12 cm. 1060. 33.8 cm. 1061. ≈ 75.4 cm. 1062. 1. $\pi R\sqrt{3}$. 2. ≈ 75.4 cm. 1065. $\frac{R\sqrt{3}}{2}$. 1066. ≈ 330 m^2. 1067. 2. 2, $\sqrt{5}$, 3. 1072. ≈ 14.23 dm. 1073. 576π cm^2. 1074. ≈ 867.8 dm^2. 1076. ≈ 183.2 cm^2. 1077. ≈ 3456 cm^2 and 1257 cm^2. 1078. 2500π cm^2. 1079. 0.25. 1080. $\frac{1}{26}$. 1081. 1. $\frac{\pi b^2}{4}(5-2\sqrt{2})$. 2. $\frac{21\pi Q}{4\pi-3\sqrt{3}}$. 1082. $400(4-\sqrt{2})\pi$. 1083. 15 cm. 1086. 12 cm^2. 1088. $R(\sqrt{3}-1)$. 1091. 2. $\sqrt[3]{36\pi V^2}$. 1092. (2) Will be reduced to $\frac{1}{64}$, $\frac{1}{27}$. 1093. 1. 3. 6 times, 6. 7 times. 2. 484 times, 10,648 times. 1094. 1,362,385 thousand km^3. 1096. Yes. 1097. ≈ 0.3 cm. 1098. (2) $\frac{32}{3}\pi$, $\frac{256}{3}\pi$, 288π. 1099. (1) 3. (2) 4.5. 1100. $\sqrt[3]{1.5}\approx 1.14$. 1101. 12 cm. 1102. 72 per cent. 1103. ≈ 33.3 per cent, ≈ 55.5 per cent, ≈ 47.6 per cent. 1104. ≈ 4.6 cm. 1105. ≈ 297.2 cm^3. 1106. 2.9 cm. 1107. $\frac{512(16-9\sqrt{3})\pi}{3}$ cm^3. 1108. ≈ 2827 cm^3. 1109. $\frac{37{,}532\pi}{3}$ cm^3 or $\frac{11{,}968\pi}{3}$ cm^3. 1110. $\frac{416\pi}{3}$ cm^3.

1111. 44. 1113. $\frac{\sqrt{5}-1}{2}$. 1114. 18,432π cm^3. 1115. $\frac{5}{16}$. 1116. $\frac{\pi R^3}{3}$. 1117. $\frac{8000\pi}{3}$ cm^3. 1118. $\approx$1206 cm^2. 1121. $\frac{S^2(6\pi R^2-S)}{24\pi^2 R^3}$. 1122. 0.2. 1123. $\sqrt{21}-4$. 1124. 2. If a circle can be circumscribed about base. 1125. 2. If the base is a rhombus and the distances between opposite faces are equal to each other. 1126. 1. If a circle can be circumscribed about the base of the pyramid. 1127. 1. Possible. 2. If it is an equilateral one. 1128. 1. $\frac{a}{2}$ and $\frac{a\sqrt{3}}{2}$. 2. $\frac{2R\sqrt{3}}{3}$ and $2R$. 1129. $2\pi R^2(3\sqrt{2}-4)$. 1130. 2304 dm^2. 1131. $8R^3\sqrt{2}$. 1132. $\approx$7794 dm^3. 1133. (1) $18R^2\sqrt{3}$, $6R^3\sqrt{3}$, (2) $24R^2$, $8R^3$, (3) $12R^2\sqrt{3}$, $4R^3\sqrt{3}$. 1134. $\frac{h^3\sqrt{3}}{2}$. 1135. $\approx$1014 cm^2. 1136. (a) $\frac{1}{5}$, (b) $\frac{1}{3}$, (c) $\frac{3}{7}$. 1137. (1) $\frac{a(\sqrt{13}-1)}{12}$, $\frac{2}{3}a$, (2) $\frac{a(\sqrt{5}-1)}{4}$, $\frac{3}{4}a$, (3) $\frac{a(\sqrt{21}-3)}{4}$, a. 1138. (1) $\frac{h\sqrt{b^2-h^2}}{\sqrt{3h^2+b^2}+\sqrt{b^2-h^2}}$, $\frac{b^2}{2h}$. (2) $\frac{h\sqrt{b^2-h^2}}{\sqrt{b^2+h^2}+\sqrt{b^2-h^2}}$, $\frac{b^2}{2h}$. (3) $\frac{h\sqrt{3(b^2-h^2)}}{\sqrt{3b^2+h^2}+\sqrt{3(b^2-h^2)}}$, $\frac{b^2}{2h}$. 1139. $\frac{a\sqrt{6}}{6}$, $\frac{a\sqrt{2}}{2}$. 1141. 16.2 cm. 1142. $\pi a^2(2-\sqrt{3})$. 1143. $\frac{a\sqrt{6}}{4}$. 1144. 1 m. 1145. $\approx$5236 cm^3. 1147. $\approx$ 606.1 dm^3. 1148. $\frac{\pi a^2}{3}$. 1149. $4\pi b^2$. 1150. πa^2. 1151. $\frac{104R^2}{3}$. 1152. 10 m. 1153. (1) $\frac{4\sqrt{2}}{3}$, $\frac{4}{3}$. (2) $\frac{2}{3}$, $\frac{2}{3}$. 1154. $\frac{3\pi R^3}{4}$. 1155. $\frac{\pi R^2(3+2\sqrt{3})}{2}$; $\frac{3\pi R^3}{4}$. 1156. 2.25 and $\frac{9}{32}$. 1157. $\frac{\pi l^2}{3}$, $\frac{\pi l^3\sqrt{3}}{54}$, $\frac{4\pi l^2}{3}$, $\frac{4\pi l^3\sqrt{3}}{27}$. 1158. $\approx$254.5 m^2. $\approx$254.5 m^3. 1159. $\frac{\sqrt{l^2-h^2}\,(l-\sqrt{l^2-h^2})}{h}$, $\frac{l^2}{2h}$. 1160. 4. 1162. 24 cm. 1163. $\frac{5V}{72}$ and $\frac{3V}{8}$. 1164. 900π cm^2, 4500π cm^3. 1165. 15 m. 1166. 1064π cm^2, 4256 π cm^3. 1167. $\frac{4S}{3}$. 1169. 18 dm. 1170. $h=R$—a hemisphere. 1171. $\frac{(9-5\sqrt{3})(2+\sqrt{2})}{8}$. 1172. $\frac{H\sqrt{\sin(\alpha+\beta)\sin(\beta-\alpha)}}{\sin\alpha\sin\beta}$, $H\cot\beta$. 1173. $\frac{a^2}{\cos\alpha}$. 1174. arc-cos $\left(\tan\frac{\alpha}{2}\right)$. 1175. 65°25′. 1177. $\frac{2}{3}\sqrt{3}\,d\cos\alpha$. 1178.

$4\sqrt{\dfrac{2Q\sin\left(30^\circ+\dfrac{\alpha}{2}\right)\sin\left(30^\circ-\dfrac{\alpha}{2}\right)}{\sin\alpha}}$. **1179.** $69^\circ18'$. **1180.** $54^\circ44'$. **1181.** $2a^2\sin\alpha$. **1182.** $\dfrac{2H\sqrt{\sin(30^\circ+\alpha)\sin(\alpha-30^\circ)}}{\sqrt{3}}$. **1183.** $\arccos\left(\tan\dfrac{\alpha}{2}\cot\dfrac{180^\circ}{n}\right)$, $42^\circ46'$. **1184.** $\arctan\left(\dfrac{\tan\alpha}{\cos\dfrac{180^\circ}{n}}\right)$. **1185.** $\arctan\left(\tan\alpha\cos\dfrac{180^\circ}{n}\right)$, 60°. **1186.** $70^\circ32'$. **1187.** $2\arctan\left(\dfrac{1}{\sin\alpha}\right)$. **1188.** $\arccos\dfrac{m-n}{p}$. **1189.** $2\arcsin\dfrac{\sqrt{2}\sin\dfrac{\alpha}{2}}{2}$, $35^\circ6'$. **1190.** $2h\cot\alpha\sin\dfrac{\beta}{2}$. **1191.** $\arccos\dfrac{a}{b}$. **1192.** $\arcsin\left(\sqrt{2}\sin\dfrac{\alpha}{2}\right)$. **1193.** $\dfrac{1}{2}\sqrt[3]{\dfrac{6V}{\sqrt{2}\cot\alpha}}$. **1194.** $\dfrac{3}{8}\sqrt{3}\,a^2\sin2\alpha\sin\alpha$. **1195.** $\dfrac{2a^2\sqrt{3}}{27\cos\alpha}\approx 16.2\ \text{dm}^2$. **1196.** $\dfrac{3a^2\cot\alpha}{16}$. **1197.** $\dfrac{b^2\cos\alpha}{2}$. **1198.** $\dfrac{H^2\tan\alpha}{2\cos\alpha}$, $157.2\ \text{cm}^2$. **1199.** $a^2\cos^3\alpha$. **1200.** $\dfrac{a^2\sin\alpha}{4\cos\beta}$. **1201.** $2d^2\sin2\alpha$. **1202.** $\dfrac{b\tan\beta}{2\sin\alpha}$. **1203.** $\dfrac{a\cos\dfrac{\alpha}{2}}{4\cos^2\dfrac{\alpha}{4}}$. **1204.** $\dfrac{(a-b)\sqrt{3}\tan\varphi}{6}$. **1205.** $\dfrac{d\cos\alpha}{2\pi}$ or $\dfrac{d\sin\alpha}{2\pi}$. **1206.** $\approx 40^\circ54'$. **1207.** $\arctan\dfrac{R}{\sqrt{R^2-d^2}}$, $59^\circ2'$. **1208.** $m^2\sin2\alpha\tan\dfrac{\alpha}{2}$. **1209.** $\dfrac{h\sin\dfrac{\alpha}{2}}{\sqrt{\cos\alpha}}$. **1210.** $4d^2\tan^2\dfrac{\alpha}{2}$, $\approx 6030\ \text{cm}^2$. **1211.** $\sqrt{2Q\cos\alpha}$, $\sqrt{Q\cos\alpha}\tan\alpha$. **1212.** $\sqrt{d^2\sin^2\alpha+R^2\cos^2\alpha}$. **1213.** $\dfrac{H^2\sin\beta}{2\cos^2\alpha}$. **1214.** $\dfrac{L^2\sqrt{3\sin(\alpha+60^\circ)\sin(\alpha-60^\circ)}}{2\sin^2\alpha}$. **1215.** $75^\circ31'$. **1216.** $2\cos^2\left(45^\circ-\dfrac{\alpha}{4}\right)$. **1217.** $\sin\beta\sqrt{Q\cot\dfrac{\alpha}{2}}$. **1218.** $\dfrac{R^2\sin\alpha\sqrt{\sin(\alpha+\beta)\sin(\beta-\alpha)}}{\cos^2\alpha\sin^2\beta}$. **1219.** $\arccos\left(\cot\dfrac{\alpha}{2}\tan\dfrac{\beta}{2}\right)$, $54^\circ44'$. **1220.** $2\pi\sin\dfrac{\alpha}{2}$. **1221.** $2\arcsin\dfrac{\alpha}{2\pi}$. **1222.** $\dfrac{d}{\sin\alpha\sin\beta}$. **1223.** $\dfrac{(R^2-r^2)\sin\alpha}{2\sin\beta}$. **1224.** $R\arcsin\dfrac{2Q\cos\beta}{(R^2-r^2)}$, $r\arcsin\dfrac{2Q\cos\beta}{(R^2-r^2)}$.

1225. $\dfrac{(a^2-b^2)\cot\dfrac{\alpha}{2}}{4}$. 1226. $-h^2\cot(\alpha+\beta)$. 1227. $\dfrac{S\sin 2\alpha}{2\pi}$.

1228. $\dfrac{2h}{2+\cot\alpha}$. 1229. $R\tan\dfrac{\alpha}{2}\sin\alpha\tan\alpha$. 1230. $\pm\sqrt{\dfrac{Q}{\pi}\dfrac{\cos\dfrac{3\alpha}{2}}{\cos\dfrac{\alpha}{2}}}$.

1231. $2\sqrt{\dfrac{Q}{\pi}}\sin^2\left(45^\circ-\dfrac{\alpha}{4}\right)$. 1232. $\dfrac{\sqrt{R^2+r^2+2Rr\cos 2\alpha}}{\sin 2\alpha}$.

1233. 116°50′. 1234. $\dfrac{\tan\dfrac{\alpha}{2}}{2\sin\alpha+\tan\dfrac{\alpha}{2}}$ $(\alpha<120^\circ)$; $\sqrt{2}-1$, $\dfrac{1}{2}$, 1, no solution. At $\alpha=120^\circ$ the conical surface is not intersected by the plane. 1235. $\dfrac{a^3\sqrt{\cos 2\alpha}}{\sin\alpha}$, $\dfrac{4a^2\sqrt{\cos 2\alpha}}{\sin\alpha}$. 1236. $ab\sqrt{a^2+b^2}\times$ $\times\tan\alpha$. 1237. $\dfrac{1}{4}d^3\sin 2\alpha\cos\alpha\sin 2\beta$. 1238. $d^3\sin\alpha\sin\beta\times$ $\times\sqrt{\cos(\alpha+\beta)\cos(\alpha-\beta)}$. 1239. $d^2\sin 2\alpha+2\sqrt{2}Q\cos(\alpha-45^\circ)$. $\dfrac{1}{2}Qd\sin 2\alpha$. 1240. $\dfrac{2a^2\sqrt{2\cos\alpha}}{\sin\dfrac{\alpha}{2}}$, $\approx 1154\text{ cm}^2$. 1241. $\dfrac{d^2\left(2+\cos\dfrac{\alpha}{2}\right)}{\sin\dfrac{\alpha}{2}}$, $\dfrac{1}{2}d^3\cot\dfrac{\alpha}{2}$. 1242. $aQ\sin\alpha\sin\beta$, $\approx 1677\text{ cm}^3$. 1243. $\dfrac{1}{4}d^3\times$ $\times\sin 2\alpha\cos\alpha\sin\beta$. 1244. $\dfrac{4d^2\tan\alpha\sqrt{\cos 2\alpha}}{\cos\alpha}$. 1245. $\dfrac{1}{4}d^3\times$ $\times\sin 2\alpha\sin\alpha$. 1246. $\dfrac{1}{2}H^3\tan^2\alpha\sin 2\beta$. 1247. $\dfrac{1}{2}Qd\sin 2\alpha$, 2955 cm^3. 1248. $\dfrac{d^3\sqrt{3}\sin 2\alpha\cos\alpha}{8}$. 1249.

$\dfrac{a^3\sqrt{3\sin\left(30^\circ+\dfrac{\alpha}{2}\right)\sin\left(30^\circ-\dfrac{\alpha}{2}\right)}}{8\sin\dfrac{\alpha}{2}}$. 1250. $\dfrac{3d^3\sqrt{3}\sin 2\alpha\cos\alpha}{16}$, $1.5d^2\sin 2\alpha$, $\approx 2.83\text{ dm}^3$, $\approx 11.95\text{ dm}^2$. 1251. $12d^2\sin\dfrac{\alpha}{2}\times$ $\times\sqrt{\sin\left(30^\circ+\dfrac{\alpha}{2}\right)\sin\left(30^\circ-\dfrac{\alpha}{2}\right)}$. $\approx 1088\text{ dm}^2$. 1252. $2a^3\sin\alpha\sin\dfrac{\alpha}{2}$, $\approx 0.6431\ m^3$. 1253. $Q\sin\alpha\sqrt{\sqrt{3}\,Q\cos\alpha}$.

1254. $12Q\sin\alpha$, $2Q\sin\alpha\sqrt{Q\cos\alpha\sqrt{3}}$. 1255. $1.5d^2\sin\alpha$, $\frac{d^3\sqrt{3}\sin\alpha\sin\frac{\alpha}{2}}{8}$. 1256. $\frac{4r^2h\cos^2\left(45^\circ-\frac{\alpha}{2}\right)}{\sin\alpha}$. 1257. $\frac{1}{8}d^3\times$ $\times\sin 2\alpha\sin 2\beta\cos\beta$, $d^2\sqrt{2}\sin 2\beta\cos\frac{\alpha}{2}\cos\left(45^\circ-\frac{\alpha}{2}\right)$, 9.754 dm³, 65.19 dm². 1258. $2Q\sqrt{2}\cos\frac{\alpha}{2}\cos\left(45^\circ-\frac{\alpha}{2}\right)$. 1259. $p^3\tan^3\left(45^\circ-\frac{\alpha}{4}\right)\tan\frac{\alpha}{2}\tan\beta$. 1260. $\frac{c^3\sin^2 2\alpha\sqrt{2}}{16\cos\frac{\alpha}{2}\cos\left(\frac{\alpha}{2}-45^\circ\right)}$.

1261. $\frac{1}{2}a^3\sin^2\alpha\tan\beta$, ≈ 582.9 dm³. 1262. $2a^3\sin\frac{\alpha}{2}\times$ $\times\sqrt{\sin\frac{3\alpha}{2}\sin\frac{\alpha}{2}}$, 23.5 dm³. 1263. $bQ\sin\alpha$. 1264. $\frac{3b^3\sqrt{3}\sin 2\alpha\cos\alpha}{8}$. 1265. $b^3\sin\frac{\alpha}{2}\sqrt{\sin\frac{3\alpha}{2}\sin\frac{\alpha}{2}}$. 1266. $\frac{a^3\cot\frac{\alpha}{2}\cos\frac{\alpha}{2}}{32\sin^2\left(45^\circ+\frac{\alpha}{4}\right)}$. 1267. $2a^3\sin^2\alpha\cos\frac{\alpha}{2}$. 1268. $\frac{a^3}{2\sin\alpha}$. 1269. $4\sqrt{2}R^2\sin 2\alpha$; $2R^3\sin 2\alpha\cos\alpha$. 1270. $1.5b^2\sin 2\alpha$. 1271. $\frac{b^3\sqrt{3}\sin 2\alpha\cos\alpha}{8}$. 1272. $0.5m^3\sqrt{3}\sin 2\alpha\cos\alpha$. 1273. $\frac{3R^2\sqrt{3}\cos^2\frac{\alpha}{2}}{2\cos\alpha}$. 1274. $2r^3\sqrt{3}\cot\alpha$. 1275. $4\sqrt{2}b^2\sin\frac{\alpha}{2}\cos\times$ $\times\left(45^\circ-\frac{\alpha}{2}\right)$. 1276. $\frac{4m^2\cos\alpha(\cos\alpha+\sqrt{1+\sin^2\alpha})}{1+\sin^2\alpha}$. 1277. $\frac{4H^3\sin^2\frac{\alpha}{2}}{3\cos\alpha}$. 1278. $V=\frac{4l^3\tan\alpha}{3\sqrt{(2+\tan^2\alpha)^3}}$, $S=\frac{4l^2}{(2+\tan^2\alpha)\cos\alpha}$. 1279. $\frac{p^3\tan\alpha}{648}$. 1280. $2m^2n\cos\alpha\cos^2\frac{\alpha}{2}\tan\frac{180^\circ}{n}$, $\frac{1}{6}m^3n\times$ $\times\sin 2\alpha\tan\frac{180^\circ}{n}\cos\alpha$. 1281. $\frac{Q\tan\frac{\varphi}{2}\sqrt{Q\cos\varphi}}{6\sqrt{2}\cos\frac{\varphi}{2}}$. 1282. $\sqrt[3]{3V\sqrt{2}\tan\alpha}$. 1283. $\frac{16R^2\cos^4\frac{\alpha}{2}}{\sin^2\alpha\cos\alpha}$, $\frac{32R^3\cos^6\frac{\alpha}{2}}{3\sin^2\alpha\cos\alpha}$. 1284.

$\dfrac{9p^3\cos^3\dfrac{\alpha}{2}}{4\sin\dfrac{\alpha}{2}\sin\left(60^\circ+\dfrac{\alpha}{2}\right)\sin\left(60^\circ-\dfrac{\alpha}{2}\right)}$. 1285. $\frac{1}{6}l^3\cot\alpha\cot\beta$.
1286. $\frac{1}{6}b^3\sin 2\alpha\sin 2\beta\cos\beta$. 1287. $\frac{1}{6}a^3\cos\alpha\cot\alpha\tan\beta$. 1288.
$\frac{1}{3}R^3\cot\left(45^\circ-\frac{\alpha}{2}\right)\cot\frac{\alpha}{2}\tan\beta$. 1289.
$\dfrac{2l^3\sin^2\alpha\cot\beta\sqrt{\sin(\alpha+\beta)\sin(\beta-\alpha)}}{3\sin\beta}$. 1290. $\frac{1}{3}b^3\sin\frac{\alpha}{2}\times$
$\times\sqrt{\sin\left(\frac{\alpha}{2}+60^\circ\right)\sin\left(60^\circ-\frac{\alpha}{2}\right)}$ 1291. $\dfrac{b^2\cot\dfrac{\beta}{2}\cos^2\dfrac{\alpha}{2}}{2\cos\alpha}$.
1292. $\frac{2}{3}b^3\sin^2\gamma\cos\gamma\sin\alpha\sin\beta\sin(\alpha+\beta)$. 1293. $\frac{1}{6}c^3\sin\frac{\alpha}{2}\times$
$\times\sqrt{\cos\alpha}$. 1294. $a^2\sin\beta\cot\left(45^\circ-\frac{\alpha}{2}\right)$. 1295. $\dfrac{d^2\cot\left(45^\circ-\dfrac{\beta}{2}\right)}{\sin^2\beta\sin\alpha}$.
1296. $\dfrac{a^3\sqrt{\cos\alpha}}{24\sin\dfrac{\alpha}{2}}$, ≈ 25.39 dm^3. 1297. $\dfrac{d^2\tan\dfrac{\alpha}{2}\cos^2\dfrac{\beta}{2}}{\cos\beta}$. 1298.
$\frac{2}{3}H\sqrt{(b^2-H^2)(c^2-H^2)}\sin\alpha$. 1299. $\dfrac{m^3\tan\alpha\sin^2(\alpha-\beta)}{3\sin^2\beta\cos^2\alpha}$.
1300. $\dfrac{2a^2\sin^2(45^\circ+\alpha)\cot\left(45^\circ-\dfrac{\alpha}{2}\right)}{\sin^2\alpha}$. 1301. $\frac{1}{3}H^3\tan\varphi\sin 4\varphi$.
1302. $\frac{1}{12}(a^3-b^3)\tan\alpha$. 1303. $8a^2\cos^2\left(45^\circ+\frac{\alpha}{2}\right)$, ≈ 186 dm^2.
1304. $\dfrac{(a^3-b^3)\sqrt{-\cos 2\varphi}}{6\cos\varphi}$. 1305. $\dfrac{12\sqrt{3}R^2}{\sin^2\alpha}$. 1306. $\frac{1}{6}\times$
$\times Q(2\sqrt{2}-1)\sin\alpha\sqrt{Q\cos\alpha}$. 1307. $\dfrac{c^3\sin 2\alpha\tan\beta}{24}$. 1308. 1.
$\frac{1}{8}\pi d^3\sin 2\alpha\cos\alpha$, ≈ 80.32 cm^3, $\frac{1}{2}\pi d^2\sin 2\alpha$, 99.26 cm^2. 2. $0.25\pi Q\times$
$\times\sqrt{Q\tan\alpha}$, ≈ 1132 dm^3. 1309. $\dfrac{8\pi d^2\sin\left(15^\circ+\dfrac{\alpha}{4}\right)\cos\left(15^\circ-\dfrac{\alpha}{4}\right)}{\cos^2\dfrac{\alpha}{2}}$,
$\dfrac{\pi d^3\sin\dfrac{\alpha}{2}}{\cos^3\dfrac{\alpha}{2}}$. 1310. $\dfrac{\pi h^3}{\sin^2 2\alpha}$, ≈ 1103 dm^3. 1311.

$\dfrac{\pi a^2\sqrt{\sin\left(\frac{\alpha}{2}+\beta\right)\sin\left(\frac{\alpha}{2}-\beta\right)}}{\sin^2\frac{\alpha}{2}\sin\beta}$. 1312. $\dfrac{2R^3\sin\beta\cos^2\frac{\beta}{2}\tan\alpha}{3}$.

1313. $\dfrac{\pi a^3\sin^3\alpha}{16\sqrt{2}\cos^3(\alpha-45^\circ)}$, $\approx 83.83\text{ cm}^3$. 1314. (1) $\frac{1}{6}\pi l^3\sin\alpha\sin\frac{\alpha}{2}$, $2\pi l^2\sin\frac{\alpha}{2}\cos^2\left(45^\circ-\frac{\alpha}{4}\right)$, (2) $\dfrac{\pi Q\sqrt{Q\cot\alpha}}{3}$, $\pi Q\cot\frac{\alpha}{2}$. 1315. $\dfrac{\pi P^2\sin\frac{\alpha}{2}}{8\cos^2\left(45^\circ-\frac{\alpha}{4}\right)}$. 1316. $\dfrac{2\pi Q\cos^2\left(45^\circ-\frac{\alpha}{2}\right)}{\sin\alpha}$, $\approx 3587\text{ cm}^2$.

1317. $2\arcsin\dfrac{S-Q}{Q}$. 1318. $\frac{1}{3}\pi r^3\cot^3\frac{\alpha}{2}\tan\alpha$, $\approx 6525\text{ dm}^3$.

1319. $\dfrac{\pi d^3\cot^2\frac{\alpha}{2}\cos\alpha}{6\sin^2\frac{\alpha}{2}}$, 100.1 cm^3. 1320. $\dfrac{\pi a^3\sqrt{\cos\alpha}}{12\sin\frac{\alpha}{2}}$, $\approx 14.32\text{ dm}^3$.

1321. $\dfrac{Q\sqrt{2\pi Q\sin\frac{\alpha}{2}}}{24\pi\sin^2\frac{\alpha}{2}\cos^3\left(45^\circ-\frac{\alpha}{4}\right)}$. 1322. $\dfrac{2\pi d^3}{3\sin 2\alpha\sin\alpha\cos^2\frac{\beta}{2}}$, $\approx 780\ 70\text{ dm}^3$. 1323. $\dfrac{a^3\alpha^2\sqrt{4\pi^2-\alpha^2}}{192\pi^2\sin^3\frac{\alpha}{2}}$. 1324. $\dfrac{\pi\sqrt{2}\,c^2\sin 2\alpha\cos(45^\circ-\alpha)}{2}$, $\dfrac{\pi c^3\sin^3 2\alpha}{12}$. 1325. $4\pi b^2\sin 2\alpha\cos\left(30^\circ+\frac{\alpha}{2}\right)\cos\left(30^\circ-\frac{\alpha}{2}\right)$, $\frac{1}{3}\pi b^3\sin^2 2\alpha$. 1326. $\dfrac{2\pi m^3\cos^2\frac{\alpha}{2}\left(3\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\cot\beta\right)}{3}$. 1327. $\dfrac{\pi d^3\sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2}}{3\sin^2\frac{\alpha-\beta}{2}}$. 1328. $\dfrac{8R^2\sqrt{2}\sin\left(45^\circ+\frac{\beta}{2}\right)}{\sin\alpha\sin\frac{\beta}{2}}$. 1329. $\dfrac{\pi m^3\sqrt{\cos\alpha}}{12\sin\frac{\alpha}{2}}$.

1330. $\dfrac{2\pi d^2\cot\frac{\alpha}{2}}{9\sin 2\alpha}$. 1331. $\dfrac{r^2\cot\frac{\alpha}{2}\cot\left(45^\circ-\frac{\alpha}{2}\right)}{\cos\beta}$. 1332. $\frac{7}{6}\pi l^3\times\sin 2\alpha\cos\alpha$. 1333. $2\pi l^2\sin\left(\frac{\alpha}{2}+15^\circ\right)\cos\left(\frac{\alpha}{2}-15^\circ\right)$. 1334. $\pi l^2\sin\alpha\tan\alpha$. 1335. $\frac{13}{24}\pi l^3\sin 2\alpha\cos\alpha$. 1336. $0.5\pi l^2\left(1+\right.$

$+\sin^2\alpha\cdot\tan^2\frac{\alpha}{2}\Big)$, $\frac{1}{6}\pi l^3\tan\frac{\alpha}{2}\left(\cos^2\frac{\alpha}{2}-\cos\frac{\alpha}{2}\cos\frac{3\alpha}{2}+\cos^2\frac{3\alpha}{2}\right)$. 1337. $\frac{8}{3}\pi a^3\sin\alpha\cos\left(30^\circ+\frac{\alpha}{2}\right)\cos\left(30^\circ-\frac{\alpha}{2}\right)$ 1338. 1. $\pi b^3\times$ $\times\sin 2\alpha\cos\alpha$, 2. $\frac{2}{3}\pi b^3\sin^2 2\alpha$. 1339. 1. $2\pi a^3\sin\alpha\sin\frac{\alpha}{2}$, $8\pi a^2\sin\frac{\alpha}{2}$, 2. $2\pi a^3\sin\alpha\cos^2\frac{\alpha}{2}$, $8\pi a^2\cos^2\frac{\alpha}{2}$. 1340. $\frac{4}{3}\pi R^3\times$ $\times\sin\left(\frac{\alpha}{2}+\beta\right)\sin\frac{\alpha}{2}$, $4\pi R^2\sin\left(\frac{\alpha}{2}+\beta\right)\sin\frac{\alpha}{2}$. 1341. $8\pi R^2\sin\times$ $\times\left(\frac{\alpha}{2}+\beta\right)\sin\frac{\alpha}{2}\cos^2\frac{\alpha}{4}$, ≈ 445.4 cm². 1342. $\frac{4\pi a^3\sqrt{3}}{27\sin^3 2\alpha}$. 1343. $\pi a^2\tan^2\frac{\alpha}{2}$. 1344. $\frac{\pi a^2}{4\sin^2\frac{\alpha}{2}\cos\alpha}$. 1345. $\pi R^3\sin 2\alpha\cos\alpha$, ≈ 200.3 dm³. 1346. $\frac{1}{3}\pi r^3\cot^3\left(45^\circ-\frac{\alpha}{4}\right)\cot\frac{\alpha}{2}$. 1347. $\frac{2}{3}\pi R^3\times$ $\times\sin^2\alpha\cos^2\frac{\alpha}{2}$. 1348. $\frac{\pi H^2\cos^2\alpha}{\cos^4\frac{\alpha}{2}}$; ≈ 1545 cm². 1349. $\frac{4}{3}Q\times$ $\times\sqrt{\frac{Q}{\pi}\sin^3 2\alpha\tan^3\frac{\alpha}{2}}$. 1350. $Q\sin\alpha\cos\frac{\alpha}{2}\cos^2\left(45^\circ-\frac{\alpha}{4}\right)$, 93.69 cm². 1351. $\frac{4\pi r^2}{\sin^2\alpha}$, $\frac{\pi r^3(7+\cos 2\alpha)}{3\sin^2\alpha}$. 1352. $2\pi R^2\sin\times$ $\times(\alpha-\beta)\cos\frac{\alpha+\beta}{2}$.

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