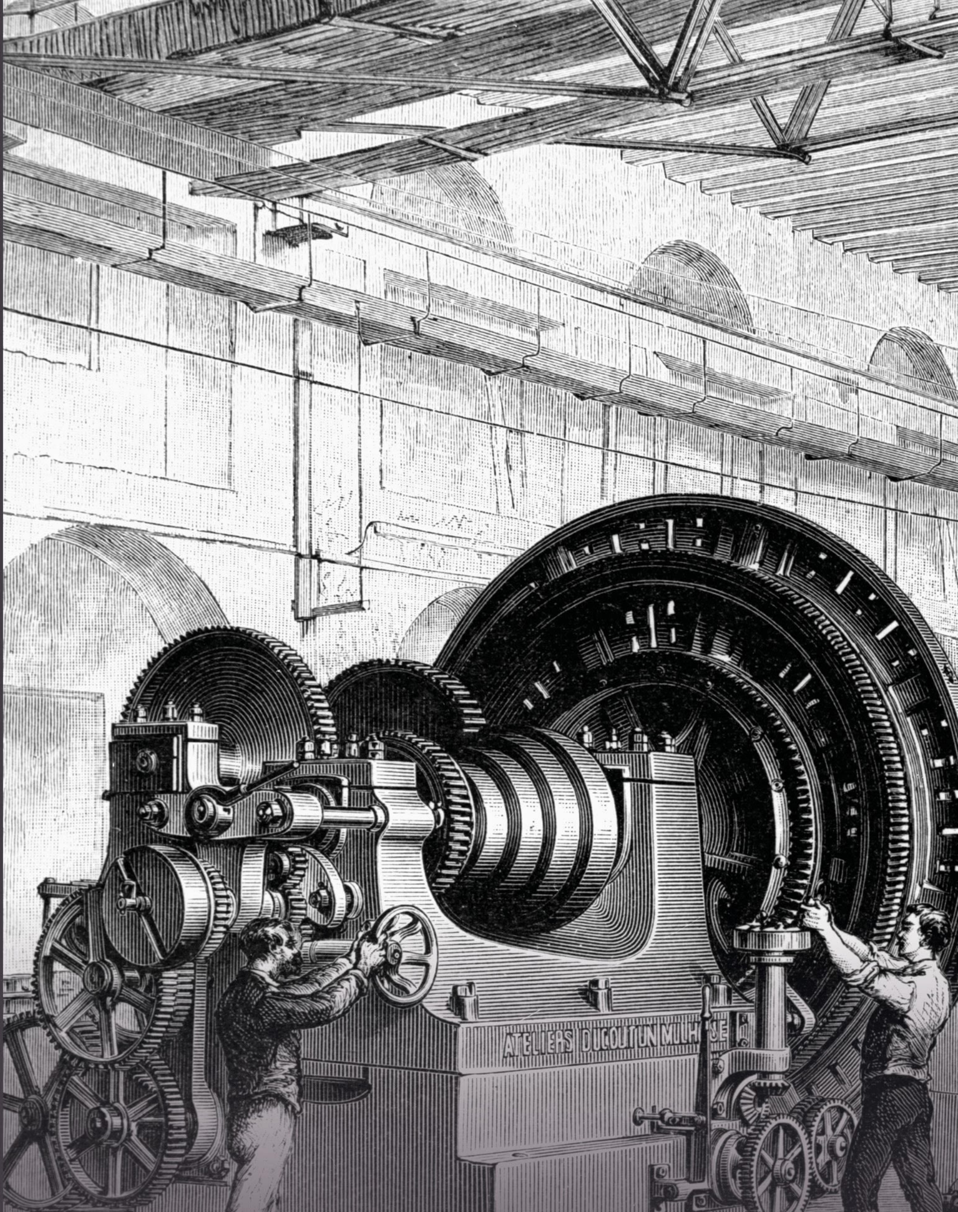

DEFENDING MATERIALISM

The Uneasy History of the Atom in Science and Philosophy

KATARINA KOLOZOVA,
WILLIAM PAUL COCKSHOTT
& GREG MICHAELSON

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INTRODUCTION

Overview

> After we came out of the church, we stood talking for some time together of Bishop Berkeley's ingenious sophistry to prove the nonexistence of matter, and that every thing in the universe is merely ideal. I observed, that though we are satisfied his doctrine is not true, it is impossible to refute it. I never shall forget the alacrity with which Johnson answered, striking his foot with mighty force against a large stone, till he rebounded from it – 'I refute it thus'. James Boswell[1]

Materialism and idealism, the twin poles of all philosophies, have contended for 2,500 years. The complexity of their opposition, both philosophical and social, is neatly encapsulated by the above exchange between James Boswell and Samuel Johnson some 250 years ago.

This argument took place in the late eighteenth century, as the British Industrial Revolution was accelerating, founded on burgeoning mathematical, physical, chemical and engineering principles. In turn, these principles threatened the age-old bases of the religious ideologies that had been the bedrock of the British ruling classes.

George Berkeley (1685–1753) was an Irish Anglican prelate who, at the start of the eighteenth century, sought to use philosophy rather than biblical authority to buttress deism. He is now seen as a founder of absolute idealism, which denies all materiality.

James Boswell (1740–95) was a Scot who came of age during the Enlightenment. Of fluid persuasion, he was raised a Calvinist and toyed with conversion to Catholicism. At the University of Edinburgh, he was taught, in part, by Adam Smith, whose economics greatly influenced both the development of manufacture and Marx. Always well connected, Boswell even discussed the possibility of an afterlife with the dying David Hume, the great sceptical philosopher, who firmly repudiated the notion. With private means, Boswell led a licentious, if self-loathing, life and is best known as Johnson's biographer.

Samuel Johnson (1709–84) was an English man of letters, who is remembered for his authoritative dictionary of the English language. Plain-speaking, Oxford-educated and widely read, he was a Tory and an Anglican. Thus, it is striking that Johnson's common-sense materialism cut through the sophistries that so perplexed Boswell.

Our materialism is equally grounded in interaction with external reality, though certainly not as accommodating of idealism as Johnson's. Indeed, the intention of this book is to defend atomistic and mechanical materialism against idealist accounts.

Our extended presentation is from a historical perspective, from the Greek atomists to the current day, as a matter of philosophical differentiation running across scientific and philosophical disciplines. We are interested in exploring the political-philosophical consequences of such differentiation.

Our critical stance is informed by the materialist doctrine of Karl Marx rather than the entire legacy of Marxism, which is not a monolithic phenomenon. Hence, we chose to extrapolate and follow Marx's original line of thinking, which, we argue, already takes shape in his doctoral dissertation on Greek atomism. Our hermeneutics is, therefore, a type of mimesis of Marx's own approach. While we dismantle the unity of dialectics and materialism, we try to follow their developments as independent yet often united or intersecting strands of thought, and analytically examine the dialectics of the two notions themselves, depending on the authors and disciplines discussed.

The book covers several parallel threads which are traced through time:

- *How the idea of atoms has both changed and remained the same across the centuries.* We see this as the point of delineation between materialism proper and materialism with a touch of idealism, or materialism as covert idealism, both in philosophy and the sciences, which include mathematics and computing;
- *The development of logic and its relationship to 'dialectics' in the work of multiple thinkers.* Here we explore the political repercussions as well as the influence of ideology on the epistemological choices made in sciences along the materialism/idealism divide;
- *The repeated appearance of the principle that nothing comes from nothing.* This originates in ancient Greek philosophy and is reprised and animated in the development of conservation laws, culminating in the understanding that these all derive from the properties of symmetry. This underpinning metaphysical dilemma is insidiously present in the sciences, political philosophy and economics. As one of the pillars of the organization of the European history of ideas, this unintended, often inadvertently present, atavism has moulded much of what we now know as contemporary science, technology and socio-economic theory.
- *The nature of time in physics, biology and reflections on human society, mainly in philosophy but also in the social sciences.* A hermeneutics of the ideas of time running through these disciplines offers a fresh view on the idea of historical materialism, disentangled from the dogmas of the diamat, that is, dialectical materialism as set in the USSR.

Our focus is the explicit idealism criticized by Lenin and Einstein and the implicit Hegelian idealism that influenced Soviet diamat. Starting with the origins of materialism in ancient Greek thinkers, we explore its revival in the physics of Newton and the Newtonian world view. We show how Marx drew on the atomists to provide the basic foundation for his critique of Hegelian idealism. We then look at the rise of historical materialism and the Darwinian transformation of biology. We further explore the wider epistemological effect of Darwinism, which can be traced in Marxism and other philosophical strands and schools of thought in a number of sciences.

The late nineteenth and early twentieth centuries saw intense philosophical controversy over atoms, something which tends to be forgotten now. Idealist epistemologies, embedded in scientific models of interpretation, prevented the acceptance of atomism as anything more than a marginal speculation. The dogma of diamat, and the authority of Hegel at its centre, had an inhibiting effect on the development of sciences, particularly mathematics and computing, in the Soviet Union. The idealist core in the deepest tenets of what purported to be a materialist epistemology affected the pace of technological development in the socialist world of the twentieth century. We explore this as an illustration of the effect of science as covert philosophy, the latter unavoidably yielding a form of idealism, as argued by Marx, on the pathways of technological and broader scientific development. This is a symptom of science suffering from its own metaphysical choices, even when it positions itself decisively as anti-metaphysical. Paradoxically, we note such an effect more in the tradition of Soviet socialist science than in Western twentieth to twenty-first century scientific development.

After the development of atomic energy, nobody doubted that atoms were real, but back in 1900 their reality was widely questioned. We trace the political and theoretical background to this philosophical controversy. We identify the aversion towards 'coarse materiality' (Marx, Cap. V. 1, Ch. 1) nesting in the 'mechanistic' world view of atomism, as opposed to any 'ethereal' principle (ἀρχή) of all existence. We then argue that this aversion is why the admittedly materialist philosophy and sciences kept an open back door, including those which did not even bother to make the metaphysical choice of idealism versus materialism. The metaphor of 'ethereal principle' refers to a plethora of notions dating from Greek antiquity to contemporary mathematics and computer sciences. This can range from an open question to simply maintaining the position that 'all is language and the outside reality does not exist', at least not in a way that should count, as in the post-structuralist mantra, for example.

Approach

Our account of materialism is drawn, as far as possible, from primary sources, using translations where original material is not in English. We have mostly avoided engaging with interpreters of the philosophers and scientists we cite. We have also mostly avoided wider historical accounts, except where we could not locate primary sources or where they offered uncontroversial summaries.

We acknowledge our 'Western' philosophical orientation and our lack of engagement with contemporary debates in the history of science. We wish to bring a fresh perspective to the understanding of materialism, which leads our readers to engage directly with original arguments as we have tried to do. If we are accused of ignorance or of revisionism, then so be it. We are happy, of course, to be corrected and to further debate our positions in the spirit of coming to a shared understanding rather than point-scoring scholasticism.

To maintain the core focus on the development of materialism in philosophy and natural philosophy, we have made a number of intentional omissions. In particular, we do not address the 'big' philosophical questions: where do we come from; why are we here; why are we like this; where are we going?

We do not consider other world views, other than where they directly contributed to or hindered materialism. In particular, overall we say little about belief or religion. Finally, we do not consider the moral, ethical, social or immediate political consequences of the materialism we elaborate. Perhaps the most significant omission is that of the nature of consciousness. Our rejection of idealism necessarily leads us to situate consciousness as a property of material systems, whether or not human.

Note that quotations are verbatim, without comment on, say, gendered language. Italics are as in the original. Dates of birth and death are often purloined from Wikipedia.

Chapters

We next briefly survey the book's contents, chapter by chapter.

Chapter 2. Philologico-philosophical examination of the conceptual material proffered by Greek Antiquity and the trans-millennial exchanges it has foregrounded

- We identify the foundations of the tension between idealism and materialism in Greek philosophy: a tension that has persevered throughout the centuries while retaining the same grounding premises identified in Antiquity. Both Aristotle and the atomists remain trapped in observing the Eleatic Principle, 'nothing comes from nothing', while trying to enable a thought of change. The atomists succeed, most notably in the work of the Epicurean Lucretius Carus, in demonstrating that 'the void' or 'nothingness' is an aspect of matter and that the principle of all existing is material – the indivisible quantity of matter, the atomoi or the uncuttables.

Chapter 3. Classical atomism

- We explore the atomists to the completion of the Newtonian world view by the eighteenth-century French mechanical school. Archimedean mechanical materialism was enormously influential on Newton, who enunciated a methodology of natural philosophy. The French school elaborated on Newton's mechanics by postulating the conservation of energy and the principle of least action.

Chapter 4. Dialectics, materialism, change from Epicurus to Marx via Aristotle

- We reconstruct the appearance of dialectics as an ontological category in European philosophy, as opposed to its origin as a category of method or ontology found in Antiquity, and in particular in Aristotle's writings on what we now call logic. The origins of materialist dialectics, as we find it in the communist doctrine of diamat, can be traced back to Hegel but not further in the past. Marx's use of the term is closer to that of Aristotle's than to the Hegel-inspired diamat. We look at dialectics and materialism as two distinct categories in Marx, but also in what served as Marx's inspiration, namely, atomism and Hegel.

Chapter 5. Historical and mechanical materialism

- We introduce materialism as applied to social and biological change. Historical materialism is treated as the independent co-discovery of Smith and Engels. Marx's *Capital* was influenced by Newtonian ideas of conservation laws and of equal and opposite actions and reactions. Engels credited Marx and Darwin with jointly and independently displaced teleology from history and biology. Pre-Darwinian anthropocentric teleological accounts of biology are contrasted with the mode of causality proposed by Darwin. We consider Darwinism's abolition of the human as distinct from the animal, and how human traits of emotion and behaviour, as much as physiology, have deep mammalian roots. Atomism revived at the start of the nineteenth century. Advances in chemistry by Dalton and others established it as the working basis of chemistry. This was the basis for understanding heat, pressure, etc., on which the steam revolution was based, and led to the development of the key concepts of thermodynamics.

Chapter 6. Idealist reprise and responses

- We cover the key dividing lines in the debate between materialistic and idealistic schools of twentieth-century physics. The revival of atomism coincided with the growth of socialism, and there was, in some countries, a fear that atheistic materialism and hence socialism would be encouraged by the new atomistic account of the world. The prominent atomist Boltzmann experienced strong opposition from the positivist/instrumentalist school of philosophy and natural philosophy headed by Mach. In 1905, Einstein published three papers that together revolutionized our understanding of the natural world and decisively repudiated Machism. These advances though led to quantum mechanics and, via the Heisenberg school, an idealist interpretation of the new physics that drew heavily on Machism.

Chapter 7. Logic and materialism

- The unity of syllogistic logic and dialectics in Aristotle's epistemology heavily influenced European thought and hence education, framed by Catholicism, until the Reformation. Thereafter, following the rejection of both logic and dialectics in favour of induction, empiricism and materialism in the British tradition, logic was rehabilitated and placed on a formal footing. It seemed that the logical foundations of all mathematics were within reach.

Chapter 8. Logic and dialectical materialism

- Hegel's emphasis on the dialectical component of Aristotelian thought strongly framed the attitude of nineteenth- and early twentieth-century Marxists towards logic. Following the publication of Engels' *Dialectics of Nature* in 1925, there was intense struggle in the Soviet Union over the balance in dialectical materialism between formalism, now associated with logic, and dialectics. This led to the anathematizing of a conflated Menshevizing Idealism and the formulaic 'diamat' that defined official communist philosophy until the death of Stalin.

Chapter 9. The crisis in logic and the apotheosis of anti-formalism

- In the early twentieth-century, advances in logic enabling it to formalize itself led to the identification of profound paradoxes at its heart. At the same time, these advances underpinned the development of computers, with profound implications for how reality may be characterized as reflective systems that model each other.

Chapter 10. Language, automata and meaning

- We present computability as a unifying framework for materialism, integrating model theory, language games, automata theory and formal languages. Markov's constructivism is considered as a materialist reformulation of intuitionism, underpinning a finitist interpretation of reality.

Chapter 11. Dialectical and stochastic materialism

- We elaborate on the implication of information theory, which is that information is something objective and physical, quite independent of human consciousness. The relationship between information and randomness is explored, demonstrating that apparent chance is governed by rules. Finally, we explore the notions of quantity, quality and phase changes.

Chapter 2

Exploring the conceptual material presented by Greek Antiquity and the trans-millennial exchanges it has influenced

A close reading of the postulates put forward by Parmenides – the father of ontology and thus of (European)[1] philosophy as we have known it for centuries – discloses the fact that the riddles posed at the birth of Western rational thought have never been resolved. Zeno's paradox is still a paradox – more so to philosophy than to the sciences – and 'nothing comes from nothing' is as much of a physical-metaphysical law nowadays as it was in sixth and fifth century BC of the European civilization. The principle was accepted by the pre-Socratics, and even those who advocated that change and movement were possible, such as the atomists, had to find a way around it instead of challenging it. In the Eleatic system, unchallenged till Aristotle, 'nothing comes from nothing' meant: change was impossible, empty space didn't exist and everything had always existed in solid and unchangeable form. 'Empty space' meant nothingness in the ontological sense, whereas movement implied coming into being in the ontological sense argued in terms of ontology instead of physical philosophy. Movement was taken to be becoming an ousia, an essence or substance, which contradicts the principle that essence is of indestructible everlasting nature – and has, thus, always already existed. The idea of change violates the principle premise, an axiom of the axiom of the Eleatics – nothing is not, nothingness does not exist and becoming would mean *and thus taking up previously 'empty space' (nothingness)*. Nothingness, non-being and empty space (or void) in the absolute sense are equated in Parmenides and continue to be so throughout antiquity, with some exceptional examples of direct confrontation with this logic that we will discuss further in this book. As for the atomist approach, it found a way to bend the Parmenidean rule rather than challenge or refute it.

Thus, the category of space as treated by the natural philosophy and sciences in the centuries to follow was absent from the Parmenidean reasoning, as well as from the reasoning of those who opposed it in Greek Antiquity but never challenged the principle and the equation at issue, namely that 'empty space equals nothingness in the ontological sense' (a paraphrase). The first to challenge this postulate, remaining a rare example, was Aristotle, and he did so on grounds of the quality or nature of the postulation itself – every predicate relates to a certain 'something' (*ousia* as *tò tí* in Greek, a 'something' or a 'what'). Also, the notion of ousia is grounded in multiplicity; according to Aristotle's categorical system, there is not only one *ousia* of Being (*tò ōn*).

> There is no such thing as motion over and above the things. It is always with respect to substance or to quantity or to quality or to place that what changes. (Aristotle, *Phys.* 200b33–201a3)[23]

Aristotle carries out a quasi-Kantian division between 'pure reason' and the *noumena* – the limits and conditions of reasoning are sought for within the very possibility of language – rhetoric, logic, the sciences of analysis and the understanding of syllogism are the result of further precision and formalization of everyday language. (A certain degree of formalization of terminology indistinguishable from its meaning in daily life – such as 'the empty', 'that which is not' etc. – has existed since the pre-Socratics.) That said, Aristotle seems to argue throughout his opus that the subject of (human) knowledge has perfected its conceptual tools serving to explain the outside reality. Nonetheless, the two are radically different things governed by their own immanent laws. That is why an *ousia* is of 'something' (a certain 'what' – *to ti* in Greek) and is always 'spoken of many'[4] – another provocation to Parmenides' Being, *to on* and *ousia* as one – but it is not the same as that something. It is for this reason that we liken Aristotle's contribution to the development of philosophical and scientific thought to that of Kant. Furthermore, we would argue that it is also a proto-scientific breaking away from the authority of metaphysics (ontology) by virtue of employing formal reasoning, and also invoking the authority of empirical proof and experimentation.

Empty space as a precondition of movement

From Parmenides to the late atomists – and, with a significant aberration from the dogma, through Aristotle – as well as throughout antiquity, for there to be movement, there has to be empty space not filled with or occupied by a body of any sort of degree. Be it a physical notion of a '(some)thing' (tò tí) or an ontological concept such as *tò ōn* – that is, the Being or substance/essence/identity of any sort (*ousia*) – to the Ancient thinkers, movement is simply an appearance of a thing or an entity at a place where there was nothing prior to it: 'for the empty is nothing' [in the original: τὸ γὰρ κενεὸν οὐδέν ἐστιν], argues Melissus (DK 30B7.7).[5] Even if the space is 'vacated' (in an absolute way) for only an infinitesimally short moment of time, the possibility of nothingness, and thus empty space, is allowed. This is considered a breach of the Parmenidean law 'nothing comes from nothing'. Such a possibility would contradict the utmost truth, the purest tautology of them all – nothing comes from nothing, or, being is being and cannot be not-being – that seems to be universally accepted in ancient philosophy even when problematized, for example, by Aristotle. Change is movement too, *as it is a matter of becoming, generation and generation*, and thus coming to being of something which is apparently not in its essence. A strictly Parmenidean objection to change, and thus to becoming or generation, would be that if nothing comes from nothing, the being (*tò ōn*) or the *ousia* is already there in its absolute form, and thus becoming is precluded. Everything, therefore, has always existed as it ever was, unchanged, unmoving, solid – no crack of empty space or nothingness is allowed. Also, everything is undying – something cannot be transformed into nothing since nothing cannot become something. Change is but an illusion; the visible world is an illusion, a dream, a hallucination, as Socrates explains at his deathbed (Plato, 66e–67a).[6] This view has persisted as the dominant one well into Neoplatonism, medieval philosophy and, more recently, in the neorationalist fetishizing of rationality, that is, of scientific thought as opposed to 'the manifest image of reality' as per Wilfrid Sellars. (We are referring here to a particular reception of Sellars[7] rather than his own writings that admit continuity between 'folk reasoning' and scientific thought.[8]) The birth of philosophy, of the notion of being and of perfectly rational thought (tautology) coincides with the birth of a world view that declares the immediately experienced reality an illusion. It coincides with the birth of somatophobia too, as the truth cannot and does not lie in the visible world of rotting flesh but in the perfect, invisible world without bodies or matter of any sort (Plato, *Phaedo*, 64c–65c). Let us consider the categories of the Parmenidean arguments that have marked centuries and millennia of idealism and anti-materialism and how they form a perfectly circumscribed and impenetrable position.

Nothing comes from nothing, a principle that is valid to this date: Scientific and philosophical-ontological accounts of it

To this date, from Newton to Hawkins, this principle – 'nothing comes from nothing' – seems undisputed by the sciences. It seems that philosophy has allowed for more possibilities: the more it has become meditational prose, independent of the sciences following the intellectual overhaul of the Enlightenment, the more paradoxical positions and those in direct contradiction with science have been permitted. Thus, 'nothing comes from nothing' is not necessarily a matter of universal consensus for modern philosophy. Nothingness in its own right is also being, it seems,and it is ontologically existent according to some philosophers, in particular with the beginning of modernity; consider existentialism, nihilism and Hegel's notion of non-being.[9]

This absence of heed for the scientific discussions and contempt for the empirical point to the continuation of the originary division between the 'visible' and 'invisible', bodily and incorporeal and physical and ideal defining Greek philosophy, as noted in the introduction to this chapter. Plato argued that the visible world is an illusion and that the empirical record does not amount to a viable account of making sense of being, at least not one seen as an absolute.

Philosophy has always been anthropocentric, from antiquity to this day: in his 'Critique of Hegel's Philosophy in General' (part of the

Economic and Philosophical Manuscripts of 1844), Marx explains that the Absolute Spirit is simply the apex of the anthropocentric pretensions of all philosophy since its inception, amounting to the notion of the universal egoist.[10] Anthropocentrism is embedded in Feuerbach's materialism too, and that is what renders it idealism – in fact, all subjectivity-centred thought – amounting to human-subjectivity-centred thought – comes down to idealism.[11] It is for this reason that, according to Marx, philosophy has always been idealist, and he thus proposes an exit from it by virtue of assuming the third party's perspective, transforming philosophy into science. Therefore, in Marx, science is premised on the abandonment of subjectivity-centred thought and miming the third party's (imagined) perspective.[12] Contemporary philosophy, the kind one calls 'continental philosophy' as opposed to 'analytic philosophy' (supposedly occupying the hegemonic position of philosophy proper), and critical theory in particular, despises empiricism. It displays interest in technology to simply theorize its social effects[13] and curiosity about theoretical mathematics (such as among the aforementioned neorationalists), but quantifiability, proof executed through empirical induction, is subject to tacit contempt. The contempt for the material (proof) is what makes it philosophy to this day, not much unlike the Parmenidean and Platonic somatophobic narratives, even when it chooses to call itself differently (theory, post-philosophy etc.).

Well into the twenty-first century, the old Cartesian and Platonist contempt for the physical is also implied or declared, as one can note in the speculative-realism-inspired neorationalism, accelerationism and posthumanist cyber theory that more often than not amounts to simply transhumanist but also xenofeminism Body, and the physical, is also of the register of movement, change and decay, and it is an illusion to be abolished so that pure Intellect – Hegel's Spirit – would rule in the form of AI and machine-human singularity. Discussions have become more elaborate and more complex since the times of pre-Socratic philosophy, but they seem to be underpinned by the same concerns since the founding premises cannot be removed without the whole edifice falling apart: Being is the central notion, ontology is what philosophy comes down to, formal reason is aligned with ontology and thus produces metaphysics and the study of the material world and its application is of inferior importance, as is the 'visible world'.

Nothing comes from nothing – or being becomes out of being – is a principle that makes sense in scientific explorations, in particular in physics (Newton, Hawking etc.) and in chemistry, and this is possible without any convoluted affirmations of paradoxes and contradictions, unlike the *aporia* to which the Eleatic proposal led. Aristotle divulges the inconsistencies of the Eleatic proposal in terms of his categories and logic and thus demonstrates that the collapse of human reasoning (application of categories) and being itself is a fallacy. By distinguishing *ousia* from quality, quantity, location and time, Aristotle demonstrates that predication is always about a certain something, and as there is a practically endless or unbound multiplicity of possible predications, there is thus a multiplicity of *ousiai*: 'the being, put simply, is always said in plural' [τὸ ὂν τὸ ἁπλῶς λεγόμενον λέγεται πολλαχῶς] (Aristotle, Meta. E.2 1026a33–34). As part of the elaboration of this argument, Aristotle examines possible contradictions, inconsistencies and paradoxes in how the human mind might work, or in the way one speaks and in the only possibly credible modes of speech that make sense – not in Being itself. This is a revolutionary break from the entire post-Parmenidean tradition – one which departs from the logical consistency of speech, that which is within the reach of human control and perfection, not from 'Being itself'. Human language reflects the consistency of reality and the outside world, but the latter seems to be in and of itself foreclosed and accessible only to the degree to which our categorical system and logic are developed. If one runs against the wall of contradiction, it is due to a problem in human reasoning, not a problem of Being, of the outside world, of the real whose essence evades human mind. In *Physics, Aristotle writes:*

> One could not easily put motion and change in another genus — this is plain if we consider where some people put it: they identify motion with difference or inequality or not being; but such things are not necessarily moved, whether they are different or unequal or non-existent. (201b19–201b23)[14]

Furthermore, he argues:

> So some say that the void is the matter of the body (they identify the place, too, with this), and in this they speak incorrectly; for the matter is not separable from the things, but they are inquiring about the void as about something separable. (214a12–214a16)[15]

Except as an abstract category, one cannot speak of matter in general as a distinct ontic entity, just as one cannot speak of the void in such fashion either. Thus, the Eleatic impossibility is refuted by Aristotle on the grounds of 'one [thus] speaks incorrectly'. Aristotle examines the 'organon' (the instrumentarium of rational reasoning and its rules) and its possibilities and its shortcomings of the era in order to make the account of *that which is* (tò ōn), as well as of that which is not, possible. That 'which is not' is not a negation of Being but rather of a particular *ousia* , of a 'some-thing'.

The above quote from Aristotle's *Physics* resonates as staunchly materialistic: matter and 'what is' – the some-thing, or *tò tí* or *ousia* – are inalienable; therefore, bodies are not something that can 'carry' or contain the void or empty space, so that the possibility of movement is foregrounded while avoiding refutation of the Eleatic, as Leucippus and Democritus have attempted to do. Quite to the contrary, Aristotle engages in such refutation. The compromise the early atomists offered was, according to Aristotle, that 'the elements are said to be full and empty, calling them either being or non-being' [στοιχεῖα μὲν τὸ πλῆρες καὶ τὸ κενὸν εἶναί φασι, λέγοντες τὸ μὲν ὂν τὸ δὲ μὴ ὄν] (Aristotle, *Metaphysics*, A.4 985b5-6).[16] This is a fallacy, an error in reasoning and most probably an ontological impossibility. We are using 'most probably' here as Aristotle does not seem to think that the outside is impenetrable just because what we can rely on is the perfection of our cognitive instruments, the *organon*. Non-being is a matter of accident or coincidence or rather contingency (*sumbebekos*) with regard to an *ousia*; it can be a falsity and, finally, a potentiality (of becoming, as an ontic phenomenon).[17] Just as matter is contingent on an *ousia*, Being is therefore a category of correct predication and actuality. Correct use of categories can lead to a correct explanation of the real or of Being. Aristotle's approach is comparable to what one might today call scientific reasoning – the perfection of the *organon* of demonstration leads to accuracy in one's account of the outside world, of the real or Being.

Aristotle remains a philosopher nonetheless – there is an ultimate truth of the ultimate *ousia*, overarching the plurality of *ousiae*, whereby real and truth establish unity insofar as the real transcends itself through truth or pure contemplation – the 'prime mover' of all existing, including the sensible world, is *noesis* (reason/ing). (Aristotle, Meta. 1072a30-32)[18] Also: 'the desired and that which is subject of thought move while remaining unmoved. They are the first [primary] and the same' (Meta. 1072a26-27) In spite of this, Aristotle's notion of the prime mover is the most direct opposition to the Eleatic argument to be elaborated throughout his oeuvre, and primarily in *Metaphysics* and *Physics*. By virtue of that stance, Aristotle can be considered either a materialist or working in favour of a development of a materialist position: movement is the cause of all existent, consequently, all existence is movement.

Aristotle's philosophy, or its materialist variant, stands on the shoulders of the pre-Socratics, in particular the atomists.

Atomists

Parmenides' prohibition on a philosophical account of nothingness, non-being, as already noted, implied the impossibility of offering an account of movement and change. Many of the pre-Socratics have sought to conceptualize an epistemic possibility for giving an account of change and movement while observing Parmenides' 'ban on non-being'. The atomists, Democritus and Leucippus, adhere to the Parmenidean axiom, according to which 'being comes from being', whereas 'non-being comes from non-being', and there is therefore no such thing as non-being, ontologically speaking. Nonetheless, they have found a way around this philosophical *aporia* in order to create an account of the physical world that allows for change and movement. Obviously, what was visible in the sensible world, such as growth and decay, emergence and disappearance and movement through space displacement, urged the atomists to unravel the hidden principles of the phenomena and offer a philosophical explication of it. Their explication was, apparently, materialist – it was formulated in terms of physics, or *physikē*, a philosophy of nature. In the time of the early atomists and throughout the period of Greek Antiquity, the word *philosophy* encompassed sciences or the scientific approach to the explication of the surrounding reality: empirical observations were formalized, and abstractions and laws of nature and Being were extracted from such empirical insights as well as corroborated by them.

Finitude – the indispensable condition for the solidity and fullness of being

Being is indeed indivisible, and Zeno's paradox is reversed: by observing the Eleatic principle, we can only claim that there must be an instance of Being where its division becomes impossible if we are to observe the principle of 'nothing comes from nothing'. Namely, there is a point of a-tomos, an element of nature that is indeed solid, unchangeable and permanent – the *atom*, the 'indivisible' (DK 67A7, DK 68A37). In *On Generation and Corruption* (325b12–325b33), Aristotle argues that the motivation for the atomists' postulation of indivisible bodies was to proffer an answer to the *aporia* brought forth by the Eleatics, namely that of the (im)possibility of movement, change and multiplicity.[19]

Thus, by observing precisely the Eleatic principle, the atomists have reached an instance that allows for the conceiving of multiplicity and movement. That is the point of an indivisible ('uncuttable'), actualized matter – *atomos*. (Aristotle, *Phys*. 206a14–206a18) If space should indeed be considered subject to limitless divisibility, the argument for solidity, stability and fixity of Being would be undermined if not contradicted entirely. Therefore, the Eleatic position is one of contradiction in terms – by seeking to demonstrate that everything has ever existed as it is, unchanging and unchangeable, it rejects the possibility of change and thus movement. Infinity is implied.

All of this is justified by the principle of 'nothing comes from nothing'. However, by endorsing the principle of 'nothing comes from nothing', the atomists have proven that change is not only possible but necessary, as finitude is actually its prerequisite – infinity implies divisibility, and infinite divisibility implies the impossibility of solidity and fullness of being of any sort. The classical Eleatic argument is ontological, as it relies on a formal argument of the being as reified abstraction. The argument of the atomists, however, is that of the natural philosophy: an empirically grounded account of *physis* – infinite division or cutting of matter implies an ever more present and overwhelming presence of the act of introducing negation or annihilation at the heart of *physis*. Introducing a point of *atomos*, or of the uncuttable, of the indivisible, is the identification of an instance and form of material reality that is beyond destruction. Thus, finitude is what vouches for the observance of the principle 'nothing comes from nothing'. Such a position is elaborated by the atomists, but it seems that the Pythagoreans held a similar stance prior to the atomists.

Aristotle offers further corroboration of the atomist stance by arguing:

> Now things are said to exist both potentially and in fulfilment. Further, a thing is infinite either by addition or by division. Now, as we have seen, magnitude is not actually infinite. But by division it is infinite. (There is no difficulty in refuting the theory of indivisible lines.) The alternative then remains that the infinite has a potential existence. But we must not construe potential existence in the way we do when we say that it is possible for this to be a statue—this will be a statue, but something infinite will not be in actuality. Being is spoken of in many ways,

> and we say that the infinite is in the sense in which we say it is day or it is the games, because one thing after another is always coming into existence. (*Phys*. 206a14–206a25)

Movement and change enabled by negative or 'empty matter'

'The elements are either full or empty, or they could be called either being or non-being', explains Aristotle in his account of the early atomists; or, in the original: 'στοιχεῖα μὲν τὸ πλῆρες καὶ τὸ κενὸν εἶναί φασι, λέγοντες τὸ μὲν ὂν τὸ δὲ μὴ ὄν.' (*Meta*. A.4 985b5-6) In this way, 'non-being' participates in Being as its presence is a hollowing out – *kenosis* – of the atoms that constitute matter. Therefore, nothingness participates in Being without being its origin and is rather nested at the heart of Being seen primarily as matter or *physis*. The atomists had to put forward the thesis that Being is multiple as well as the thesis that emptiness or void (the absence of space or of anything for that matter) is an aspect of material reality. This attempt to overcome the frustration of scientific and philosophical examination of the question of movement, change, generation and degeneration, created by the observance of the Eleatic law, resembles Heraclites's solution: the One is in fact multiple, and even though it might go virtually extinguished, it never fully goes out, instead rekindling 'according to certain measures', that is, internal laws of nature and Being, scientific and ontological. (DK22B30) Or, in the original: κόσμον τόνδε, τὸν αὐτὸν ἁπάντων, οὔτε τις θεῶν οὔτε ἀνθρώπων ἐποίησεν, ἀλλ' ἦν ἀεὶ καὶ ἔστιν καὶ ἔσται πῦρ ἀείζωον, ἁπτόμενον μέτρα καὶ ἀποσβεννύμενον μέτρα (DK22B30). Unlike Heraclitus, who assumes a virtual extinction of existence, the atomists speak of an absolute void inside of an atom.

The trouble with conceiving movement in Greek philosophy, and not only among the pre-Socratics, lies in its ontological definition – it is a matter of Being and becoming, of movement from one *ousia* into another. Speaking purely ontologically, such movement seems impossible and does involve the premise of the emergence of Being from non-being. Being has been, is and will always be – becoming is an impossibility. It is rarely linked with space and natural philosophy or exact reasoning concerning the behaviour of matter. With the pre-Socratics, and the atomists more specifically, even when it is, empty space becomes a possibility for non-being, and the fullness and limitless being-there of Being is compromised. Melissus argues that 'the empty', apparently conceived as a spatial category, is nothing: 'as the empty is nothing', or, in the original, τὸ γὰρ κενεὸν οὐδέν ἐστιν (DK 30B7.7). In order for something to become, the previous absence of *ousia* is assumed; in order for an *ousia* to 'displace' itself, one ought to imagine an evacuation of space and re-emergence in an absolutely empty prior space which is understood as non-being. This understanding of movement among the atomists is noted by Aristotle in his *Physics*: 'these people reasoned that the empty is a fissure through which non-being is in the way of a sensible body' (*Physics* IV.6 213a27-9), or, in the Greek original: οἱ δὲ ἄνθρωποι βούλονται κενὸν εἶναι διάστημα ἐν ᾧ μηδέν ἐστι σῶμα ἰσθητόν.

Aristotle was the first to place space, and therefore movement, among the categories that can become a part of a predicate but do not participate in the 'essences' or 'substances', that is, *ousiai*. In this way, Aristotle offers a solution both ontological as well as in terms of natural philosophy, allowing for exact and mathematical explorations of the material phenomenon of movement.

Aristotle's intervention and the centuries of scientific enquiry that followed render Epicurus' task of offering a convincing account of atomism easier or less contrived by the Eleatic dictum. At least, that is how it reads in Lucretius Carus' *De Rerum Natura*:

nec porro augendis rebus spatio foret usus
seminis ad coitum, si e nilo crescere possent.
nam fierent iuuenes subito ex infantibus paruis
e terraque exorta repente arbusta salirent.

(Liber I, 184–7)[20]

Marginal pagination is used as per the standards of classical philology in order to enable easier navigation through different editions and translations.

Or in English translation: 'Would space be needed for the growth of things; Were life an increment of nothing: then; The tiny babe forthwith would walk a man, And from the turf would leap a branching tree — Wonders unheard of; for, by Nature, each; Slowly increases from its lawful seed, And through that increase shall conserve its kind'.[21] The Epicureans did not need the conjecture of 'atoms containing void' in order to proffer an account of movement and change – atoms are 'solid' matter, and growth and decay observe the laws of its preservation:

sunt igitur solida ac sine inani corpora prima.
praeterea quoniam genitis in rebus inanest,
materiem circum solidam constare necessest.
nec res ulla potest uera ratione probari
corpore inane suo celare atque intus habere,
si non, quod cohibet, solidum constare relinquas.
id porro nihil esse potest nisi materiai
concilium, quod inane queat rerum cohibere.

(Titus Lucretius Carus, *De Rerum Natura* , Liber I, 510–15)

Or in English translation: 'Thus primal bodies are solid, without a void.; But since there's void in all begotten things; All solid matter must be round the same; Nor, by true reason canst thou prove aught hides; And holds a void within its body, unless; Thou grant what holds it be a solid. Know; That which can hold a void of things within; Can be naught else than matter in union knit'.[22] For the matter to constantly move and grow within its finite or limited yet unbound existence[23] – 'as there is nothing beyond existence, it is naught and therefore it is not to be found outside the realm of material reality – space is required and nothingness contained within the material universe: Such huge abundance spreads for things around— / Room off to every quarter, without end. Lastly, before our very eyes is seen / Thing to bound thing: air hedges hill from hill, / And mountain walls hedge air; land ends the sea, / And sea in turn all lands / but for the All / Truly is nothing which outside may bound'.[24]

A move away from abstract ideality, be it a conceptual or mathematical ideal, *ousia*, or Plato's *eidoi* (forms), is the indispensable condition for the existence of nature as it is the product of collision between atoms, of 'strife' (cf. Liber II of *De Rerum Natura* by Lucretius Carus) or confrontation, not unlike Heraclitus' *polemos*. In the second book of *De Rerum Natura*, we read that a very small (paulum) swerve from the ideal 'downright movement' of atoms is indispensable (II, 214–20) in order for the collision to take place, which gives birth to new combinations of atoms and thus new forms of nature:

> Be borne along, either by weight their own, Or haply by another's blow without. For, when, in their incessancy so oft They meet and clash, it comes to pass amain; They leap asunder, face to face: not strange— Being most hard, and solid in their weights, And naught opposing motion, from behind. (Lucretius Carus, *On the Nature of Things: Liber II*, 83–88)[25]

It is important to note that *chance* – accident or unpredictable contingency – is the condition of the creative collision. The swerve appears at an uncertain time and location – in the original, 'incerto tempore ferme incertisque locis spatio deflectere paulum' (Lucretius Carus, Liber II, 219–20) – which then causes collision, dispersion and reorganization in the atomic linkages, one structure falling apart and a new one emerging. Movement is constant, as the absoluteness of the void does not allow for the atoms to stand still – there is no location that they could occupy in the void that is empty in an absolute way, disallowing position, as it does not create any opposition, resistance or matter that would constitute such supposed location.

> Nor is there any place, where, when they've come, Bodies can be at standstill in the void, Deprived of force of weight; nor yet may void; Furnish support to any,— nay, it must, True to its bent of nature, still give way. Thus in such manner not at all can things; Be held in union, as if overcome; By craving for a centre. (Lucretius Carus. Liber I, 1078–83)

or in the original:

> nec quisquam locus est quo corpora com venerunt
> ponderis amissa vi possint stare in inani,
> nec quod inane autem est, ulli subsistere debet,
> quin, sua quod natura petit, concedere pergat.
> haud igitur possunt tali ratione teneri
> res in concilium medii cuppedine victae. (1077–1082)[26]

The void is in fact nothingness, an absolute naught, and it therefore exists only conceptually, not ontologically – it is rather that against the annihilating power of the void life and thus existence that could be found only in a physical form are born and perpetually reborn. Space and location do not have a physical representation, they are unbound and pure extension: 'Space has no bound nor measure, and extends; Unmetered forth in all directions round. (Titus Lucretius Carus, *On the Nature of Things, Liber II*, 91–2) Or in the original: 'quoniam spatium sine fine modoquest, inmensumque patere in cunctas undique partis'. The fact that space has no measure points to its understanding as an aspect of the void and of that enabling nothingness that does not engender anything – 'nothing comes from nothing' is the atomist principle. Yet it is a category that enables change and movement to be grasped. The potentiality of annihilation is intrinsic to matter itself, and an aspect of the behaviour of the atoms and that very negativity is what sustains life. However, it does not constitute an ontological reality in and of itself – nothing is what is not, and it exists only in the fissures of collisions of atoms, amidst the otherwise solid matter the atoms are. Lucretius Carus uses the following metaphors to explain the role of space (with regard to void) in the occurrence of material (physical) processes:

> Amid a void in the very light of the rays, And battling on, as in eternal strife, And in battalions contending without halt, In meetings, partings, harried up and down. From this thou mayest conjecture of what sort; The ceaseless tossing of primordial seeds; Amid the mightier void – at least so far. (Lucretius Carus, Liber II, 118–23)

Or in the original:

> multa minuta modis multis per inane videbis
> corpora misceri radiorum lumine in ipso,
> et velut aeterno certamine proelia pugnas
> edere turmatim certantia nec dare pausam,
> conciliis et discidiis exercita crebris,
> conicere ut possis ex hoc, primordia rerum
> quale sit in magno iactari semper inani—
> dumtaxat rerum magnarum parva potest res
> exemplare dare et vestigia notitiai. (Liber II 116–24)

Nothingness is an ontological concept but not an ontic reality. Yet it occasions the curve, or the swerve or *clinamen*, and thus it is not only an abstraction. It is real insofar as it is an effect of processes of decay and destruction that occur due to the atoms' collisions. However, causally, the swerve precedes the collisions and is a property of the void itself. Yet again, the void does not exist spatially, nor does it have temporal fixed point. One might conjure that it is the property of atoms' movement and does not constitute temporal causality whereas the latter is not linear. The void is there because atoms must collide, and they do so as there is no spatial void with a centre that can ensure a static position. In that sense, the void is absent from matter and the constitution of atoms and yet again makes itself present as an 'operator' in the never-ceasing atomic actions of composition and decomposition. One might liken it to what is nowadays known as anti-particles or the abstract category of measurement of space, since it has the capacity to cause the swerve and the collision that engenders existence while being mere concept without factual, temporal and special position or body of matter, albeit with a material effect on the atomic reality.

Chapter 3

CLASSICAL ATOMISM

Before the steam engine's invention, mechanics and the mechanical world view were based on Archimedes' simple machines. These machines were amenable to mathematical analysis through geometry. Archimedes' works provided this analysis, with levers as the central example, viewing other machines as lever generalizations.

Atomistic materialism posits that the fundamental constituents of reality are particles of matter, referred to as atoms, corpora or other terms. Mechanical materialism builds on this by stating that matter adheres to strict conservation laws. Lucretius elaborated the principle 'nothing comes from nothing', which, although philosophical in his context, is given precise expression in mechanical materialism through conservation laws and symmetries.

Archimedes

The ancient thinker whose surviving works expressed this most clearly is Archimedes,[1] who opened his book *On the Equilibrium of Planes* as follows:

I POSTULATE the following:

1. Equal weights at equal distances are in equilibrium, and equal weights at unequal distances are not in equilibrium but incline towards the weight which is at the greater distance.
2. If, when weights at certain distances are in equilibrium, something be added to one of the weights, they are not in equilibrium but incline towards the weight to which the addition was made.
3. Similarly, if anything be taken away from one of the weights, they are not in equilibrium but incline towards the weight from which nothing was taken.

Principle (1) states a conservation law that the distance times weight on each side of the fulcrum must be conserved. He later states this more explicitly as:

> Two magnitudes, whether commensurable or incommensurable, balance at distances reciprocally proportional to the magnitudes.

Principles (2) and (3) express the *ex nihilo nihil fit* principle in the context that without the addition (or subtraction) of matter from an equilibrium position, there will be no motion.

We have missed other propositions, which he needed in order to make detailed analyses of the centres of gravity of various geometrical shapes. He proves some propositions, like: 'In any triangle the centre of gravity lies on the straight line joining any angle to the middle point of the opposite side' by classical geometrical techniques. In other parts of his work, however, he uses what seem to be remarkably modern techniques: recursive decomposition and what is effectively integration. Some of the formulae that children have had to learn by heart these last two millennia – πr^2 for a circle's area and $4\pi r^2$ for a sphere's – were first discovered by him using these techniques. The first formula he stated in the form:

> The area of any circle is equal to a right-angled triangle in which one of the sides about the right angle is equal to the radius, and the other to the circumference.

A moment's thought suffices to convince one that the area so described must be πr^2 in our notation. He then goes on to prove it by inscribing and circumscribing the circle, initially with squares, then regular octagons and then sixteen-sided polygons etc.

Polygons inscribed in and circumscribed around a circle provide upper and lower bounds for its area. Using recursive subdivision, he proved a proposition by constructing convergent series for the bounds which converge on the area of a triangle with base set by the circle's circumference and height set by its radius. He also deduced, by explicitly calculating the areas of 96-sided polygons, the upper and lower bounds for π:

$$3\frac{10}{71} < \pi < 3\frac{1}{7}$$

In modern decimal notation, this is equivalent to deducing

$$3.1408 < \pi < 3.1428$$

A modern calculator gives it as 3.141592654.

Concerning Archimedes' proof that the area of a sphere is four times the area of its great circle, the Victorian mathematician and editor of the Cambridge edition of *Archimedes*, Thomas Little Heath, says that the derivation is essentially identical to 'true integration'. Archimedes was using methods equivalent to Newton's calculus almost two thousand years before the English philosopher.

Archimedes' approach shows a close and mutually supportive relationship between geometry and mechanics. For example, his derivation of the volume of a sphere as $\frac{4}{3}\pi r^3$ uses explicitly mechanical arguments. His first step is to show

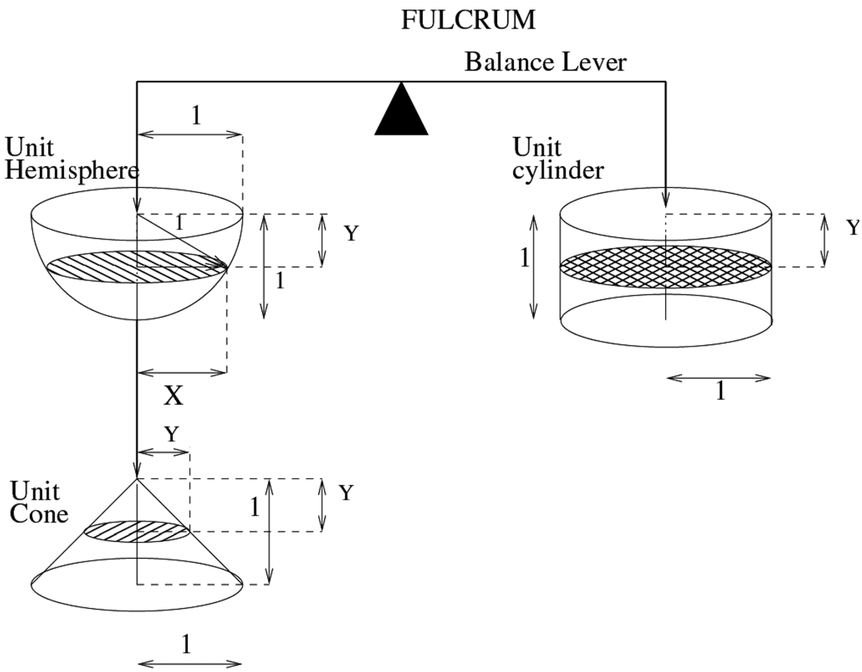


Figure 3.1 Archimedes proved that a unit cylinder has the same volume as a unit cone and a unit hemisphere. Figure by authors.

that a unit cylinder has the same volume as the sum of the volumes of a unit hemisphere plus a unit cone.

A unit cylinder is one whose height and radius are each 1. A unit cone is one whose height and radius are 1. Of course, a unit hemisphere is the hemisphere with a radius of 1.

His proof uses what he called his 'mechanical method'. Assume that the cone, hemisphere and cylinder are weights made of the same material. He asserts that if we attach them at equal distances from a fulcrum, as shown in Figure 3.1, with the cone and hemisphere on one side and the cylinder on the other, then they will exactly balance.

The proof is that if you take infinitesimally small circular slices through all three solids, moving down in step from the top of each, then at each level the slices of the cone and hemisphere will be the same area as the slices of the cylinder.

1. The area of the cylinder slices is easy. Since they are unit circles, they have an area of $\pi = \pi 1^2$.
2. Suppose we are at distance Y from the top of each solid. The radius of the cone at Y will also be Y, since it is a cone whose height is the same as its radius. So the area will be πY^2.

3. By Pythagoras' theorem, the radius X of the slice through the hemisphere must be $\pi\left(1^2 - Y^2\right) = \pi\left(1 - Y^2\right)$
4. Thus the sum of the circles in the hemisphere and cone must be $\pi\left(1 - Y^2\right) + \pi Y^2 = \pi$, Q.E.D.

Archimedes has independently of this argument proven that the volume of a unit cylinder is three times the volume of the unit cone, so the volume of the hemisphere must be $\frac{2}{3}\pi$, and that of the unit sphere must be $\frac{4}{3}\pi$. Scaling for a sphere of arbitrary radius r we get the familiar $\frac{4}{3}\pi r^3$.

There is an intimate relationship here between the mathematical abstraction and the real mechanics. Prior axiomatic mechanics postulates about balances are used as a stepping stone to derive what amounts to an integral calculus proof.

He did not content himself with just cones, spheres and cylinders. He moved on to integrate the areas under parabolas, for which proof he imagines more complex arrangements of the balance, with triangles and areas under a parabola suspended at different distances from the fulcrum. But why his concern with curves like parabolas?

Russo[2] argues that Archimedes' research, particularly his invention of hydrostatics as a discipline, was closely connected to the economic and political order of late Hellenistic civilization. Russo points to the economic and military objectives of Syracuse, the state in which Archimedes lived, as an explanation. Syracuse was a major naval power, striving to build ever larger ships. That is why Archimedes was so concerned in his hydrostatic analysis to derive the angles of stable repose of floating bodies. This included an analysis of how bodies described in terms of parabolas would float. Parabolic curves are possible hull forms.

The parabolic shape had already been analysed by one of Archimedes' predecessors, Apollonius, as segments of cones. The method of constructing and drawing such curves was understood. From these rules, shipwrights could cut out the forming ribs of a ship's hull.

During the Hellenistic period, competing merchant states funded scientific research for economic and political reasons. Archimedes' new science of hydrostatics was driven by economic and military necessity, and he used it to deduce the maximum stable load that hulls could bear. However, the Roman conquest ended funding for science, and Archimedes was famously killed. The establishment of a single state over the inland sea removed the sources of funding for science. The French Academy and British Royal Society resumed state funding of science in the seventeenth century, driven again by the existence of mercantile sea-borne trading economies in mutual competition.

In Archimedes' proof above, he is using equivalence to deduce the properties of the sphere. But this equivalence argument in maths is relying on prior axioms about mechanics. These in turn assume that mechanical laws themselves are defined in terms of equivalence relations.

The same applies to his much better-known discoveries in hydrostatics. The key laws here are that a floating body displaces its own weight of liquid, whereas one that is submerged displaces its own volume of liquid, this being less than its own weight. Here again we have equivalences of volumes and equivalences of weights, the themes in the proof we have run through.

Archimedes' geometrical proof methods for determining centres of mass and buoyancy demand ingenuity and depth of understanding that few can master today. Instead, we use finite element methods, essentially numerical integration, as a substitute. However, without electronic computational aids, geometry remains one of the only tools to solve these problems. With classical geometry, there is on the one hand a mechanical material process: the drawing of straight lines and intersecting circles; on the other hand, there is actual calculation in terms of rational numbers, as exemplified by the explicit numerical bounds Archimedes set for π. Let us reflect for a bit on what is involved here.

Suppose you were an architect constructing a temple. You draw plans in charcoal, perhaps on papyrus or perhaps on a white marble slab. These plans constitute a material model of the temple to come.[3] The plans are drawn with mechanical aids, ruler and compass. But suppose the drawing is one made using just these tools, with lengths set by, for example, the 'square doubling technique' or the drawing of a rectangle in the Golden Ratio. In that case, you also know the approximate linear dimensions of the final temple. These will also be in ratios, ultimately derivable from Pythagoras' theorem, that must exist between the lengths of the elements of the design. The theoretical geometry establishes a similarity between the necessarily approximate measurements on the scale drawing and the much more precise relative dimensions of the final building. Depending on the accuracy required, the same constructions can be done while increasing scales, again using straight lines and circles. Straight lines can be marked by visual alignment of posts and large circles traced out with taut chains anchored to a fixed point.

Mechanical materialism allows one material system, the plan, to act as a model for another, the temple. Both are material. The similarity between plan and building is assured by the repetition of the same constructive steps on an enlarged scale. The constructed image is both objective knowledge and *telos* of the building. The physical modelling process allows the coordination of the multiple craft workers, each constructing subcomponents, pillars, plinths etc., whose individual dimensions are coordinated by golden ratios.

Criticism of atomism by ancient materialists

In ancient times, there existed materialists who did not subscribe to atomism. The atomic theory, which is fundamentally valid, suffered a loss of credibility during the classical era. This was due to its attempts to account for phenomena that were far too complex for the rudimentary form of atomism available at the time. Epicurus knew of electrostatic and magnetic phenomena. He knew that a rubbed amber rod would attract dust and chaff. He knew that lodestone attracted iron.

He interpreted this in terms of the attracting object (the amber or lodestone) emitting tiny particles, ten thousand times smaller than the atoms of other things. The shapes of these tinier atoms were such that they became entangled with the object being attracted and bound the two together.

We now know that electrostatic attraction does involve tinier particles: electrons being transferred when amber is rubbed. An electron is indeed of the order of tens of thousands of times lighter than the atoms making up the amber. But without the notion of positive and negative charge and without the notion of action-at-a-distance forces, the Epicurean explanation seemed implausible. Applied to magnetism, it was still more incredible.

Atomism was a significant philosophical approach in classical Roman medicine, chiefly advocated by Asclepiades. This perspective suggested that diseases resulted from the constriction or dilation of tiny body pores that allowed molecules to circulate. Diseases were classified as 'stasis' if the pores were constricted and as 'flux' if they were dilated.

The hypothesis was not totally absurd, since modern medicine still recognizes vaso-constriction and vaso-dilation as possible pathologies. But when applied to the details of anatomy, it could become absurd. Asclepiades held that water passed from the blood into the bladder via invisible tiny channels in the wall of the bladder.

A rival medical teacher, Galen, who was an expert on anatomy, had no difficulty in refuting these claims about the bladder. He describes vivisection experiments which demonstrated that the bladder was filled via a flow of urine through the ureters.

> Now the method of demonstration is as follows. One has to divide the peritoneum in front of the ureters, then secure these with ligatures, and next, having bandaged up the animal, let him go (for he will not continue to urinate). After this one loosens the external bandages and shows the bladder empty and the ureters quite full and distended – in fact almost on the point of rupturing; on removing the ligature from them, one then plainly sees the bladder becoming filled with urine.[4]

Having criticized the Epicureans on anatomical grounds, Galen then goes on to cast doubt on their explanations of electrostatic and magnetic attraction. Epicurus, he said, believes that iron is attracted to the lodestone and chaff to amber. He explains that the atoms flowing from the stone easily interlock with those flowing from the iron, causing entanglement and attraction. Despite his unconvincing hypotheses, Epicurus acknowledges the existence of attraction. But it's hard to believe that particles from a lodestone can attract iron. Even if we accept this, it still doesn't explain why iron sticks to other iron. How does this happen?

Do the particles collide with the iron and bounce back, suspending it?

Or do they penetrate the iron and collide with another piece without being able to pass through?

> Then do they return to the first piece and cause entanglements like before?
>
> The hypothesis here becomes clearly refuted by its absurdity. As a matter of fact, I have seen five writing-stylets of iron attached to one another in a line, only the first one being in contact with the lodestone, and the power being transmitted through it to the others. Moreover, it cannot be said that if you bring a second stylet into contact with the lower end of the first, it becomes held, attached, and suspended, whereas, if you apply it to any other part of the side it does not become attached. For the power of the lodestone is distributed in all directions; it merely needs to be in contact with the first stylet at any point; from this stylet again the power flows, as quick as a thought, all through the second, and from that again to the third. Now, if you imagine a small lodestone hanging in a house, and in contact with it all round a large number of pieces of iron, from them again others, from these others, and so on,—all these pieces of iron must surely become filled with the corpuscles which emanate from the stone; therefore, this first little stone is likely to become dissipated by disintegrating into these emanations.[5]

The magnetism of iron was not understood until the mid-twentieth century. A proper understanding of magnetism required a whole set of further advances: action-at-a-distance forces, Faraday's unification of electric and magnetic phenomena and the discovery of electrons and electron spin.

Galen was concerned to promote a rival materialist theory of physiology which allowed transformation of substances, one into another. He believed that food was converted into blood via a faculty of the hepatic portal vein. He ridiculed the notion that bread contained within it the elements of flesh and bone, arguing instead that it was a faculty of the body to transmute substances.

Galen believed that the nerves worked pneumatically. This was a theory initially advanced by Erasistratus, who held that nerves contain a cavity through which *psychic pneuma*[6] passed. That air pressure was transmitted down the nerves and caused muscles to expand and thus contract. Galen and Erasistratus believed in 'pneuma' in physiology. Inhaling air transforms into vital pneuma in the lungs that mixes with blood and reaches the brain. It's converted into psychic pneuma, 'spiritus animalis', which regulates brain and nerve function.[7]

That life depended on air was clear. If an animal had its nose and mouth stopped, it suffocated. His dissection experiments had demonstrated that if he cut a nerve, the animal he was dissecting lost the power of movement in the area supplied by the nerve. Moreover, the ancients knew how to build pneumatically powered machines, or even simple pneumatic automata. Pneumatics was the advanced technology of the age.[8]

Galen's materialism formed the basis for idealistic themes in Catholic discourse. He denied the existence of an immortal soul and believed that consciousness depended on the brain. In the Christian period that followed, a modified theory emerged, stating that the immortal spirit resided in the brain's ventricles, animating the body. This transformed the original materialist theory into its opposite.

Instead of the animal spirit being manufactured by the body from the material air (pneuma) inspired into the lungs, we get the Catholic Catechism stating:

> The Christian vision of the human person made in the image of God with a spiritual soul as well as a body is of central importance. The soul, the seed of eternity we bear in ourselves, irreducible to the merely material, can have its origin only in God.
>
> . . .
>
> The human body is human precisely because it is animated by a spiritual soul.

The Christian doctrine is the distorted memory or misunderstood relic of a past experimental science. The actual pneumatic mechanical meaning of spiritus (Latin translation of Galen's pneuma) is totally lost. The word, detached from its original real meaning, became something entirely abstract, to which the theologians became free to attach any imaginary properties.

Newton

In Newton's day, if someone referred to the classical thinkers, they meant the Greeks and the Romans. Now, with the passage of time, Newton is seen as the archetypal philosopher of mechanical materialism, so much so that the terms *classical mechanics* and *Newtonian mechanics* have effectively become synonymous.

Via Lucretius, the idea of a universe made up of constantly moving particles had survived the fall of classical civilization. Via Archimedes, what later came to be known as static mechanics had also survived. We now know that Archimedes had also developed mathematical techniques very close to calculus. But the manuscript by which we know these things was not published in print till the late nineteenth century. Russo[9] speculated that perhaps Newton had access to this and other classical texts, which, through mould and decay, have subsequently been lost to us. Whatever the truth of these speculations, it is via Newton and Leibniz that the differential calculus was subsequently promulgated. Newton transformed mechanics from a science of static bodies to one of moving bodies.

The task of philosophy

Newton sets out the task of philosophy as follows:

> *We offer this work as the mathematical principles of philosophy; for all the difficulty of philosophy seems to consist in this – from the phænomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phænomena;*

Start with observations of motion. Infer from these the forces and laws of motion. This allows you to look at new situations and predict what will occur. Every space probe arriving at Mars or Jupiter confirms this approach.

We noted that Lucretius also based his atomism on deductions from what was empirically observable. As a poet, these simply come out in the telling. As a philosopher, Newton systematized this in a set of four explicit rules of reasoning, which we quote below:

RULE I

> *We are to admit no more causes of natural things than such as are both time and sufficient to explain their appearances.*
>
> To this purpose, the philosophers say that Nature does nothing in vain, and more is in vain when less will serve, for Nature is pleased with simplicity and affects not the pomp of superfluous causes.[10]

This is what is colloquially called the KISS principle: Keep It Simple, Stupid. Before that, it was known as Occam's razor, which holds that entities should not be multiplied beyond cause. Here Newton is simply repeating, as he says, the consensus of previous philosophers that simple explanations should always be preferred over complex ones.

In terms of mathematics, this means that we should always prefer the simplest mathematical formula over the more complex when trying to discover a natural law. Leibniz, Newton's contemporary, argues the same point

> Thus, let us assume, for example, that someone jots down a number of points at random on a piece of paper, as do those who practice the ridiculous art of geomancy. I maintain that it is possible to find a geometric line whose motion is constant and uniform, following a certain rule, such that this line passes through all the points in the same order in which the hand jotted them down.
>
> But when a rule is extremely complex, what is in conformity with it passes for irregular. Thus, one can say in whatever manner God might have created the world, it would always have been regular and in accordance with a certain general order. But God has chosen the most perfect world, that is, the one which is at the same time the simplest in hypotheses and the richest in phenomena, as might be a line in geometry whose construction is easy and whose properties and effects are extremely remarkable and widespread.[11]

The precise means by which Leibniz intended to construct a smooth curve through all the random points is not specified. In the eighteenth century, perhaps the means would have been to fit a polynomial like

$$a + bx + cx^2 + dx^3 + ex^4 + ...$$

to the points. Fourier would later discover a technique by which you can arbitrarily fit a curve through points using sine and cosine functions. We will return to this.

The most obvious example that Newton could invoke in terms of simplicity was how much simpler his explanation of the solar system was than that advocated by Ptolemy.

When dealing with the solar system via Newtonian mechanics, we need the angular momentum, radius and rotational phase for each planet at time 0. The laws of motion give us all the rest. This amounts to three numbers per planet. I am disregarding knowledge of the mass of the Sun and the gravitational constant, as I am assuming these are given from the outset and we are just considering the extra information needed for each planet.

If we take the Ptolemaic model, each planet rotates around the Earth with one cycle upon which one epicycle rotates. So for Ptolemy, we needed at least two radii, two rates of rotation and two starting phases: six numbers per planet. That amounts to a twofold redundancy compared to Newton. So on the grounds of the information economy, Newton was to be preferred. His theory had the added bonus that it could also account for the motion of comets – for which no law had existed in the older astronomy.

If we are allowed to add epicycles on epicycles, then it is in principle possible to fit observations, whether of planets or any other data, to any degree of accuracy we want. As Figure 3.2 illustrates, the Ptolemaic approach amounts to adding cosine waves together. We illustrate it as a simple projection down onto the horizontal axis. For Ptolemaic astronomical observations, the x-axis is the 360° of the ecliptic in the sky against which he plotted the planetary positions. It is clear that what is going on here is the same as Fourier analysis.[12] Fourier decomposition of a signal

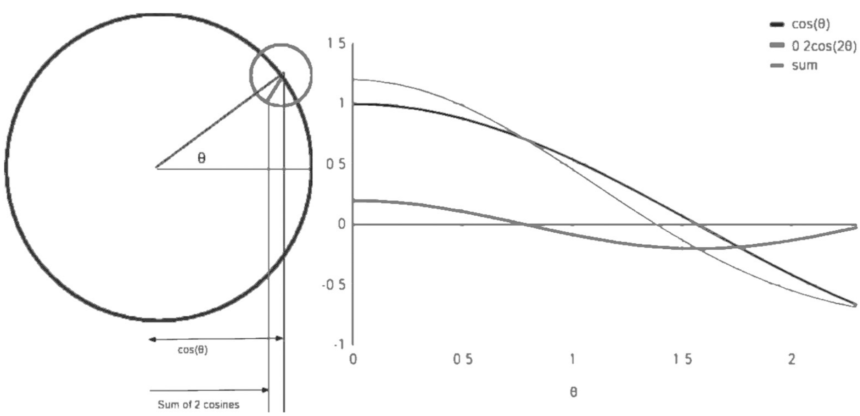


Figure 3.2 The projection of the orbital of a body with an epicyclic path onto the x-axis is a sum of cosine curves. The epicycle has a diameter of 0.2 and rotates at twice the rate of the large circle. The right-hand graph plots the cosines and their sums. Figure by authors.

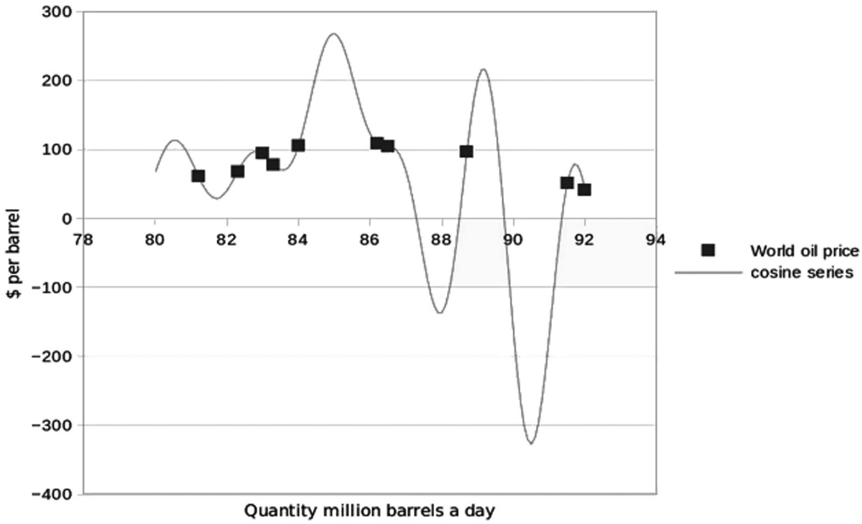


Figure 3.3 A continuous curve that goes through the points representing world oil price/shipment data sampled between 2007 and 2016. The curve was produced by applying the epicycle method used in Ptolemaic astronomy. Figure by authors.

into cosine waves of different frequencies is what image compression schemes like JPEG and MPEG use. When you view a YouTube video, you are seeing the Ptolemaic method in action. A computer has analysed the picture into what amounts to epicycles with different rotational velocities. The video file contains what amounts to the radii of these epicycles, from which the video codec can reconstruct a good approximation of the original image.

We can, as Leibniz implies, apply the method to any data set we want. Figure 3.3 applies it to the relationship between the number of barrels of oil sold (in millions) and the price per barrel at which they were sold.

The formula

$$p = 136\cos(2+\theta) + 27.7\cos(2+2\theta) + 39.5\cos(2+3\theta) +$$

$$19.6\cos(2+4\theta) + 62\cos(2+5\theta) +$$

$$52\cos(2+6\theta) + 50.1\cos(2+7\theta) +$$

$$37\cos(2+8\theta) + 70.5\cos(2+9\theta) + 19\cos(2+10\theta)$$

with $\theta = \pi(q-80)/10$, where p is the price of oil and q the quantity sold, does what Leibniz described when he said: '*I maintain that it is possible to find a geometric line whose motion is constant and uniform, following a certain rule, such that this* *line passes through all the points in the same order in which the hand jotted them down'.*

But it would clearly be absurd to identify this formula with a law regulating the price of oil as a function of quantity supplied. The rule is 'extremely complex' and the data in conformity with it is 'irregular'. In contrast, Newton's laws of motion are simple and apply with great regularity to astronomical phenomena. As such, they count as real laws of nature.

RULE II

> *Therefore to the same natural effects we must, as far as possible, assign the same causes.*
>
> As to respiration in a man and in a beast; the descent of stones in Europe and in America; the light of our culinary fire and of the Sun; the reflection of light in the Earth and in the planets.

The examples are a bit old and homely, but both the examples and the principle are still accepted. When temperatures of distant stars are estimated from their spectra, we assume that the same black body radiation laws apply there as on Earth. When astronomers attempt to deduce the composition of asteroids from their albedo, using the albedos of mineral samples here on Earth, they apply this principle.

Underlying it is the deliberate exclusion of supernatural or religious causes. Newton's contemporaries would have understood that reference was being made to Anaxagoras, who first posited that the Moon was a rock reflecting light of the Sun, itself a burning rock, before being exiled from Athens for such an impiety. Newton implicitly sides with the ancient materialist in Rule II.

RULE III

> *The qualities of bodies, which admit neither intension nor remission of degrees, and which are found to belong to all bodies within the reach of our experiments, are to be esteemed the universal qualities of all bodies whatsoever.*

Science needs this rule if it is to be able to generalize from experiment. Newton carried out experiments using pendulums to investigate what he formulated as a law of conservation of momentum. If the conservation of momentum applied in experiments, he inferred that the same applied to the Moon and planets. When Archimedes observed water being displaced as he stepped into a bath, he inferred that a ship likewise displaces a volume of water as it slides into the sea.

RULE IV

> *In experimental philosophy we are to look upon propositions collected by general induction from phænomena as accurately or very nearly true, notwithstanding*

> *any contrary hypotheses that may be imagined, till such time as other phænomena occur, by which they may either be made more accurate, or liable to exceptions.*
>
> This rule we must follow that the argument of induction may not be evaded by hypotheses.

This is easily misunderstood given the current associations with the word 'hypothesis'. A common presentation of science is that it advances hypotheses about reality, which are then put to the test of observation and experiment. Why did Newton famously remark that he made no hypotheses?

In this context, he meant a priori assumptions about how the world must work. Instances of this would have been the hypothesis that the Earth is fixed at the centre of the universe, or the hypothesis that forces could only act by direct physical contact. Either of these would, if insisted upon, have resulted in an evasion of the arguments from induction that he builds up in his *System of the world*.

We will later touch on how 'evading the argument of induction by hypotheses' has caused controversy in historical materialism.

A remark on mechanics

We said above that Newtonian mechanics rested on the foundation of classical mechanics, and in the section on Archimedes we recounted how geometry itself was for the ancients a mechanical art to be used in architecture. Newton relies throughout the *Principles* on classical geometry. He extends its methods towards the calculus, but he relies on Euclid as the place to stand while he studies the movement of worlds.

He makes some interesting observations on the relationship between mechanical arts and geometry:

> To describe right[13] lines and circles are problems, but not geometrical problems. The solution of these problems is required from mechanics; and by geometry the use of them, when so solved, is shown; and it is the glory of geometry that from those few principles, brought from without, it is able to produce so many things.

He is saying that describing straight lines and circles, that is to say, describing in the sense of drawing or laying out, is not a problem for geometry. It is a problem for the mechanical practice of the draftsman or the surveyor. Axiomatic and deductive geometry then takes these simple results of mechanics and shows how to compose them to 'produce so many things'.

> Therefore geometry is founded in mechanical practice, and is nothing but that part of universal mechanics which accurately proposes and demonstrates the art of measuring. But since the manual arts are chiefly conversant in the moving of bodies, it comes to pass that geometry is commonly referred to their magnitudes, and mechanics to their motion. In this sense rational mechanics will be the science of motions resulting from any forces whatsoever, and of the forces required to produce any motions, accurately proposed and demonstrated.

The science is to be about the motion of material bodies. It is by geometry that he will analyse the motions and forces which impel motions. He goes on to define the terms he will use in this geometric treatment of matter and motion.

Definitions

DEFINITION I.

> *The quantity of matter is the measure of the same, arising from its density and bulk conjunctly.*
>
> Thus, air of a double density, in a double space, is quadruple in quantity; in a triple space, sextuple in quantity. The same thing is to be understood of snow, and fine dust or powders, that are condensed by compression or liquefaction; and of all bodies that are by any causes whatever differently condensed. I have no regard in this place to a medium, if any such there is, that freely pervades the interstices between the parts of bodies. It is this quantity that I mean hereafter everywhere under the name of body or mass. And the same is known by the weight of each body; for it is proportional to the weight, as I have found by experiments on pendulums, very accurately made, which shall be shewn hereafter.

He is establishing weight as the measure of matter. He is distinguishing the weight of matter from its volume. This is pretty obvious empirically, but it is nevertheless significant in two important senses:

1. He explicitly excludes the hypothetical luminiferous ether from mass with his allusion to a 'medium, if any such there is, that freely pervades the interstices between the parts of bodies'. If this ether exists, he says it is massless or immaterial.
2. He is asserting the equivalence of inertial and gravitational mass. Although this will not become apparent until he later states his law of gravitation, he refers forward to the experiments with pendulums by which he demonstrated this.

DEFINITION II.

> *The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjunctly.*
>
> The motion of the whole is the sum of the motions of all the parts; and therefore in a body double in quantity, with equal velocity, the motion is double; with twice the velocity, it is quadruple.

What Newton called motion is what we now call momentum, so he is giving in words the defining equation $p = mv$. There is no distinct concept of kinetic energy yet present in the *Principles*. We return to this later.

DEFINITION IV.

> *An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line.*

This is a qualitative definition that is given quantitative meaning by his Laws I & II lower down. It does, however, depend in large part on the laws for its meaning. It is a prelude to:

DEFINITION V.

> *A centripetal force is that by which bodies are drawn or impelled, or any way tend, towards a point as to a centre.*

Here he introduces, for the first time, directionality to force. It is interesting that prior to this, in Definition IV, force is not yet a vector, not yet having directionality. When he introduces directionality, it is in the specific sense of a centripetal attractive force. It is possible that absent any introduction of a frame of reference, he felt it impossible to define directionality. One point allows directionality to be defined towards that point, but it also allows the definition of a force away from the point – centrifugal. In most of the book, Newton is careful to deal with different kinds of force laws, considering laws that vary inversely with distance, inversely with square of distance, cube of distance etc. It is surprising that he restricts the definitions to attractive ones rather than defining repulsive ones as well. Even if the repulsive electrostatic force was not studied in his day, repulsive magnetism was.

DEFINITION III.

> *The vis insita, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, endeavours to persevere in its present state, whether it be of rest, or of moving uniformly forward in a right line.*

He is here defining in a qualitative way the notion of inertial mass. However, in what is now the accepted presentation of Newtonian mechanics, inertia would not be described as a force.

He has three definitions (VI to VIII) giving different ways of quantifying centripetal force. The first defines it as 'proportional to the efficacy of the cause that propagates it from the centre, through the spaces round about', which is rather obscure since it is not clear how one is to measure the efficacy of a cause or in what units he intends to measure it. The next defines the accelerative quantity of centripetal forces as being proportional to the velocity they induce per unit time. In the $f = ma$ relation, this amounts to abstracting from the inertial mass of the body being accelerated, something which is perfectly reasonable when dealing with different small masses in orbit around a much larger one. In this case, measuring the 'accelerative quantity' of the gravitational force is equivalent to what we now term the intensity of the gravitational field: newtons per kilogram. He finally defines the motive quantity of a centripetal force as proportional to the motion it generates in a given time. He uses the term *motion* for what we would now term momentum (mv), so he appears to mean something that in SI units would be measured in kilogram metres per second squared.

Time and space

The conceptual pair (bodies, void) was fundamental to classical atomism. For Newton, a conceptual definition of time and space had to precede any systematic theory of the motion of bodies. His definitions of time and space are widely quoted because it is on this issue that post-Einstein physics departs from him.

TIME

> Absolute, true, and mathematical time, of itself, and from its own nature flows equably without regard to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year.

Newton here applies a metaphor: that absolute time flows. But plausible as the metaphor seems, it is surely contradictory. When a stream flows, it moves. Its motion is defined in the context of time.

We throw a twig down one side of a bridge; a few seconds later, it appears on the other side, and we then conclude that the river flows. But to measure the flow, we need two observations at distinct times. At time t_0, the twig lands in the stream, and at time $t_0 + 8$ seconds, we see the twig emerge, 4 metres downstream on the other side of the bridge. So we conclude that the water is flowing at half a metre per second.

This is a flow in space with respect to time. But what is time supposed to flow with respect to?

To speak of the flow of time makes no more logical sense than it would to speak of spatial distance flowing.

If I plot a graph for $y = x^2$, I can say that the line flows upwards with respect to the x axis, and I can take the derivative of this, $2x$, which tells me how fast y flows up as x changes. But I can only do these things because two orthogonal dimensions are involved. Similarly, given an analogous equation of motion for a falling body, $y = 4.9t^2$ and its derivative $v_y = 9.8t$, I can determine the downward flow through space of the object at time t. Again, this is because time, in the equation, is an orthogonal dimension. But time cannot flow with respect to itself.

But he no sooner introduces his flowing absolute time than he puts it to one side to introduce what he actually works with – relative time.

His sensible or relative time is measured by motions, giving as examples units like the year or the hour. These are indeed measured by motions: the motion of the Earth around the Sun for the year and the spin of the Earth on its axis for the day. By

Newton's day, human arts had constructed clocks whose motions modelled those of the Earth, with two rotations of the hour hand for each rotation of the Earth.

According to Newton, motion cannot be measured in relation to absolute time. Instead, it can only be measured in relation to other motions. In the field of astronomy, a classical astronomer would determine the moment of occultation of a star by the Moon by measuring the angle with respect to the celestial zenith of either the same star or a reference star.

The astronomer could then convert this to sidereal time[14] for their records.

Newton aimed to explain planetary motion, observing correlations of motions and positions without an independent measure of time. He believed that all motions are conceptually defined with respect to time, creating a time dimension. The Earth rotates with absolute time, while the Moon has an angular rotation around us. Newton inferred from the existence of stable ratios that there is a common absolute objective time dimension for these varied rotations.

SPACE

> Absolute space, in its own nature, without regard to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces; which our senses determine by its position to bodies; and which is vulgarly taken for immovable space; such is the dimension of a subterraneous, an æreal, or celestial space, determined by its position in respect of the Earth.

Newton's hypothesized absolute space is something that was dropped by twentieth-century physics with the theory of relativity. But no sooner has Newton raised the idea of absolute space than he immediately moves on to relative space. He was aware of and accepted Galilean relativity, which is to say that the laws of motion remain the same in a constantly moving frame of reference.

He later elaborates on this relativity, giving the example of a ship:

> A clear proof of which we have from the experiment of a ship; where all motions happen after the same manner, whether the ship is at rest, or is carried uniformly forwards in a right line.

But given that he accepts Galilean relativity, why does he persist with the notion of absolute space? The rationale, we think, is provided in the last section of his definition of motion.

MOTION

> Absolute motion is the translation of a body from one absolute place into another; and relative motion, the translation from one relative place into another. Thus in a ship under sail, the relative place of a body is that part of the ship which the body possesses; or that part of its cavity which the body fills, and which therefore moves together with the ship: and relative rest is the continuance of

> the body in the same part of the ship, or of its cavity. But real, absolute rest, is the continuance of the body in the same part of that immovable space, in which the ship itself, its cavity and all that it contains, is moved. Wherefore, if the Earth is really at rest, the body, which relatively rests in the ship, will really and absolutely move with the same velocity which the ship has on the Earth. But if the Earth also moves, the true and absolute motion of the body will arise, partly from the true motion of the Earth, in immovable space; partly from the relative motion of the ship on the Earth; and if the body moves also relatively in the ship; its true motion will arise, partly from the true motion of the Earth, in immovable space, and partly from the relative motions as well of the ship on the Earth. As if that part of the Earth, where the ship is, was truly moved toward the east, with a velocity of 10010 parts; while the ship itself, with a fresh gale, and full sails, is carried towards the west, with a velocity expressed by 10 of those parts; but a sailor walks in the ship towards the east, with 1 part of the said velocity; then the sailor will be moved truly in immovable space towards the east, with a velocity of 10001 parts, and relatively on the Earth towards the west, with a velocity of 9 of those parts.

But he goes on to explain his belief as to why there is such a thing as absolute motion as well as relative motion.

> But we may distinguish rest and motion, absolute and relative, one from the other by their properties, causes and effects. It is a property of rest, that bodies really at rest do rest in respect to one another. And therefore as it is possible, that in the remote regions of the fixed stars, or perhaps far beyond them, there may be some body absolutely at rest; but impossible to know, from the position of bodies to one another in our regions whether any of these do keep the same position to that remote body; it follows that absolute rest cannot be determined from the position of bodies in our regions.
>
> *The effects which distinguish absolute from relative motion are, the forces of receding from the axis of circular motion. For there are no such forces in a circular motion purely relative, but in a true and absolute circular motion, they are greater or less, according to the quantity of the motion.*

The final point above is very important.

Absolute translation is discredited by the Michaelson-Morely experiment, but Newton was already saying that absolute space is immeasurable except for rotation. This is proven through a thought experiment. He demonstrates that absolute space is immeasurable except for rotation by hanging a half-filled bucket with water from a tree with a rope. The water initially remains stationary, but as the bucket spins, its motion is transmitted to the water, which starts spinning. This causes the water's surface to curve up at the sides, indicating that the water was not initially spinning with respect to absolute space. The constraint of the bucket wall creates a centripetal force, causing the water to pile up at the edges.

This thought experiment is quite convincing.

Laws

His laws of motion set the standard for what we understand natural laws to be. There had been previous formulations – Archimedes law of displacement by floating bodies, and Galileo's law of acceleration[15] – but no previous philosopher had formulated a complete set of motion laws.

> Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.[16]
>
> The alteration of motion is ever proportional to the motive force impressed and is made in the direction of the right line in which that force is impressed.[17]
>
> To every action, there is always opposed an equal reaction, or the mutual actions of two bodies upon each other are always equal and directed to contrary parts.
>
> If you press a stone with your finger, the finger is also pressed by the stone. If a horse draws a stone tied to a rope, the horse (if I may so say) will be equally drawn back towards the stone: for the distended rope, by the same endeavour to relax or unbend itself, will draw the horse as much towards the stone as it does the stone towards the horse, and will obstruct the progress of the one as much as it advances that of the other.[18]

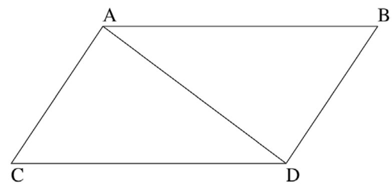
Law III is a clear equivalence relation. As such, it builds on the equivalence relations of Archimidean mechanics. With his example of the pressure of a finger on a stone, Newton is repeating a more limited point made by Galileo

> When one holds a stone in his hand does he do anything but give it a force impelling [virtù impellente] it upwards equal to the power [facoltà] of gravity drawing it downwards? And do you not continuously impress this force [virtù] upon the stone as long as you hold it in the hand?

Galileo only gave this equality of action and reaction in the special case of a body balanced against gravity. Newton extended it to a general principle of moving and accelerating bodies, irrespective of the nature of the forces acting on them.

Newton is the first to present mechanics of change and motion in which all the changes and motions are subject to certain universal equivalences and constraints, equivalences and constraints which can be treated mathematically. In Newton's case, as with Archimedes and Galileo, the mathematical treatment is still mainly in terms of classical geometry. His text is constantly interspersed with geometric diagrams to illustrate its arguments. Dealing with the composition of forces, he writes:

> A body by two forces conjoined will describe the diagonal of a parallelogram, in the same time that it would describe the sides, by those forces apart.



> And hence is explained the composition of any one direct force AD, out of any two oblique forces AC and CD; and, on the contrary, the resolution of any one direct force AD into two oblique forces AC and CD: which composition and resolution are abundantly confirmed from mechanics.

He formulates a set of explicit conservation laws. First, a conservation of momentum law:

> The quantity of motion, which is collected by taking the sum of the motions directed towards the same parts, and the difference of those that are directed to contrary parts, suffers no change from the action of bodies among themselves.[19]

The above is a conservation law – conservation of aggregate momentum of a collection of bodies – deduced from the laws. Recall that by 'motion', Newton means mv, the product of velocity and mass in motion, what we would now call momentum. He is saying that given a collection of bodies, if you choose an arbitrary direction and compute the sum of their momenta in that direction, then no interaction of those bodies, whether mutual gravitation or impacts between them, can alter the aggregate momentum of the bodies in the specified direction.

He next extends this to their centres of gravity. He argues that the common centre of gravity of two or more bodies remains constant regardless of their actions, ensuring that all bodies acting upon each other, excluding outward actions and impediments, are either at rest or moving uniformly in a right line. Moreover in a system of two bodies acting on each other, their relative motions, whether approaching or receding from the common centre of gravity, will be equal due to reciprocal distances between the bodies.

This extends Archimedes' work on centres of gravity to moving bodies interacting by mutual forces. This will later be used to deal with problems of planetary orbits. It means, for instance, that the centre of gravity of the Earth and the Moon is not altered as the Moon orbits the Earth. He is careful to restrict this to the effect of the bodies 'acting upon each other', since to the extent that the Earth and Moon are embedded in a solar gravitational field, the effect of this will alter the mutual centre of gravity of the Earth-Moon system.

> The motions of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line without any circular motion.
>
> A clear proof of which we have from the experiment of a ship; where all motions happen after the same manner, whether the ship is at rest, or is carried uniformly forwards in a right line.[20]

Since, in orbiting the Sun, the Earth and Moon do not move 'uniformly forward in a right line without any circular motion', he is explicitly excluding planet-satellite systems from his rule that the 'motions of the bodies are the same among themselves'. But more generally, he is asserting what is now known as Gallilean relativity – that is to say that the mutual motions of bodies in an inertial frame of reference are unaffected by the uniform motion of that frame of reference. The analogy with a ship is borrowed from Gallileo.[21]

Newton explicitly excludes circular motion since according to him such circular motion distinguishes relative from absolute motion through space. He explicitly refers to Galilleo for authority on the fact that falling bodies traverse a distance proportional to the square of their time of fall:

> By the first two Laws and the first two Corollaries, Galileo discovered that the descent of bodies observed the duplicate ratio of the time, and that the motion of projectiles was in the curve of a parabola; experience agreeing with both, unless so far as these motions are a little retarded by the resistance of the air. When a body is falling, the uniform force of its gravity acting equally, impresses, in equal particles of time, equal forces upon that body, and therefore generates equal velocities; and in the whole time impresses a whole force, and generates a whole velocity proportional to the time. And the spaces described in proportional times are as the velocities and the times conjunctly; that is, in a duplicate ratio of the times.[22]

The relations here are $v = at$ and $d \approx t^2$.

He then recounts actual experiments he has undertaken with steel balls as pendulums that collide to verify the relations, with careful allowance being made for the effects of air resistance.

His method of proof of gravity

From the basic laws of motion, Newton in book I of the *Principles* examines many possible trajectories of bodies interacting under different types of forces. He does not only look at inverse square law forces like his law of gravitation. Indeed, he does not introduce the law of gravitation until book III. He instead deduces from his initial laws of motion what orbits and trajectories would be expected under different sorts of forces. He comes up with the following generalizations:

> If the periodic times are as the radii, and therefore the velocities equal, the centripetal forces will be reciprocally as the radii; and the contrary.
>
> If the periodic times are in the *sesquiplicate* ratio of the radii,[23] and therefore the velocities reciprocally in the subduplicate ratio of the radii,[24] the centripetal forces will be in the duplicate ratio of the radii inversely, and the contrary.
>
> And universally, if the periodic time is as any power R^n of the radius R, and therefore the velocity reciprocally as the power R^{n-1} of the radius, the centripetal force will be reciprocally as the power R^{2n-1} of the radius; and the contrary.[25]

He now has a general rule which relates orbital periods to the nature of forces. He is also able to deduce Kepler's law that orbital bodies sweep equal areas in equal times as a special case that holds if centripetal forces have an inverse square law. In book III, he introduces six key facts, or, as he puts them, Phænomena:

1. That the orbital periods of the moons of Jupiter are in *sesquiduplicate* ratio to their orbital radii.
2. That the orbital periods of the moons of Saturn are similarly in *sesquiduplicate* ratio to their orbital radii.
3. That the five primary planets: Mercury, Venus, Mars, Jupiter and Saturn rotate around the Sun.
4. That they have periodic times in sequiduplicate proportion of their mean distances from the Sun.
5. That while the areas which the planets describe by radii drawn to the Sun are proportional to the times of description, no such relation exists in the areas that are swept by radii connecting the planets to the Earth.
6. That the Moon, by a radius drawn to the Earth's centre, describes an area proportional to the time of description.

In all cases, he cites what were then the best-known estimates of orbital radii and periods to justify his claims. From these phenomena and his previous results about orbital periods, he deduces that the moons of Jupiter, the primary planets and the Moon must all be drawn of rectilinear motion by a force subject to an inverse square law. He concludes

> The force which retains the celestial bodies in their orbits has been hitherto called centripetal force; but it being now made plain that it can be no other than a gravitating force, we shall hereafter call it gravity.

He then argues that this gravitational force must not only operate inversely to the square of distances but must also operate in proportion to the masses of the bodies, irrespective of the body towards which the bodies are drawn. His proof is that were it the case that the gravitational attraction towards the Sun experienced by Jupiter and its satellites was not in proportion to their masses, then the orbits of Jupiter's moons would be very evidently distorted by the Sun's gravity, which is not the case. His final formulation is a law of attraction that is proportional to the masses and inversely as the square of distances.

Recapitulating the whole structure of the argument,

- He initially states laws of motion that have been verified by experiment.
- He then, by lengthy geometrical proofs, demonstrates the possible trajectories consistent with these laws of motion and various conceivable force laws.
- From observation of all the moons and planets then visible, he shows that these follow the trajectories that would follow from inverse square attraction.
- He then infers that since all observed bodies follow this law, it is a universal law of nature.

This is a circular process where initial experiments provide laws, astronomical data helps in infer external forces and a new law, gravity, is inferred. This law allows for deductive reconstruction of phenomena like Earth's aspherical distortion and Lunar orbit irregularities. The process also involves determining the forces exerted by the Sun and Moon on the sea.

Book II

It is books I and III of Newton's *Principles* – containing the general laws of motion and the theory of gravity – that have had a lasting impact. Book II attempted to deduce the laws of resistance that bodies would experience due to passage through gases and liquids. His treatments have subsequently been proven to be much too simple to account accurately for hydrodynamic forces.

On one point, however, his ideas in book II have stood the test of time. He has a section titled *Of Motion Propagated Through Fluids*, which deals with compressive sound waves passing through a fluid, and his account is explicitly atomistic. He views a fluid as being made up of atoms, which he draws as spheres. He argues that it is impossible for a pulse to pass through a fluid in a straight line, his argument being that if a pulse was being transmitted through atoms *a, b, c, d* and *e*, it would only continue so long as the atoms were perfectly lined up. In his diagram, atom *e* would next pass it on to *f* and *g* causing the pulse to diverge into two components. From this, he was able to deduce that sound waves passing through a fluid must exhibit diffraction when they pass through a narrow cavity.

On this he was right.

The Hegelian critique of Newton

The German philosopher Georg Wilhelm Friedrich Hegel was very critical of Newton, whom he compared unfavourably to 'our great countryman Kepler, blessed with the gift of genius'. In his critique of Newton, Hegel claims that the inverse square law of gravity could have been deduced by Kepler directly from the latter's law of orbits. He writes:

> Indeed, the law which he gave (ie that the areas measured by the vector radii of the bodies in circular motion are proportional to the times) he would have been able to transmute into the form (species) of a physical law (ie that gravity is in proportion (in ratione) to the area belonging to equal sectors); and since the total surfaces of the circles A and a stand in the same ratio as the squares of their radii R and r, we know that $1/A:1/a$ is equivalent to r^2/R^2. Since $1/A$ and $1/a$ express the quantity of motion, and, if you wish, the quantity of the centripetal force, he could have said that the force of gravitation or centripetal force stands in inverse ratio to the radii, or distances.[26]

This shows a misunderstanding both of Newton and of Kepler. Kepler's law of equal areas applies to segments of the elliptical orbit of one planet, meaning that the angular velocity must be greatest at perihelion. Hegel is misapplying it to compare two distinct circular orbits of different planets. Kepler's law does not state that the planets sweep out equal areas in equal times. In fact, the further out a planet is, the less the area it sweeps out each 24 hours. We can see this by looking at the parameters of the orbits of Venus, Earth and Mars:

Planet	Radius in AU (semi major axis)	Period	Area per day square AU
Venus	0.723	225	0.0202
Earth	1	365	0.0172
Mars	1.524	687	0.0139

Newton showed that you can still have Kepler's law with a gravitational force law that is of different form, but the absolute velocities would be different in the two cases.

Newton worked out the properties of Keplerian orbits with gravitational laws of the form $f = mMG/r^n$ for $n = 1,2,3$ and finds that all three are consistent, but that the predicted velocities differ.

He then used actual observations of the orbits of the Jovian satellites to show that only the inverse square law was consistent with observation. We can illustrate the compatibility of Kepler's law with different gravitational laws with a simple example.

Consider a satellite in orbit around the Earth such that the most distant point A (apogee) is twice as far from the Earth as the closest point P (perigee) Assume further that Kepler's law of equal areas holds, so that if the velocity at point A is v, then velocity at point P must be $2v$.

Let the satellite have mass m; then kinetic energy at point A is $0.5mv^2$, and at point P, it is $2mv^2$.

The difference in potential energy between P and A, assuming Newtonian gravity and mass of Earth M, will be $\int_1^2 mMG/r^2 dr = mMG/2$. In other words this is the integral over the change in radial distance with respect to gravitational force, and an integral of force with distance is of dimension energy. The conservation of energy then requires us to have $1.5mv^2 = mMG/2$, so $v = \sqrt{MG/3}$.

Suppose instead that the gravitational law was such that force was inversely proportional to the distance rather than the square of the distance, but that Kepler's law still holds.

Call the new velocity at A in this case v_1, and the velocity at P is now $2v_1$.

The kinetic energies now differ by $1.5mv_1^2$, which now has to equal the difference in gravitational potential under a different force law (acceleration $\propto \frac{1}{r}$) so that $\int_1^2 mMG/r\,dr = mMG\log(2)$ we can again solve for velocity such that $v_1 = \sqrt{\frac{MG\log(2)}{1.5}}$.

Since $\log(2) \neq \frac{1}{2}$, it follows that velocities will differ with the new gravity law.

The velocities are a free variable in the models. Only observation of actual velocities can tell you which model of gravity is correct.

Newton's argument about orbital periods

Consider the centripetal acceleration experienced by a body in a clockwise circular orbit.

Clearly, the velocity v of the body is proportional to the radius of the orbit and inversely proportional to period of the orbit. If at time t_0 the body has velocity vector $[0,v]$, then at time t_{90} a quarter rotation later its velocity will be $[-v,0]$. Its entire initial velocity in the X direction has been cancelled. The longer the period of rotation, the more gradual this deceleration will be. So the centripetal acceleration is directly proportional to radius and inversely proportional to the square of the orbital period.

$$A_c \propto \frac{r}{p^2}$$

This centripetal acceleration has to be equal to the acceleration due to gravity at radius r. Newton considers three possible gravity laws:

1. Acceleration is inversely proportional to distance. This would not be implausible given that Hooke's then recently discovered law of elasticity also had a linear dependence on displacement.

$$A_g \propto \frac{1}{r}$$

2. Acceleration is inversely proportional to the square of distance, as seen in gravitational force spreading in successive spheres. The surface of these spheres grows as the square of radius, so the proportion of the area intersected by the orbiting body will have an inverse square relation.

$$A_g \propto \frac{1}{r^2}$$

3. Acceleration is inversely proportional to the cube of the distance. This is also a conceivable law if you think of the gravitational field expanding to fill larger volumes; the share of the volume of the Sun's field at radius r occupied by a planet would then have this relation.

$$A_g \propto \frac{1}{r^3}$$

If gravitational acceleration is the centripetal acceleration, we have $A_c = A_g$. In the case of gravity being inversely proportional to distance, we then have

$$\frac{1}{r} \propto \frac{r}{p^2}$$

If you plot of orbits of planets from Venus to Saturn (Figure 3.4) against the predictions of hypothetical gravity laws considered by Newton, it is clear that inverse square laws are the best fit. The fit is almost perfect for the most circular orbits. See the data for eccentricity in Table 3.2. Mars is the most prominent outlier, as it has a relatively more eccentric orbit.

So it follows that $p^2 \propto r^2$ and $p \propto r$. We can construct a table of similar relations for the three laws (Table 3.1).

We can then compare this with actual orbital data for the planets and see that although each of the hypothetical gravity laws had plausible prior justifications, only the inverse square law fits the data well (Table 3.2).

In contrast, the idealist Hegel wants to deduce the laws of motion from pure thought, disparaging experiment. He argued that Newton and his followers confirm the hypothesis of centrifugal force by giving the example of a stone that is

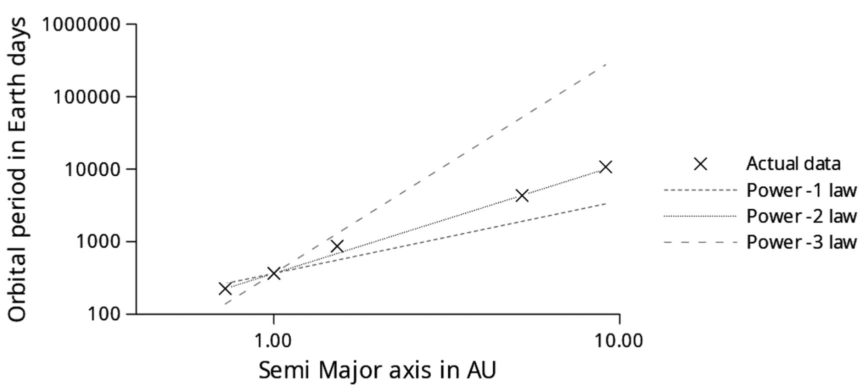


Figure 3.4 Orbital semi-redius against period. Figure by authors.

Table 3.1 The Three Gravity Laws Considered by Newton and Their Predicted Relations of Orbital Period to Orbital Radius

Power law	Centripental acceleration	Gravity acceleration	Period law	
−1		$A_g \propto \frac{1}{r}$	$p^2 \propto r^2$	$p \propto r$
−2	$A_c \propto \frac{1}{p^2}$	$A_g \propto \frac{1}{r^2}$	$p^2 \propto r^3$	$p \propto \sqrt{r^3}$
−3		$A_g \propto \frac{1}{r^3}$	$p^2 \propto r^4$	$p \propto r^2$

Table 3.2 Actual Orbital Data from the Solar System Compared with the Predictions of the Three Possible Gravity Laws Considered by Newton

Power law	Actual period	Radius	Period predicted if $A_g \propto \frac{1}{r}$	Period predicted if $A_g \propto \frac{1}{r^2}$	Period predicted if $A_g \propto \frac{1}{r^3}$	Eccentricity
Venus	225	0.72	264	224	138	0.01
Earth	365	1.00	365	365	365	0.02
Mars	867	1.52	556	687	1292	0.09
Jupiter	4331	5.22	1906	4355	51960	0.05
Saturn	10747	9.11	3324	10032	275723	0.05
Neptune	59800	30.18	11016	60518	10033936	0.01

whirled about in a sling. The stone endeavours to recede from the hand that turns it, distending the sling, and as soon as it is let go, it flies away. But he says:

> It may be, however, that philosophy *a priori* deduces what the experimental method, which calls itself philosophy, undertakes to know falsely and with unfelicitous success from experiments, seeking as it does with blind zeal and by means of the senses the simulacrum of the true concepts of philosophy.[27]

Hegel criticizes the experiments Newton conducted with pendulums to establish the equality of inertial and gravitational mass.[28] Hegel claims that all that Newton has done is show that the weights of different bodies are equal. Hegel completely misses the point of the experiments, that is, that we have no *a priori* reason to assume that gravitational mass as weight should equal inertial mass: resistance to acceleration. It is an empirical fact that the two are the same, but it does not necessarily have to be the case.

Physicists speculate that 'dark matter' has inertial mass but only interacts with ordinary matter through gravity. Conversely, charged particles with inertial mass could, if Newton was wrong, interact with ordinary matter through electrostatic force, but not via gravity. However, the existence of these particles is still an unanswered empirical question. Newton's pendulum experiments, which verified the equivalence of gravitational and inertial masses, sit in a historical sequence of such tests.

Forces other than gravity were known in Newton's time, such as the tension on a cord measured by weight or a spring balance. Newton experimented with pendulums using gravitational force, but clock balance wheels could have been used. They rely on the force of a spring working against the mass of the wheel to set the period of oscillation.

That a philosopher with Hegel's repute for conceptual analysis should have so completely failed to understand the conceptual distinction between weight and inertial mass tells us something.

His work on planetary orbits is a farrago of nonsense and national prejudice in which he understands neither Kepler nor Newton nor observed facts. He claims, for example, that the Earth is flattened at the equator:

> Lastly, let us say that those things agree remarkably well with the figure of an Earth wider elsewhere than at the equator, the diameter of which is shorter than the axis.[29]

Among other oddities, Hegel held that it was not the great mass of the Sun that accounted for its gravitational attraction. He instead thought it exerted attraction because it was luminous:

His argument went as follows: Newton believed that the centre of gravity should not be placed in the sun as it shifts slightly due to planetary attractions. However, a centre of orbits is required to prove propositions concerning curvilinear motion. Newton assumed the reciprocal attraction of attracting and attracted bodies, where both revolve as if about a common centre of gravity. The common centre of gravity is a purely mathematical point. The sun's centre of force is not due to necessity but rather due to its massive size, which is based on the hypothesis that every force depends on mass. Physical philosophy suggests that the true centre of forces is the source of light, where the sun's true force and power must be posited.[30]

Given his odd and counterfactual views, it is no surprise that Hegel's account of astronomy soon passed into obscurity. Insofar as he had a positive reputation in the twentieth century, it is down to his being seen as having played a role in the formation of modern communism. We will recount later how, in the period around the Russian Revolution, he was credited with having contributed one of the component parts of communist doctrine. Lenin said that Marx and Engels defended philosophical materialism. Their views were exemplified in Engels, *Ludwig Feuerbach* and *Anti-Dühring*, which served as handbooks for class-conscious workers. Marx, he said, expanded on eighteenth-century materialism, enriching it with German classical philosophy, particularly Hegel's system, which had led to Feuerbach's materialism. Lenin said Marx's main achievement was dialectics, the doctrine of development and the relativity of human knowledge.

Recent discoveries in natural science, such as radium and electrons, confirmed Marx's dialectical materialism.[31]

This stamp of approval encouraged Marxists to pay attention to him, and in the process, Hegel's otherwise forgotten objections to the materialist science of the seventeenth and eighteenth centuries were partially revived in the twentieth century. We will later look at the debate between the Soviet Mechanists and the Hegel-influenced Deborin school. In this later debate, we see the Deborin side echoing some of Hegel's objections that mechanics relies on 'external' forces. In Hegel's case, the argument against what he saw as external forces was explicitly religious since God's actions are not external, mechanical, arbitrary or fortuitous. According to experimental philosophy, the forces that God gave to matter truly dwell within it and constitute its nature. However, mechanics cannot understand this idea since it deals only with external causes and does not conceive nature through reason.[32]

This same harping on about 'external' forces reappears in the 1930s communist philosophical literature. For example, the American Marxist Emery writes:

> This was what happened to the atom. When it was discovered that it is a system of complex movements, the mechanically minded natural scientist went on a search for the 'last unit' of the system (electrons, subelectrons etc), and the force which holds the system in equilibrium. If all material points exercise only one force, contradictions can only be external. Therefore no inner contradiction, no unity of opposites, is known in mechanics. All contradictions which seem to be immanent are cut in two, externalised into the conflict of forces embodied in different material particles. This is the heritage of Newton's mechanics. But the force which acts in one point is external in relation to that point, too. Mechanics recognises force as the only attribute of the material point, but not as a necessary attribute. It might be abstracted, and matter as such, matter in complete rest, might be conceived. Since rest is taken as the immanent mode of existence of matter, movement has to come from an outside source – force.[33]

Emery was almost as confused about Newtonian mechanics as old Hegel was. Not only does he echo Hegel's old criticism about force being 'external', he also misstates the Newtonian definition of force.[34] Newtonian mechanics also does not deal with 'material points'; it deals with material bodies. Points only arise as a mathematical tool for approximating the behaviour of orbiting bodies. His 'material points' confuse the mathematical treatment which approximates a body by a point at its centre of mass, with the understanding of physicists that this is just an approximation. The approximation that treats a body as a point mass is only valid up to relatively low spatial derivatives of the gravitational field.

If a satellite, for example, were to orbit within what is called the Roche limit, then tidal forces would tear it apart – as happened to whatever solid bodies gave rise to Saturn's rings. The subatomic physics was also half-digested nonsense. There was and is no proposal that there are sub-electrons.

> The material parts of which the world consists are not processes, but atoms in the old sense of the word, rigid, isolated points which have no other attribute besides the forces they possess. Each point can exercise only one force, and therefore, only forces of different points are different in direction. If a hypothetical point would exercise several forces, the mechanist would divide it into several points.[35]

This is again complete nonsense. In classical mechanics, masses or charges have the ability to exert forces on all other masses or charges. When dealing with the orbits of multiple bodies, each body exerts a force on each other, and these forces can be combined through vector addition. For instance, the Earth, Moon and Sun each exert forces on one another, and there is no need to divide the centre of the Earth into two points to handle the forces attracting it to the Moon and the Sun.

Physicists discovered additional nuclear forces to explain atom stability. They predicted hydrogen fusion at high temperatures, which was confirmed by experiments. Sakharov used calculations of electrostatic and strong nuclear forces, not mystical concepts, to build his H-bomb. Disregard for nuclear forces would have hindered Soviet nuclear energy progress.

We continue our critique of the Hegelian interpretation of Marx in later chapters.

Du Châtelet

The Marquise du Châtelet (1706–49) produced the first French translation of Newton's work. She also published the first French physics textbook, *Institutions Physiques*.[36] Along with her companion Voltaire, she was one of the most important advocates of the Newtonian system in France at a time when there existed considerable national controversy between what were seen as national champions in philosophy: Descartes, Newton and Leibniz. In the introduction to her textbook, she rejects this nationalism and writes:

> The search for truth is the only thing in which the love of your country must not prevail, and it is surely very unfortunate that the opinions of Newton and of Descartes have become a sort of national affair.[37]

We are including du Châtelet in this account because, in addition to translating Newton, she contributed to clarifying Newtonian physics as it is now understood. Her textbook retains considerable influence from Leibniz[38] and includes a number of what would now be seen as rather metaphysical arguments about methodology.

She starts out by defending the use of hypotheses in physics, differentiating herself from Newton in this. She argues that it is only hypotheses in the form of the unintelligible and meaningless jargon of the religious schoolmen that must be rejected.

As an example of this, she gives the medieval scholastic hypothesis that plants grow because they have a vegetative soul. This hypothesis, she says, gives an apparent cause for growth, but when one pursues it further, it is meaningless: 'because it contains nothing that helps us to understand how the vegetation of which I seek the cause operates'.[39] But she held that hypotheses remain essential. To the Newtonians, she objects that advance in astronomy has depended heavily on hypotheses. She says that if astronomers had waited for 'the true theory' of what regulated planetary motion, no progress would have been made. Instead, astronomers took heliocentrism as an initial working hypothesis:

> Thus, they began to explain and to predict phenomena by this hypothesis, called Ptolemy's hypothesis, until the insurmountable difficulties of the consequences that derived from it when compared with observations, and the impossibility of constructing tables according to this hypothesis which were in accord with the phenomena of the sky, brought Copernicus to abandon it entirely and to test the opposite hypothesis, which is so much in agreement with the phenomena, that its certitude is at present not far from demonstration; and that no astronomer dares adopt that of Ptolemy.
>
> Hypotheses must then find a place in the sciences, since they promote the discovery of truth and offer new perspectives; for when a hypothesis is once posed, experiments are often done to ascertain if it is a good one, experiments which would never have been thought of without it. If it is found that these experiments confirm it and that it not only explains the phenomenon that one had proposed to explain with it but also that all the consequences drawn from it agree with the observations, its probability grows to such a point that we cannot refuse our assent to it, and that is almost equivalent to a demonstration.[40]

She constructs several rules for hypotheses to be useful.

1. It must not contradict the principle of sufficient reason.
2. You must have certain knowledge of the facts as they stand so as not to base your hypothesis on a narrow foundation. Otherwise, your hypothesis will soon be overthrown by facts that you neglected to pay attention to.
3. A hypothesis must not be passed off as the truth itself before one has irrefutable evidence to support it. You have to be sober in your estimate of the probability of your hypotheses and not insist that others must accept it.

These are views which would be accepted by most scientists today. On the other hand, she retains a number of essentially metaphysical ideas from Leibniz, holding that the principle of continuity, introduced by Leibniz, was a fundamental concept in physics. It stated that nature does not change suddenly, and any being cannot move from one state to another without passing through all the intermediate states between them.

Classical physicists, like Leibniz, confused the abstract continuity of ideal curves with the properties of the material world. The 'real' continuum and continuous differential functions are abstractions necessary for differential calculus. These abstractions allowed classical physicists to make accurate predictions about the motion of substantially sized bodies. They assumed that the material world also had ideal properties, like the mathematical abstractions they used.

The advent of quantum mechanics in the twentieth century showed that nature does in fact make jumps, but one cannot fairly criticize du Châtelet for her lack of foresight. On this, she was just expressing a view with which most scientists would have agreed until 1905.

Du Châtelet argues that atoms, insofar as they are of finite size, cannot be the fundamental elements of matter since the fundamental elements would have to be point particles.

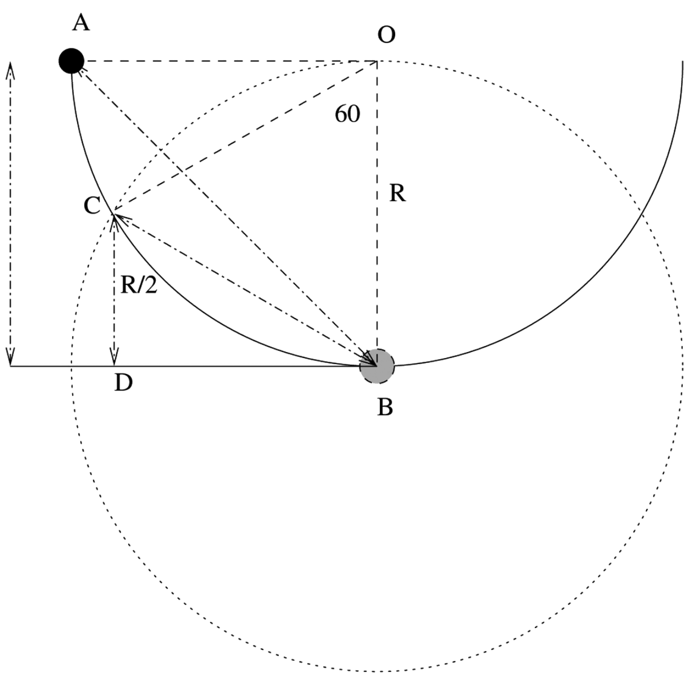
> Now, as there is no contradiction in the divisibility of extended beings, the indivisibility of atoms cannot be accepted as necessary; thus, one must come to simple beings.[41]

Her great contribution was introducing a clearer idea of energy than there is in Newton.

In the course of describing experiments with pendulums, Newton makes the striking remark '*For it is a proposition well known to geometers, that the velocity of a pendulous body in the lowest point is as the chord of the arc which it has described in its descent*'.

It is clear that from this relationship between chords and velocities, one could deduce another equivalence relation/conservation law – that of the equivalence of potential and kinetic energy. But Newton fails to formulate this explicitly as a conservation law. The conservation relation can readily be demonstrated using the form of geometric argument that Newton uses in the *Principles*.

Consider the following diagram.[42]



Consider the pendulum with centre O. The diagram shows two possible starting points for its swing, A and C.

If it starts at A with the angle AOB being 90 degrees, it will, according to the above, have a velocity at position B proportional to the chord AB. If it starts at C, with COB being 60 degrees, then it will have a velocity proportional to chord CB.

It is easy to demonstrate that if the radius of the pendulum is R, then the height CD = R/2.

Further, since COB is equilateral, we know CB = R, and since AOB is an isosceles right triangle, then AB = $\sqrt{2}$R .

So we have the relation between chords $\frac{AB}{CB} = \sqrt{2}$, but that between heights of fall $\frac{OB}{CD}$ =2.

If the velocity is proportional to the chords, then the velocities are proportional to the square root of the heights. If we postulate kinetic energy e, du Châtelet's *forces vives*, such that $e \approx v^2$ we have the kinetic energy proportional to initial potential energy. Since Newton establishes that pendulums regain their original height in the absence of air resistance, he has a conservation law here. The earlier statement in the Scholium about falling bodies, that 'the spaces described in proportional times are as the velocities and the times conjunctly; that is, in a duplicate ratio of the times', could also be used to deduce the same conservation relationship.

Newton only explicitly identifies a conservation of momentum, not a conservation of energy. He said that the force applied by a screw is proportional to the force applied by the hand that turns the handle. The force applied by a mallet to a wedge is proportional to the progress of the wedge. These principles apply to all machines. More generally:

> I was only willing to show by those examples the great extent and certainty of the third Law of motion. For if we estimate the action of the agent from its force and velocity conjunctly, and likewise the reaction of the impediment conjunctly from the velocities of its several parts, and from the forces of resistance arising from the attrition, cohesion, weight, and acceleration of those parts, the action and reaction in the use of all sorts of machines will be found always equal to one another.

This is again close to a conservation of energy principle for ideal simple machines. He is saying

$$f_1 v_1 = f_2 v_2$$

but velocity is distance over time $\frac{d}{t}$, so over a fixed time interval we have

$$f_1 d_1 = f_2 d_2$$

which corresponds to the later definition of work as force times distance, saying work in equals work out for an ideal simple machine (disregarding friction etc.).

What Newton had was conservation of momentum and equivalence of forces, but without seeing through to the modern formulation of work or energy.

Du Châtelet advanced on Newton and was a proponent of what she called *living force*, what we would now call energy. She overloads the term *force* here, but she gets right that it must be proportional to the square of velocity:

> *Forces vives* may be the only point of physics which some still dispute while acknowledging the experiments that prove it; for if you ask those who reject them what would be the effects of two bodies equal in mass on two equal obstacles, but the speeds of which are 4 and 3, they will answer that one will be an effect, as 16 and the other as 9. Now, it is easy to see that, whatever distinction and whatever modification they next bring to this acknowledgment that the force of truth draws from them, it always remains certain that the effect being squared, there must have been a squared force to produce it.[43]

She says that Newton had concluded that there was no overall conservation law regarding motion and that he believed instead that:

> motion is constantly diminishing in the universe; and lastly that our system will some day need to be formed anew by its Author, and this conclusion was a necessary consequence of the inertia of matter, and the opinion held by M. Newton that the quantity of force was equal to the quantity of motion.[44]

She claimed that in this Newton was demanding continuous miracles to sustain motion, asserting instead that:

> But when the product of the mass by the square of the speed is taken as force, it is easy to prove that the forces vives always remain the same, although the quantity of motion varies perhaps at each instant in the universe, and in all the cases, and especially in that which I have just cited from M. Newton, the forces vives stay invariable.[45]

This is the first explicit formulation of the principle of energy conservation. Lucretius' nothing from nothing extended to motion. Du Châtelet's successors Lagrange (1736–1813) and Rankine (1820–72) extended the idea, Lagrange by systematizing the mechanical dynamics of frictionless systems in terms of kinetic and potential energy. Finally, Rankine explicitly states the conservation of energy as a law of nature. The obstacle to generalizing it as a law had been in dealing with frictional losses. While Lagrange could formulate a law governing the planets' orbits in terms of potential and kinetic energy, it was far less obvious that it could be applied to systems on the Earth, where friction slowed things down. The recognition that heat was another form of energy which balanced the loss of kinetic energy, allowed a general principle of energy conservation to be established. These ideas had been well established by the time Marx wrote *Capital*, and we will later show how conservation principles underpin his analysis.

Mechanical determinism

Aristotle categorized causes into efficient and final. Efficient causes work from present to future, while final causes are states in the future that cause events in the present.

Newton's mechanical materialism relies on efficient causes. The current positions and momenta of all celestial bodies fully dictate their future positions and momenta. The existence of other celestial bodies may make the calculations more complex, but they do not alter determinism. Other forces, such as electrostatic and magnetic forces, were discovered in the late nineteenth century, but the mechanics remained the same.

Mechanical determinism is paradoxical because it implies both efficient and final cause. The position and momentum of the Earth today determine where it will be tomorrow and constrain where it must have been yesterday. Forces t_0 dictate the rate of motion. Astronomers can infer future planetary positions by integrating forward and past positions by integrating backward.

The laws of mechanics possess time symmetry. This means that reversing the time dimension and movements of the bodies cause time to run backward. The distinction between efficient and final cause becomes irrelevant. The present state of the solar system constrains its past and future. This concept becomes clearer when using formulations of mechanics by Lagrange or Hamilton, which reason about complete trajectories. Variational calculus, an extension of differential calculus, is used to reason about these trajectories.

The Hamiltonian and Lagrangian mechanics arose by analogy with optics.

If a light beam is shone into water at an angle, it bends. Conversely, if you place a straight ruler partially under water, the ruler appears bent. This is a consequence of the bending of light on the air/water boundary. The classical mechanists knew that although light from the ruler to your eye takes a bent path, the path it 'decides' to take is such that it minimizes the time a photon will take to traverse the distance. Since water is denser than air, light travels more slowly through it, and in consequence the quickest route to your eye is one in which the photon spends more of its time going through air than it would were it to go on a straight line.

This law of optics, known as Fermat's principle, already seemed to undermine naive notions of time, cause and effect. How could the light ray setting out from the tip of the ruler know what path to take to reach your eye fastest?

Euler's calculus of variation can be used to show[46] that the trajectory the light follows turns out to be the quickest that it could take. From the standpoint of time, light follows the simplest possible path. One of Euler's students, Lagrange, then had the insight that perhaps the trajectories laid down by Newton's laws might be treated the same way. But in this case the simplest path could not simply be the quickest. It had to be simple in another sense.

The answer he came up with was that nature follows the course of least action. Action here is a scientific metaphor whose precise mathematical definition is non-obvious to say the least.[47] Lagrangian mechanics is a mathematical implication of Newton's laws. Using Lagrange's method, computations for many problems are easier. It shows how nature imposes constraints on trajectories, similar to light's path. While it's easy to visualize a single particle's trajectory, it's harder to understand a pair or a multiplicity of particles.

In school maths, one is taught about trajectories in which y is a function of x. We learned to plot parabolas and learned that these have characteristic equations of the form $y = ax^2 + b$. This is all simple and easy to visualize since x, y represent distances along the horizontal and vertical axes on a sheet of graph paper. It is also straightforward to express movement in two dimensions mathematically in terms of x, y coordinates. We have an intuitive notion of what two dimensions or three dimensions means. We accept that we live in three-dimensional space and that we draw our pictures on two-dimensional sheets of paper. But what would a six-dimensional or a twelve-dimensional space be?

What is a twelve-dimensional trajectory?

When the classical physicists dealt with systems of many particles or multiple coupled independently moving bodies, they resorted to an abstract form of space they called *configuration space*.

Consider a single moving particle. Its position and velocity in three-dimensional space can be written down as six numbers, three to give its x, y, z coordinates and another three, $\dot{x}, \dot{y}, \dot{z}$, which mean its velocity components in the three cardinal dimensions. Mathematically, this is a six-dimensional system. If you have two particles, you have twelve numbers etc. This set of abstract dimensions is called configuration space.

What does the abstract space of mechanics have in common with the real space that we learn about from Euclid?

In what sense can we speak of dimensions and trajectories when all we actually have are signs on paper, $x_1, y_1, z_1, \dot{x}_1, \dot{y}_1 .... \dot{z}_n$?

How do the signs and squiggles relate to real space?

If we return to the two-dimensional example, we can see that there is an operational basis for the equivalence of positions on graph paper and pairs of numbers. You can count grid lines in the two dimensions to convert a dot on the paper to x, y coordinates or, conversely, you learn to plot a point on the paper given two written numbers. Architectural and surveying practice created a link between classical geometry and the physical world. Architectural diagrams were used to build larger buildings sharing Euclidean similarity. This correspondence was established through practice.

In the *Principia*, Newton still used classical Euclidean geometry. By the mid-eighteenth century it becomes standard to reason about physics in terms of Cartesian coordinates. This involves a shift in the material practice of modelling. Instead of using the ruler and compass, one uses algebra. With Euclidean techniques one can get analogue results. You follow the geometric construction using specific radii and then obtain a result measuring along new lines that you have drawn. If you want the result to be more accurate than you can manage with a ruler, you have to devise some ultimately Pythagorean process of construction in terms of right triangles, after which you explicitly compute squares and square roots.

The physicist used the Cartesian method to derive an equation for a planetary trajectory. The astronomer used Lagrangian methods to parameterize the actual orbit. While the relationship between equation and orbit isn't immediately clear, we can learn to use methods that are more accurate and efficient than old geometric approaches. In both cases, we have a physical paper model of the Moon's motion on Earth.

Let us return to what a dimension means.

In the context of human practice, dimensions are related to bodily movement. Look at a map of the eighteenth-century New Town of Edinburgh.[48] It took an Enlightenment sensibility to construct the urban environment according to the geometric postulates of Descartes, but the same procedure, writ large, was to be employed to map whole nations, plotting every road junction, hill or bridge on Cartesian grids.

Walking east along George Street gives me one dimension. I can then turn left into North Castle Street. This gives me a second dimension, the north-south one. What makes the dimensions distinct is that walking north does not move me in any way east. If my destination was St Andrew Square, I would still have to go three blocks along Queen Street and then turn left into North St David Street. My walk to the north did not reduce the number of blocks I still had to go east.

What makes the north-south and east-west dimensions distinct is their independence. Movement on one axis gets me no further along towards the other. The two dimensions are my degrees of freedom in a street stroll. The variables in the equations of Lagrange, $x_1, y_1, z_1, \dot{x}_1, \dot{y}_1 \ldots x_2, y_2 \ldots \dot{z}_n$, are also independent degrees of freedom for the multi-particle system. The x_1 denotes position of the first particle in the x direction, y_2 the position of the second particle in the y direction, and $\dot{z}_n$ the motion of the nth particle in the z direction. All of these can vary independently, and so in an abstract metaphorical sense they are dimensions. A point in configuration space is therefore a particular combination of positions and velocities for all particles. A trajectory in configuration space must then be a sequence of these points.

Now here is the crucial point: *the equations of Lagrange specify a unique time trajectory of the entire multi-particle system through multi-dimensional configuration space.* Lagrangian determinism specifies the totality of all movement just as strongly as it specifies the movement of one body. The entire universe has a configuration space – the positions and velocities of all the atoms. The universal trajectory through configuration space is the product of all the individual trajectories in real space.

If you are new to mechanics and the mechanical approach, this takes some time to sink in.

It follows that the universe too has a unique trajectory through configuration space, a path of least action throughout time. This is a determinism to shame Calvin or Knox.

On 15 February 2013, over the major industrial city of Chelyabinsk in central Russia, an intense ball of fire appeared in the sky. Brighter than the Sun, citizens out in the open could feel its intense heat on their skin. Luckily for the residents of Chelyabinsk, the 500 kiloton detonation took place at an altitude of about

30,000 metres. Nonetheless, many were injured and hospitalized. Flash blindness, radiation burns, peeling the skin off, blast injuries and cuts from flying debris were reported. Shock-wave damage spread over hundreds of square kilometres. Thousands of blocks of flats, schools, hospitals and factories had to be repaired.

The cause? Not one of the American Minuteman missiles pointed at Chelyabinsk since the 1960s, but a 20-metre lump of rock from the asteroid belt. Its path of least action, looping through the solar system for countless ages, terminated over the city. Witnesses saw it emerge in the low Western sky, from the direction of the rising sun. Fleeing its closest solar approach on its way back to the asteroid belt, bound by Kepler's law, it had to be travelling fast. It raced along at 60,000 km/h on a hyperbolic path that, had it not grazed the atmosphere, would have led it back into outer space. Instead, kinetic energy flashed into heat, radiation and blast. Had the angle of approach been steeper and the explosion lower, Chelyabinsk would have been flattened.

But in orbital dynamics, there are no might-have-beens. There was no way that its path could have been any different. To revert to classical language, it was *fated* to narrowly spare the city. Though Chelyabinsk lived, it was still a salutary reminder. Zeus may hurl his bolts, or the Phaethon may fall without warning.[49]

NASA tries to grant warnings. Their 'Sentry Earth' project searches out dangerous asteroids. Yet Chelyabinsk's superbolide had not been spotted by NASA.

Searches are bounded by the available telescopes.

Even for the ones they have spotted, no exact predictions are made. They instead give probabilities. You can go to their website and view the risks they have spotted. For example, an asteroid called 2020 VV is estimated to have a 0.23 per cent chance of striking us sometime between 2044 and 2111. Luckily, this would only be a 36 kiloton hit. But why, given the determinism of orbits, is the estimate only in terms of probabilities? In 2021, NASA sent Phoenix Lander to Mars using Lagrangian mechanics. It landed on time, within a few metres of its intended destination. If NASA can do that, why are asteroid collisions a game of dice?

NASA has made seventy-nine observations of 2020 VV, an object 12 metres across, between October and December 2020. These observations were made using telescopes from millions of kilometres away. NASA must estimate the object's positions and velocity based on these observations, but the exact position on the screen is uncertain. As a result, the initial information is fuzzy, and the predictions made based on this information will also be fuzzy.

To the best of our understanding, the macroscopic laws of motion are deterministic. But given our ignorance, both in terms of things we have not yet observed and in terms of the imprecision of our observations, there appears to be an element of chance. An unobserved meteor may hit at any instant. For the observed ones, we know at most a window through which they will pass on a given day.

Lagrangian materialism is absolutely deterministic, and apparent randomness is the product of our ignorance. Whether modern materialism necessarily implies determinism, and what such determinism means, is something we will return to. For now, let us examine the structure of knowing created by classical mechanics.

What we have is a multilayered process of algorithmic modelling.

The differential calculus and the calculus of variations are procedures by which formulae can be transformed. Using them, Newton's initial laws could be transformed into Euler-Lagrange equations of motion, including terms for potential and kinetic energy. In modern language, these equations are a class of software.

They are relatively high-level software since they involve functionals, or functions which transform other functions. They are rules for taking functions which give kinetic and potential energies in terms of generalized coordinates and transforming these into new functions of motion.

The new functions for motion themselves need to be parameterized by initial boundary conditions. Once the full parameterization has occurred, you then derive functions with only one free variable – time.

At this point, you can switch your algorithmic operations from the algebraic term rewriting of Euler to the original *algorithismes* that we are taught in school as long multiplication and addition.[50] With these familiar techniques, detailed calculations were done using decimal arithmetic.

Nowadays, the parameterized functions of motion would be encoded in Fortran or some similar language, and NASA computers would use the code to compute trajectories.

When combined with a process of material computation – either the paper and pencil labour process of a human mathematician or the cycling of an electronic computer – the parameterized functions constitute a model for the system under study, be it Jovian moons, near-Earth asteroids etc.

The modelling relationship that exists is between the real-time motion of the moons or asteroids and the simulated computational time of the computing machine or old-style human computer.

In both cases, we have material systems, with moons and planets on one side and computational systems on the other. They both evolve through time. The time evolution of the computational system will typically be faster than that of the real world. The model/system equivalence is itself established within certain accuracy constraints by observational labour processes. People use tools – telescopes, notepads, keyboards, processors etc. – to determine initial positions and check final positions.

Scientists use practice to verify knowledge. In rocket science, practice has verified the ontological reality of simulated systems. Mars was once just a red dot in the sky, but now we watch videos of his rocks and dunes up close and hear Martian breezes.

For modelling the real world, procedures for constructing Lagrangians or Hamiltonians are means of production for models, not models themselves. Actual models are built through collective labour processes and observation. They are not mere 'thought objects', but localized physical processes that consume energy and evolve through time as computations are performed.

Chapter 4

Dialectics, materialism, change from Epicurus to Marx via Aristotle

Dialectics before its Hegelian reinvention

It is presumably a widely known fact that, in Greek antiquity, the term *dialectics* (*dialektikê*) referred to the 'art', as Aristotle calls it in *Rhetoric*,[1] of correct thinking, one that ought to be the prerequisite and the inevitable characteristic of scientific or philosophical inquiry.[2] Any examination of a matter that could lead to a truthful or accurate conclusion, a 'logical one' (or 'syllogical') as Aristotle would put it,[3] any pursuit of truth that could call itself philosophical, operates on the premises of what we would nowadays call logic. Aristotle would have called it dialectics – *logikê* does not appear until late antiquity, and when it does, it is still not in the sense we use the word in our era of modernity. In *Rhetoric*, Aristotle makes sure to distinguish a good argument in the sophist sense – thus not necessarily driven by 'the love of wisdom' – from a properly dialectical argument. The latter is concerned with values, or ethics, to be more precise, the former with the skill or art – the *technē*, Aristotle would say[4] – of accurate argumentation itself, which can be entertained by any individual and not necessarily a professional philosopher (or its equivalent in Greek antiquity).

In this chapter, we will attempt to reconstruct the appearance of dialectics as an ontological category on the scene of European philosophy. It seems that the origins of materialist dialectics as we find it in the communist doctrine of Diamat can be traced back to Hegel but not quite to Marx himself. This may seem counterintuitive, or as something that goes against everything we have been taught and have gotten used to thinking on the matter regarding Marxism and Marx's own body of work and legacy. Nonetheless, a closer look at Marx's writings on the dual topic of materialism and dialectics, seen as a unity as well as two separate categories, will show us that his treatment of dialectics has always been closer to that of the Ancients than to that of Hegel.

We must note a slight exception to this rule in his doctoral thesis, in which we can detect a form of reasoning that operates to a certain (and arguable) degree similarly to the Hegelian model of dialectics, whereas his materialism grounded in the atomist theory contradicts the logic of the Hegel-inspired Diamat. Marx's reading of Greek atomism informs his uncompromisingly materialist stance, something we will inspect more closely further on in the chapter. At this point, suffice it to say that the contradiction that underpins Epicurean atomism, according to young Marx, is the product of the properties that define an atom materially and the form that is the atom's conceptual determination. The latter, however, does not constitute a higher form of truth or purpose of existence, *causa finalis*. It is nothing that would go beyond quite simply being the concept of a material phenomenon – the atom, always already embodied and never endowed with an ideality or participation in a greater and all-encompassing idea.[5] Ideality is thus not an idea but a perfect form of the material whose predication in terms of property is necessarily an aberration from the ideal form – or simply the concept, as Marx puts it. The concept is abstracted from the concrete but does not reside in a realm of ideality, and it cannot inhabit any other universe but that of materiality – because there is no such world.

We will, therefore, look at the two categories, materialism and dialectics, as distinct and unilateral to one another,[6] without the canonically presumed unity of the two when it comes to the Marxist tradition of thought, be it 'scientific' (Marxism) or philosophy. Marx will lead us retrospectively to look for the scientific origins of materialism – or for the origins of the materialism that is closer to a scientific mode of inquiry than to a philosophical-theological one – in Greek atomism, more specifically that of Epicurus and the famous Epicurean Lucretius Carus. Before we do so, let us revisit the question of dialectics in Greek antiquity and the possible routes of its evolution to the Hegelian understanding of the term.

Dialektikê, mechanicity and materiality of cognition

In encountering philological-philosophical discussions of the Greek notion of dialectics, in particular the one in use in Aristotle's works, one often becomes bewildered. How could the practice of 'debating', or rather the culture of public polemical exchanges typical of the Greek *polis*, be confused with 'logic' (the art of thinking worthy of the attribute philosophical, or nowadays scientific)? Yet we do not believe it is a matter of confusion but a formalization of the observed practice of dialectical exchanges through the abstraction of its rules or simply the laws of thinking accurately or in line with a consistent pursuit of truth. Thus, similarly to Marx's treatment of the 'concept of the atom' – or its form – Aristotle abstracts the ideal form of what we would nowadays call logical thinking. It is as if a mould of the observed material reality is detached from its foundation and, by way of applying 'the art' of dialectical thinking, its image (or form) further formalized as a 'concept' (like Marx's concept of the atom), very much like Wittgenstein's Maßstab in the *Tractatus*.[7] The concept is the product of a 'correct way of thinking', as Aristotle argued in *Physics* (214a12–214a16), or the dialectical or syllogistic way of thinking discussed below through a close reading of passages from Aristotle's *Rhetoric*. The art consists in being capable of discerning the 'real syllogisms', which are those of dialektikê, from the illusionary syllogism, which belongs to rhetoric: ὥσπερ καὶ ἐπὶ τῆς διαλεκτικῆς συλλογισμ όν τε καὶ φαινόμενον συλλογισμόν (from Aristotle's *Rhetoric* 1: 1),[8] or in English (W. Rhys Roberts' translation): 'art to discern the real and the apparent means of persuasion, just as it is the function of dialectic to discern the real and the apparent syllogism'.[9]

Dialectics, as used in Aristotle, concerns the pure form of 'syllogical' thinking, universally applicable to any subject matter. As already said, it is a meta-science that he in fact calls 'art' (craft or skill – *technē*) of thinking that can lead one to truthful conclusions. Emptied of the pursuit of truth, it will remain rhetoric, whereas what makes it dialectical and apt to serve proper scientific exploration would be its ethical concern – the interest in truthfully explaining matters of surrounding reality.

> There are, then, these three means of effecting persuasion. The man who is to be in command of them must, it is clear, be able (1) to reason logically [in the original: λαβεῖν τοῦ συλλογίσασθαι δυναμένου; or, in other words – by way of syllogism], (2) to understand human character and goodness in their various forms, and (3) to understand the emotions – that is, to name them and describe them, to know their causes and the way in which they are excited. It thus appears that rhetoric is an offshoot of dialectic and also of ethical studies. Ethical studies may fairly be called political; and for this reason rhetoric masquerades as political science, and the professors of it as political experts – sometimes from want of education, sometimes from ostentation, sometimes owing to other human failings. As a matter of fact, it is a branch of dialectic and similar to it, as we said at the outset. Neither rhetoric nor dialectic is the scientific study of any one separate subject: both are faculties for providing arguments. (Aristotle, *Rhetoric* 1: 2)[10]

The presence of ethical investment – or the lack of it – affects not only the content of a syllogism and of the art of argumentation itself but also its form, rendering rhetoric an art inferior to that of dialectics, producing sophists instead of philosophers:

> It is clear, then, that rhetoric is not bound up with a single definite class of subjects, but is as universal as dialectic; it is clear, also, that it is useful. It is clear, further, that its function is not simply to succeed in persuading, but rather to discover the means of coming as near such success as the circumstances of each particular case allow. In this it resembles all other arts. For example, it is not the function of medicine simply to make a man quite healthy, but to put him as far as may be on the road to health; it is possible to give excellent treatment even to those who can never enjoy sound health. Furthermore, it is plain that it is the function of one and the same art to discern the real and the apparent means of persuasion, just as it is the function of dialectic to discern the real and the apparent syllogism. What makes a man a 'sophist' is not his faculty, but his moral purpose. In rhetoric, however, the term *rhetorician* may describe either the speaker's knowledge of the art, or his moral purpose. In dialectic it is different: a man is a 'sophist' because he has a certain kind of moral purpose, a

> 'dialectician' in respect, not of his moral purpose, but of his faculty. (Aristotle, *Rhetoric*: 1: 1)[11]

Aristotle invents the craft of abstraction and the formalization of its processes – such as syllogisms, later to become algorithms – as the practice of pure and semantically empty form of thought. He invents the *technē* of algorithmic thinking. Both his use of abstraction (or form) and his dialectics are a matter of *technē*, not of ontology – the outside reality does not need to be dialectical or 'syllogical', but our reasoning of it must be, just as an algorithm does not need to reflect any nature of outside reality but can nonetheless convey it and mime it too. The invention of formal thinking and of logic and algorithmic disciplining of thought is the product of a practice extracted from the material and the concrete. It took thousands of years for this realization to be rediscovered and elucidated, first by Marx (defending the thought of the concrete in *Grundrisse*, discussed below) and then by Saussure, who exclaims in his *Course in General Linguistics*:

> For the first time we have broken away from abstraction. Now for the first time we have found the concrete, irreducible units that occupy a place and correspond to a bent in the spoken chain: p was nothing except an abstract unit linking the common characters of p> and p<, the only units that actually exist. In the same way, the still higher abstraction of 'labiality' links together P, M and B. We may speak of P as if it were a zoological species; there are male and female representatives of the species, but there is no ideal specimen. Before, we had been singling out and classifying the abstractions; but we had to go beyond the abstract to reach the concrete. Phonology made a great mistake in considering abstractions real units without examining more carefully the definition of the unit. (1959, 53)[12]

Saussure demonstrates not only that any formal procedure of analysis, which is the path to scientific truth, must depart from the material but also that the material is concrete and that the abstracted concepts ought not to be mistaken for self-standing realities. This is a vindication of materiality, of abstraction as the (material) practice of scientific examination, and, finally, of the necessarily mechanistic proceduralism of scientific thought. However, it also discloses the mechanistic aspect of all organicity – in this case, language. Marx's explanation of the nature of true materialist inquiry and the method of his political economy is very similar:

> if we were to begin with the population, this would be a chaotic conception [Vorstellung] of the whole, and we would then, by means of further determination, move analytically towards ever more simple concepts [Begriff], from the Imagined concrete towards ever thinner abstractions until we had arrived at the simplest determinations. From there the journey would have to be retraced until we had finally arrived at the population again, but this time not as the chaotic conception of a whole, but as a rich totality of many determinations

> and relations. [. . .] The concrete is concrete because it is the concentration of many determinations, hence unity of the diverse. (*Grundrisse*, 41)[13]

Mechanical acts of extrapolating form from a material subject of study at hand (Marx, partly Aristotle) or the act of signification in its most basic instance of 'a radical dyad' – here we are referring to the relation between two signifiers, which does not necessarily need to be considered a unity, a couple or a binary, as conventional structuralist linguistics would treat it – come down to mere material differentiation, always already using a material mark (phonemes, writing traces or otherwise) between two signifying units. We have argued elsewhere that within the 'radical dyad', a term borrowed from François Laruelle,[14] the two elements are unilaterally related to one another, the one being the real and outsideness to the other. The idea of a unity of two, of a binary as the primary element of signification, is a matter of philosophization or philosophical hallucination, to paraphrase Laruelle, as the relation within a chain is differentiation between one and another unit, between one and zero, and they do not exist as a binary in the materiality of signifying practice. To quote Paul Cockshott from our private correspondence in August 2020: 'A simple binary is not even a relation since it is unordered. It is { +,-} == {-,+}, simply a set with two distinguishable signs. No different from {α,ω} or {■,□} etc. Without some other semantics, there are just two distinguished marks.' Nonetheless, these acts of mechanicity amount to multiple forms of organicity: the signifying automata of natural and artificial languages, or, as Yuk Hui has argued recently, even in nature itself.

In Hui's *Contingency and Recursivity*, the latter is also treated as a process of signification, and, as Hui argues, it is the procedure of recursion one also finds in organic life that enables mechanicity to always already turn into organicity, the two being mere sides of a Moebius stripe rather than mutually exclusive categories. Contingency is bereft of telos; it is almost nonsensical, and mechanicity as mechanicity is senseless too (it is 'whatever works', which is in its nature contingent). Nature resorts to recursion in order to include the contingent, just as AI does. According to Yuk Hui, there is only a non-teleological telos, circularity of life sustaining itself:

> The natural end is something that cannot be observed objectively. We can see such and such a tree or such and such an animal, but we cannot grasp nature as a whole through mechanical rules. Reason can only understand the natural end through reflective judgment, meaning that it recursively arrives at a self-organising being. Teleological thinking is in this sense circular: A→B→C→A. (Hui, 41)[15]

Mechanicity and materialism are conditioned upon one another, and we are using the term here quite differently from, if not in the opposite sense to, that deployed by Descartes in reducing the body to a lifeless machine, while mind – *res cogitans* – is superior and not of the realm of materiality in any way whatsoever. Therefore, everything that seems to breathe life is moved by Mind; all immanent, seemingly moved by and of itself, is moved by mind; and the organic is spiritual. Materialism that draws from atomism and Marx, from Saussure, Turing and Boltzmann, is mechanistic (Cockshott Turing refers to his chapter here on Mechanicity plus the paper/s on Turing).[16] Yet it is the movement of the uncuttable smallest particles of matter enabled by their constant mechanic encounters – similarly to the constant collisions caused by Epicurus' swerve – that can transform itself into waves, languages spoken by humans and gods, social relations and human productivity. All that is organic and self-regulated is originated or premised on the crude mechanicity of coarse materiality. We argue elsewhere in the book that the forms of materialism we consider uncompromised by idealist philosophization are either mechanicist or they embrace atomism, or both: Newton and Lagrange, and Boltzmann and Markov, to name a few. If mechanicity is reduced to materiality as an inferior reality allowing for an entire realm of existence that is explained neither mechanically nor organically (by way of 'recursive' integration of 'nonsensical occurrence' of mechanical default), we are dealing not only with idealism but also with a notion of mechanicity that does not play the role of the primary modality of materialism. We are quite evidently not dealing with materialism as the *metaphysical* foundation in the search of the *archē* (sometimes in plural) of the all-existing. Here, one author might differ from the co-authors of this book in the use of the term *metaphysical*. As a proto-scientific mover of thought, and of all human productivity for that matter – consisting in questions such as *Why do we exist on this planet? What is good? Why is there an outsideness and am I separated from it?* and *Does the universe have a beginning?* – any metaphysical question is relevant and can be tackled in a way that circumvents philosophy and the effects of the 'principle of sufficient philosophy'.[17] Mechanicity – very often combined with atomism, elevating movement and change among its first principles – is what defines materialism proper. The residues of the Eleatic origin of rationalism and philosophy, which consist in seeking the principles or the *archai* in what is unchanging and undying, are unavoidable in all forms of idealism, including materialism indebted to philosophy, as argued by Marx in his passages on Feuerbach in *German Ideology* and elsewhere. Consider the following passage:

> The chief defect of all hitherto existing materialism – that of Feuerbach included – is that the thing, reality, sensuousness, is conceived only in the form of the object or of contemplation, but not as a sensuous human activity, practice, not subjectively. Hence, in contradistinction to materialism, the active side was developed abstractly by idealism – which, of course, does not know real, sensuous activity as such. (Marx, *Theses on Feuerbach*)[18]

What all of the materialists mentioned have in common, a shared constant across centuries, is that unlike the greatest part of the authoritative philosophy of their era, they argued that movement is not an 'apparition' behind which a static substance resides – the truth of Being, with the Being itself unmovable and eternal – but rather that which is both the apparent and the hidden truth of existence: constant movement and change.

Radical movement and historical materialism rather than dialectical materialism

Historical materialism should be seen as a radically materialist understanding of movement and change, more indebted to the early atomists than to Hegel. In his interpretation of Epicurean atomism, Marx displays a Hegelian influence insofar as he seeks a dialectical solution to the contradiction between the concept of the atom and the concrete properties that make it material. We discussed at the beginning the way Marx solves this contradiction, which amounts to the foundation for his materialist philosophy, building on the legacy of Epicurus. According to Thomas Nail, Marx began to develop his materialist philosophy and scientific world view prior to his exposure to Hegel, leading him to lay the foundations for what Nail calls kinetic materialism (rather than dialectical materialism):

> Marx's theory of motion is nowhere more evident and focused than in the earliest writings of his doctoral dissertation and Epicurean notebooks. Long before Marx had read Hegel, he had already been working out a critique of religion and a new theory of materialism through ancient atomism. After his exposure to Hegel, Marx used his dissertation to simultaneously critique the reactionary modernist and enlightenment interpretations of ancient atomism, materialism, idealist theories of freedom, religion, and the Hegelian philosophy of nature. Thus, the theory of motion and materialism put forward in Marx's dissertation is not just a Hegelian residue; it is his first effort to develop a materialist and kinetic theory of dialectics. (Nail, 20–1)[19]

We concur with Nail that we can speak of kinetic materialism in Marx and that it is derived from Epicurean atomism, as it is precisely the atomist theory of movement – and the invention of the 'declination from the concept', or the swerve, meaning the imperfection of movement – that enables a purely materialist grounding for his philosophy and what is to become science. However, we argue that Marx did not arrive at materialism to provide a theory of dialectics but instead quite the opposite: he used the dialectical method, mainly as developed by the Greeks and Aristotle in particular, to create a materialism premised on the idea of incessant movement, or as Nail has termed it, kinetic materialism. As already noted, one can detect Hegel's influence when dialectics is no longer only a method but also builds an ontological argument. For example, the tension between the concept and the material determinants or properties of the atom is presented as material, or at least real, in the sense of the notion of 'real abstraction' developed by the Marxist epistemologist Alfred Sohn-Rethel.[20] The 'concept' is indeed always already materialized, inescapably so, since, as Marx explains (as discussed above), there would be no concept without the material properties, and it nonetheless seems to be treated as an active agency – ontological entity:

> The contradiction between existence and essence, between matter and form, which is inherent in the concept of the atom, emerges in the individual atom

> itself once it is endowed with qualities. Through the quality the atom is alienated from its concept, but at the same time is perfected in its construction. It is from repulsion and the ensuing conglomerations of the qualified atoms that the world of appearance now emerges.[21]

Marx and Epicurus do not postulate separate realms that are independent from one another. The contradiction is material, and its dialectical solution is not a resolution; the contradiction is perpetual, and it is what maintains material reality in constant movement. The atom is 'invisible', yet its movement and other properties, such as *gravity*, enable the visible world (or the one that appears to us), the phenomenon of an all-moving and all-changing existence. In his interpretation, Marx parts ways with the greater share of European philosophy and its inherent idealism, as he does not relegate the visible world to a realm of falsities, apparitions and hallucinations – it is 'no less true' than the invisible one. Quite to the contrary, 'the invisible atoms are realised' only through their phenomenal form (the properties that include movement). A true materialist affirms the reality of the phenomenal world and also treats it as the departure point for accessing the invisible one – the one is no less real or true than the other:

> The one [Democritus] is a sceptic, the other [Epicurus] a dogmatist; the one considers the sensuous world as subjective semblance, the other as objective appearance. He who considers the sensuous world as subjective semblance applies himself to empirical natural science and to positive knowledge, and represents the unrest of observation, experimenting, learning everywhere, ranging over the wide, wide world. The other, who considers the phenomenal world to be real, scorns empiricism; embodied in him are the serenity of thought satisfied in itself, the self-sufficiency that draws its knowledge *ex principio interno*. But the contradiction goes still farther. The sceptic and empiricist, who holds sensuous nature to be subjective semblance, considers it from the point of view of necessity and endeavours to explain and to understand the real existence of things. The philosopher and dogmatist, on the other hand, who considers appearance to be real, sees everywhere only chance, and his method of explanation tends rather to negate all objective reality of nature. There seems to be a certain absurdity in these contradictions.[22]

Scepticism and any variant of subjective idealism – we would include here the poststructuralist understanding of the 'Real's radical foreclosure' – are predicated on the expectation that human thought has full access to, if not fusion with, the real, amounting to the equation truth = real (and vice versa). This, according to Laruelle, is what constitutes the grounding fallacy of all philosophy.[23] In the opening pages of *Philosophy and Non-Philosophy* ([1989]; 2013), Laruelle terms said fallacy the amphibology of real and thought[24] whose principle *par excellence* is the principle of all European philosophy – the idea of *being* or tò ón (τὸ ὄν) as the unity of truth and reality – also termed as the principle of sufficient philosophy (PSP).[25] Marx admires Epicurus for not falling into the trap of the same principle that Laruelle identifies as the self-defeating, autophagist foundation of philosophy – dismissing the world of 'appearances' as phenomena deceiving instead of affirming that its materiality is the only blueprint of the invisible foundation (of the atoms) and perhaps also the necessity of their realization. The 'apparition' or the 'deceiving phenomena' of the visible world (as Parmenides, Plato, Berkeley, Hegel, Judith Butler and, in one way or another, all philosophy has argued, at least as per the analysis of Laruelle and Marx we subscribe to here) are the 'sensuous world' Marx writes about as the only foundation a true materialist science – or a philosophy rid of the PSP, we would add – should ground itself in. 'Objective appearance' is an instance of the real in and of itself; it is as objective as social relations or values, as elaborated in materialist epistemological detail by Alfred Sohn-Rethel, exemplified in the notion of 'real abstraction'.[26] At the end of the paragraph, Marx concludes that there is an absurdity in the contradictions underpinning Epicurus' philosophy. These contradictions can be explained, but through the insidious idealism in all philosophy, as Marx explains in his writings on Feuerbach and the German ideology and its expectations. As already noted, form or concept and the atom's properties – or simply the properties of the invisible material foundation and *archē* (ἀρχή) of everything – constitute a dialectical continuity, a dialectics that works as a perpetual recursion of the continent or 'chance' rather than a sublimation (das Aufgehobene) amounting to transcendence and the Hegelian understanding of dialectics.

The foundations of Diamat and their contradictions

After more than a century of Marxism and the centrality of the notion of dialectical materialism (or the dogma of Diamat),[27] it is quite a challenge to demonstrate that the compound notion of dialectics and materialism was not the cornerstone of Marxism in its original form, that is, Marx's form, and that he viewed dialectics and materialism as distinct categories not necessarily and unavoidably constituting a unity. His method of moving away from an abstraction, which is in fact a vague philosophical generalization, to arrive at the concrete, only to extrapolate another new form of abstraction (determined by the concrete), would not allow for a mixture and an amphibology of method, nor a claim about an ontological foundation of reality, that is, of dialectics and method, respectively. Much of what followed in this strange detour from what we argue was Marx's original intention is probably indebted to the legacy of Diamat (dialectical materialism), both as consolidated by the Comintern and as the official party doctrine of the USSR consolidated after Lenin's death.

In *Materialism and Empirio-criticism*,[28] Lenin praises Joseph Dietzgen for coining the notion of 'dialectical materialism', thus having produced a proper materialism, one practiced by way of the method of dialectics, or as materialist dialectics, to be considered as an important addition to Marx's and Engel's original doctrine.[29] In this treatise penned by Lenin, we see one thing very clearly – Marx is not presented as the thinker who developed dialectical materialism but rather as the one who offered grounds for it to be developed by Dietzgen. However, Lenin's use of the method of 'dialectical materialism' is not centred on constituting an ontology of the classical, Stalinist Diamat type but rather serves to refute subjectivism as inherently idealist and proffer a defense of Marxism as uncompromising materialism. The triad of thesis, antithesis and synthesis, and thus the reconciliation of materialism and idealism, is neither Lenin's object of interest nor his objective in the title at issue:

> A red thread that runs through all the writings of all the Machists is the stupid claim to have 'risen above' materialism and idealism, to have transcended this 'obsolete' antithesis; but in fact this whole fraternity is continually sliding into idealism and it conducts a steady and incessant struggle against materialism.[30]

Lenin's *Materialism and Empirio-criticism* is first and foremost a stark defense of materialism, and it is hardly – if at all – a defense of the notion, then still novel, called 'dialectical materialism', which was never really accepted by Marx and his immediate circle.

Evald Ilyenkov's reading of both Marx and Lenin proves that the kernel of Marx's – and, for that matter, Lenin's – materialism is the creation of an episteme allowing for thinking in terms of the concrete and constituting thought of objectivity.

> The most important aspect of Marx's definition of the concrete is that the concrete is treated first of all as an objective characteristic of a thing considered quite independently from any evolutions that may take place in the cognising subject. [. . .] Concreteness is not created in the process of reflection of the object by the subject either at the sensual stage of reflection or at the rational-logical one. In other words, 'the concrete' is first of all the same kind of objective category as any other category of materialist dialectics, as 'the necessary' and 'the accidental', 'essence, and 'appearance'. It expresses a universal form of development of nature, society, and thinking. In the system of Marx's views, 'the concrete' is by no means a synonym for the sensually given, immediately contemplated.[31]

In Ilyenkov, concrete is, therefore, a form of conceptualization, one determined by the material and shaped objectively, which is achieved through assuming a third party's perspective as per Marx's method, while its 'identity of the last instance'[32] can be an 'abstraction', akin to Sohn-Rethel's notion of real abstraction (discussed above). Put differently, in Laruellian terms, its material can be of transcendental nature, similar to the notion of Laruelle's transcendental or philosophical 'material',[33] whereas its effects are real. Not only are they real, they are also underpinned by a compound of material determinants grounding a structure of meaning similar to what Wittgenstein would call Maßstab, or, in English, scale ('of the real'),[34] or to the notion of 'syntax of the real' as developed by Laruelle in *Introduction to Non-Marxism*.[35] This is the type of dialectics Ilyenkov refers to in the passage cited above, whereby the gap between cognition and the concrete object of cognizing retains its status of a productive contradiction and implies a movement of sublation understood as it is in Marx's doctoral dissertation rather than as a redeeming reconciliation (through unity) of contradicting instances. In other words, the concrete ought to remain concrete instead of undergoing self-transcendence culminating in some form of ideality, even if presented as materialist, such as in Dietzgen's synthesis.[36] The following passage from Ilyenkov offers further corroboration of the comparative interpretation just presented:

> This use of the term 'abstract' is not a terminological whim of Marx's at all: it is linked with the very essence of his logical views, with the dialectical interpretation of the relation of forms of thinking and those of objective reality, with the view of practice (sensual activity involving objects) as a criterion of the truth of the abstractions of thought. Still less can this usage be explained as 'a throwback to Hegelianism': it is against Hegel that Marx's proposition is directed to the effect that 'the simplest economic category, eg exchange value . . . cannot exist except as an abstract, unilateral relation of an already existing concrete organic whole'.[37]

The categories in which materialists think are, by definition and unavoidably, 'abstract', but that abstraction is neither substituted for the real nor do they, the real and thought insofar as abstraction (by definition), constitute a unity of reality and truth – a philosophical realization or actualization of the Being, be it material or ideal. The concrete is not sensual, as both Ilyenkov and Lenin[38] explain, but conceptual. On the other hand, the abstract is not ideal but rather a human product of cognition, an entity of the transcendental order which can be unilaterally positioned vis-à-vis the concrete, and through it the real as the concrete is the closest to the 'scale of reality' mentioned above. In Ilyenkov's analysis, we can see the unilateral duality at work, which is the same one we already noted in Marx's own writings, his dissertation more specifically, as we did in Laruelle's method too, discussed above.

In conclusion, we would like to point out to the fact that, in the era of the rise of the poststructuralist episteme, we witness a perpetuation of the very logic that Sohn-Rethel (and similarly, Ilyenkov too) sought to defeat: without discussing individual authors, we can safely argue that the old division between manual (the physical) and intellectual (immaterial) labour was reinforced, as well as the implication of the superiority of the latter in terms of its emancipatory potential. We are referring to the reception of Sohn-Rethel discussed in Alberto Toscano's paper, *The Open Secret of the Real Abstraction*, focusing mainly on Paolo Virno and Lorenzo Cillario.[39] 'General intellect' seems to be transcending the value form of capitalism, whereas praxis is dismissed by reducing the concrete to the sensuous, physical and sensory, committing the fallacy Ilyenkov warns against, as presented above. The objective of Sohn-Rethel's project, similarly to Ilyenkov's, was to convey an accurate reading, and, based on that, a productive expansion, of the epistemological foundation of Marx's project. In a letter to Adorno, cited by Toscano, Sohn-Rethel writes:

> 'fetish-concept of logic has a different social referent with regard to the fetish-concept of value. The latter refers to the antagonism between capital and labor, the former to the antithesis between intellectual and manual labor.[40]

The two are connected in a 'genetic' sense, as Sohn-Rethel puts it,[41] and in arguing so, he retains orthodox fidelity to Marx: the exchange value form is affirmed as abstraction, albeit treating it as a determining reality (akin to the notion of 'social relations', for example). The abstraction in question does not become real through any kind of (general) 'Intellect' being objectified or realized. Such reasoning would imply the fallacy of reification. The dialectics between use and surplus value is at the core of capitalism; it determines wage labour as commodified. It determines every commodity for that matter, and its very logic of auto-acceleration (M-M' superceding M-C-M) demonstrates that it culminates in the exchange of pure value that transcends commodity or anything material (think of speculative economy).[42] This would be the determination of the last instance of capitalist universe. It is in that way indeed that economic production determines everything in society – it is the beating heart of a universe of social relations that recreates the same model of subjectivation in all forms, ranging from art and science to the organization of kinship and intimacy.

Chapter 5

HISTORICAL AND MECHANICAL MATERIALISTS

Smith and Engels

The materialists we have dealt with up to now concerned themselves with the natural world. They described a world made up of atoms whose movements were governed by natural laws. We now call these laws classical mechanics, and we may thus consider these thinkers to have been mechanical materialists. In Soviet literature, the term 'mechanical materialism' tended to have a rather negative connotation, as something primitive that was later to be replaced by dialectical and historical materialism. As this book goes on, we will argue that these negative connotations are unjustified. Leaving that aside, this section will concern itself with the ideas of two of the most influential Historical Materialists: Adam Smith and Frederick Engels.

Why do we group together two authors who are seen respectively as the arch advocates of capitalism and communism?

It is because their basic materialist approach to history was almost the same. Engel's most influential book was *The Origin of the Family Private Property and the State*,[1] which was published in 1884. Smith's main work on the subject is his *Lectures on Jurisprudence*, which date from the 1760s but were not published until 1978.[2] The similarities in the reasoning of the two thinkers were thus the result of independent discovery, though Engels had undoubtedly some familiarity with Smith's *Wealth of Nations*.

State derives from private property

Government, Smith said, arises due to class inequality.

Smith conceives of four stages of society: hunting, herding, farming and commercial society. The first three correspond fairly closely to the stages that Engels referred to as savagery, barbarism and civilization. Engels, having access to Darwin and at least some archaeology by the 1880s, divides the hunting stage of society into sub-phases. In the first, man still lived in tropical forests and was primarily at least a tree dweller. In the second phase, stone tools and fire are put to use. In the final phase of hunter-gatherer society, hand weaving and bows and arrows are known.

Smith saw the shift from hunting to herding as the first important transition. Engels agrees with this with respect to the Old World but says that the lack of domesticated sheep and cattle in the Americas led to distinct development paths in the Eastern and Western Hemispheres.

Both agreed that the state cannot arise among a population whose mode of subsistence lacks herding or farming.

> The population is extremely sparse; it is dense only at the tribe's place of settlement, around which lie in a wide circle first the hunting grounds and then the protective belt of neutral forest, which separates the tribe from others. The division of labor is purely primitive, between the sexes only. The man fights in the wars, goes hunting and fishing, procures the raw materials of food and the tools necessary for doing so. The woman looks after the house and the preparation of food and clothing, cooks, weaves, sews.[3]

Smith similarly emphasizes the effect of a limited population density.

> In a nation of hunters and fishers few people can live together, for in a short time any considerable number would destroy all the game in the country, and consequently would want a means of subsistence. Twenty or thirty families are the most that can live together, and these make up a village. But as they live together for their mutual defence, and to assist one another, their villages are not far distant from each other. When any controversy happens between persons of different villages, it is decided by a general assembly of both villages. As each particular village has its own leader, so there is one who is the leader of the whole nation. The nation consists of an alliance of the different villages, and the chieftains have great influence on their resolutions[4]

Both agree that it is the transition to herding that gives rise to inequality. Smith described this process as follows:

> In a nation of hunters there is properly no government at all. The society consists of a few independent families who live in the same village and speak the same language, and have agreed among themselves to keep together for their mutual safety, but they have no authority one over another. The whole society interests itself in any offence; if possible they make it up between the parties, if not they banish from their society, kill or deliver up to the resentment of the injured him who has committed the crime. But this is no regular government, for though there may be some among them who are much respected, and have great influence in their determinations, yet he never can do anything without the consent of the whole.
>
> The appropriation of herds and flocks which introduced an inequality of fortune, was that which first gave rise to regular government . Till there be property there

> can be no government, the very end of which is to secure wealth, and to defend the rich from the poor.[5]

That last phrase that the function of government is to secure wealth and defend the rich from the poor is as clear an expression of historical materialism as you would get from Lenin.[6]

Smith directly links the domestication of animals to private property saying: 'the rich had made the game, now become tame, their own property'. The very process that makes previously wild cattle domesticated eliminates them as potential prey animals for hunters. Smith is deploying a concept of 'primitive accumulation' as simultaneously appropriation by one class and pauperizing another that will recur in Marx's analysis of capitalism.

According to the *Lectures*, in a herding society, wealth is all portable and accumulates in the form of animals. Unlike later rulers who can consume wealth as luxuries, a society without settled manufactures provides few luxury goods. The wealth of the chiefs thus takes the form of influence over dependents who lack sufficient animals of their own. The poor are fully dependent on the benevolence of the chiefs. The combination of nomadism and chiefly authority generates incessant warfare.

> The exploits of hunters, though brave and gallant, are never very considerable. As few of them can march together, so their number seldom exceeds 200 men, and even these cannot be supported above fourteen days. There is therefore very little danger from a nation of hunters. Our colonies are much afraid of them without any just grounds. They may indeed give them some trouble by their inroads and excursions, but can never be very formidable. On the other hand a much greater number of shepherds can live together. There may be a thousand families in the same village. The Arabs and Tartars, who have always been shepherds, have on many occasions made the most dreadful havoc. A Tartar chief is extremely formidable, and when one of them gets the better of another, there always happens the most dreadful and violent revolutions. They take their whole flocks and herds into the field along with them, and whoever is overcome loses both his people and wealth. The victorious nation follows its flocks, and pursues its conquest, and if it comes into a cultivated country with such numbers of men, it is quite irresistible. It was in this manner that Mahomet ravaged all Asia.[7]

So during the ages when cattle herding was the predominant form of wealth, you had chiefdoms that were prone to raiding and incessant minor warfare. In his opinion, this corresponded not only to the historic societies of the Tartars (Mongols) and the Arabs but also to the heroic Greek society described by Homer. He notes that all property disputes described by Homer are over cattle.

As agriculture becomes more significant, nomadism becomes impossible, but the most significant store of wealth is still cattle. Society is now able to produce a surplus, which can potentially be traded with neighbouring tribes, but cattle are constantly subject to the risk of being stolen by neighbouring tribes. This forces the construction of fortified settlements into which the people and beasts can move in times of conflict. A fortified town has a bigger population than those that local chiefs ruled over, typically comprising several small tribes. While the first form of government of the town places one chief in a supreme position it is soon replaced by a republican form of state.

> A people inhabiting such a country, when the division of land came to take place and the cultivation of it to be generally practised, would naturally dispose of the surplus of their product among their neighbours, and this would be a spur to their industry. But at the same time it would be a temptation to their neighbours to make inroads upon them. They must therefore fall upon some method to secure themselves from danger, and to preserve what it formerly cost them so much trouble to procure. It would be more easy to fortify a town in a convenient place than to fortify the frontiers of the whole country, and accordingly this was the method they fell upon. They built fortified towns in the most convenient places, and whenever they were invaded took shelter in them with their flocks and moveable goods, and here they cultivated the arts and sciences. Agreeable to this, we find that Theseus fortified Athens and made the people of Attica carry into it all their goods, which not only increased his power over them, but also the authority of that state above others. When people agreed in this manner to live in towns, the chieftains of the several clans would soon lose their authority, and the government would turn republican, because their revenue was small, and could not make them so conspicuous and distinguished above others as to retain them in dependence. The citizens gradually increase in riches, and coming nearer the level of the chieftain, become jealous of his authority. Accordingly we find that Theseus himself was turned out.
>
> After this nine regents were set up who were at first to have authority for life, but were afterwards continued only for ten years. Thus Athens, and in like manner all the Greek states, came from a chieftainship to something like monarchy, and from thence to aristocracy.[8]

Engels describes the same process, though he misses out on some of the economic and military processes that Smith identified. Remember that although Smith's work was earlier, Engels did not have access to it, and Engels was not an economist of the stature of Marx or Smith.

> The constitution ascribed to Theseus was introduced. The principal change which it made was to set up a central authority in Athens – that is, part of the affairs hitherto administered by the tribes independently were declared common affairs and entrusted to the common council sitting in Athens. In taking this step, the Athenians went further than any native people of America had ever done: instead of neighboring tribes forming a simple confederacy, they fused together

> into one single nation. Hence arose a common Athenian civil law, which stood above the legal customs of the tribes and gentes.
>
> By a second measure ascribed to Theseus, the entire people, regardless of gens, phratry or tribe, was divided into three classes: eupatridai, or nobles, geomoroi, or farmers, and demiourgoi, or artisans, and the right to hold office was vested exclusively in the nobility. Apart from the tenure of offices by the nobility, this division remained inoperative, as it did not create any other legal distinctions between the classes. It is, however, important because it reveals the new social elements which had been developing unobserved. It shows that the customary appointment of members of certain families to the offices of the gens had already grown into an almost uncontested right of these families to office; it shows that these families, already powerful through their wealth, were beginning to form groupings outside their gentes as a separate, privileged class, and that the state now taking form sanctioned this presumption.[9]

According to Smith, while the Greeks and the Romans initially had aristocratic forms of government, with the aristocratic families inherited from the pre-urban class of chiefs, two economic processes tended to shift the state towards a more republican form.

The first was the existence of slavery. This allowed free, but non-aristocratic citizens to attend the assembly while their slaves minded the farm or workshop. He contrasts this with the more modern Italian and Dutch republics which, he says, were exclusively run by the aristocracy.

Secondly, the development of the division of labour and the production of luxury goods meant that the aristocrats supported fewer retainers, choosing instead to spend their wealth on luxuries. Since they spent less on their clients, their ability to swing votes in the assembly diminished. Populists pushed to allow plebs to be elected to office. As a result of these two economic processes, in both Rome and Greece, elected offices became open to non-noble citizens.

What we have here is a model of society in which political forms arise and change as a result of the mode of material life. Not only that, the history of these urban, civilized societies is, to paraphrase Engels and Marx, a history of class struggles.

Smith was also aware that any form of constitution or government was historically transitory. Each political and economic form generated within itself the processes which would ultimately lead to its downfall. Speaking of the fall of the Roman Empire, he wrote:

> We come now to show how this military monarchy came to share that fated dissolution that awaits every state and constitution whatever.[10]

For each period, the specific conflicts that led to the dissolution were different, but certain common themes recur. The development of the division of labour allows the upper classes to spend more on luxuries. Under feudalism, this growth in luxury consumption meant that the aristocrats had less to spend on retainers. As their retinues declined, so too did their military influence, allowing the growth of the absolute monarchies of the early modern period. His argument being that the revenues of the Crown were so large that it was hard for a king to spend it all on luxuries. Some was always left over for the army, and as the barons built luxury homes rather than maintaining hordes of retainers in simple halls, the military Power Royal was triumphant.

This same theme of economic development leading to military change was deployed in his account of the fall of Rome. As society became more 'polite' and people became acquainted with domestic luxury, 'they become less fond of going to war'. An economically developed state finds that its tax revenues are hurt if it mobilizes its manufacturing population for war, while a barbarian society or a purely agricultural one can send its men off to war during the summer gap between sowing and harvest. Nomadic herdsmen were the most able to mobilize their entire populations for conquest, since they had no immovable possessions and the entire nation could venture forth to conquer. So it was that soldiers could be recruited more cheaply from among the barbarian nations nearby than from among the Roman citizens, and this could be done without harm to the industries of the Romans. The leaders of the barbarian armies, in due course, found that they could turn their arms against the Roman government and make themselves masters of the country.[11] This process, he claimed, happened repeatedly:

> In the same manner all the Asiatic governments were dissolved. Their soldiers were hired from Tartary, arts and manufactures were carried on, the people made more by their trades than by going to war.

There are echoes of Khaldun[12] in the account of the fall of 'Asiatic governments', Smith's analysis, however, is set within a much broader historical materialist theory, going from hunting society to capitalism, a vast perspective that reappears in the work of Engels and Marx. When one materialist appears to extend ideas that an earlier one started to work on, whether it be the use of infinitesimals by both Archimedes and Newton, labour value in Khaldun and Smith or the progression of modes of production in Smith and Engels, there is a temptation to suppose that transmission not reinvention was involved.[13] Since the Al-Muqaddimah was only available in Arabic in the eighteenth century, and the Lectures on Jurisprudence only became available long after Engels was dead, this seems unsupported.

Diderot's non-anthropocentric materialist world view

Diderot's materialism is what one might call, following Thomas Nail's use of the term, kinetic materialism: *motion* is behind the creation of organic matter, or the transition from inert to organic (living) matter. Differently from the traditional philosophical views, including Aristotle's, Diderot does not define motion and change as 'displacement in place' premised on the equation of empty space and ontological nothingness which precluded, or paralysed as in the case of Aristotle, any attempt to think about change and movement (discussed in Chapter 2 of this book). Physical processes linked to heat – a particular 'thermodynamic' thesis,[14] at the centre of Diderot's dialogue with D'Alembert – lead to the creation of organic life in its ever-increasing complexity, while the difference between human and animal is seen only as a matter of degree. The foundation of it all is material, and the first and indivisible building bloc is the atom.[15]

> But take an egg. This is what refutes all the schools of theology and all the temples on earth. What is this egg? A mass that is insensible until the embryo is introduced into it, and when this embryo is introduced, what is it then? An insensible mass, for in its turn, this embryo is only an inert and crude liquid. How does this mass arrive at a different organisation, arrive at sensibility and life? By means of heat. And what produces heat? Motion.[16]

Furthermore, sensation, argues Diderot in the already cited dialogue with D'Alembert, is built into matter itself. In short, Diderot explains that all cognition depends on the organization of the material living components as they ensue from sensation, which amounts to different forms of organisms (animal, for example, vversus humans) with different cognitive faculties. To quote Diderot himself (also as the character appearing in the cited dialogue, responding to D'Alembert): 'a simple supposition which explains everything, namely, that the faculty of sensation is a general property of matter, or a product of its organisation'.[17] As far as the difference between human and animal is concerned, as far as cognition is concerned, Diderot is explicit:

> Would you maintain with Descartes that this is a simple imitating machine? Little children will laugh at you, and the philosophers will reply that if this be a machine then you too are a machine. If you admit that the difference between these animals and you is only one of organisation, you will prove your common sense and sagacity, you will be right. But from this will follow a conclusion against you; namely, that from inert matter organised in a certain way, impregnated with another bit of inert matter, by heat and motion—sensibility, life, memory, consciousness, emotion, and thought are generated. One of the two, continues Diderot, either admit some 'hidden element' in the egg, that penetrates to it in an unknown way at a certain stage of development, an element about which it is unknown whether it occupies space, whether it is material or whether it is created for the purpose – which is contradictory to common sense, and leads to inconsistencies and absurdities; or we must make a simple supposition which explains everything, namely, that the faculty of sensation is a general property of matter, or a product of its organisation. (Diderot to D'Alembert)[18]Little children will laugh at you, and the philosophers will reply that if this be a machine then you too are a machine. If you admit that the difference between these animals and you is only one of organisation, you will prove your common sense and

> sagacity, you will be right. But from this will follow a conclusion against you; namely, that from inert matter organised in a certain way, impregnated with another bit of inert matter, by heat and motion—sensibility, life, memory, consciousness, emotion, and thought are generated.

Contemporary experimental philosophy, operating with knowledge from the cognitive sciences including evolutionary biology, demonstrates that 'abstracting' or pattern recognition happens on the level of protein mass and cell organization,[19] as it does on the level of, for example, bee's ability of counting.[20] The ability of the higher forms of 'physical organization', to paraphrase Diderot, that is, of the animals other than humans, to 'abstract' or to recognize mathematical patterns and even count, in the twenty-first century, is by no means a controversial idea.[21] Apart from the metaphysical reveries of Reason among the so-called neo-rationalists,[22] hardly any philosophy of science or the sciences themselves linked with the interdisciplinary and multidisciplinary area of cognitive sciences believes in Reason as the exclusive faculty of humanity. Reason has been rendered obsolete as a notion by contemporary science just as its opposites, such as nature or emotion, have been – they are all part of the same continuum of informational ecosystems.

Sense, cognition and mental design are derived from the material foundation, and thus the 'purpose' of social relations of the 'species being of humanity', to borrow Marx's term,[23] is the product of material necessity to sustain and expand life or its 'form' as design of particular 'automaton', self-moved and self-sustained system, as Aristotle would define it (my paraphrase or rather interpretation of Marx's definition). In Marx's own words:

> Man is a species-being [Gattungswesen], not only because in practice and in theory he adopts the species (his own as well as those of other things) as his object, but – and this is only another way of expressing it – also because he treats himself as the actual, living species; because he treats himself as a universal and therefore a free being. The life of the species, both in man and in animals, consists physically in the fact that man (like the animal) lives on organic nature; and the more universal man (or the animal) is, the more universal is the sphere of inorganic nature on which he lives. Just as plants, animals, stones, air, light etc constitute theoretically a part of human consciousness, partly as objects of natural science, partly as objects of art – his spiritual inorganic nature, spiritual nourishment which he must first prepare to make palatable and digestible – so also in the realm of practice they constitute a part of human life and human activity. Physically man lives only on these products of nature, whether they appear in the form of food, heating, clothes, a dwelling etc. The universality of man appears in practice precisely in the universality which makes all nature his inorganic body – both inasmuch as nature is (1) his direct means of life, and (2) the material, the object, and the instrument of his life activity. Nature is man's inorganic body – nature, that is, insofar as it is not itself human body. Man

> lives on nature – means that nature is his body, with which he must remain in continuous interchange if he is not to die. That man's physical and spiritual life is linked to nature means simply that nature is linked to itself, for man is a part of nature.[24]

The material activity which is life treats organic and inorganic matter (nature or 'nature', as this notion of nature is far from the romantic understanding of German idealism, as well as from Feuerbach's materialism) as an extension of the substance or definition of humanity or rather constitutes an ecological unity with it. Thus, the essence and the purpose – the being of the species – consist in sustaining and expanding life, whereas life is both organic and inorganic and it is determined by activity both human and its inhuman extension or expansion into the co-determining ecosystem of nature and technology. Life is mechanic, and it is thus inorganic in that sense and not in the sense of the Cartesian reductionist equation of the animal and the contempt-worthy clock-like machine (as opposed to the organic, natural as if moved by a sublime or godlike substance). It is the product of the *mekhane* or *techne* of piecing bits of information any form of matter is endowed with and perpetually executes – ranging from protein folding to phonetics of the so-called natural languages – and thus the 'rganic' is merely a matter of degree, not of substantive difference from the 'mechanic'. Still, there is a continuity and also difference of mere degree and not of substance between what we commonsensically and in some more archaic scientific language mean by 'life'. This is what the contemporary science teaches us as did the atomists, in particular the Epicurean Lucretius, as well as young Marx and Lenin.[25]

The 'purpose' or the telos one might speak of in this sense is apparently not linear, it does not operate as *causa finalis*, and we are, therefore, not certain that one can speak of 'telos'. We will use the term on a provisory basis, in the absence of a more fitting one. In *Contingency and Recursivity*, Yuk Hui explains that the 'telos' of the organic or nature, understood in line with German idealism and Schelling in particular, comes down to a perpetual contingency and recursion akin to the ones we find as founding principles of computing automata. Despite Hui's allegiance with Schelling and idealism, the operations his analysis reveals demonstrate that the 'ideal' is inextricable from nature. Also, one can read his analysis as demonstrating a certain continuity between mechanicity and organicity, the dichotomy is redundant and serves no purpose in the analysis.[26] Or in Hui's own words:

> The natural end is something that cannot be observed objectively. We can see such and such a tree or such and such an animal, but we cannot grasp nature as a whole through mechanical rules. Reason can only understand the natural end through reflective judgment, meaning that it recursively arrives at a self-organising being. Teleological thinking is in this sense circular: $*A \to B \to C \to A$[27]

The set of functions that amount to maintaining the design (form and function) of the 'species being' of humanity, which is built of social relations and with the goal of sustaining its existence and enhancing its effectiveness, is circular.

In other words, the 'being' is perpetually self-organizing in production and reproduction while improving the ecosystem between technology and nature (that which is not artificially built, which is not the product of the human *technē* or artfulness, the literal meaning of *technē*). One seems to be talking of an automaton here, in Aristotle's sense – a being unmoved by an external cause, or by a mover, but also bereft of *causa finalis*, or, as Aristotle explains, it operates as if it had a telos nonetheless it doesn't have one (as it is void of Reason).[28] Only the 'unmoved mover' is self-moved, and it is an Idea,[29] the 'why' of it all – that is why the automaton, the self-moved material reality resembles a purposeful and meaningful reality, yet it is not – in Aristotle's universe.

It is often said that Aristotle's influence on Marx has been considerable, and if one abstracts the notion of the perfect Idea contemplating itself or the 'unmoved mover', much of the dialectics between form and matter developed by Aristotle can be seen as reflected in Marx's thought, including his doctoral dissertation. In said thesis, Marx puts forward the following argument:

> Through the qualities the atom acquires an existence which contradicts its concept; it is assumed as an externalised being different from its essence. It is this contradiction which mainly interests Epicurus. Hence, as soon as he posits a property and thus draws the consequence of the material nature of the atom, he counterposits at the same time determinations which again destroy this property in its own sphere and validate instead the concept of the atom. He therefore determines all properties in such a way that they contradict themselves. Democritus, on the other hand, nowhere considers the properties in relation to the atom itself, nor does he objectify the contradiction between concept and existence which is inherent in them.[30]

There is no external idea to aspire to or seek to fulfil – by returning to it through self-annihilation of the material as in Hegel's vision – but rather 'the concept' of the atom is unavoidably materialized and validated as such through its objectivization via material properties. This dialectic of 'form' (concept) and matter resembles Aristotle's metaphysics (in its original, Aristotelian sense) far more than that of Hegel or the other idealists. Similarly, social (including technological) design is the product of the immediate and material needs and class *interests* of the proletariat – it is not ideal(ist) in any sense of the word, including 'ethical' or moral(ist).[31]

Idealism is subjectivity-centred, it is anthropocentric, not only according to Marx but also according to Diderot:

> Those philosophers are called idealists who, being conscious only of their existence and of the sensations which succeed each other within themselves, do not admit anything else. An extravagant system which, to my thinking, only the

> blind could have originated; a system which, to the shame of human intelligence and philosophy, is the most difficult to combat, although the most absurd of all.[32]

In 'Critique of Hegel's Philosophy in General' (part of *The Economic and Philosophic Manuscripts of 1844*)[33] Marx discusses the concept of subjectivity as being what defines philosophy, and a resort to subjectivity as the key factor of undermining any attempt of 'Marxist' thought to establish itself as scientific.[34] The type of objective thought Marx advocates is in no way similar to August Comte's positivism. Quite the opposite, it is premised on decentring thinking from the (individual) human subject. To quote Marx:

> To be objective, natural and sensuous, and at the same time to have object, nature and sense outside oneself, or oneself to be object, nature and sense for a third party, is one and the same thing.[35]

The use of 'sensuous' and 'natural' should be linked to Diderot's heritage rather than the German vitalists as it is anti-anthropocentric – even when radically humanist – and the use of both terms, including that of 'the physical and the real' is far more common in Marx when he makes a materialist argument (rather than 'matter' or 'material'). I find the reasons for this terminological choice in his avoidance to be equated with the other young-Hegelian materialism, that of Feuerbach, a question I discuss in detail in my bookKaterina Kolozova, (Punctum Books, 2015).[36] *Toward Radical Metaphysics of Socialism*. Let us note the similarity in this closing note from Diderot (the already cited dialogue with d'Alembert):

> d'Alembert: For instance, your system doesn't make it clear how we form syllogisms or draw inferences.
>
> Diderot: We don't draw them; they are all drawn by nature. We only state the existence of connected phenomena, which are known to us practically, by experience, whose existence may be either necessary or contingent; necessary in the case of mathematics, physics, and other exact sciences; contingent in ethics, politics and other conjectural sciences.[37]

We believe we have demonstrated in these passages dedicated to Diderot's materialism and its links with Marx that both are neither naturalism in the philosophical sense nor some sort of animistic vitalism, but rather non-anthropocentric materialist world views. At the same time, they do propose 'humanist' ideals and visions of a more just universe of (human) social relations, but the emphasis on them being the result of ecosystems that transcend the capitalist exploitative attitude towards 'natural resources' is more than clear in both authors.

The Newtonian Marx

> One nation can and should learn from others. And even when a society has got upon the right track for the discovery of the natural laws of its movement – and it

> is the ultimate aim of this work, to lay bare the *economic law of motion* of modern society – it can neither clear by bold leaps, nor remove by legal enactments, the obstacles offered by the successive phases of its normal development. But it can shorten and lessen the birth-pangs.[38]

The aim, stated above from the preface to Capital, was to establish a law of motion for modern society. In other words, to do for modern society what Newton had done for the natural world. How does the method of Marx in Capital fit into the Newtonian scientific practice?

> for all the difficulty of philosophy seems to consist in this – from the phænomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phænomena. (Newton, introduction to the *Principia*)

The method of Newton was,

1. From experiment and observation to formulate laws of motion.
2. By deduction, via geometry to then derive general properties of centripetal forces.
3. From the periods and orbital radii of the satellites of Jupiter to deduce that the gravitational force must be inverse square.
4. From an inverse square law of gravitation to deduce more accurate predictions about the planetary orbits.

When we look at the process of argument from the abstract to the concrete in Marx, we see a very similar process.

- Formulation of conservation laws in both cases.
  - Newton: equal and opposite action, conservation of momentum,
  - Marx: equal values in exchange, sum of values conserved in trade
  - These initial axioms are observationally based.
- Movement from abstract laws to concrete conclusions about motion in both cases.
  - Newton: centripetal forces sweep equal area in equal time, gravitational force inverse square
  - Marx: rate of profit falls as capital accumulation intensifies

Right from the start of Capital, Marx is concerned to establish that commodity exchange involves a conservation relation:

> Let us take two commodities, eg corn and iron. The proportions in which they are exchangeable, whatever those proportions may be, can always be represented by an equation in which a given quantity of corn is equated to

> some quantity of iron: eg 1 quarter corn = x cwt. iron. What does this equation tell us?
>
> It tells us that in two different things – in 1 quarter of corn and x cwt.[39] of iron, there exists in equal quantities something common to both. The two things must therefore be equal to a third, which in itself is neither the one nor the other. Each of them, so far as it is exchange value, must therefore be reducible to this third.
>
> A simple geometrical illustration will make this clear. In order to calculate and compare the areas of rectilinear figures, we decompose them into triangles. But the area of the triangle itself is expressed by something totally different from its visible figure, namely, by half the product of the base multiplied by the altitude. In the same way the exchange values of commodities must be capable of being expressed in terms of something common to them all, of which thing they represent a greater or less quantity.[40]

So, if an exchange takes place between $\frac{1}{4}$cwt of corn = x cwt Iron both sides contain the same amount of something: Newton's principle of action producing an equal and opposite reaction.

The equivalence is even explained in terms of Greek geometric approach, used by Archimedes, of proving the equivalence of areas by breaking them down into triangles. Archimedes used this to derive the area of a circle as being πr^2 even though, at a surface level, a circle and a triangle are very different. Marx's argument is directly in the Archimedes/Newton tradition.

The form of exchange value is something that is empirically observed. From the properties of this observation, Marx deduces that there is a conservation process involved. There is some substance, *value*, that is conserved in an exchange of equivalents. This maps onto Newton's deduction of the conservation of *motion*, or as we now say momentum, from the form of motion of colliding pendulums.

He says exchange is an equivalence relation. Formally, we would now say that an equivalence relation is a binary relation that is reflexive $a \equiv a$, symmetric $a \equiv b \Rightarrow b \equiv a$ and transitive $a \equiv b, b \equiv c \Rightarrow a \equiv c$.

Writing before modern symbolic logic notation, Marx does not set his argument for exchange being an equivalence relation in exactly this form, but using his own invented notation, he says the same things. He states transitivity with what he terms his Form B of value, which he writes down symbolically as:

> **B. Total or Expanded Form of value**
> z Com. A = u Com. B or = v Com. C or = w Com. D or = x Com. E or = etc.[41]

He explicitly establishes reflexivity in a subsequent passage that directly gives in words the implication $a \equiv b \Rightarrow b \equiv a$.

> The expanded relative value form is, however, nothing but the sum of the elementary relative expressions or equations of the first kind, such as:

> 20 yards of linen = 1 coat
> 20 yards of linen = 10 lbs of tea, etc.
> Each of these implies the corresponding inverted equation,
> 1 coat = 20 yards of linen
> 10 lbs of tea = 20 yards of linen, etc.[42]

The symmetry relation (x commodity A = x commodity A) was too obvious to need stating.

His deduction from these observed properties is that because there is an equivalence relation, there is some abstract third thing behind the equivalence. An equivalence relation is an indicator of conservation. The deduction that this common substance exists is structurally equivalent to the deduction that a common substance *energy* is exchanged between potential and kinetic forms during a planet's orbit around the Sun. We only know of the existence of energy via the equivalence relationships that it imposes on motions.

By the time Marx was writing Capital, work on the conservation of energy had gone much further than just kinetic and potential energy. The science available to him was discussing energy conservation in electricity, chemical energy, heat and light. The idea of a common substance, energy present in multiple forms, was being actively discussed between him and Engels prior to writing Capital.

> Another result that would have delighted old Hegel is the correlation of forces in physics, or the law whereby mechanical motion, ie mechanical force (eg through friction), is, in given conditions, converted into heat, heat into light, light into chemical affinity, chemical affinity (eg in the voltaic pile) into electricity, the latter into magnetism. These transitions may also take place differently, backwards or forwards. An Englishman [Joule] whose name I can't recall has now shown that these forces pass from one to the other in quite specific quantitative proportions so that eg a certain quantity of one, eg electricity, corresponds to a certain quantity of each of the others, eg magnetism, light, heat, chemical affinity (positive or negative — combining or separating) and motion.[43]

From the deduction that there must be a common substance conserved in exchange, Marx immediately identifies this with labour.

> This common 'something' cannot be either a geometrical, a chemical, or any other natural property of commodities. Such properties claim our attention only in so far as they affect the utility of those commodities, make them use values. But the exchange of commodities is evidently an act characterised by a total abstraction from use value. . . .
>
> If then we leave out of consideration the use value of commodities, they have only one common property left, that of being products of labour
>
> . . .

> Along with the useful qualities of the products themselves, we put out of sight both the useful character of the various kinds of labour embodied in them, and the concrete forms of that labour; there is nothing left but what is common to them all; all are reduced to one and the same sort of labour, human labour in the abstract. Let us now consider the residue of each of these products; it consists of the same unsubstantial reality in each, a mere congelation of homogeneous human labour, of labour power expended without regard to the mode of its expenditure. All that these things now tell us is, that human labour power has been expended in their production, that human labour is embodied in them. When looked at as crystals of this social substance, common to them all, they are – Values.

Modern commentators, raised on marginal economics, see this as a leap too far. While they may be willing to accept his conclusion that exchange is an equivalence relation, they believe Marx's conclusion that labour is the conserved quantity to be unsound. We now have considerable empirical evidence that Marx was correct.[44] This new evidence relies on modern economic statistics like I/O tables. Marx did not have this data, but at the time he was writing, the determination of value by labour was already taken as an established truth of political economy. The determination of value by labour had been established by Smith in the Wealth of Nations.[45] Just as Newton accepted Galileo's findings as a given, Marx could assume the determination of prices by labour content was well understood. It was only after the disruptive implications of Marx's analysis became widely recognized that the economic consensus moved away from Smith's conclusions. To contemporary readers, the reference to labour as the common element may appear arbitrary. However, Marx's initial step of proving that exchange is an equivalence relation with a conserved substance is crucial for the subsequent phase of his argument.

He symbolically analyses the circulation of commodities with the formula:

$$C_1 \to M \to C_2$$

A seller starts out with commodity 1 (C_1) exchanges it for money M of equal value, and then uses the money to purchase an amount of commodity 2 (C_2). Since exchange is an equivalence relation and as such is transitive, the value of the start and finish are the same $C_1 \equiv C_2$. All is fair and good until he points out that this is not what capitalists do. Their activity takes the form

$$M \to C \to M'$$

Where the final quantity of money is greater than what he starts out with: $M' > M$. The difference $S = M' - M$ is what Marx calls the capitalist's *surplus value*. It is evident that $M \to C \to M'$ violates the equivalence property of commodity exchange.

Previous economic writers had not analysed commodity exchange as a formal system subject to conservation laws. By using a formal analysis, Marx was able to spot a contradiction or symmetry breaking inherent in the existence of profit.[46] How is this possible?

If value is preserved in exchanges, then profit or surplus value cannot be accounted for at the level of commodity exchange. It must be explained by factors external to the exchange, specifically, the actual process of commodity production. The capitalist spends their initial M on two components that Marx labels constant capital c and variable capital v. So $M = c + v$.

Constant capital consists of the typical commodities that a capitalist acquires, such as cotton yarn and coal for the steam engine. Variable capital, a term coined by Marx, refers to the labour power purchased by the capitalist. It is deemed 'variable' because it is the component of the capital that increases in value; unlike constant capital, its value is not fixed but grows. The value of constant capital remains preserved, simply transferring its existing value to the final product without alteration.

Of constant capital, he says:

> this will have no influence on the creation of value or on the variation in the quantity of value. What Lucretius says is self-evident; 'nil posse creari de nihilo,' out of nothing, nothing can be created. Creation of value is transformation of labour-power into labour. Labour-power itself is energy transferred to a human organism by means of nourishing matter.[47]

What a great passage! It brings together Atomism that was the subject of Dr Marx's PhD with concepts of power, work and energy.

Where a historical materialist of the eighteenth century like Smith just saw in wages a payment for labour, Marx could make the distinction between power and work. If labour were a commodity and if commodities exchanged in proportion to their value, their labour content, then we have a contradiction.

What is the value of labour itself?

Work and power

At first glance, one might say the value of an hour's labour is simply one hour. However, political economists argue that its value is actually determined by the amount of labour needed to produce the sustenance – food and clothing – that a worker requires for that hour. This perspective was maintained without acknowledging the inherent contradiction.

Adam Smith and his successor, Ricardo, developed their theories during a time when labour was predominantly human-powered. The concept that Marx later introduced, distinguishing labour from labour power, was inconceivable then. By the time Marx was writing, after over a century of mechanical progress, steam engines had become the primary source of energy for British industry. To differentiate their products, steam engine manufacturers rated their machines in terms of horsepower, providing a standard measure to compare against competitors' engines.

The initial task of steam engines was hauling coal from mines, or pumping water from mines. It was reckoned that a horse on a windlass could raise a 550 lb weight at 1 foot a second. If a steam engine could raise the same weight of coal 10 feet in a second then it was a 10 horsepower engine, etc.

As a result of practical mechanical engineering, the distinction between work done and power as the ability to do work became evident. A little dimensional analysis revealed that work done was the same as energy.[48]

Work was equated with energy, and power was defined as the amount of work done in a specific time unit. In Victorian England, energy or work was quantified in foot-pounds, while power was measured in horsepower. A specialized field of engineering emerged to accurately measure the power output of steam engines through indicator diagrams. These diagrams were graphs automatically drawn by connecting the engine's piston to the x-axis of a paper, while a pencil, linked to the pressure gauge, moved vertically to record the variations. These diagrams were used to prove, to dubious buyers, the power of the engine.

In the twenty-first century, interpreting indicator diagrams or gauging the power of steam engines may seem like obscure subjects. However, in the nineteenth century, steam was the cutting-edge technology of the era. Knowledge of steam was akin to our understanding of computers, with steam-era concepts and metrics as familiar to Victorians as gigabytes and pixels are to us today.

Marx's differentiation between labour power – the capacity to work – and labour itself – the work performed – would have been instantly comprehensible to a readership in the 1860s. These were commonplace distinctions within the context of mechanical engineering. Engels, owning a steam-powered mill, would have been well-versed in these concepts. Marx and Engels, in essence, were the steampunk communists, articulating in a vernacular of energy, force and power.

Take the example of a mine owner who hired a 10-horsepower steam engine. Whether he actually got $10 \times 60 \times 60 \times 550 = 19{,}800{,}000$ foot-pounds of coal lifting done each hour would depend on how intensively he worked the engine. If there were not enough truckloads of coal brought to the foot of the shaft, there would be periods when the engine stood idle. So the power of the engine and the work actually done were understood to be two different things.

From the very moment that Marx introduces labour, work done, as the substance of value he equates this with the expenditure of labour power – the ability to work. From this, he derives his resolution to the contradiction implicit in $M \to C \to M'$.

According to Marx, a capitalist employs a labourer for a day, with the labour power's value determined by the sustenance needed to keep the worker alive for that period. However, the actual hours worked can vary. Similar to a mine owner utilizing a steam engine for the longest possible duration, the capitalist does the same with his workers. They are paid just enough for daily survival. Marx suggests that if six hours of labour suffice to produce the necessities for a worker's survival, there is no compulsion for the capitalist to release the workers after six hours. Instead, the capitalist aims to extend the working day to its fullest extent.

> The working day is thus not a constant, but a variable quantity. One of its parts, certainly, is determined by the working-time required for the reproduction of the labour-power of the labourer himself. But its total amount varies with the duration of the surplus labour. The working day is, therefore, determinable, but is, per se, indeterminate.
>
> This maximum limit is conditioned by two things. First, by the physical bounds of labour-power. Within the 24 hours of the natural day a man can expend only a definite quantity of his vital force. A horse, in like manner, can only work from day to day, 8 hours. During part of the day this force must rest, sleep; during another part the man has to satisfy other physical needs, to feed, wash, and clothe himself. Besides these purely physical limitations, the extension of the working day encounters moral ones. The labourer needs time for satisfying his intellectual and social wants, the extent and number of which are conditioned by the general state of social advancement. The variation of the working day fluctuates, therefore, within physical and social bounds. But both these limiting conditions are of a very elastic nature, and allow the greatest latitude. So we find working days of 8, 10, 12, 14, 16, 18 hours, ie of the most different lengths.[49]

Marx defined the additional time a worker must labour beyond what is needed to earn their wages as surplus labour. He asserted that there is a direct correlation between the surplus labour done and the surplus value seized by the capitalist. From this fundamental principle, Marx inferred a range of empirical characteristics of the capitalist society he observed.

- Since the capitalists try to maximize profit, and since this depends on surplus labour, they constantly push to extend the working day to the limits of human endurance. This he termed absolute surplus value.
- The other way they can maximize profit is by reducing the hours required to produce wage goods. If, through mechanization of weaving, the work required to clothe the workers can be performed in fewer hours, then the share of any given working day falling to capital will increase. From this stemmed the constant drive towards mechanization. This he called relative surplus value.
- Analogous to Newton's orbital mechanics, he formulates (see Figure 5.1 drawn from data computed in Cockshott, Cottrell and Michaelson[50]) a rotating process by which the dominance of capital over labour is maintained. This he called the General Law of Capital Accumulation. It was the governing law of motion of capitalist society.

The progression of planetary systems, as studied by Newtonian mechanics, was largely cyclical. Various cycles overlapped: the Earth's rotation cycle influenced daily tides, while the lunar orbit altered the timing of high tides. The annual cycle dictated the seasons, and even longer cycles controlled the precession of Earth's axis and its orbital major axis. It is now understood that these extended Milankovitch cycles are responsible for initiating Ice Ages.

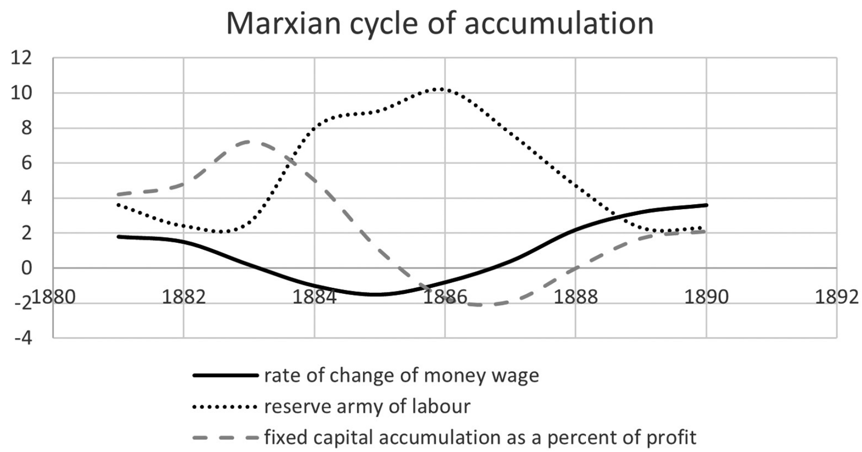


Figure 5.1 Illustration of Marx's General Law of Capital Accumulation using a Victorian economic cycle. The law describes a cyclical process whereby a period of relatively full employment (1881–3) allows wages to rise. The capitalists respond by investing in labour-saving machinery (1882–5). This throws workers onto the reserve army of labour and causes wages to fall (1883–7). At this lower wage, the capitalists again take on more workers, wages start to rise and the cycle repeats. The cycle typically took just under a decade. Figure by authors.

For Marx, the economic world was a similar superposition of cycles: the commodity cycle $C \to M \to C$ and the basic capital cycle $M \to C \to M'$ interleaved and were fast, of the order of days or weeks. On top of this came the cycle driven by his General Law, which lasts a decade or so. Beyond that, there was a protracted process through which other trends emerged: a declining rate of profit and the steadily increasing social and political influence of the industrial working class. The timeline for the rise and fall of the capitalist system, from the so-called primitive accumulation in the sixteenth century to a final revolutionary crisis, spanned hundreds of years. The modes of production fit into a Grand Cycle, evolving from primitive to advanced communism. For Marx, these cycles were upward spirals, with the future communism surpassing its predecessor, propelled by what he termed, in another mechanical metaphor, the productive forces.

> No social order ever dissappears before all the productive forces, for which there is room in it, have been developed; and new, higher relations of production never appear before the material conditions of their existence have matured in the womb of the old society. Therefore, humanity always takes up only such problems as it can solve, since, looking at the matter more closely we will always find that the problem itself arises only when the material conditions necessary for its solution already exist or are at least in the process of formation. In broad outline we can designate the Asiatic, the ancient, the feudal, and the modern bourgeois methods of production as so many epochs in the process of the

> economic formation of society. The bourgeois relations of production are the last antagonistic form of the social process of production – antagonistic not in the sense of individual antagonism, but of one arising from conditions surrounding the life of individuals in society; at the same time the productive forces developing in the womb of bourgeois society create the material conditions for the solution of that antagonism. This social formation constitutes, therefore, the closing chapter of the prehistoric stage of human society.[51]

Rhetoric versus discovery

Those of our readers who are familiar with current debates in Marxian philosophy will see that the account we have given of a Newtonian and mechanistic Marx is radically different from some neo-Hegelian presentations of Marx that have gained ground in recent decades. These accounts are multiple, but a particularly clear example is given by.[52]

Starosta is honest. He only claims to be dealing with Marx's exposition in Capital, not the sources of his ideas. It is a reasonably competent account of the form of presentation in one chapter of that book. The Marxist philosopher most strongly associated with non-Hegelian accounts of Marx was Althusser with his claim that there was a fundamental conceptual break from Hegelianism in the 1840s.

As an arguer against the Althusserian thesis that there was an epistemological break from Hegel at the time of the German Ideology, Starosta is useless. He just blithely assumes a continuity, for example:

> and as he clearly states in the Marginal Notes on Adolf Wagner, Marx takes as a point of departure neither the concepts of political economy nor any concept whatsoever (Marx, 1975, 198), in order thereby to discover alienated labor[53]

Well, there is no mention whatsoever of alienated labour in notes on Wagner.[54] This was originally translated into English in 1972 by an Althusserian journal (Theoretical Practice) precisely in order to demonstrate the completeness of the epistemological break. The author would have been wise to read the original introduction to the English translation by Athar Hussain.[55]

He then goes on to simply assume that the notion of alienated labour is used in Capital:

> In this sense, Marx's exposition in Capital does not advance towards the discovery of alienation but starts from what the analytic stage of the dialectical inquiry revealed as its most abstract and general form

In the English translation of Capital I the word 'alienation' only occurs in the old Scottish legal sense that Adam Smith uses – meaning sale or gift of property. You alienate your house if you sell it. This is the sense in Capital. For example, these passages from[56]

> It became real money, by the general alienation of commodities, by actually changing places with their natural forms as useful objects, and thus becoming in reality the embodiment of their values.
>
> But with the development of circulation, conditions arise under which the alienation of commodities becomes separated, by an interval of time, from the realisation of their prices.

The German original of Capital Chapter 3 uses veräußerung or sale, not entfremdung, the word that is translated as alienation or estrangement in English translations of the early writings. This is by no means the problematic of 'alienated labour' 'entfremdete Arbeit' in the EPM, even if they may be given the same English translation.

Starosta also propagates the currently fashionable, but rather odd, interpretation that when discussing commodities, Marx was only talking about capitalist society:

> As has now been widely acknowledged, this starting point is not an ideal–typical – or worse, historically existent – simple commodity–producing society, as in the orthodoxy derived from Engels and popularised by authors such as Sweezy and Meek.

That is highly disputable. The basic distinction between use value and exchange value that Marx presents is drawn from Aristotle as was shown by Meikle.[57] Marx explicitly credits many of his ideas to Aristotle in footnotes and body text. Particularly relevant, because it relates directly to the value form, Marx was quite specific that the equivalent form of value that he presents in Capital was derived from Aristotle:

> The two latter peculiarities of the equivalent form will become more intelligible if we go back to the great thinker who was the first to analyse so many forms, whether of thought, society, or Nature, and amongst them also the form of value. I mean Aristotle.
>
> In the first place, he clearly enunciates that the money form of commodities is only the further development of the simple form of value – ie of the expression of the value of one commodity in some other commodity taken at random; for he says:
>
> > 5 beds = 1 house
> > is not to be distinguished from
> > 5 beds = so much money.
>
> He further sees that the value relation which gives rise to this expression makes it necessary that the house should qualitatively be made the equal of the bed, and that, without such an equalisation, these two clearly different things could not be compared with each other as commensurable quantities. 'Exchange,' he says, 'cannot take place without equality, and equality not without commensurability'.

> . . . Here, however, he comes to a stop, and gives up the further analysis of the form of value. 'It is, however, in reality, impossible , that such unlike things can be commensurable' – ie qualitatively equal. Such an equalisation can only be something foreign to their real nature, consequently only 'a makeshift for practical purposes.'
>
> Aristotle therefore, himself, tells us what barred the way to his further analysis; it was the absence of any concept of value. What is that equal something, that common substance, which admits of the value of the beds being expressed by a house? Such a thing, in truth, cannot exist, says Aristotle. And why not? Compared with the beds, the house does represent something equal to them, in so far as it represents what is really equal, both in the beds and the house. And that is – human labour.
>
> There was, however, an important fact which prevented Aristotle from seeing that, to attribute value to commodities, is merely a mode of expressing all labour as equal human labour, and consequently as labour of equal quality. Greek society was founded upon slavery, and had, therefore, for its natural basis, the inequality of men and of their labour powers. The secret of the expression of value, namely, that all kinds of labour are equal and equivalent, because, and so far as they are human labour in general, cannot be deciphered, until the notion of human equality has already acquired the fixity of a popular prejudice.

So Marx is saying:

- That his analysis of the contradiction of the value form is derived from Aristotle.
- That commodity production and commodity value existed in ancient Athens – long before capitalism.
- That the secret of value, that it is human labour, was inconceivable to a member of the slaveholding class of that time – even though the source of value even then was human labour. It was the class blindness of an advocate of slavery that blocked him from realizing this.

Marx and Aristotle were justified in treating exchange as an equivalence relation because they recognized the necessity of a common element in the exchange of dissimilar commodities. The criticism that Marx fails to establish labour as this common substance is contested by Starosta, who argues that Marx does indeed identify a common substance and dialectically introduces labour as such, justified by the characteristics of labour conducted privately for societal needs. However, this approach by Marx can be seen as a dialectical manoeuvre that, while plausible, is not necessarily scientifically rigorous. Historical examples, such as the phlogiston[58] theory of combustion, demonstrate that plausible and materialist explanations can ultimately be proven incorrect:

> INFLAMMABLE fossils[59] abound with phiogifton, do not unite with water, but when pure dissolve in oils; exposed to the fire, they smoke, generally inflame, are for the most part confumed, and sometimes, totally vanish.
>
> METALS when perfect do not diffolve at all in water ; only a few of them in oils, and then only when in part deprived of their phlogiston. They are the heaviest of all known substances, the lightest of them weighing more than fix times its bulk of water.
>
> Phlogisticated vitriotic ACID (volatile vitriolic àcid[60]) is frequently thrown out by the craters of volcanoes ; its finell suffocating and penetrating, The union to phlogiston and the matter of heat gives it an aerial form, but does not prevent its union with water.
>
> Nitrous ACID is by some excluded from the foffil kingdom, because they suppose it to be produced from the putrefaction of organic bodies. But these bodies when deprived of life are again received amongst the folds, from whence their miore fixed parts were originally derived.
>
> In the most concentrated state that art can procure it, its specific gravity is 1, 580. Colourless when pure; but its strong attraction*to phlogiston renders particular management necessary to procure it so *. With different proportions of phlogiston it forms phlogisticated acid and nitrous air.
>
> The most highly coloured and fuming nitrous acid[61] may readily be rendered colourlets by boiling it hastily in an open veffel. Part of the acid fies off, carrying the fuperabundant phlogiston along with it, in the form of nitrous air[62]

The justification given by eighteenth-century chemists is structurally very similar to Marx's dialectical presentation. Common properties (combustibility, exchangeability) and forms of expression (flame, price in gold) – dialectical inference there must be a common substance (phlogiston, value). But we know that the phlogiston theory was wrong, that there is no common substance to things that combust; instead, fire is produced by the oxidation of the fuel. Priestly's pure phlogiston was actually hydrogen. The difference between a metal and its 'calc' is due to oxidation, not the loss of phlogiston. The shared characteristic eventually discovered was that combustible substances could transition to a lower energy state through oxidation.

While Marx may have identified value as a common substance in exchangeable commodities and argued that labour is abstractly quantifiable and essential for a commodity's existence, this alone does not confirm labour as that substance.

It is not a matter of contesting Marx's correctness; rather, it is conceivable that there exists another common substance he overlooked. Energy is a prime candidate, possessing the qualities of abstract quantifiability and the ability to manifest in various concrete forms, such as heat, electricity, kinetic energy and chemical energy.

The correctness of Marx's derivation rests not on its intellectual plausibility, but on whether in fact commodity values are proportional to labour content. If it turns out when we measure things that prices correlate much more closely with the number of watt-seconds = Joules needed to make a commodity rather than the number of person-seconds of human effort, then Marx would be wrong. We now know that empirically Marx was right. Commodity prices do correlate much more closely with person-seconds of labour than with Joules of energy used. In that case, Marx's laws of capitalist motion would just be a special case of the conservation of energy rather than higher order statistical mechanics laws.[63]

Of course, Marx was not arbitrarily selecting labour as the value substance. He was following a tradition from Khaldūn[64] to Ricardo[65] who had observed, by rough and ready means, that the requisite empirical relation actually existed.

Marx's approach, as a scientific procedure, was indeed a shortcut. It lacked the modern standards of scientific rigour, relying on dialectical sleight of hand. Ideally, he should have provided his own empirical data to substantiate the assertion that labour is the essence or at least referenced the observational arguments of his predecessors that supported the notion that labour content dictates price.

These sources, his predecessors, are not concealed. The entire fourth volume of Capital, titled Theories of Surplus Value, is dedicated to his predecessors and their thoughts. Within, we can discern which ideas he deemed correct, the conceptual disputes among them, and where he found their reasoning flawed.

However, dialectical methods might offer benefits in a work that is both scientific and polemical, such as Capital. When the goal is to persuade readers, dialectical rhetoric can be judged as an effective strategy. The issue arises when later generations regard these methods as a standard for investigation worthy of emulation.

Marx's argument only works because he stood on the giant shoulders of his predecessors.[66] He could introduce into his dialectical argument something which had been prepared earlier by Smith and Ricardo.

This is also why further attempts to apply the dialectical method are futile. The Hegelian school of Marxist economists has failed to innovate or generate new knowledge through this method, as it is not designed for such purposes. It merely reformulates pre-existing knowledge into rhetorical constructs.

This is particularly evident in the non-sensical writings on natural dialectics. Proponents of these theories have not uncovered anything novel. Instead, they have repackaged scientific knowledge, obtained through conventional research methods, in the terminology of dialectical concepts like the unity of opposites and the transition from quantity to quality.

Engels[67] is only able to use the transition of water into steam as an example of a change of quantity into quality and as an example of dialectical leap because of prior work in thermodynamics. The investigations of Black and his assistant Watt had to have shown that heat itself was quantifiable[68] and that there was such a thing as a latent heat of steam vapourization. The dialectical method was not essential for their discoveries. Indeed, Professor Black discovered latent heat in 1761, well before Hegel's time. We reference Black for two reasons: his ideas were later showcased by Engels as examples of natural dialectics, and they pertain to a subsequent stage in Marx's discourse in 'Capital'—the mystery of surplus value.

Marx addresses Aristotle's dichotomy between use value and exchange value by positing labour as the shared essence of value, an insight he gained from earlier political economists. He then resolves another significant dialectical contradiction: the enigma of surplus value. If you recall, he first presents the paradox that the circuit of capital m-c-m' exists, which appears to conflict with the general principle of equivalent exchange of commodities. How, he asks, can you have m<m' if all commodity exchanges along the way are exchanges of equal value?

As we explain in Section 5.3.1, Marx addressed the issue with a conceptual distinction between labour and labour power. He defined labour power as the capacity to work. According to Marx, it is labour power, not labour itself, that the capitalist purchases with wages. The mystery of surplus value is unveiled by the fact that the value of labour power is set by its labour cost of reproduction, similar to all commodities, yet it requires less than a full day's labour to sustain the worker for a day. The excess time, which is the difference between the working day's length and the labour value of the wage goods, becomes the source of surplus value.

This differentiation between labour and labour power was an innovation in political economy. Predecessors like Smith and Ricardo had equated the wage with the value of labour itself, with Ricardo determining that the wage matched the labour needed to sustain the worker. Furthermore, Ricardo identified the inverse relationship between wages and the proportion of value that forms the capitalist's profit.

Marx's innovations here were twofold:

1. Generalization from profit to surplus value – which included also rent and interest.
2. The distinction between labour and labour power.

It can be argued that the mode of presentation of the issue in Capital uses Hegelian rhetorical tricks: deriving a contradiction between c-m-c and m-c-m' and then resolving the contradiction in a new concept, labour power. But that does not explain where the dialectical conjuration produces the idea of labour power from.

This idea certainly did not originate from Hegel, nor can it be found in Smith or Ricardo. Rather, as previously mentioned, it stemmed from Watt's concepts. Watt's work laid a dual foundation for Marx's. Watt, having discovered with Black that steam had latent heat, realized he could significantly enhance steam engine efficiency by adding separate condensers. This insight not only secured his success as an engineer but also propelled capitalist industrialization. Marx noted that the hand mill yielded the feudal lord, while the steam mill brought forth the industrial capitalist. Practically, Watt's invention was pivotal in generating what Marx termed relative surplus value. It enabled the widespread adoption of steam-powered machinery, supplanting human labour and providing Marx with the theoretical tools needed to unravel the mystery of surplus value. Marx observed that Aristotle failed to recognize labour as the common substance of value due to the ideological constraints of slave society. It was only through economic advancement that this became apparent to Smith.

So by the time Marx is writing, engineering practice has made the conceptual distinction between work and power, between work and the ability to perform work, part of the common basic knowledge of industrial England. Similarly, capitalist commercial practice in the renting of steam engines had established the principle of hiring out the ability of these engines to perform work for a monthly rate.

When Smith was writing, his acquaintance Watt was still tinkering with his prototypes,[69] and even in Ricardo's time steam engines were relatively rare compared to water mills. But by the time Marx was writing, thermodynamics and the practice of capitalist civilization had provided the conceptual abstractions necessary for him to make the distinction between labour and labour power and thus solve the problem of surplus value.

Additionally, there is a parallel concept – the rate of surplus value corresponds to the steam engine's coal return rate: the proportion of coal consumed versus coal hauled. Known as the energy return on investment, this represents the essential thermodynamic or energy limitation of industrial capitalist society. A positive surplus is crucial for a positive rate of relative surplus value. The correlation between these rates warrants further investigation by political economists as we near the fossil fuel capitalism crisis.

Furthermore, we should not confuse the dialectical method's rhetorical manoeuvres with actual historical or ideational structures. A dialectician's success hinges on the availability of pre-existing concepts to utilize. The effectiveness of their method depends on unveiling these concepts in an appropriate sequence and assumes the audience has some understanding of the presented ideas. In the mid-nineteenth century, concepts such as labour as a value source and the distinction between work and power were presumably familiar enough for the typical reader of 'Capital' to grasp Marx's points.

Marx's originality in adapting certain concepts to political economy is not under scrutiny. However, the foundational ideas originated not from Hegel but from earlier social and engineering studies. Advancing Marxist science today necessitates studying tangible patterns in economic and social data and drawing extensively from the sciences and engineering for analytical tools. Dialectics study, at best, offers a way to craft compelling narratives around your findings. However, this approach is fraught with risk, as it requires an extraordinary rhetorical talent akin to that of Marx to succeed. In the hands of individuals lacking Dr Marx's writing prowess and extensive knowledge, the outcome is likely to be dull and uninspiring.

Darwin and the end of teleology

> Darwin, by the way, whom I'm reading just now, is absolutely splendid. There was one aspect of teleology that had yet to be demolished, and that has now been done. Never before has so grandiose an attempt been made to demonstrate historical evolution in Nature, and certainly never to such good effect.[70]

> Darwin's work is most important and suits my purpose in that it provides a basis in natural science for the historical class struggle. . . . Despite all shortcomings, it is here that, for the first time, 'teleology' in natural science is not only dealt a mortal blow but its rational meaning is empirically explained.[71]

Two significant themes in Darwin's work are particularly noteworthy. The first is the dismantling of teleology through his theory of natural selection, a point that Engels and Marx emphasized. The second is the challenge to human exceptionalism, viewing humans simply as another species of mammal. Darwin's fundamental approach was to interpret human behaviour through the lens of animal behaviour. Rather than shying away from attributing human-like qualities to animals, he posited that understanding human traits requires acknowledging our shared mammalian heritage.

The resistance to this perspective likely stems from the ideologies tied to earlier economic systems. For instance, Aristotle's assertion that some individuals were naturally slaves mirrored the societal conditions of his time. Similarly, an economy reliant on animal labour, such as that of horses and oxen, necessitated a clear demarcation between humans and animals. Humans, akin to slave masters, were deemed rational beings, a quality supposedly absent in animals or slaves.

Darwin, whose family background was in the rising Quaker industrial class, was a passionate opponent of slavery,[72] and he wrote when steam was displacing horsespower as the motive force of society. He could work both to assert the common humanity of white and black and the common mammalian character of human and animal.

Ending teleology

In the mid-nineteenth century, there was a significant challenge in explaining the vast expanse of time and the realization that Earth's species had undergone complete transformations repeatedly. This process was clearly directional, not cyclical. The question arose: how could this be explained? Similarly, from the perspective of capitalist industrial civilization, history appeared to have a direction. What causal model could explain this phenomenon?

Teleological theories were one solution. Marx and Engels saw Darwin's rejection of teleology as a pivotal development. To comprehend this, it's essential to understand the intellectual milieu in which Darwin formulated his ideas. What teleological concept did he dismiss?

The most developed version of a teleological theory of the historical evolution of species had been put forward by Chambers in his *Vestiges of the Natural History of Creation*, first published in 1844. Vestiges was extremely controversial as it popularized Hutton/Miller/Lyell's geological deep time reconciled with a Christian perspective, breaking decisively with Bishop Usher's creation in 4004 BC. Chambers found it hard to get it reviewed, but it paved the way for Origin of Species.

What is interesting about the book is just how much was known about astronomy and geology at this point. Chambers had a pretty good estimate of the size not only of the solar system but also of the distance to Alpha Centauri, a fair idea of the shape of our galaxy. He was aware that there were multiple other galaxies. His account of how the solar system was formed was not so far off from what you would now get in a school astronomy book.

The broad outlines of geological time are recounted. It is fascinating how the ages: Cumbrian (not Cambrian), Devonian, Silurian, Carboniferous and Cretaceous are not presented as abstract epochs with start and end dates as they are now, but rather are directly related to the rocks in specific parts of Britain, such as Devonian in Devon, Cumbrian in Wales and Cretaceous in Sussex.

He had a rough understanding of the chronological appearance of major animal and plant classes, but he held what might have seemed plausible, yet now appears to be wild ideas about geological causes. For example, he believed that the seas where the Old Red Sandstone was deposited were hundreds of miles deep and that dry land did not exist until the Carboniferous period. He proposed the intriguing hypothesis that before the Carboniferous, CO_2 levels were too high for land animals to exist. According to his theory, the sequestration of carbon allowed land animals to emerge during what he referred to as 'the new red sandstone', which we now identify as the Permian to Jurassic periods.

The teleology is evident in his suggestion that the *function* of the forests giving rise to the coal beds was both to purify the air for animal life and to provide the fuel needed for our civilization.

This was tied to his theory of how animal forms arise. He emphasized that life as he put *it presses in* on any opportunities. To a modern ear, this sounds like the idea of life rapidly evolving to fit ecological niches, but in his case, it was seen as the development of the laws of form. He was a pronounced naturalist. Life arises by natural laws and the forms of life are driven by them as well. He was convinced by the early nineteenth century that electricity was the source of life. This echoed the same themes as Mary Shelly's Frankenstein.

As an example, he claimed that the dendritic or branching pattern that you get in electric discharges was the same as you get in trees because the same electric laws operate between the negatively charged Earth and the positively charged atmosphere. This electric potential drove plant growth and gave rise to the characteristic shapes of trees, palms, etc. He even cited a Frankenstein-like experiment by a Mr Crosse, who claimed to have observed the spontaneous generation of insect life by passing strong electric currents through 'silicate of potash'. This was apparently replicated by Mr Weekes using 'ferro-cyanet of potash'.

The insects were apparently rather minute, 'a species of ascuraus minute and semi transparent and furnished with long bristles'.

The citation for this series of experiments is not very convincing: 'see a pamphlet circulated by Mr. Weekes in 1842'.

The basic way Chambers reconciles natural law with 'the Mosaic texts', is to say that God willed that there be life, not that he created each line of organisms. Instead, natural laws were so designed by The Great Architect of the Universe as to ensure that first planets would come into existence, and then that life would spontaneously be generated by electricity from inorganic compounds. Then, as soon as new physical conditions arose, the appropriate form of life would spontaneously develop to fill the space.

To the modern reader, it is disconcerting that he repeatedly uses the idea of species being links. We now think of links in Darwinian terms – *Archaeopteryx* being the link between reptiles and birds in the sense that it was descended from reptiles and that subsequent birds descend from it (or one of its close relatives).

However, the links he refers to are now considered to have little credibility. He believes that Devonian placoderms with bone coverings are a transitional form between crustaceans and fish. He suggests that ostriches, with their urinary bladder and diaphragm similar to mammals and feathers resembling hair, are a link between mammals and birds. He also views the platypus as an intermediary between birds and mammals. His proposed sequence is Fish, Reptiles (including amphibians), Birds and then Mammals. Thus, the notion that ostriches are evolving into mammals aligns with this sequence. He frequently employs temporal terms, placing fish first and reptiles either higher or subsequent, reflecting a teleological sequence that coincides with the geological record of species emergence. The central teleological theme of his work was the notion that there was an innate 'law of development' that led to the generation of higher forms of life. Using the analogy with honey bees, the feeding of whose larvae determines whether they emerge as queens, workers or drones, Chambers hypothesized that changed conditions in embryo will lead to more developed offspring. He claimed that once removed from the hardships of barbarism, Africans who lived in European conditions would assimilate not only to the mode of life of Europeans but also to their appearance:

> The coarse features, and other structural peculiarities of the negro race only continue while these people live amidst the circumstances usually associated with barbarism. In a more temperate clime, and higher social state, the face and figure become greatly refined. The few African nations which possess any civilisation also exhibit forms approaching the European.[73]

He was an extreme environmentalist. That third-generation descendants of Africans resident in Europe might look more 'refined' was not put down to intermarriage, but to living conditions. Chambers unhesitatingly assumed that those features he was most familiar with among his own countrymen were the most advanced. The whole law of development of nature aimed, it seemed to him, to produce as its goal a well-fed member of the British upper classes.

Using the embryological similarity between mammals, fish, reptiles, etc., he argues that the differentiation of a mammalian embryo is the result of an inbuilt developmental law, which, given the appropriate opportunities, a fish embryo is also capable of.

> A human foetus is often left with one of the most important parts of its frame imperfectly developed: the heart, for instance, goes no farther than the three-chambered form, so that it is the heart of a reptile. There are even instances of

> this organ being left in the two-chambered or fish form. Such defects are the result of nothing more than a failure of the power of development in the system of the mother, occasioned by weak health or misery. Here we have apparently a realisation of the converse of those conditions which carry on species to species, so far, at least, as one organ is concerned. Seeing a complete specific retrogression in this one point, how easy it is to imagine an access of favourable conditions sufficient to reverse the phenomenon, and make a fish mother develop a reptile heart, or a reptile mother develop a mammal one. It is no great boldness to surmise that a super-adequacy in the measure of this under-adequacy (and the one thing seems as natural an occurrence as the other) would suffice in a goose to give its progeny the body of a rat, and produce the ornithorynchus, or might give the progeny of an ornithorynchus the mouth and feet of a true rodent, and thus complete at two stages the passage from the aves to the mammalia.[74]

He was convinced that life existed not only on Mars and Jupiter but that, thanks to this developmental law, it looked very much like Earth life, and that similar organisms exist on countless planets around other stars. A conceit which is shared, in our day, by the producers of Star Wars.

Chambers' book presents a vision of a relentless, progressive ascent of organisms, guided by an inherent law of development – a concept that is undeniably teleological, especially considering Chambers' interpretation of it as the manifestation of God's design. However, this notion of progress bears a resemblance to certain common portrayals of Marxism, where the inevitable advancement from fish to reptiles, birds and ultimately mammals parallels the progression through different modes of production. Sayers[75] for example, claims that

> Marxism comprehends the progressive patterns of historical development as the outcome of class struggle and the conflict between the forces and relations of production. Marx – like Darwin – is best understood, not as repudiating teleological notions, but rather as using a naturalistic version of them that is consistent with modern science.

The problem with this is that the influential pre-Darwinian account of Chambers could also be described as presenting a naturalistic version of teleology consistent with modern science as it stood in the mid-nineteenth century. That Marx applauded Darwin for breaking with this paradigm indicates that he saw both his own work and Darwin's as radically distinct from teleology.

An outline of the causal mechanism proposed by Charles Darwin is:

- Variation exists in all animal and plant species.
- Variable traits can be inherited.
- Selection for variable traits increases their frequency, as evidenced by the selective breeding of animals and plants by farmers and gardeners.
- The struggle for survival selects those best adapted to produce offspring.
- This can lead to sufficient change that new species arise

He gives a good one-paragraph summary of his theory in the third chapter of *Origin of the Species*:

> Owing to this struggle for life, any variation, however slight and from whatever cause proceeding, if it be in any degree profitable to an individual of any species, in its infinitely complex relations to other organic beings and to external nature, will tend to the preservation of that individual, and will generally be inherited by its offspring. The offspring, also, will thus have a better chance of surviving, for, of the many individuals of any species which are periodically born, but a small number can survive. I have called this principle, by which each slight variation, if useful, is preserved, by the term of Natural Selection, in order to mark its relation to man's power of selection. We have seen that man by selection can certainly produce great results, and can adapt organic beings to his own uses, through the accumulation of slight but useful variations, given to him by the hand of Nature. But Natural Selection, as we shall hereafter see, is a power incessantly ready for action, and is as immeasurably superior to man's feeble efforts, as the works of Nature are to those of Art.[76]

Darwin's theory is frequently contrasted with Lamarckian evolution, where the key distinction is thought to be the inheritance of acquired traits, such as a seal's improved swimming ability, by offspring. Labelling this inheritance as Lamarckian and its rejection as Darwinian is, however, anachronistic. Before genetics was understood, it wasn't obvious that learnt traits were separate from inherited ones, and Darwin never ruled out the inheritance of acquired characteristics. In places, in[77] he speaks of learnt habits of emotional expression being inherited by offspring.

In[78] he emphasizes that all he requires is that variation exists, irrespective of the circumstances that cause it or give rise to it. Origins of variation:

> It has been disputed at what period of life the causes of variability, whatever they may be, generally act; whether during the early or late period of development of the embryo, or at the instant of conception. Geoffroy St. Hilaire's experiments show that unnatural treatment of the embryo causes monstrosities; and monstrosities cannot be separated by any clear line of distinction from mere variations. But I am strongly inclined to suspect that the most frequent cause of variability may be attributed to the male and female reproductive elements having been affected prior to the act of conception. . . . in fact, 'sports' support my view, that variability may be largely attributed to the ovules or pollen, or to both, having been affected by the treatment of the parent prior to the act of conception.[79]

Sports is the old term for what we now call mutations in plant varieties. Although the concept of genetic mutation was not yet available, this account is functionally analogous. He is saying that environmental stress affecting the sex organs of a species may induce variation. If we rate exposure to background radiation, etc., as such stress, then this would still be acceptable.

If additional variation could be induced by learnt habits, that makes little difference to his theory of selection. Whatever the source of the variation, the important thing is the selection process.

He introduces variation and selection in the context of domesticated animals and plants. His first chapters contain a wealth of examples showing what vast range of variation there is in our domestic species. He emphasizes that domestic dogs, for example, vary far more widely than wild ones. He went so far as to join pigeon fancier and pigeon breeding societies to educate himself on the range of variation of these birds and the lore of their breeders. Whenever he attempts to establish a theoretical point, he brings in a wealth of supporting observations. Some of his observations seem strikingly modern for nineteenth-century books. For instance, he was aware of traits associated with domesticated animals that were confirmed in the late twentieth century.

> Not a single domestic animal can be named which has not in some country drooping ears; and the view suggested by some authors, that the drooping is due to the disuse of the muscles of the ear, from the animals not being much alarmed by danger, seems probable.[80]

The empirical observation is borne out by modern research,[81] but Darwin's theory of the causes of domestic traits is not supported. Floppy ears, shorter muzzles, neotonous behaviour and patchy coat are all traits that in combination are now called the domestication syndrome (DS). Long-term experiments begun in the USSR in the late 1950s, which involved selecting captive foxes for domestication, have provided insight into this process. Over fifty years of selective breeding, the research programme led by Lyudmila Trut has shown that a full suite of DS traits can quickly emerge through selecting solely for tameness in a species without prior domestication history. Comparable results have been seen in rats and mink when subjected to the same selection pressures by the same research team.[82]

By repeatedly selecting for desired traits, breeders have been able to create enormously variable domestic stock. The variation is so great that were the different breeds of dogs to be observed in the wild, they would appear to be different species. If deliberate selection can create the appearance of speciation, what form of selection produces actual speciation?

He then introduces the struggle for existence in nature. At its base is the exponential reproductive potential of any living organism.

> There is no exception to the rule that every organic being naturally increases at so high a rate, that if not destroyed, the Earth would soon be covered by the progeny of a single pair.[83]

If exponential growth continues, an organism will quickly outgrow the limits of its habitat. Even the most fertile environments are finite. For every organism, there is a potential competitor. The competitor may be trying to eat you, or may compete for food or space. As he said, the face of nature often beams with joy, and food seems plentiful; yet we overlook the fact that the carefree birdsong around us comes from creatures that survive on insects or seeds, perpetually ending lives. We also disregard the extent to which these singers, their eggs or their young fall prey to other predators. Moreover, we fail to remember that this abundance of food is not consistent throughout the year's cycles.[84]

Life is a ceaseless struggle. Animals struggle against the environment. They are threatened by periodic drought or frost. They compete against rival species for their ecological niche. Individuals of a species compete within that niche to populate it with their offspring.

> Lighten any check, mitigate the destruction ever so little, and the number of the species will almost instantaneously increase to any amount. The face of Nature may be compared to a yielding surface, with ten thousand sharp wedges packed close together and driven inwards by incessant blows, sometimes one wedge being struck, and then another with greater force.[85]

Darwin, while not using modern ecological terms like niche, habitat or food chain, still bases his arguments on similar concepts. He describes the pollination of red clover by bumblebees; an abundance of these bees leads to thriving clover. However, field mice prey on the bees' underground hives, reducing the clover as their numbers increase. In villages, domestic cats hunt the mice, resulting in bountiful bumblebees and flourishing clover in village gardens. Thus, in every ecosystem, intricate interactions exert constant pressure, allowing only a few offspring to survive. In each species, the best adapted offspring survive to reproduce themselves.

> Can it, then, be thought improbable, seeing that variations useful to man have undoubtedly occurred, that other variations useful in some way to each being in the great and complex battle of life, should sometimes occur in the course of thousands of generations? If such do occur, can we doubt (remembering that many more individuals are born than can possibly survive) that individuals having any advantage, however slight, over others, would have the best chance of surviving and of procreating their kind?[86]

Natural select acts constantly to hone the fitness of every species. Over time, fitness improves and species change. Old species die out as new better-adapted ones replace them.

> It may be said that natural selection is daily and hourly scrutinising, throughout the world, every variation, even the slightest; rejecting that which is bad, preserving and adding up all that is good; silently and insensibly working, whenever and wherever opportunity offers, at the improvement of each organic being in relation to its organic and inorganic conditions of life. We see nothing of these slow changes in progress, until the hand of time has marked the long lapse of ages, and then so imperfect

> is our view into long past geological ages, that we only see that the forms of life are now different from what they formerly were.[87]

But in what sense can we speak of direction? Is the old language of higher and lower orders of animals not just an anthropocentric projection?

Is there any real sense in which we can say that wild animals around today are more 'advanced' or better than those from fifty million years ago?

Perhaps the apparent direction of evolution is just an illusion prompted by our knowledge of ancestral lines. Is the horse really 'better' than *Eohippus*?

Might it not be that Eohippus was perfect for the kind of forested Eocene world that it inhabited?

If we take that view, natural selection gives no long-term direction. There is no overall improvement, just more or less rapid adaptation to climate change.

Darwin was convinced that improvement is real. That in any given environment there is room for progressive adaptation. He cites the way that invasive species outcompete natives as proof of the room for improvement.

> No country can be named in which all the native inhabitants are now so perfectly adapted to each other and to the physical conditions under which they live, that none of them could anyhow be improved; for in all countries, the natives have been so far conquered by naturalised productions, that they have allowed foreigners to take firm possession of the land. And as foreigners have thus everywhere beaten some of the natives, we may safely conclude that the natives might have been modified with advantage, so as to have better resisted such intruders.[88]

Gould[89] argues that the evidence is incontestable. He refers to the marsupial species of South America, where natural selection has fostered the convergent evolution of horse-like creatures and carnivores akin to sabre-toothed cats. However, following the creation of the Panama isthmus, these species were outcompeted and eradicated by the more dominant North American fauna. The true felines were indeed superior to their marsupial counterparts. European rabbits introduced in Australia proliferated to pestilential levels. Conversely, wallabies released in Europe only survive on isolated, small islands.

Thus, we observe improvement, enhancement and the semblance of design, yet without teleology or a designer. Darwinian biology delineates several hierarchies: lineage, trophic levels and adaptability. From a pure Darwinian standpoint, despite the presence of improvement, one cannot categorize animals into lower and higher orders. The hesitation to use terms like 'higher' and 'lower' stems from the potential insinuation that 'higher' species are more adept at survival than 'lower' ones. Commonly, seagulls are perceived as superior to sea urchins. Although gulls may soar over, feed on, sea urchins, they do not compete for the same ecological niche. Biologists now prefer to describe echinoderms as more basal than birds. Basal refers to a position on an ancestral tree. Sea urchins and echinoderms, in general, are closer to the common ancestor of echinoderms and vertebrates than birds are. Gulls are higher in the food chain and less basal in terms of descent, but they are no better adapted to their own peculiar mode of life than sea urchins.

The accusation of teleology against Darwin falls on the marsupial hurdle. What about the hierarchy of modes of production advanced by Smith or Engels?

Recall that Smith's sequence went from hunting society to pastoral society to agricultural civilization. Engels had the sequence: savagery, barbarism, slave society, feudalism, capitalism and communism. The first two terms he used in the same sense as Morgan.[90] Savagery maps roughly onto hunting society. Barbarism maps onto those economies that practice horticulture or herding but not plough agriculture.

Engels's framework views societal development as both ancestral and progressive. Capitalism is seen as more advanced than feudalism, and barbarism as more advanced than savagery. Modern sensitivity often rejects terms like 'barbarian' or 'savage' due to their Euro-centric or Sino-centric biases, which can border on racism. However, from a Marxist perspective, the advancement of a barbarian society engaged in horticulture and animal husbandry over a society solely reliant on hunting is undeniable. Similarly, capitalism is deemed more advanced than feudalism. This is based on the same criterion Gould identifies: the ability to compete. Historically, when barbarian nations came into contact with savage ones, they often displaced them, taking over large areas of their original territories. This was evident as populations from West Africa and Anatolia, equipped with domesticated animals and agriculture, pushed back the earlier hunter-gatherer societies.

Industrial capitalism likewise spread out across the world, subduing barbarian nations and forcing those at a feudal level to modernize. The more advanced social order displaced the less advanced just as the placental felids replaced the marsupial borhyaenids in South America.

Let's go from the question of teleology to what it means to be human. Darwinism undermined the idealist or religious world view by establishing the common descent of humans and apes. In the place of a categorical distinction between rational humans made in the image of God and brute animals, there came the conception of humans as naked apes. What seems specially human is often just a development of earlier animal traits.

When looking at any human trait, Darwin's starting point is to treat it in the context of our mammalian ancestors. Whether he is recounting the use of simple stone tools by chimps or projectile weapons by baboons in Ethiopia,[91] he constantly emphasizes that supposedly unique human traits are just part of a common heritage.

> It has often been said that no animal uses any tool; but the chimpanzee in a state of nature cracks a native fruit, somewhat like a walnut, with a stone. (37. Savage and Wyman in 'Boston Journal of Natural History', vol. iv. 1843-44, p. 383.) Rengger (38. 'Saugethiere von Paraguay', 1830, s. 51-56.) easily taught an American monkey thus to break open hard palm-nuts; and afterwards of its own accord, it used stones to open other kinds of nuts, as well as boxes. It thus

> also removed the soft rind of fruit that had a disagreeable flavour. Another monkey was taught to open the lid of a large box with a stick, and afterwards it used the stick as a lever to move heavy bodies; and I have myself seen a young orang put a stick into a crevice, slip his hand to the other end, and use it in the proper manner as a lever. . . . Brehm (40. 'Thierleben', B. i. s. 79, 82.) states, on the authority of the well-known traveller Schimper, that in Abyssinia when the baboons belonging to one species (C. gelada) descend in troops from the mountains to plunder the fields, they sometimes encounter troops of another species (C. hamadryas), and then a fight ensues. The Geladas roll down great stones, which the Hamadryas try to avoid, and then both species, making a great uproar, rush furiously against each other. Brehm, when accompanying the Duke of Coburg-Gotha, aided in an attack with fire-arms on a troop of baboons in the pass of Mensa in Abyssinia. The baboons in return rolled so many stones down the mountain, some as large as a man's head, that the attackers had to beat a hasty retreat; and the pass was actually closed for a time against the caravan. It deserves notice that these baboons thus acted in concert.

When discussing the psychology of animals, he likewise attributes to them the full gamut of human emotions:, shame, anger, courage and affection.

> There can, I think, be no doubt that a dog feels shame, as distinct from fear, and something very like modesty when begging too often for food. A great dog scorns the snarling of a little dog, and this may be called magnanimity.

Against the prejudice that this is anthropomorphism, he counterposes the opposite principle. Insofar as we have an emotional life, we can only feel these emotions because of our common mammalian heritage. Emotions themselves are treated as states and postures of the body. Their expression, he claims, is govererned by the principle of opposition. Opposing emotions bring into play opposing sets of muscles.

He illustrates this with the behaviour of cats or dogs. When a cat feels aggressive, it assumes a crouching position, flattens its back, tucks its ears and lashes its tail. These postures offer a survival benefit. The low stance slightly alters the pre-pounce posture, staying inconspicuous with hind legs ready to spring. Flattened ears prevent bites, while a thrashing tail and hissing serve as deterrents. Conversely, a friendly cat stands tall, with extended legs, an arched back and a raised tail. Emotions are intrinsically linked to neuromuscular actions; without corresponding physical expressions, emotions do not manifest. Darwin used animal behaviours to illustrate this concept before demonstrating its applicability to human emotions and expressions, presenting a fundamentally materialistic viewpoint.

It is backed up by a conception of the nervous system that sits halfway between Galen and Hebb.[92] Darwin writes of *nerve-force* being transmitted along and through the nerves. This is only one step removed from Galen's *psychic pneuma*.

But it is combined with an awareness of reflexes and the concept that habitual action reinforces interconnection between parts of the nervous system.

> Experience shows that nerve-force is generated and set free whenever the cerebro-spinal system is excited. The direction which this nerve-force follows is necessarily determined by the lines of connection between the nerve-cells, with each other and with various parts of the body. But the direction is likewise much influenced by habit; inasmuch as nerve-force passes readily along accustomed channels.[93]

Darwin had the right basic intuitions on heredity and nervous action, though he did not know the actual mechanisms by which it came about: DNA and the synapse. Darwin uses a language of 'force', which is very physical, to describe the nervous system. But in its own day, Galen's *pneuma* was a physical theory of the nervous system. In time, this materialist theory of Galen became its opposite, so that translated into the Latin *spiritus* and intoned by priests, it became something mystical. We can see the same potential ambiguity in the period between Darwin and Hebb. Cajal and Pavlov explored the mechanical materialist path, elucidating basic mechanisms leading to the point where Hebb could declare:

> Modern psychology takes completely for granted that behaviour and neural function are perfectly correlated, that one is completely caused by the other. There is no separate soul or life-force to stick a finger into the brain now and then and make neural cells do what they would not otherwise Actually, of course, this is a working assumption only as long as there are unexplained aspects of behaviour. . . . Our failure to solve a problem so far does not make it insoluble One cannot logically be a determinist in physics and chemistry and biology and a mystic in psychology.[94]

In this, Hebb was implicitly criticizing the Freud-to-Jung trajectory in psychology that starting out from actual research into the nervous system by the young Freud, later branched out along a path of mystical neuro-energetics.[95] Working in a US context in which Freud was still highly influential, Hebb had to be tactful in his criticism, but the thrust is no less clear.

> Ego, Id, and Superego are conceptions that help one to see and state important facts of behavior, but they are also dangerously easy to treat as ghostly realities, as anthropomorphic agents that want this or disapprove of that, overcoming one another by force or guile, and punishing or being punished. Freud has left us the task of developing these provisional formulations of his to the point where such a danger no longer exists. When theory becomes static it is apt to become dogma; and psychological theory has the further danger, as long as so many of its problems are unresolved, of inviting a relapse into the vitalism and indeterminism of traditional thought.[96]

Unfortunately, this lapse into Freudian indeterminism and vitalism had, by the 1960s, so influenced French Marxist theorists that Timpanaro[97] could remark that it was their main distinguishing feature.

The new atomism

> *The remarkable facts that chemists daily discover are as many new proofs of atomism.*[98]

Newton was a follower of Lucretius, that is, an atomist. His account of the propagation of sound waves is explicitly based on the idea that bodies are made of atoms. From this, he was able to correctly predict that diffraction of sound should exist,[99] a remarkable early demonstration of the power of the atomic hypothesis. But it was through chemistry rather than what we would now call physics that atomism became concrete.

Dalton

While the term 'atom' continued to be used during the eighteenth century, it was mentioned mainly in the context of historical accounts of the Epicureans[100] or as a literary metaphor in popular prose, the assumption being that educated people had read Lucretius. It was not used seriously by chemists until the work of Dalton.

He developed his thoughts on atoms in the context of experiments he did on the quantities of different gases that could be dissolved in water.[101] From this, he developed the idea of partial pressures of gases in solution, a notion that continues to be used in contemporary chemistry. The theoretical model he had for partial pressure was that when a gas, let us say oxygen dissolves in water, then the oxygen atoms are interspersed with water 'atoms' in a regular grid.

Dalton discovered the notion of partial pressure in a solution, which led him to estimate the proportions of the principal gases in the atmosphere and formulate 'Dalton's Law'. He also introduced the concept of atomic weights by utilizing the idea of partial pressure and measuring the densities of gases. He was the first to clearly distinguish what he termed mechanical mixtures of atoms, for example, oxygen dissolved in water, from chemical compositions.

> Now it is one great object of this work, to shew the importance and advantage of ascertaining the relative weights of the ultimate particles, both of simple and compound bodies, the nuimber of simple elementary particles which constitute one compound particle, and the number of less compound particles which enter into the formation of one more compound particle[102]

He got the relative order of molecular weights right. His estimates of the molecular weights of nitrogen and oxygen are out by a factor of at leas two; for compound gases, his errors are much smaller. The larger errors for Nitrogen and Oxygen molecules are probably due to his assumption that these gases were monatomic.

> Before we can apply this doctrine to find the specific heat of elastic fluids, we must first ascertain the relative weights of their ultimate particles. Assuming at present what will be proved hereafter, that if the weight of an atom of hydrogen be 1, that of oxygen will be 7, azote 5, nitrous gas 12, nitrous oxide 17, carbonic acid 19, ammoniacal gas 6, carburetted hydrogen 7, olefiant gas 6, nitric acid 19, carbonic oxide 12,[103]

The estimates that he arrived at are summarized in Table 5.1, which compares them with the modern values, and translates his names for compounds into modern chemical formulae.

In[104] he elaborated the hypothesis that atoms came in several orders. The primary atoms were the elements – of which he was already able to identify twenty. Pairs of these could be bound together to form what he called binary atoms, triples to form ternary atoms etc. These higher orders correspond pretty closely to what we now know as molecules. As Figure 5.2 shows, some of his formulae are now known to be wrong. For instance, he believed water was made of one hydrogen and one oxygen atom, another consequence of the assumption that hydrogen gas was monatomic.

The idea he had for a water 'atom' was in some ways strikingly modern. We now think of atoms as being made of a nucleus of neutrons and protons surrounded by a cloud of electrons. He thought that binary and ternary, etc., atoms had a nucleus of elementary particles surrounded by what he called an atmosphere of heat.

The shift from Dalton's model of 1808 to the mid-twentieth-century model came down to two points:

1. Dalton took the chemical elements to be elementary particles. Mid-twentieth-century theory took protons and neutrons to be the 'elementary' particles. Late twentieth-century theory decomposed even these into quarks and gluons.

Table 5.1 Comparisons of Dalton's Estimates of Relative Molecular Weights to Those We Know Today

	Dalton's name	**Actual molecular weight**	**Ratio to hydrogen molecule**	**Dalton's estimate**
H_2	Hydrogen	2.01594	1	1
N_2	Azote	28.0134	13.89595	5
O_2	Oxygen	31.9988	15.87289	7
N_2O	Nitrous oxide	46.0055	22.82087	17
NO	Nitrous gas	30.0061	14.88442	12
CO_2	Carbonic acid	44.0098	21.83091	19
NH_3	Amoniacal gas	17.03061	8.447975	6

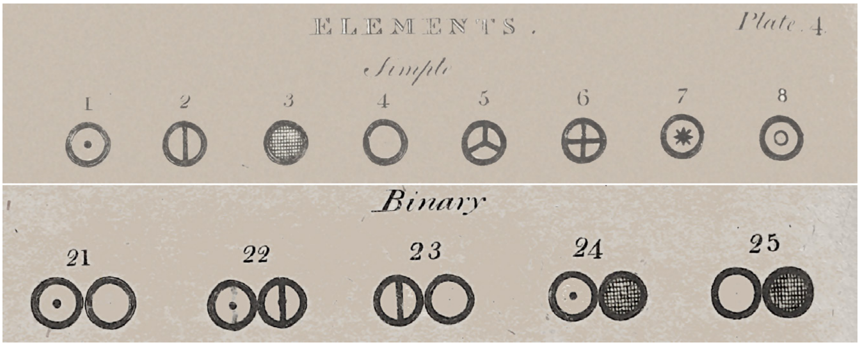


Figure 5.2 Dalton's symbolic notation for the atoms of elements. [1] Hydrogen, [2] Azote (Nitrogen), [3] Carbone, [4] Oxygen, [5] Phosphorous, [6] Sulphur, [7] Magnesia, [8] Lime (Calcium); also for some compounds: [21] Water, [22] Ammonia, [23] Nitrous oxide, [24] Olefiant gas (Ethylene C_2H_2), [25] Carbonic oxide (Carbon monoxide). Note that he did not realize that gaseous hydrogen was H_2, hence the odd composition of water, etc. From John Dalton, *A New System of Chemical Philosophy*.

2. Dalton attributed the elasticity of gases to an *atmosphere* of *caloric* or heat around a nucleus made up of chemical elements. As a gas was heated, caloric was added to the atoms, which grew larger, thus making the gas expand.

   This has obvious analogies to the *cloud* of electrons that modern elementary textbooks describe around the nucleus. We accept that it is the electrostatic repulsion of the electron clouds that is responsible for at least the elasticity of solids. We even had the concept in the Bohr model of the atom that when you added energy to an atom, its electrons moved to higher levels with a consequent expansion, which is not far from Dalton's concept that the atmosphere of caloric expanded with the addition of heat.

There is a repeated process by which materialist understanding discovers what it takes to be atomic or indivisible particles, *primordia* in Lucretius terms, only to find that they are composed of even more basic ones. The Standard Model in use now has a set of elementary particles: quarks, gluons, leptons and neutrinos. It remains to be seen whether a further level of detail will be detected below these.

Heat, matter and time

Dalton's ideas, much like compounds, are built on the foundational work of others. Torricelli's discovery and measurement of atmospheric pressure in 1643 laid the groundwork for understanding partial pressures. In the years that followed, chemists identified new elemental gases: hydrogen in 1766, nitrogen in 1772 and oxygen in 1777, which were crucial for conceptualizing the various partial pressures that constitute the atmosphere. However, Dalton's work still operated under the notion of heat as a separate entity, known as 'caloric'. This concept was based on the pioneering work of Black and Watt, who quantified heat, specific heat and latent heat, thereby setting the stage for the era of steam-powered industrial capitalism.

We have already commented on how this new technical basis for society prepared key conceptual tools for the understanding of the economy.[105] But through the necessities of engineering efficiency, it also prepared the study of thermodynamics and motivated the next stage in the development of atomism: the understanding of heat as motion.

> If the steam-engine had not been invented, we should assuredly stand below the theoretic level which we now occupy. The achievements of Heat, through the steam-engine, have forced, with augmented emphasis, the question upon thinking minds: What is this agent, by means of which we can supersede the force of winds, of rivers, of horses and men ?[106]

In the late eighteenth and early nineteenth centuries there were two schools of philosophy on heat. On the one hand, there were those who were styled *materialists*, like Dalton and many other chemists, who thought that heat was a distinct form of matter. On the other hand, there were what were termed *mechanists* for whom heat was the mechanical motion of atoms. The mechanistic ideas, which eventually triumphed, went back at least as far as Bacon,who believed that heat expanded only in the smaller particles of a hot body. This expansion was checked, repelled and beaten back by other particles, creating a back-and-forth motion that caused the fury of heat and fire.[107]

But the idea that heat was mechanical motion had to contend with what initially seemed an equally plausible materialist view.

- Heat flows. This seemed very like the behaviour of a liquid. If you bring a wet cloth into contact with a dry one, the dampness spreads into the dry cloth. If you bring a hot piece of metal in contact with a cold one, the heat spreads into a similar way.
- When a gas is compressed, it heats up. This was interpreted as the heat being squeezed out of the gaps between the atoms.
- Friction was interpreted as releasing heat that had hitherto been too tightly contained in objects. If one observed a spark being struck from a flint and steel, this seemed a plausible account. The blow appeared to release the spark from the steel, implying that the heat was latent within it.
- Thermometers allowed scientists to study specific heat, which refers to the amount of heat needed to raise the temperature of different substances by the same number of degrees. Water can absorb thirty times more heat than mercury for each degree of increase in temperature. The 'heat capacity' of a compound was taken to indicate how much space there was between the atoms for heat to enter.

According to Tyndall, a crucial observation discredited the materialist account. It was Count Rumford's experience with cannon boring. Rumford observed that cannons got very hot when being bored.

> Being engaged, lately, in superintending the boring of cannon, in the workshops of the military arsenal at Munich, I was struck with the very considerable degree of heat which a brass gun acquires, in a short time, in being bored; and with the still more intense heat (much greater than that of boiling water, as I found by experiment,) of the metallic chips separated from it by the borer.
>
> The more I meditated on these phenomena, the more they appeared to me to be curious and interesting. A thorough investigation of them seemed even to bid fair to give a farther insight into the hidden nature of heat; and to enable us to form some reasonable conjectures respecting the existence, or non-existence, of an igneous fluid,: a subject on which the opinions of philosophers have, in all ages, been much divided.[108]

As we have said, the standard account of friction was that it released heat latent in the subject being rubbed. The materialists held a 'nothing from nothing' principle. The total amount of heat remained constant. If heat was released in a flint on steel spark, then it must have been hidden in the steel before the flint struck it. Analogously, the heat of a bored cannon must have been latent in the bronze before it was ground to dust by the boring machine. Rumford was not convinced and experimented.

> From whence comes the heat actually produced in the mechanical operation above mentioned ?
>
> Is it furnished by the metallic chips which are separated by the borer from the solid mass of metal ?
>
> If this were the case, then, according to the modern doctrines of latent heat, and of caloric, the capacity for heat of the parts of the metal, so reduced to chips, ought not only to be changed, but the change undergone by them should be sufficiently great to account for all the heat produced.
>
> But no such change had taken place; for I found, upon taking equal quantities, by weight, of these chips, and of thin slips of the same block of metal separated by means of a fine saw, and putting them, at the same temperature, (that of boiling water,) into equal quantities of cold water, (that is to say, at the temperature of 59° F .) the portion of water into which the chips were put was not, to all appearance, heated either less or more than the other portion, in which the slips of metal were put.[109]

He was able to reject the hypothesis that the heat of friction was some substance released from the brass. Other experiments proved that there was no weight change associated with heating or cooling. Instead, the heat must have come from the work put into the borer by the unfortunate yoked horse driving it. He held that heat could be produced by a horse's strength and indeed used for cooking, but never more heat than could be obtained by using the horse's food as fuel.

> In reasoning on this subject we must not forget that *most remarkable circumstance*, that the source of the heat generated by friction in these experiments appeared evidently to be *inexhaustible* It is hardly necessary to add, that anything which any *insulated* body or system of bodies can continue to furnish *without limitation* cannot possibly be a *material substance*; and it appears to me to be extremely difficult, if not quite impossible, to form any distinct idea of anything capable of being excited and communicated m those experiments, except it be MOTION.[110]

There are obvious echoes here of Marx discussing labour power and wage goods as the necessary source of energy for productive labour. In Chapter [5], we referred to the dependence of Marx's theory of exploitation on concepts relating to the physical equivalence of work and energy. We have cited above from Tyndall's lectures on these topics. It turns out that Marx actually attended these lectures in which Tyndall summarized the work of Rumford.

> Marx and Engels had some familiarity with and in some cases had closely studied the works of many of the scientists involved in the development of thermodynamics (both the first and second laws) – including Hermann von Helmholtz, Julius Robert Mayer, James Prescott Joule, Justus von Liebig, Jean-Baptiste Joseph Fourier, Sadi Carnot, Rudolf Clausius, William Thomson, Peter Guthrie Tait, William Grove, James Clark Maxwell, and Ludwig Eduard Boltzmann. In addition, we knew that Marx had attended numerous public lectures on natural science in the years leading up to and following the publication of Capital, Volume I in 1867, and that among these was a series of lectures by the English physicist John Tyndall, author of Heat Considered as a Mode of Motion. Tyndall, a major figure in the developing physics in his own right, was the principal advocate of the ideas of J.R. Mayer – one of the co-discoverers of the conservation of energy (the first law of thermodynamics). Marx followed Tyndall's research on the Sun's rays, particularly as it related to heat.[111]

Tyndall's lectures taught him that heat is a result of atomic or molecular motion, and that the expansion, melting and vaporization of ice occur due to molecules' motion overcoming cohesion forces, and he could estimate the actual velocities of gas molecules in the atmosphere.[112] Engel's familiarity with these topics of the science of his time is obvious from the notes and fragments for *Dialectics of Nature*.[113]

But we now move to two questions that pass beyond the boundary of what Marx and Engels seem to have known about the mechanics of their day, questions that Maxwell and Boltzmann could pose but not yet give an adequate answer to.[114]

1. Why does time appear to have a direction?
2. How does knowledge or information relate to work?

Historically, the questions were posed in the opposite order, but we will deal with times arrow first. It relates to issues already raised in Sections 5.1 and 5.4 since both Engels and Darwin implicitly accept not only that time is directional, but also provide distinct hypotheses for that directionality.

The difference between the past and the future, and the 'flow' of time has been obvious since antiquity. Birth, growth, death and decay were obvious. People knew from experience that snow melts in the spring, that hot pans cool down, etc. This direction of time was unremarkable. Time's flow and passage might be regretted but seemed self-evident. Even a careful philosopher like Newton took time-flow as an axiomatic assumption, which we criticized earlier.[115] The development of Lagrangian and Hamiltonian mechanics, which focus on complete trajectories constrained by conservation laws, emphasized the inadequacy of viewing time as a flow, revealing that the basic laws of mechanics were symmetrical with respect to the time axis.

> IN abstract dynamics an instantaneous reversal of the motion of every moving particle of a system causes the system to move backwards, each particle of it along its old path,-and at the same speed as before when again in the same position – that is to say, in mathematical language, any solution remains a solution when t is changed into $-t$.
>
> If, then, the motion of every particle of matter in the universe were precisely reversed at any instant, the course of nature would be simply reversed for ever after. The bursting bubble of foam at the foot of a waterfall would reunite and descend into the water : the thermal motions would re-concentrate their energy and throw the mass up the fall in drops reforming into a close column of ascending water. Heat which had been generated by the friction of solids and dissipated by conduction, and radiation with absorption, would come again to the place .of contact and throw the moving body back against the force to which it had previously yielded. Boulders would recover from the mud the materials required to rebuild them into their previous jagged forms, and would become reunited to the mountain peak from which they had formerly broken away.[116]

Tracing the paths of two colliding billiard balls, the scenario appears identical if the paths are reversed. On a real table, balls decelerate due to friction, whereas atoms glide without friction through space, allowing perfect reversibility. However, thermodynamics has uncovered fundamentally irreversible processes. Heat naturally flows from hot to cold bodies, and the entropy of closed systems does not decrease. While theoretical mechanics may not differentiate between past and future, thermodynamics confirms our innate sense of time's directionality.

Thermodynamics starts as an empirical science, an outgrowth of the practical needs of production.[117] Engineers perfecting steam engines were its initial impulse. It was, in Marx's terms, a science driven by the new mode of material production. A child of capitalist industry's drive to economize on labour time, thermodynamics asserted the inexorable objectivity of time's direction. Given this, the challenge for the atomic theory of heat was to demonstrate that the laws of mechanics themselves set time's compass pointing to the future.

Thomson, whose pointing out of the theoretical reversibility of time is cited above, gave an explanation in the following way.

Instead of exploring the more complex instances of time reversal, such as waterfalls flowing in reverse or friction working backwards, he chose to focus on the simpler case of heat transfer. Imagine a metal bar, he proposed, uniformly heated on the left side and uniformly cooled on the right, all encased in perfect insulation. Over time, the heat would spread throughout the bar, eventually leading to a consistent temperature from one end to the other.

He acknowledges that while it was accepted that heat in a bar of metal was due to the motion of the bar's atoms, in 1875, physics had an imperfect understanding of how atoms vibrated within metal. He proposed a cavity with warm and cold gas, separated by a permeable barrier with small holes. If these holes could be opened or closed remotely, hot fast molecules from the left would pass through the barrier, while cold slow ones would pass from right to left. This exchange would equalize the average speeds of the molecules on both sides.

Thompson was explicitly considering finite systems, with fixed numbers of gas molecules. But his finite systems still involved very large numbers of molecules: 8,000,000,000,000 molecules of nitrogen in one example.

We shall illustrate the principle with much smaller numbers: just eighty molecules. Instead of looking at molecules moving freely in three dimensions at varying velocities, we will take a much simpler model. We will restrict the motion to two dimensions to allow us to draw simple pictures. We will further restrict the directions of motion of the molecules to be either up/down or left/right and assume velocities are constant. In other words, we will quantize or reduce the number of degrees of freedom of the model, obtaining what is termed a lattice gas model.

These simple lattice models have been proven to be adequate for many practical purposes – computer simulation of aerodynamics being an example.[118] Lattice gas models for the plane, since they involve tessellation, can either be hexagonal or the right-angle one we will use (Figure 5.4). In both types, the behaviour is specified by a set of rules that relate incoming particles on the lattice to outgoing ones in such a way as to conserve mass, momentum and energy. The rules we will use in this example are displayed diagrammatically in Figure 5.3.

Lattice gas models lend themselves to efficient digital implementation. The authors have constructed specialized machines for this purpose.[119] Since they work with particles moving at constant velocities, we cannot model temperature differences as in Thomson's original example, but we can illustrate a similar effectively irreversible process using pressure differences.

Look at Figure 5.4. You can see how initial order ends up with gas atoms spread through the area of lattice in what appears to be a random fashion(Figure 5.4.b).

The two images effectively illustrate the concept of increasing entropy. Image (a) depicts a highly ordered state with all eighty atoms positioned to the left, moving either right or down. Image (b) shows these atoms spread out across the area and,

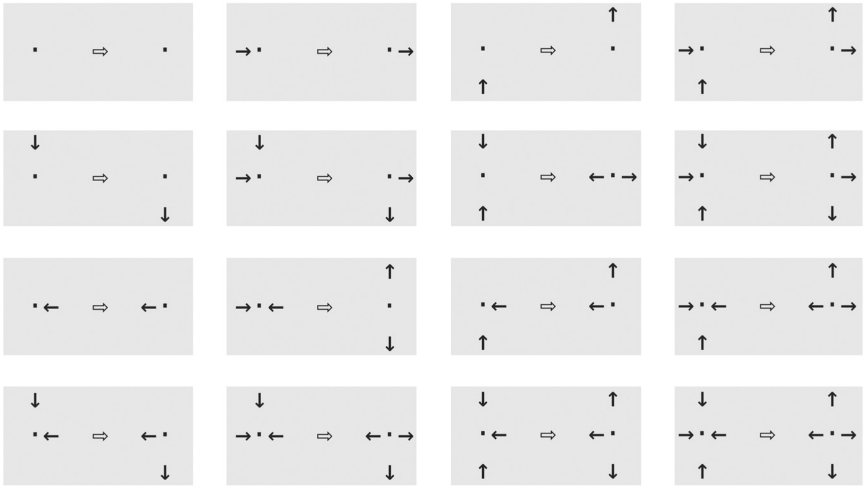


Figure 5.3 The evolution rules of a rectangular 2D lattice gas system are illustrated, with each grey rectangle representing a rule indicating the configuration of incoming and outgoing molecules, conserving matter, energy and momentum. Figure by authors.

in terms of momentum, with an equal likelihood of moving in any direction—up, down, left or right.

Once the atoms are randomly distributed among lattice states, the chances of finding them all on one side upon further observation are exceedingly slim. If we record how many atoms were on the left as the process proceeds (Figure 5.5), we see the left count starts at eighty but rapidly evolves to oscillate noisily around forty.

The simple state structure of lattice gases lends them to easy probability analysis. The snapshots in Figure 5.4 are taken as atoms leave lattice nodes. There are eighty lattice positions from which atoms can depart, and each lattice node can have between zero and four atoms leaving it.

Since there are eighty atoms in the gas, on average there is one atom leaving each lattice node. Each exit path thus has a $\frac{1}{4}$ chance of being occupied by a departing atom and a $\frac{3}{4}$ chance of being empty. Obviously, these chances would be different if the overall pressure of the gas, in terms of atoms to grid positions, was higher or lower.

There are 160 different exit paths on the right-hand side of the grid. The probability of there being no atoms anywhere on the right-hand side must therefore be $\left(\frac{3}{4}\right)^{160}$ which is about 1.022×10^{-20} or $\frac{1}{100{,}000{,}000{,}000{,}000{,}000{,}000}$. Suppose you had the physical apparatus we built in the 1990s that could update a lattice gas twenty million times a second, then you would have to run it for around 150,000 years to have a good chance of the gas molecules all returning simultaneously to

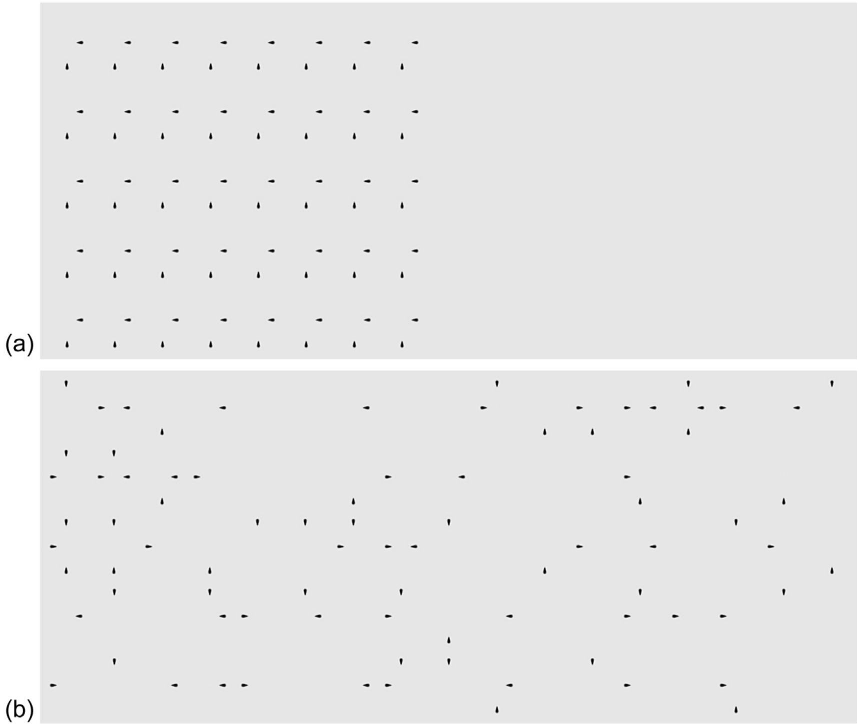


Figure 5.4 Gas lattice of dimension 5 × 165. Motion is indicated by the teardrop shape. The lattice gas is initially configured in a low-entropy state, with all atoms on the left side and only two of four possible atomic momentum states. After 1,000 time steps, the atoms are evenly distributed. Figure by authors.

the left-hand side. This is with a tiny number of atoms. Thomson in his examples assumed vastly greater numbers 8,000,000,000,000 molecules, and these molecules had much more freedom of movement. They could move in any direction and at a continuous range of velocities. In such an example, the chances of all molecules ending up on one side would be so low that it would not have occurred in what we now estimate to be the entire duration of the universe.[120] If dark energy is real, and phase space consequently expands remorselessly, then even the age of an infinite universe is not going to give us recurrence.

So, starting from a very ordered state of a volume of air, whether in terms of temperature or in terms of relative distribution of the oxygen and nitrogen molecules, for Thomson used both examples, a return to that order after it was allowed to freely evolve is vanishing unlikely.

Theoretical processes, such as heat flow from hot to cold, are thus explained in a way compatible with Newtonian atomic motion. However, the possibility of mechanical reversal, as described by Thomson, raises questions about what would happen if atoms' motion suddenly reversed after reaching equilibrium.

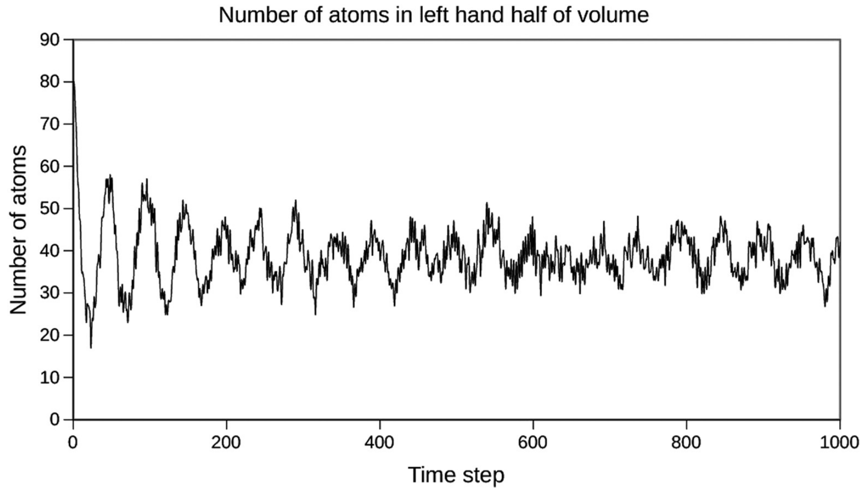


Figure 5.5 At the start, all eighty atoms are on the left. The system settles to a higher entropy state with oscillations around a mean of forty, with around half of the atoms in the left half of the space. This entropy fluctuation is a basis for Thomson's argument. Figure by authors.

The collisions and diffusion that had previously occurred must now play backwards. After the exact same time that the diffusion had originally taken, the initial starting point must be recovered with hot, fast molecules to the left and cold, slow ones to the right, except that in each case now, the movements must be in the reverse sense. Similarly, were we in our lattice gas simulation to apply the inverse rules of Figure 5.3 at step 1,001 and continue doing this 1,000 times then all our atoms would end up on the left. But this time all momenta would be towards the left or upwards. Order would have been recreated from disorder.

But this prospect of a return to Eden is exquisitely sensitive. Change the momentum of one atom in picture (b), and the application of the reversed evolution rules will fail to recreate picture (a), yielding instead another apparently disordered state with atoms all over the place. Moreover, even should the reversal work perfectly, the result would be evanescent:

> Suppose now the temperature to have become thus very approximately equalised at a certain time from the beginning, and let the motion of every particle become instantaneously reversed. Each molecule will retrace its former path, and at the end of a second interval of time, equal to the former, every molecule will be in the same position, and moving with the same velocity, as at the beginning; so that the given initial unequal distribution of temperature will again be found, with only the difference that each particle is moving in the direction reverse to that of its initial motion. This difference will not prevent an instantaneous subsequent commencement of equalisation, which, with entirely different paths for the individual molecules, will go on in the average according to the same law as that which took place immediately after the system was first left to itself.[121]

So, according to Thomson, we would have a time symmetry(Figure 5.6). Viewed from the perspective of our normal time direction, the gas moves from a low-entropy improbable state to a high entropy probable state. If we reverse all motions, it evolves back from high entropy to low entropy, but would then pass through the low-entropy state and again move into a more probable high entropy state.

What does the thermodynamic arrow of time then amount to, given that it possesses this deeper time symmetry?

Look back at Figure 5.6. From time 10, all atomic motions reverse, so the system recapitulates its history in reverse until time 20, it arrives at a mirror image[122] of its state at time 0. Since this low-entropy state at time 20 is inherently improbable, the overwhelming likelihood is that from time 20 on the system evolves into a more disordered state. But this argument implies the prehistory before time 0.

By the time symmetry of the laws of mechanics, the state at time −1 must be the mirror image of that at time 21. Period A is the reflection of period D. As such, the entropy must be inferred to be the same at time −1 and time 21, and by induction, the entropy for period A leading up to time 0 *should have been falling!*

So the explanation that time asymmetry arises from probability turns out not to be final. If we assume that an isolated system is in a low-entropy state now, then it is indeed probable that it will be in a higher entropy state in an hour's time. But it must also be probable that an hour ago it was in a higher entropy state than it is now.

If you had a finite model system like a lattice gas that evolves under symmetrical dynamical laws, ones which have conservative properties, that is indeed what you

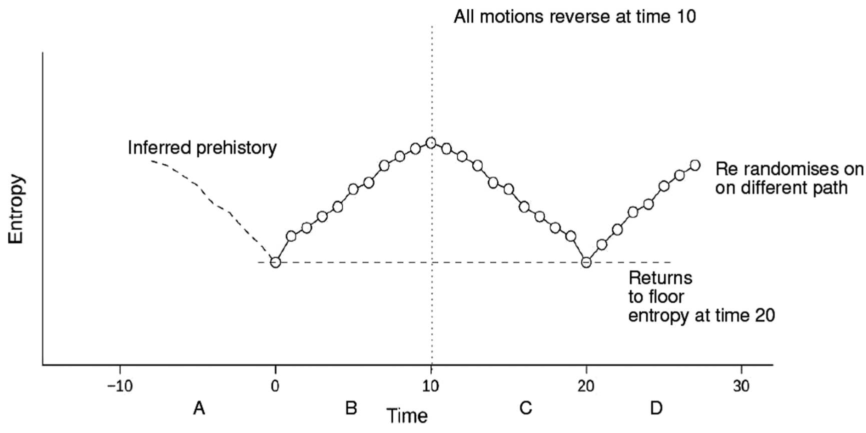


Figure 5.6 The time evolution of a finite system as described by Thomson. At time 0, the system is in a low-entropy state and evolves towards increasing entropy. At time 10, all particle motions are reversed. The system now evolves back to its original low-entropy state, attaining it at time 20. Entropy then starts to increase again but follows, at the microscopic level, a different path. Figure by authors.

would observe. If we applied reverse evolution rules 1,000 times to picture 5.4.a, we would get an apparently random picture *c* with atoms throughout the volume. Apply the forward rules to picture *c* and we would end up again with picture 5.4.a.

When applying this concept globally, the implications are unsettling. Should we infer, based on probability, that a seemingly improbable low-entropy state of the world one second ago was likely preceded by a higher entropy state two seconds prior?

Is it plausible to believe that the current rise in entropy originates from a fleeting low-entropy condition a few seconds or days earlier, before which time flowed in reverse?

Could it be that our recollections of the past are merely deceptive?

Thomson's argument about closed systems would seem to imply this possibility.

He says that during an entropy-reducing phase (times −10 to 0 or times 10 to 20 in Figure 5.6) consciousness would reverse.

> if also the materialistic hypothesis of life were true, living creatures would grow backwards, with conscious knowledge of the future, but no memory of the past, and would become again unborn.[123]

We would know the future but, remembering it, think it was the past.

He avoids the conclusion that this is happening because he says he was just considering finite and isolated systems. He is careful to state that it does not apply to systems communicating with the rest of the universe:

> It is a strange but a true conception of the old well-known law of the conduction of heat to say that it is very improbable that in the course of 1,000 years one half the bar of iron shall of itself become warmer by a degree than the other half; and that the probability of this happening before 1,000,000 years pass is 1,000 tines as great as that it will happen in the course of 1,000 years, and that it certainly will happen in the course of some very long time. But let it be remembered that we have supposed the bar to be covered with an impermeable varnish. Do away with this impossible ideal, believe number of molecules in the universe to be infinite ; then we may say one half of the bar will never become warmer than the other, except by the agency of external sources of heat or cold. This one instance suffices to explain the philosophy of the foundation on which the theory of the dissipation of energy rests.[124]

In the account of Thomson's 1875 paper above, our commentary has freely used the notion that less probable states have lower entropy. Proof of this was due to Boltzmann's paper of 1872 (translated as[125]) in which he proved in terms of statistical arguments that the entropy in a closed system must tend to increase.[126] In 1876, Boltzmann's close friend Loschmidt made a similar reversibility argument to Thomson[127] where he explicitly pointed out that in the event of particle reversal then entropy must decrease as shown in Figure 5.6.

The fine detail of Boltzmann's 1872 proof rested on the assumption that when two molecules (A and B) of a gas collided at location *x*, the probability that each molecule would arrive at position *x* was statistically independent.[128] But immediately after two molecules collide, their position and velocities are not independent. The act of collision has correlated both their positions – they must have been very close to one another to collide.

If you reverse all motions at time 10 in Figure 5.6, then the motions of the molecules are most definitely not independent. They are predetermined to be exactly such as will return the system to its starting position. In that case, the assumption that Boltzmann had relied upon to deduce an arrow of time is violated, and the time arrow could reverse – entropy could decline with time.

But we do experience a rise in entropy as an empirical given. How is one to explain this in terms of the nature of the universe and its initial state?

> One has the choice of two kinds of pictures. One can assume that the entire universe finds itself at present in a very improbable state. However, one may suppose that the eons during which this improbable state lasts, and the distance from here to Sirius, are minute compared to the age and size of the universe. There must then be in the universe, which is in thermal equilibrium as a whole and therefore dead, here and there relatively small regions of the size of our galaxy (which we call worlds), which during the relatively short time of eons deviate significantly from thermal equilibrium. Among these worlds the state probability increases as often as it decreases. For the universe as a whole, the two directions of time are indistinguishable, just as in space there is no up or down. However, just as at a certain place on the Earth's surface we can call 'down' the direction towards the centre of the Earth, so a living being that finds itself in such a world at a certain period of time can define the time direction as going from less probable to more probable states (the former will be the 'past' the latter the 'future') and by virtue of this definition he will find that this small region, isolated from the rest of the universe, is 'initially' always in an improbable state. This viewpoint seems to me to be the only way one can understand the validity of the second law and the heat death of each individual world without without invoking an unidirectional change of the entire universe from a definite initial state to to a final state. The objection that it is uneconomical and hence senseless to imaging such a large part of the universe as being dead in order to explain why a small part is living – this objection I consider invalid. I remember only too well a person who absolutely refused to believe that the Sun could be 20 million miles from the Earth, on the grounds that it is inconceivable that there could be so much of space filled only with aether and so little with life.[129]

Modern physics generally just accepts that the whole universe just started in a low entropy big bang – the first of Boltzmann's alternatives.[130] But variants of his second alternative, with multiple local regions or island universes, are still discussed.[131] Modern 'heat death' scenarios involve stars collapsing into black holes, which eventually evaporate to radiation.[132]

Boltzmann's account of entropy can be used to explain how structure initially emerges. Lucretius attributed this to the *clinamen* or unpredictable swerve of atoms. Lucretius envisaged the atoms as initially 'falling', so he assumed parallel downward paths. We now know that, as Boltzmann said, 'in space there is no up or down'. So we cannot assume initial parallel falling. The initial big bang is assumed to have had no preferred direction, but just a mass of electrons, protons, neutrons and electromagnetic radiation (photons). These were equivalent to the *corpora* or first bodies of which Lucretius wrote.

If we assume these were in thermal equilibrium, which the relative uniformity of the cosmic background radiation suggests, then the problem that Lucretius faced remains. What brought the 'first bodies' together?

Why did the universe not remain a sea of subatomic particles?

This, in its most elementary form, is the old philosophical question as to why order emerges from chaos.

Frautschi[133] constructs a simple argument based on Boltzmann's S formula for this. Applying it to the whole universe with N initial particles, we have $S = k\log($ number of N particle states). Next, he shows that since logs, even for cosmological quantities, are less than 100, you can approximate the entropy of the initial neutron-proton gas by $S(n, p \text{ gas}) \approx kN$.

If four nucleons combine to form an alpha particle, a first agglomeration of Lucretian corpora, a 7 MeV (γ) photon is released. The entropy of the resulting alpha and photon gas, or plasma, can be written:

$$S(\alpha, \gamma \text{ gas}) \approx k\left(N_\alpha + N_\gamma\right) \approx k\left(\frac{N}{4} + \frac{N7MeV}{3kT}\right)$$

The important point here is that the entropy contribution of the photons is inversely proportional to temperature T. If $kT > 7$ MeV, the entropy of the disassociated neutrons and protons is higher than that of the alpha particles plus gamma rays. As the universe expanded, the temperature of the neutron-proton gas fell below a critical level[134] at which the entropy of the helium nuclei plus gamma rays became greater than that of the isolated nucleons, and primordial helium was formed. Further expansion cooled the γ photons down to the 2.4 Kelvin microwave background we now see.

Frautschi's[135] demonstration of order from chaos relies upon several things that Boltzmann himself did not know: sub-atomic particles, the binding energy of α particles, Einstein's mass-energy equivalence. All were discovered within two decades of Boltzmann's premature death by suicide, in the absence of which he might reasonably be expected to have seen them. In the century and a half since Boltzmann's original paper, the application of his ideas have spread. Our understanding of information,[136] the economy[137] and even gravity[138] have been shown to be derivable from his statistical theory of entropy.

Chapter 6

IDEALIST REPRISE AND RESPONSES

Opposition to Boltzmann

Boltzmann spent a large part of his career as a physicist. His final years, though, were as the professor of philosophy at the University of Vienna. In this post, he replaced Mach, who also straddled physics and philosophy.

In terms both of philosophy and physics the two were rivals, proponents of polar opposite positions. Boltzmann, the materialist atomist, against Mach, the idealist.

Although Boltzmann's atomism is now completely uncontroversial, that was far from the case in his lifetime. A lot of prominent continental physicists were highly sceptical about the existence of atoms. Poincaré, for example, referred to the atomistic account of heat propounded by Boltzmann, Thomson and Maxwell as the 'English[1] kinetic theory' and doubted it as an explanation of the law of increasing entropy.

> I do not know if it has been remarked that the English kinetic theories can extricate themselves from this contradiction. The world, according to them, tends at first toward a state where it remains for a long time without apparent change; and this is consistent with experience; but it does not remain that way forever, if the theorem cited above is not violated; it merely stays there for an enormously long time, a time which is longer the more numerous are the molecules. This state will not be the final death of the universe, but a sort of slumber, from which it will awake after millions of millions of centuries. According to this theory, to see heat pass from a cold body to a warm one, it will not be necessary to have the acute vision, the intelligence, and dexterity of Maxwell's demon; it will suffice to have a little patience.[2]

The opposition to atomism by Poincaré was founded on strictly mathematical objections. They were ones founded on abstract Lagrangian or Hamiltonian mechanics rather than philosophy or ontology. In essence, he translated the heuristic explanation given by Thomson in the passages quoted earlier into formal maths using Liouville theorem. This can be used to show that if you take a small volume δ_t in a $6N$ dimensional phase space at time t and allow it to evolve along a course of least action for some small time ϵ it maps to a new region in phase space $\delta_{t+\epsilon}$ whose volume is unchanged. Thus he argued that if you start with a closed system in some low-entropy state, that is, in some δ_t that is a small part of the possible phase space positions, it will always end up in an equivalent volume. That is to say, the resultant volume of phase space is equally improbable, while similar arguments were being made by Zermelo. They rested on the apparent necessity for recurrences to occur in any finite system of particles operating by deterministic laws. While this is certainly valid for simple relatively small models like our lattice gas example, the argument fell if either the universe is infinite – which was essentially Thomsons point – or that such recurrences would take such a long time for even modest volumes of gas, that we might as well treat the system as infinite – Boltzmann's response.

> Thus when Zermelo concludes, from the theoretical fact that the initial states in a gas must recur – without having calculated how long a time this will take –, that the hypotheses of gas theory must be rejected or else fundamentally changed, he is just like a dice player who has calculated that the probability of a sequence of 1000 one's is not zero, and then concludes that his dice must be loaded since he has not yet observed such a sequence![3]

There is also an implicit assumption in the Poincaré reccurence argument that the positions of the atoms are objectively defined to arbitrary precision at all instants. If this is abandoned, if we allow for any absolute, objective uncertainty in the positions of atoms, then the collision process magnifies the scale of that initial uncertainty.

We have discussed this earlier[4] in our treatment of Fredkin's hypothetical reversible computers[5] which are in a sense a variant of the Poincaré argument. In terms of modern information theory, which we will discuss later, the assumption of arbitrarily precise particle positions implies a system with infinite information content and thus a zero volume in phase space. Since entropy is defined in terms of a logarithm over volumes in phase space, and since log(0) is an undefined function, the very conceptual setup of the Zermelo/Poincaré recurrence paradox renders the concept of entropy meaningless.

One either concludes, as Poincaré did, that atomism and the 'English kinetic theory' are false or that the arbitrary precision mathematics of position assumed by classical analysis is unphysical. The development of quantum mechanics in subsequent decades pointed to the latter conclusion. If positions are not arbitrarily divisible, then you can still have deterministic models like the lattice models presented earlier, but you could, in principle, also have non-deterministic lattice models.

Machism

The final years of Boltzmann's life were consumed by what likely felt like a futile struggle to persuade the German-speaking scientific community to embrace his atomic theories. In addition to previously mentioned critics, the prominent figure of Mach stood against him. Mach, a professor of Philosophy at Vienna until 1901, propagated anti-atomistic and anti-realist philosophies that greatly impacted a generation of European scientists. His philosophical stance still resonates in today's debates on quantum mechanics interpretations. Following Mach's retirement in 1901, Boltzmann succeeded him as the Chair of Philosophy at Vienna, where he began teaching the philosophy of science. Sadly, his tenure was brief.

> With his wife and daughter he visited a place at the Bay of Duino near Trieste on September 6, 1906. When his wife and daughter were enjoying swimming he hanged himself. He had made an earlier suicide attempt when he was at Leipzig. Physicists everywhere were devastated by the news that Boltzmann, in deep depression, had committed suicide. His act was difficult for many physicists to understand. His suicide seems to have been due to factors in his personal life (depressions and decline of health) rather than to any academic matters. Depressed and in bad health, he committed suicide just before experiment verified his work. In a tribute to Boltzmann, Ostwald described Boltzmann as a victim of the immense sacrifices of health and strength demanded of those who struggle for scientific truth. Some physicists ascribed Boltzmann suicide to mental instability. According to Chandrasekar, he was greatly depressed by the violent attacks made on his ideas by Ostwald and Mach. This made him to commit suicide.[6]

He died just as two more famous figures were entering the battle of ideas over materialism: Einstein and Lenin. Before we go into how Einstein vindicated Boltzmann scientifically in 1905 and Lenin vindicated him philosophically in 1908, we must run through the objections raised by Mach to the atomic theory.

In Chapter 5, we recounted how, from the seventeenth to the early nineteenth centuries, there were two competing theories of heat. The mechanical theory held that heat was the motion of small particles; the alternative theory held that heat was a substance, often called *caloric*. Incredible as it now seems, Mach was a late proponent of what amounted to the caloric theory of heat.

One would have imagined that all this would have been settled by the mid-nineteenth century, but in[7] (originally published 1872), he gave arguments as to why he considered the mechanical theory of heat unjustified.

If you recall, the key arguments against the caloric theory had been that experiments proved that heat was not a conserved quantity. Heat could be used up. Steam engines and the ideal heat engines of Carnot were able to use up heat to do work. Heat could also be created from work by friction, as Rumford had shown. The conclusion of Tyndall, Maxwell, Thomson and Boltzmann was that heat was just a form of mechanical energy. Mach tried to argue that this was an illegitimate conclusion.

The analogy he made was with electricity.

> In 1785 Coulomb constructed his torsion balance, by which he was enabled to measure the repulsion of electrified bodies. Suppose we have two small balls, A, B, which over their whole extent are similarly electrified. These two balls will

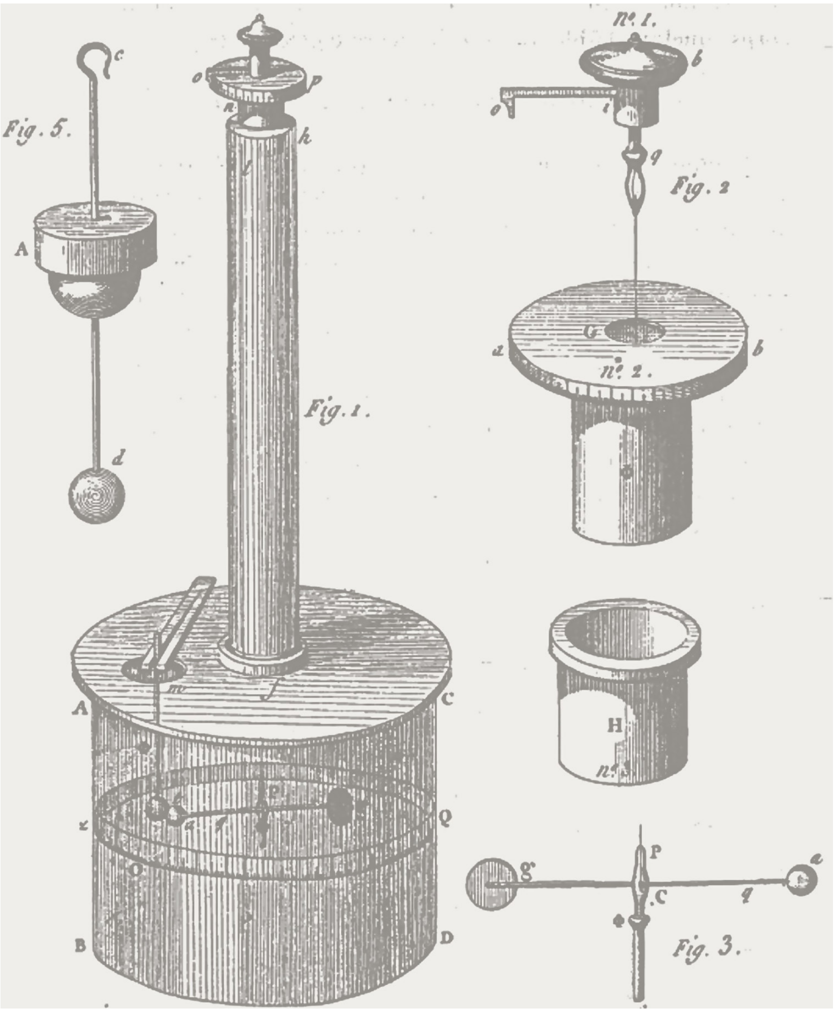


Figure 6.1 Coulomb balance as used in Mach's argument. From Coulomb 1785.

exert on one another, at a certain distance r of their centres from one another, a certain repulsion p. We bring into contact with B, now, a ball C, suffer both to be equally electrified, and then measure the repulsion of B with A and of C from A at the same distance r. The sum of these repulsions is again p. Accordingly something has remained constant. If we ascribe this effect to a substance, then we infer naturally its constancy. But the essential point of the exposition is the divisibility of the electric force p and not the simile of substance.

In 1838 Riess constructed his electrical air-thermometer (the thermoelectrometer). This gives a measure of the quantity of heat produced by the

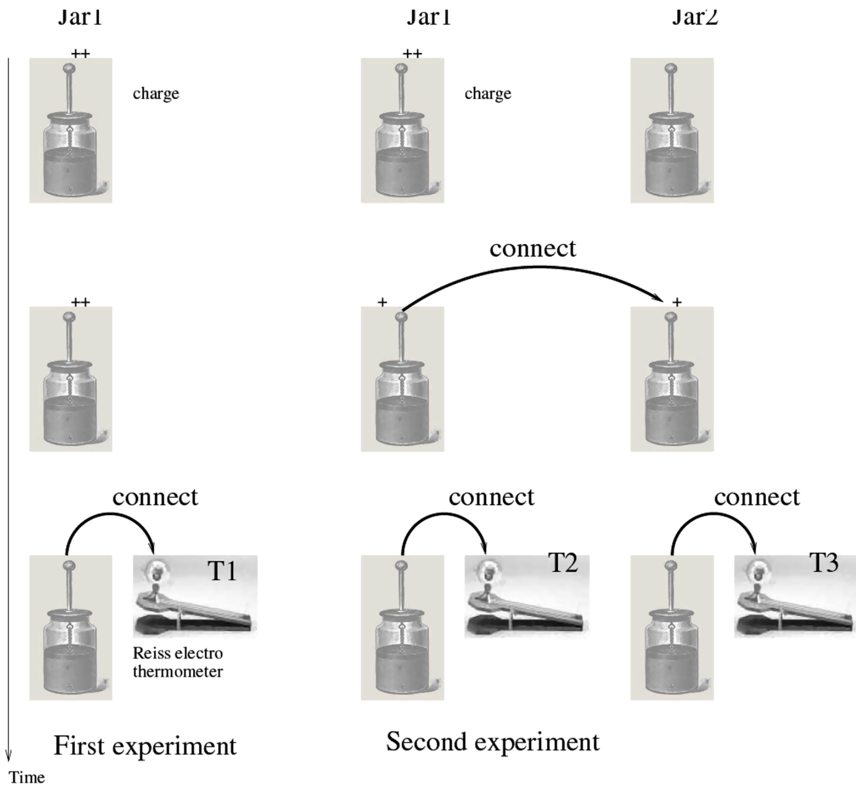


Figure 6.2 The Leiden jar experiments as described by Mach. Figure by authors.

> discharge of jars. This quantity of heat is not proportional tc the quantity of electricity contained in the jar by Coulomb's measure, but if q be this quantity and s be the capacity, is proportional to q^2/s, or, more simply still, to the energy of the charged jar. If, now, we discharge the jar completely through the thermometer, we obtain a certain quantity of heat, W. But if we make the discharge through the thermometer into a second jar, we obtain a quantity less than W, But we may obtain the remainder by completely discharging both jars through the air-thermometer, when it will again be proportional to the energy of the two jars. On the first, incomplete discharge, accordingly, a part of the electricity's capacity for work was lost.[8]

He then argues that it was just a matter of historical accident that the Coulomb[9] experiment was performed first. If the Leiden jar experiment had been done first, then the way of measuring electricity would have been in terms of quantities of electrical energy, not electrical charge.

Mach proposed that our belief in the conservation of electricity, now understood as charge conservation, stems from Coulomb's initial experiments.

Had we defined electricity in terms of energy, we might have observed that electric transfer between capacitors, which Coulomb's spheres also functioned as, is not conservative. Mach drew parallels between this hypothetical loss of 'electricity' and the non-conservation of heat in Carnot's heat engines.

He argued that there is as much reason to consider electric charge conserved as there is to consider caloric conserved. Electrostatics acknowledges that a specific amount of charge can represent varying amounts of energy depending on its potential. Similarly, a certain amount of caloric might contain different energy levels based on its potential. Thus, temperature could be seen as a thermal equivalent to electrical voltage, where voltage represents the potential of a charge, and temperature represents the potential of caloric within a body.

If science was willing to accept the notion of electric charge, then it had no reason to reject caloric.

What are we to make of this argument?

Critique of Mach's electrostatic analogy

Look again at how Mach concludes his first, Coulomb analogy: 'If we ascribe this effect to a substance, then we infer naturally its constancy. But the essential point of the exposition is the divisibility of the electric force *p* and not the simile of substance'. Notice that he is hedging his bets here. He is not saying outright that he even agrees that electric charge is substantial. He calls the notion of charge a simile, saying that what is real is the divisibility of the electric force. This is in keeping with his general instrumentalist approach to science, whereby science is about relations between measurements on instruments rather than about discovering the what the things are that cause dials to move.

But if we examine the sequence of actions performed in the Coulomb experiment we find that 'divisibility of force' is not an accurate description of what is going on.

In the first step, an experimenter charged up a ball (shown as *d* in Figure 6.1) and inserted it into the torsion balance. He then read off the electrostatic force it exerted on the ball *a* using the dial at the top of the balance. Ball *d* was then removed from the balance and brought into contact with another ball *b* (which is not shown in Coulomb's diagram). Balls *d, b* are then successively placed in the balance, and the deflections they induce read off on the dial.

It is not really valid to call this a measurement of division of forces since what we have is three distinct measurements of forces at three different times. An actual division of force demonstration requires that the forces being divided act simultaneously, as in Figure 6.3. What Coulomb had observed was that there appeared to be something which could be transferred between balls in such a way that the forces that these distinct balls could induce at distinct times were conserved. He deduced that there was something, call it electric charge, which was being divided between the balls.

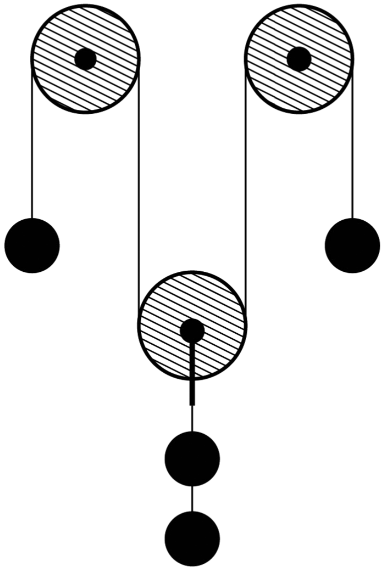


Figure 6.3 Diagram of an actual division of forces setup. The balancing forces of the peripheral weights act simultaneously with the central ones. Figure by authors.

Mach's assertion that Coulomb witnessed a division of force is a distortion. Coulomb actually noted a division of something that could potentially cause force. Mach's Kantian bias against the knowledge of things in themselves prompted him to reject this notion and incorrectly assert that a division of forces was observed. In mechanics, force is an immediate agent of acceleration. Therefore, what can be accumulated on pith balls or in Leiden jars is not force per se, but rather a substance that has the potential to generate force.

If we now look at Mach's account of the Leiden jar experiment, we find it equally faulty. It helps to draw a circuit diagram of it (Figure 6.4). With this, we can see that a key step is left out of his analysis. When the first leiden jar is connected to the second, it is done via a connection which has some resistance (R1 in the diagram). The heat lost through this resistor is left out of Mach's account.

If we did not measure h_1 the heat released in shifting half the charge to capacitor C2, we would have an experiment similar to the Coulomb one, but we would note that $h_0 > h_2 + h_3$. It would thus appear that electricity was not conserved.

The analogy he wants to make is between the voltage of a charge, and the temperature of a hypothetical *caloric*. But he does this by a thought experiment in which electric energy (charge × voltage^2) is converted to heat by discharge through a resistor. In Figure 6.4, the heat lost through resistor R1 is supposed to be analogous to the work out of a Carnot heat engine but this an absurd analogy on thermodynamic grounds. Mach is matching a loss of useful energy to heat through a resistor, with a

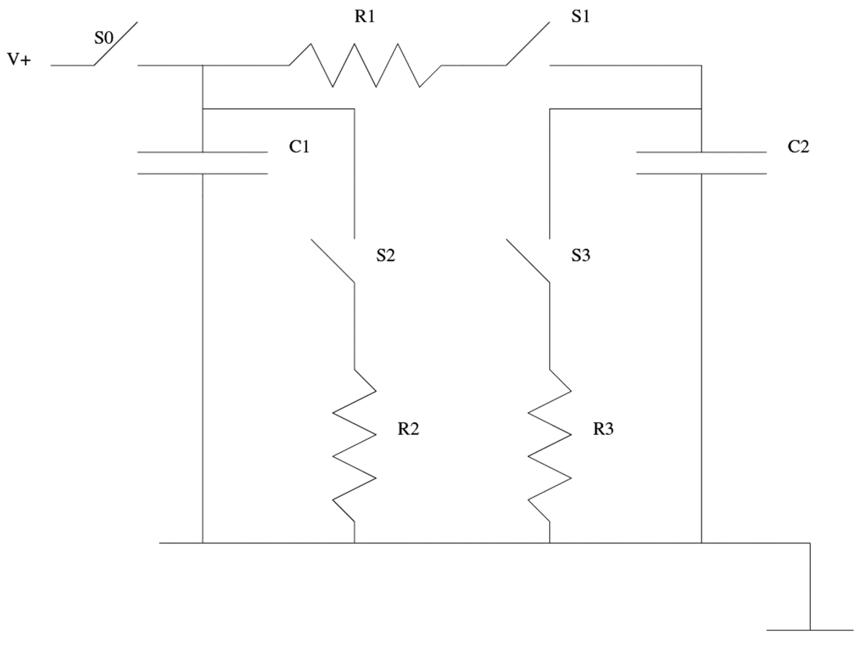


Figure 6.4 The Leiden jar experiment described by Mach as a modern circuit diagram. Steps: (1) close switch S0 and charge C1; (2) open switch S0, then close switch S2 and measure heat dissipated in R2. Call this h_0; (3) repeat step (1); (4) close Switch S1 and measure heat dissipated in R1, call this h_1; (5) open S1 and then simultaneously close S2 and S3, measuring heat in R2 (h_2) and R3 (h_3). Figure by authors.

conversion of heat to useful energy. But the second law of thermodynamics tells us that such transfers are not symmetric. You can transform electrical energy into heat with 100 per cent efficiency, but the conversion of heat to useful energy, whether this is electricity or mechanical work, is always less than 100 per cent efficient.

Suppose we had a thermal equivalent diagram to Figure 6.4 with thermal reservoirs substituting for capacitors and heat engines where the resistors occur in the original diagram.

In operations described in the original diagram, all the electrical energy is eventually dissipated as heat. In the thermal version, only a part of the starting thermal energy gets transferred into work out.

A close examination of Mach's attempted analogy shows that it simply does not work. As we go through Mach's argument here in detail it's full absurdity sinks in. He was attempting to refute the atomic theory by making analogies in thought experiments in which he effectively ignored the law of increasing entropy. It is Maxwell and Boltzmann's atomic theory that explains increasing entropy. Mach tried to refute it with analogies, just ignoring entropy increase.

In mitigation, one might say that the first publication date of the book, 1872, was quite early, the same year as Boltzmann's landmark paper on entropy. It was before Maxwell's book on the Theory of Heat,[10] but the key elements of Maxwells account had already been published.[11] Even supposing that Mach be allowed a few years to absorb these ideas, the excuse wears thin. Mach did not reverse his objections in subsequent years. He had the text published again in English in Vol. 5, No. I, of The Monist, October, 1894, with minor modifications repeating the same electrostatic analogies against the mechanical theory of heat.

It may be objected that up, until that date, the evidence for the existence of atoms was inductive. Clausius, Maxwell and Boltzmann had, from accepted principles of mechanics, constructed a theory that explained:

1. Temperature and pressure.
2. Entropy increase.
3. Viscosity in gases.
4. The 'law of equivalent volumes' in gases.
5. Was consistent with the atomic theory of chemistry.

Einstein demolishes Machist opposition to atoms

> Who could think that, only in 1900, peoples were battling, one might say to the death, over the issue whether atoms are real or not. The great philosopher Ernst Mach in Vienna said, NO. The great chemist Wilhelm Ostwald said, NO. And yet one man, at that critical turn of the century, stood up for the reality of atoms on fundamental grounds of theory. He was Ludwig Boltzmann. . . . The ascent of man teetered on a fine intellectual balance at that point, because had the anti-atomic doctrines then really won the day, our advance would certainly have been set back by decades, and perhaps a hundred years.[12]

Shortly before Boltzmann's suicide, an obscure patent clerk published an astonishing sequence of four papers that both completely vindicated Boltzmann and set twentieth-century physics on a revolutionary new path.

1. June 1905,
   a. *On the movement of small particles suspended in stationary liquids required by the molecularkinetic theory of heat.*[13]
   b. *On a heuristic point of view concerning the production and transformation of light.*[14]
   c. *On the electrodynamics of moving bodies*[15]
2. September 1905, *Does the inertial of a body depend on its energy content?*[16]

Paper [a] sets the stage by showing that the Brownian motion of small particles could be quantitatively predicted from the molecular kinetic theory.[17] In a section entitled '*Formula for the Mean Displacement of Suspended Particles. A New Method of Determining the Actual Size of Atoms*', he showed that if one assumes Avogadro's number was 6×10^{23}, then spheres of 1-micron diameter could be expected to wander about 6 microns per minute due to random bombardment by water molecules. Alternatively, if one were by experiment to obtain accurate measurements of the movement of such spheres one could get a more precise estimate of Avogadro's number. The prediction was quickly validated in experiments by[18] who, in the process, refined the estimate of Avogadro's number to 7.15×10^{23}.

On its own, this paper would have been enough to permanently refute the Machist arguments against atoms, molecules and the kinetic theory of heat. But if we look at the whole volley of papers that Einstein published over that summer, we see a developing theme of defending a corpuscular view of reality against the contenting energeticist school favoured by Mach and his supporters.

Einstein[19] lays out an argument as to why light should be viewed as corpuscular. He introduces it by saying:

> The wave theory of light with its continuous spatial functions has proven to be an excellent model of purely optical phenomena and presumably will never be replaced by another theory. Nevertheless, we should consider that optical experiments observe only time-averaged values, rather than instantaneous values. Hence, despite the perfect agreement of Maxwell's theory with experiment, the use of continuous spatial functions to describe light may lead to contradictions with experiments, especially when applied to the generation and transformation of light.
>
> In particular, black body radiation, photoluminescence, generation of cathode rays from ultraviolet light and other phenomena associated with the generation and transformation of light seem better modeled by assuming that the energy of light is distributed discontinuously in space. According to this picture, the energy of a light wave emitted from a point source is not spread continuously over ever larger volumes, but consists of a finite number of energy quanta that are spatially localised at points of space, move without dividing and are absorbed or generated only as a whole.[20]

How does he justify his suggestion?

Initially, he examines a gas within a cavity at thermal equilibrium, presuming it to be corpuscular in nature, composed of atoms and electrons. He posits that the electrons, confined by various local electrostatic potentials, can serve as resonators for electromagnetic radiation. Employing molecular kinetic theory, he deduces a formula for the average energy of a resonator, indicating that it should be directly proportional to the gas temperature.

Up to this point, he was outlining the accepted theory of light emission. However, he demonstrates that the theory's predictions deviate from experimental results. Considering increasingly higher frequencies of radiation, the predicted radiant energy within a cavity would theoretically become infinite. He derives an equation for what the entropy of monochromatic light within a cavity v_0 would be and how this entropy would change if all the energy of the light were concentrated in some sub-volume v of the cavity.[21] Of this, he observes:

> This equation shows that the entropy of monochromatic radiation of sufficiently low density varies with volume according to the same law as the entropy of an ideal gas or that of a dilute solution. In the following the equation just found will be interpreted in terms of the principle introduced by Mr. Boltzmann that says that the entropy of a system is a function of the probability of its state.[22]

This immediately indicates, though Einstein does not labour the point, that the light is behaving in a corpuscular way like a collection of atoms.

He then uses Boltzmann's entropy formula for the molecules in a gas of volume v_o and derives a formula for what the change in entropy would be if all the molecules were within a sub-volume $v < v_0$ – basically a generalization of the half cylinder examples given in the previous chapter. It turns out that the formula for the change in entropy of the gas is functionally the same[23] as the formula he derived for the change in the entropy of monochromatic radiation. He then concludes:

> In terms of heat theory monochromatic radiation of low density (within the realm of validity of Wien's radiation formula) behaves as if it consisted of independent energy quanta of the magnitude $\frac{R\beta\nu}{N}$. >8*

Reflect on his achievements. He applied Boltzmann's statistical interpretation of entropy twice: initially in the context of radiation, and subsequently in the context of gas molecules. He demonstrated that the entropy of light changes with volume in the same manner as the entropy of gas. This led him to deduce that light is particulate, consisting of what would later be termed photons. Notably, he did not presuppose the particulate nature of light in his derivation of light's entropy within a cavity; rather, he deduced it from the empirically validated formula for black body radiation. Thus, Boltzmann's statistical mechanics was not only correct regarding atoms, as evidenced by Einstein's paper on Brownian motion, but it was also applicable to light, inferring its particulate nature.

Given that Boltzmann's atomic account of gases was still being treated with great scepticism, this was an audacious extension. It implied that the physics community must both drop their doubts about the gas theory and face a perhaps harder challenge – to accept a particulate theory for electromagnetic radiation, the continuous wave account of which had up until then seemed a triumph of nineteenth-century physics.

This would merely be a bold hypothesis by itself, but Einstein provided experimental evidence to support it. The photoelectric effect, already known, is a phenomenon where incident light causes electrons to be emitted from a cold cathode in a vacuum tube, moving towards the anode. This movement generates a detectable current. In a more contemporary application, this principle is how solar panels operate. By applying a reverse bias to the anode, electrons are repelled,

* The modern notation for this is $h\nu$ for h = Plank's constant and ν frequency. He wrote $\frac{R\beta\nu}{N}$ but his $R, \beta, N \frac{n!}{r!(n-r)!}$ constants ensure $\frac{R\beta}{N} = h$.

preventing current flow. The 'stopping voltage' – the point at which current ceases – indicates the energy, in electron volts, that the electrons possess when emitted from the cathode.

The previous wave theory of light predicted that the stopping voltage would be determined by the intensity of the incident light. Einstein's account said that it should be determined by the frequency of the light. High-frequency light quanta had more energy and would thus excite electrons to higher voltages.

The experimental data aligns with Einstein's quantum theory over the wave theory. The stopping voltage is dependent on frequency, not intensity. It was for his work on the photoelectric effect that Einstein received the Nobel Prize in Physics.

A century ago, the photoelectric effect was an obscure phenomenon known only to experimental physicists. Now, it's a part of our daily lives. Solar panels, LED light bulbs and the cameras in our smartphones all utilize it. In fact, the sensors in modern digital camera chips are so sensitive that the statistical fluctuation in the number of discrete photons striking pixels becomes visible 'shot noise' against a uniform background. Modern technology has made it possible to observe the quantization of light on our cell phones.

It could be objected that his photo-electric was a post-diction on his part. The experimental evidence for the photoelectric effect was well-established. Einstein's theory provided an explanation for these observed results, which the continuum theory could not. However, this was not a prediction. The 1922 discovery that X-rays could be scattered by electrons, resulting in a gain of momentum by the electrons, proved to be a crucial confirmation of Einstein's theory.

His final two 1905 papers address the theory of special relativity[24] and derive his famous $e = mc^2$ formula.[25] His arguments in deriving the theory of relativity are only tangentially relevant to the issue of materialism, so we will not go into them here.[26] The conclusion that matter and energy were equivalent to one another did have philosophical as well as immense practical relevance. Philosophically, it undermined the ability to appeal to 'energetism' as an alternative to materialism.[27] Historically, it opened up the possibility of atomic power with all that entails. If anything established the idea of atoms in the popular mind, it was atomic energy.

Bächtold[28] summarizes Mach's view on atoms as: 'Mach's view on atoms can be described as consisting of five main ideas: (a) atoms do not exist in themselves, (b) the notion of atom is only a thought-symbol for a complex of sensations, (c) atomism is an unfruitful hypothesis, (d) atomism is an artificial hypothesis and (e) atomism is an unjustified hypothesis'.

By the mid-twentieth century, the Machist objection to the idea of atoms just seemed absurd and obscene. Hiroshima was not destroyed by a 'complex of sensations'.

Philosophical Machism and Lenin's intervention

Mach's influence did not just extend to holding back the acceptance of the atomic theory. His philosophical teaching conditioned half of a generation of scientists.

It established instrumentalism, the doctrine that science was just about formulae used to correlate instrument readings and sense impressions, as the starting point from which many of the original quantum physicists thought through the conceptual revolution induced by quantum mechanics.

Like Newton and Leibniz,[29] Mach emphasized the virtues of simplicity in scientific thought.

> The communication of scientific knowledge always involves description, that is, a mimetic reproduction of facts in thought, the object of which is to replace and save the trouble of new experience. Again, to save the labor of instruction and of acquisition, concise, abridged description is sought. This is really all that natural laws are. Knowing the value of the acceleration of gravity, and Galileo's laws of descent, we possess simple and compendious directions for reproducing in thought all possible motions of falling bodies. A formula of this kind is a complete substitute for a full table of motions of descent, because by means of the formula the data of such a table can be easily constructed at a moment's notice without the least burdening of the memory.
>
> . . .
>
> In algebra we perform, as far as possible, all numerical operations which are identical in form once for all, so that only a remnant of work is left for the individual case. The use of the signs of algebra and analysis, which are merely symbols of operations to be performed, is due to the observation that we can materially disburden the mind in this way and spare its powers for more important and more difficult duties, by imposing all mechanical operations upon the hand. One result of this method, which attests its economical character, is the construction of calculating machines. The mathematician Babbage, the inventor of the difference-engine, was probably the first who clearly perceived this fact, and he touched upon it, although only cursorily, in his work, The Economy of Manufactures and Machinery.[30]

The problem is that this economic principle was tied up with what amounted to an individualist or psychological conception where the information economy occured – a confusion of individual perception with the social activity of science.

> In mentally separating a body from the changeable environment in which it moves, what we really do is to extricate a group of sensations on which our thoughts are fastened and which is of relatively greater stability than the others, from the stream of all our sensations. Absolutely unalterable this group is not. Now this, now that member of it appears and disappears, or is altered. In its full identity it never recurs. Yet the sum of its constant elements as compared with the sum of its changeable ones, especially if we consider the continuous character of the transition, is always so great that for the purpose in hand the former usually appear sufficient to determine the body's identity. But because

> we can separate from the group every single member without the body's ceasing to be for us the same, we are easily led to believe that after abstracting all the members something additional would remain. It thus comes to pass that we form the notion of a substance distinct from its attributes, of a thing-in-itself, whilst our sensations are regarded merely as symbols or indications of the properties of this thing-in-itself. But it would be much better to say that bodies or things are compendious mental symbols for groups of sensations—symbols that do not exist outside of thought.[31]

He is saying that objects in the world do not exist, only our thoughts or sensations of them. This is almost Berkeleyan in its idealism. We do have a symbolic representation for the asteroid that caused the KT extinction event. But it was not this symbolic representation that killed the dinosaurs. It was a real mass of rock and metal that created the Chicxulub crater.

Why would it be better to say that things are just mental symbols for groups of sensation symbols that do not exist outside of thought?

How does he reconcile this individualism with what he knows about the collective nature of science which he acknowledges elsewhere?

> By communication, the experience of many persons, individually acquired at first, is collected in one. The communication of knowledge and the necessity which every one feels of managing his stock of experience with the least expenditure of thought, compel us to put our knowledge in economical forms. But here we have a clue which strips science of all its mystery, and shows us what its power really is. With respect to specific results it yields us nothing that we could not reach in a sufficiently long time without methods. There is no problem in all mathematics that cannot be solved by direct counting. But with the present implements of mathematics many operations of counting can be performed in a few minutes which without mathematical methods would take a lifetime. Just as a single human being, restricted wholly to the fruits of his own labor, could never amass a fortune, but on the contrary the accumulation of the labor of many men in the hands of one is the foundation of wealth and power, so, also, no knowledge worthy of the name can be gathered up in a single human mind limited to the span of a human life and gifted only with finite powers, except by the most exquisite economy of thought and by the careful amassment of the economically ordered experience of thousands of co-workers.[32]

The above is a good recognition of information economy along with the collective character of science. But in other places, we get individualistic and psychological accounts. Mach was quite willing to recognize that science advanced by collective effort so there must have been real people in the past who came up with the laws of motion. There was a real Newton whose laws of motion Mach used. If Mach could accept the reality of other scientists, why did he reject the reality of the planetary bodies whose motions Newton explained?

His opposition to atoms sprung directly from this conception of the scientific process.

> When a geometer wishes to understand the form of a curve, he first resolves it into small rectilinear elements. In doing this, however, he is fully aware that these elements are only provisional and arbitrary devices for comprehending in parts what he cannot comprehend as a whole. When the law of the curve is found he no longer thinks of the elements. Similarly, it would not become physical science to see in its self-created, changeable, economical tools, molecules and atoms, realities behind phenomena, forgetful of the lately acquired sapience of her older sister, philosophy, in substituting a mechanical mythology for the old animistic or metaphysical scheme, and thus creating no end of suppositious problems. The atom must remain a tool for representing phenomena, like the functions of mathematics. Gradually, however, as the intellect, by contact with its subject-matter, grows in discipline, physical science will give up its mosaic play with stones and will seek out the boundaries and forms of the bed in which the living stream of phenomena flows. The goal which it has set itself is the simplest and most economical abstract expression of facts.

This viewpoint, which abandons efforts to comprehend the truths behind phenomena, fails to explain how Einstein could predict unobserved phenomena based on atomic theory. If atoms are non-existent, how could Einstein accurately forecast the diffusion rate of pollen grains? While a formula that economically accounts for past observations is useful, without a theory that accurately captures reality, predicting previously unobserved phenomena would be unfeasible. This applies as much to Einsteins Brownian motion and photoelectric predictions as it does to grander astronomical ones. Le Verrier's prediction of Neptune's location, using Newton's law of gravity, was based on the need to explain the perturbations in other planets' orbits. This law indicated where to search, and upon looking there, Neptune was indeed observed. To consider Neptune merely as a tool to represent these perturbations would be non-sensical. Similarly, general relativity was not just a simple representation of pre-existing facts; it was a theory describing the real universe's geometric properties. It forecasted numerous phenomena that were unknown in 1915 and only observed much later. Einstein's theory predicted Edingtons's observations during the 1919 solar eclipse that the apparent positions of stars would be shifted by the Sun's gravity. In more recent years, the gravitational waves and black holes predicted by the theory have been observed.

In machine learning, it's a common understanding that a model well-fitted to the training set might perform poorly on new data. This phenomenon is so recognized that evaluating a model on a separate dataset from the training set is a standard practice in research. Only models that perform well on previously unseen examples are deemed trustworthy. Similarly, the atomic theory of entropy, the photon theory of light and general relativity are considered robust models. Their strength lies not just in fitting existing data but in encapsulating objective causal mechanisms.

Just after Einstein's papers came out, but before they had yet made much impact, a Russian socialist with the pen-name Lenin, published a book attaching Machism.[33] As it transpired, this socialist later went on to become the first Soviet Premier,[34] and as a result, his criticism of Mach was repeatedly reprinted and had a big influence on Soviet philosophy.

As a politician and political theorist, Lenin's critiques of Mach were significantly less technical compared to Einstein's. While Einstein indirectly contested Mach by disputing his physical claims about atoms, Lenin directly confronted the philosophical bases of Mach's theories.

Mach's profound scepticism regarding the existence of atoms, which appears contradictory to the progress of nineteenth-century science, was an element of a broader anti-materialist stance. Atomism, originating from ancient Greek materialism, traditionally countered this perspective. Mach did not just deny the existence of atoms but of all material objects: 'it would be much better to say that bodies or things are compendious mental symbols for groups of sensations – symbols that do not exist outside of thought'.[35]

He asserts that sensations are fundamental, and material bodies merely represent mental or conceptual symbols for clusters of sensations. Consequently, if one consistently doubts the material world, it logically extends to doubting the existence of matter's minutest elements – atoms.

Mach considered physical science a branch of psychology, placing psychological phenomena and personal experience at the core of reality. To him, everything else was an illusion, albeit a coherent one. Lenin remarked on this passage:

> An old song, most worthy Professor! This is a literal repetition of Berkeley who said that matter is a naked abstract symbol. But it is Ernst Mach, in fact, who goes naked, for if he does not admit that the 'sensible content' is an objective reality, existing independently of us, there re- mains only a 'naked abstract' I, an I infallibly written with a capital letter and italicised, equal to 'the insane piano, which imagined that it was the sole existing thing in this world'[36]. If the 'sensible content' of our sensations is not the external world, then nothing exists save this naked I engaged in empty 'philosophical' fancies. A stupid and fruitless occupation!

Mach pursued the metaphor that 'things' were just symbols for sensations all the way down to atoms, giving as an example sodium.

> The physicist who sees a body flexed, stretched, melted, and vaporised, cuts up this body into smaller permanent parts; the chemist splits it up into elements. Yet even an element is not unalterable. Take sodium. When warmed, the white, silvery mass becomes a liquid, which, when the heat is increased and the air shut out, is transformed into a violet vapor, and on the heat being still more increased glows with a yellow light. If the name sodium is still retained, it is because of the continuous character of the transitions and from a necessary instinct of economy. By condensing the vapor, the white metal may be made to reappear.

> Indeed, even after the metal is thrown into water and has passed into sodium hydroxide, the vanished properties may by skilful treatment still be made to appear; just as a moving body which has passed behind a column and is lost to view for a moment may make its appearance after a time. It is unquestionably very convenient always to have ready the name and thought for a group of properties wherever that group by any possibility can appear. But more than a compendious economical symbol for these phenomena, that name and thought is not. It would be a mere empty word for one in whom it did not awaken a large group of well-ordered sense-impressions. And the same is true of the molecules and atoms into which the chemical element is still further analysed.[37]

So, for Mach the sodium atom was just a symbol for a set of sense impressions. Ultimately, there were no sodium atoms, no atoms in general; these were just ideas we used to impose consistency on our impressions.

For Lenin, the crux of materialism lay in its claim that the material world is independent of our perception. This assumption is one shared by all pragmatic individuals, with only idealist philosophers questioning it. Lenin argued that Mach subverted this idea, elevating human experience above all else. Despite Mach's background in physics and his contributions to the philosophy of science, Lenin contended that his philosophy merely echoed that of Bishop Berkeley.[38] As a socialist politician, Lenin was keenly aware of the deep connection between philosophy and politics. Berkeley had made it clear that his idealism aimed to counter the atheists of his time. Diderot, the Encyclopedist and philosopher of the French Revolution, had sharply criticized Berkeley, viewing him as a defender of the Church and Crown. In Lenin's era, the monarchies of Central and Eastern Europe were under threat from revolutionary movements that would, between 1917 and 1918, ultimately topple them. To Lenin, the revival of Berkeleyian ideas by Mach was another philosophical intervention into politics designed to reinforce Church and Crown against the atheistic socialist movement. It is perhaps worth noting that Mach's position, professor of philosophy in Vienna, was one that had to be personally approved by the Austro-Hungarian Emperor Franz Joseph.

Mach, Lenin said, was only halfhearted in his idealism. He cites Mach as saying that with the advance of science, it should in principle be possible, by examining a brain, to say what experiences it was undergoing. This is indeed now possible, Shen et al.[39] show that using a combination of functional magnetic resonance and adversarial neural nets the end-to-end model can learn a direct mapping between brain activity and perception. When a subject is looking at a picture, the neural net, using as inputs fMR data, can generate on a screen a passable representation of what the person is seeing. For examples see Figure 6.5. But Lenin asks how this can be consistent with Mach making sensations primary, since it is effectively an admission that sensations are a result of organized matter – the brain.

> This means that outside us, independently of us and of our minds, there exists a movement of matter, let us say of ether waves of a definite length and of a definite velocity, which, acting upon the retina, produce in man the sensation

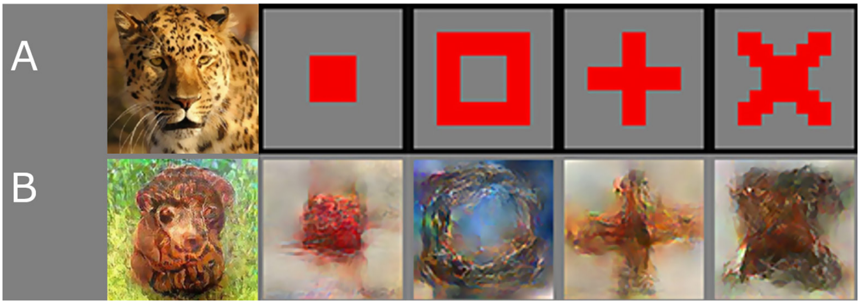


Figure 6.5 It is possible, using brain scanning to reconstruct what a person is seeing. Row A: images presented to a subject; row B: computer reconstruction of what they are perceiving using real-time information about the activity of their brain. From the work of Shen et al.

> of a particular colour. This is precisely how natural science regards it. It explains the sensations of various colours by the various lengths of light-waves existing outside the human retina, outside man and independently of him. This is materialism: matter acting upon our sense-organs produces sensation. Sensation depends on the brain, nerves, retina etc, ie on matter organised in a definite way. The existence of matter does not depend on sensation.[40]

Idealism and materialism in the quantum theory

Lenin was writing during a revolutionary period in the atomic theory of matter. Meanwhile, Einstein had introduced a contradiction to classical physics by proposing that light consists of quantized photons, seemingly at odds with the well-established wave theory of light, which was supported by considerable empirical evidence. Then Rutherford[41] showed, by the scattering of beta radiation from thin gold foil that atoms, which had previously been thought of as solid, were in fact mostly void with only a tiny nucleus. In the same year Nicholson,[42] proposed that the bulk of the volume of atoms was occupied by electron orbits, with the allowable angular momenta of the electrons being multiples of Plank's constant h.

> If, therefore, the constant h has, as Sommerfeld has suggested, an atomic significance, it may mean that the angular momentum of an atom can only rise or fall by discrete amounts when electrons leave or return. It is readily seen that this view presents less difficulty to the mind than the more usual interpretation, which is believed to involve an atomic constitution of energy itsef.[43]

By the following year, Bohr[44] had integrated this into a theory of the atom, which could explain the observed emission spectra of the elements. In his model, the atom resembled a miniature solar system with electrons orbiting a positively charged nucleus. It was a semi-quantum approach to atomic structure. While orbits were analysed classically, they were restricted by Planck's constant, which quantized the possible angular momenta for the electrons. Bohr, however, was dissatisfied with Einstein's idea that light was particulate, finding it contradictory to the known interference capabilities of light. Along with others, he attempted to construct a theory[45] of radiation that would integrate his model of the atom with classical electrodynamics, obviating the need for the photon. While the details of his 1924 theory were soon contradicted by experimental data, his philosophical remarks in the paper opened up what later, and indirectly, became a recurrence of essentially Machian philosophical views of physics.

> Although the correspondence principle makes it possible through the estimation of probabilities of transition to draw conclusions about the mean time which an atom remains in a given stationary state, great difficulties have been involved in the problem of the time-interval in which emission of radiation connected with the transition lakes place. In fact, together with other well-known paradoxes of the quantum theory, the latter difficulty has strengthened the doubt, expressed from various sides whether the detailed interpretation of the interaction between matter and radiation can be given at all in terms of a causal description in space and time of the kind hitherto used for the interpretation of natural phenomena[46]

Bohr's causal description implied that energy and momentum conservation were maintained. In his 1924 paper, he tried to bridge the gap between classical and quantum mechanics by proposing that energy and momentum conservation applied on average, statistically. He theorized that while momentum would be conserved across many electron energy transitions, it might not be in each separate electron interaction with a hypothesized virtual electromagnetic field. However, he was quickly disproven as experiments with photon scattering by electrons confirmed that momentum was conserved in each interaction, leading to the widespread acceptance of Einstein's photon hypothesis.

However, the philosophical points about causality in quantum systems continued to be raised by Bohr.

> Notwithstanding the difficulties which hence are involved in the formulation of the quantum theory, it seems, as we shall see, that its essence may be expressed in the so-called quantum postulate, which attributes to any atomic process an essential discontinuity, or rather individuality, completely foreign to the classical theories and symbolised by Planck's quantum of action.
>
> This postulate implies a renunciation as regards the causal space-time co-ordination of atomic processes. Indeed, our usual description of physical phenomena is based entirely on the idea that the phenomena concerned may be observed without disturbing them appreciably.
>
> . . .
>
> Now the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected.

> Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation. After all, the concept of observation is in so far arbitrary as it depends upon which objects are included in the system to be observed. Ultimately every observation can of course be reduced to our sense perceptions.[47]

While the last sentence is redolent of the Machist theory of knowledge, Bohr hedged his statement against subjectivism somewhat with the previous reference to 'agencies of observation'. An agency of observation of photons could, for instance, be a photographic plate. In the light of what would latter be called the collapse of the wave function, a crystal of silver iodide would be as effective as a molecule of visual purple in the retina. He goes on:

> As regards light, its propagation in space and time is adequately expressed by the electromagnetic theory. Especially the interference phenomena in vacuo and the optical properties of material media are completely governed by the wave theory superposition principle. Nevertheless, the conservation of energy and momentum during the interaction between radiation and matter, as evident in the photoelectric and Compton effect, finds its adequate expression just in the light quantum idea put forward by Einstein.
>
> As is well known, the doubts regarding the validity of the superposition principle on one hand and of the conservation laws on the other, which were suggested by this apparent contradiction, have been definitely disproved through direct experiments. This situation would seem clearly to indicate the impossibility of a causal space-time description of the light phenomena. On one hand, in attempting to trace the laws of the time-spatial propagation of light according to the quantum postulate, we are confined to statistical considerations. On the other hand, the fulfilment of the claim of causality for the individual light processes, characterised by the quantum of action, entails a renunciation as regards the space-time description.[48]

Bohr was criticized for the obscurity of his statements, so let's look a bit more closely at what he seems to have meant.[49] There are a lot of implied meanings in the language.

First, what does he mean by the superposition principle? In contemporary discussions about, for example, quantum computing, one speaks of superposition of base states, but in the mid-1920s, the term was used in physics to refer to the superposition of waves producing interference, so by the superposition principle, he means optical interference. The doubts to which he refers, about combining interference phenomena with the conservation laws involved in Einstein's treatment of the photon, are therefore his own doubts expressed in Bohr, Kramers and Slater.[50]

When he talks about attempting to trace the time-space propagation of light, he is talking about the solution to Maxwell's equations, which give the amplitude of the light wave. But this solution only gives an amplitude for the light arriving at, for example, a given point on a photographic plate. The square of this electromagnetic field amplitude gives the flux of photons arriving at that point, but the individual photons arrive randomly with Poisson statistics. Thus, his statement that 'we are confined to statistical considerations'[51].

When he then contrasts this with 'the fulfilment of the claim of causality', he is referring, as he did in his 1924 paper, to the conservation laws. The whole causal structure of classical physics was structured around the conservation of energy and momentum. The space-time paths followed by the planets and their satellites were those which conserved energy and momentum. But in quantum mechanics, he said that if we want to talk about energy conservation, we are forced to move from Maxwell's model of light to Einstein's and all you have is an emission of a photon from one atom at time t_0 and an absorption of the photon by another atom at time t_1. In between those to events we can assign no definite path to the photon.

If you set up a spectrometer using a diffraction grating, expose it to a very low flux of monochromatic light such that on average only one photon at a time is passing through the grating, you still get a clear spectral line on long exposure.[52] The photons must each have passed through multiple grating lines at once to produce this effect, so their positions between emission and absorption are undefined.

> The limitation in the classical concepts expressed through relation (2)[53] is, besides, closely connected with the limited validity of classical mechanics, which in the wave theory of matter corresponds to the geometrical optics, in which the propagation of waves is depicted through 'rays'. Only in this limit can energy and momentum be unambiguously defined on the basis of space-time pictures.[54]

Bohr is concerned with the extent to which you can localize a photon or other particle if you use the wave model.

If we view the passage of a particle through space from the standpoint of the wave account, then it can only be localized on its trajectory to the extent that we view it as a 'wave packet'. This wave packet can be viewed as the sum of waves of different frequencies. The sharper the localization we want to achieve, the more different frequency components we need to take into account.

His relation (2) above refers to his equation

$$\Delta t \Delta E = \Delta x \Delta I_x = \Delta y \Delta I_y = \Delta z \Delta I_z = h$$

where Δt refers to the time interval for a wave packet to pass a point. This gives the uncertainty about when a photon will arrive.

The relation $\Delta t \Delta E = h$ comes from Einstein's result that the energy and frequency of photons are inversely related by Plank's constant h such that $h = E\tau$ where τ is the period of vibration of the light wave.

If we want to have more certainty about precisely when a wave packet will arrive, we have to have a wave packet whose Fourier sum has more component frequencies. Because each frequency has a different associated energy, greater certainty about arrival time means less certainty about photon energy. This is the complementary relationship that Bohr is addressing with respect to the space time picture and the conservation of energy.

$\Delta x, \Delta y$ is the length of the wave packed in the x, y directions, etc. This gives the spatial uncertainty about where it will arrive.

ΔI_x is the range of momenta in the x direction, etc. So $\Delta x \Delta I_x = h$ expresses what was later known as Heisenberg's uncertainty principle.[55] Bohr has derived this from Einstein's formula for the momentum of photons $I\lambda = h$, where I in this notation means momentum.

This is what Bohr termed the complementary principle. You could either view a process as having a definite space time evolution or as being one governed by conservation laws, but not both at the same time.

Bohr and Heisenberg were later to be associated with what became known as the Copenhagen interpretation of quantum theory, though it is arguable that they differed substantially in their views and the popular account of the Copenhagen theory is somewhat of a myth.[56] As we have said, for Bohr a causal interpretation meant one in terms of the conservation laws. For Heisenberg a *causal* interpretation was in terms of the deterministic operation of wave functions. The wave function evolved in a deterministic fashion with respect to time. The function assigns amplitudes to the orthogonal bases of some state space such that the square of these amplitudes gives the probability of observing the system in the corresponding basis state. This is formally the same as the optical case, where the square of the field amplitude gives the photon flux. For Heisenberg, the actual detection of a particle, or more generally, the detection of a basis state of the system, constituted a cut or *schnit* between the quantum world governed by the evolution of the wave function and the classical world in which the result was observed.

Until recently, this would have seemed a rather arcane topic, but with the advance of technology, it is now being harnessed for practical purposes in quantum cryptography. For instance, Liao et al.[57] were able to demonstrate that they were able to use quantum cryptography to transmit data to an orbiting satellite.

It is now possible to use quantum cryptography to distribute long keys for ordinary digital cryptography, so long that effective brute force decryption is ruled out. We will give an explanation of how this technology works and then use its operation to illustrate different idealist and materialist accounts of QM.

Consider the setup in Figure 6.6. It shows how an initial polarizer can be used to prepare a vertically polarized light beam. If you place a second polarizer at 45° in this beam, then only half of the photons in the first polarized beam will get through the second one.

Figure 6.6 shows simple polarizing sheets, but more sophisticated polarizing devices exist which will sort an initially polarized stream of photons into two, one of vertically polarized and one of horizontally polarized light. This provides a potential way to transmit information. Suppose we treat vertically polarized

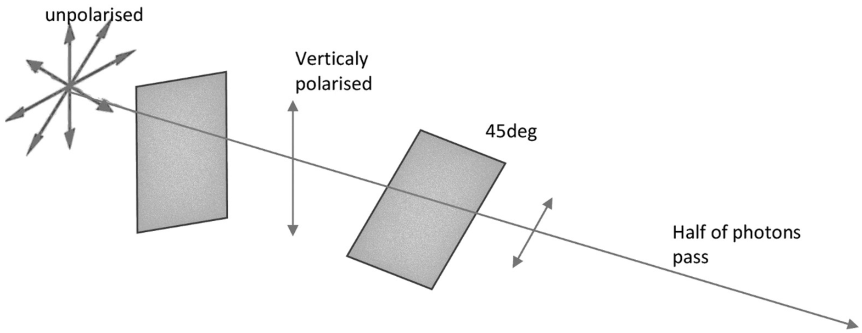


Figure 6.6 If light is passed through a polarizer a vertically polarized beam can be created. If this is then passed through a second polariser at 45° to the first, then a beam with 45° polarization and half the number of photons results. Figure by authors.

photons as 1 and horizontally polarized ones as 0, then this can be used as a transmission code.

We can complicate this further by allowing the sender to encode their message with either of two polarizers at 45° to one another.

Suppose Alice wants to send to Bob the bit sequence 1001011001 and she randomly selects whether to send individual bits with vertical or diagonal polarizers. In what follows, we indicate a vertical polarizer as + and a diagonal one as x.

```
1001011001  Bit stream Alice tries to send
xx+x++x++x  Encoding polarizer she used
```

Bob does not know either the initial bit-stream or the sequence of polarizer settings. He can guess which polarizer setting was used. If, for a given bit, he makes the correct choice, then by using a beam splitting polarizer he can correctly read the 1 or the 0 that Alice sent for that bit in the sequence. If he makes the wrong choice, then he will read either a 0 or a 1 with a 50 per cent probability of whatever Alice sent. For example, we might have:

```
1001011001  Bit stream Alice tries to send
xx+x++x++x  Encoding polarizer she used
xx++x+++xx  Bob's polarizer settings
101**1*0*1  What Bob recieves, * indicates random 1 or 0
```

After the quantum transmission is finished, Bob uses an ordinary digital channel to tell Alice which polarizes he used, she replies with a bit stream telling him which bits in the original message were received with the correct polarizer. Bob discards the bits in his received message that used the wrong polarizer. At the end of the procedure, Bob and Alice share a known bit-string that can be used as a cryptography key.

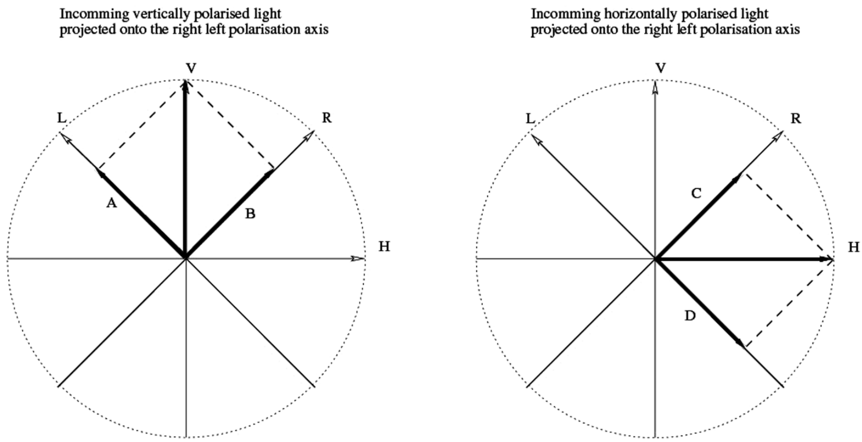


Figure 6.7 Left picture shows how the amplitude of vertically polarized light gets projected onto the Left and Right axes of a polarizer at 45° to the original axis of polarization. The vertical vector V representing the initial amplitude projects down two equal length component vectors A,B on the new frame of reference. The right picture shows the corresponding situation for horizontally polarized light. Figure by authors.

The important step here is that each incorrect selection of a polarizer gives a random reading unrelated to what was sent.

The nice thing about using polarized light as an example is that it introduces the linear algebra that provided the mathematical foundation of quantum theory from Neumann[58] in a simple context that can be understood with no more than classical Euclidean geometry. Consider Figure 6.7. The images show two frames of reference, the H, V or Horizontal and Vertical axes, and another frame of reference, whose axes we have labelled L, R, meaning Left and Right. The L, R axes are rotated 45° relative to the H, V axes. We assume that L, R, H, V are all vectors of the same length. Without loss of generality, we assume length 1.

Clearly, with pairs of axes at right angles like [H,V] or [L,R] we can express any position within the unit circle using either coordinate system. That includes the endpoints of the axes themselves.

So, the point labelled V in the diagram has coordinates $V=\left[0,1\right]_{HV}$ in Horizontal Vertical system and coordinates $\left[A,B\right]_{LR}$ in the Left, Right system. Similarly, $H=\left[1,0\right]_{HV}=\left[D,C\right]_{LR}$. By Pythagoras theorem you can readily solve for A, B, C, D as

$$V=\left[0,1\right]_{HV}=\left[\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]_{LR}$$

and

$$H=\left[1,0\right]_{HV}=\left[\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]_{LR}$$

Up till now this has just been an example of changing the frame of reference in Cartesian geometry, using a style of argument that would have been familiar to any seventeenth-century scientist. We can evolve it to one about quantum measurement in two steps.

1. Consider the frames of reference to be actual physical polariers capable of splitting an initial beam into two component beams whose electric fields are at right angles to one another. The light passes through a first beam splitter and the vertically polarized beam (V) enters a second one at 45° which splits it into L and R component beams. If the amplitude of the beam V is v then the amplitude of the L and R beams will be $\frac{v}{\sqrt{2}}$.

   Because the brightness of a beam is proportional to the square of the field amplitude, then L and R beams will be half as bright as the V beam. All of this follows from the basic geometric arguments given earlier.
2. Now consider what is happening at the level of individual photons. If we are sending information on single particles with two possible (polarization) states, then instead of bits we have qubits.

   In describing qubits and other quantum states, the physicists use a specialized notation, the Dirac notation, for the geometry that we described earlier. This can look a bit strange if you are unfamiliar with it, but the basic principle is simple. Where we used standard Cartesian notation for the initial frame of reference, writing the vertical axis vector as $\left[0,1\right]_{HV}$ they would write it as $0|H\rangle+1|V\rangle$, but this is still just an explicit way of writing down coordinates in a frame of reference. The bracketing construct $|\rangle$ is referred to as a *ket*.

   In the standard account of QM what is happening when a qubit is sent in one of two orthogonal pure states $1|V\rangle+0|H\rangle$ or $0|V\rangle+1|H\rangle$. The notation $|H\rangle$ indicates a labelled state, H standing for horizontal polarization in this case. If we perform a 45° rotation and project these pure states onto the basis states[59] formed by the new polarizer which we will label L, R we get

$$0|H\rangle+1|V\rangle \to \frac{1}{\sqrt{2}}|L\rangle+\frac{1}{\sqrt{2}}|R\rangle$$

   This is evidently just a notational translation of equation (6.1), but what does it mean in practice?

The experimental evidence does not of course change as a result of which view we take. We still observe that the brightness going down L and R is half of what we would have gotten if we removed the second polarizer and measured the brightness of the beam V. The classical picture gives an intelligible account of this in terms of the geometric decomposition of the electric field into equal independent components in the new 45° coordinate system.

The Bohr interpretation is that the $\frac{1}{\sqrt{2}}$ amplitude coefficients in front of the *kets* in the expression $\frac{1}{\sqrt{2}}\left|L\right\rangle + \frac{1}{\sqrt{2}}\left|R\right\rangle$ express the square root of the probability that the photon will be found going down this path. This clearly fits the experiment just as well as the classical account. Whatever ontological semantics you associate it with the $\frac{1}{\sqrt{2}}, \left(\frac{1}{\sqrt{2}}\right)^2$ is still $\frac{1}{2}$ as bright.

The maths certainly works.

Bohr's complementary view, initially expressed in terms of how the wave packet view involved an ambiguous position to the packet in Fourier terms, becomes much more pronounced with a setup like a beam splitter. Until one of the two detectors in the setup fires, the photon could be in either channel. Since the arms of the two beams can be arbitrarily extended, the spatial position of the photon also becomes arbitrarily indeterminate.

Bohr's complementary approach effectively says that the maths is well defined and empirically correct, but that you cannot meaningfully speak of the position of the photon before it is detected. You just have to use the calculations based on treating light as a wave to predict the relative probabilities of final positions. The photon abstraction is inherently tied to the process of emission or absorption of quanta of energy at particular places. It allows us to understand what happens as something that conserves energy, but it prohibits us from simultaneously talking about the space-time process that connects the two events. These have to be dealt with in terms of light waves, which tell us the relationship between expected brightness in different places.

Heisenberg's approach was subtly different. The wave (or wave function), evolves through time along all possible paths and then, when a measurement is performed and one of the detectors fires, then there is a 'cut' between the quantum world and the classical world. This is sometimes spoken of as there being a collapse of the wave.

There is nothing inherently idealistic about the Bohr statement of things. If one is a strict materialist and accepts that all that exists are the atoms and the photons[60] by which they exchange force and energy, then speaking of where the photons are between when they influence atoms is meaningless idealism. We may like to imagine the photon as having a position in between emission and absorption, but that is just imagination. It is just an attempt to form an ideal picture in our minds. Since the photons and the atoms are the fundamental components of reality, an ideal position of a photon independent of atomic interaction is by definition unreal.

Heisenberg's account does lead to potentially idealist accounts though, now often known as the Copenhagen interpretation,[61] in which it is the participation of the knowing subject, the observer, that causes the wave function to collapse and a definite result to be seen. Heisenberg[62] claims: 'such concepts as "objective reality" have no immediately evident meaning, when they are applied to the situation which one finds in atomic physics'. This process of giving a privileged position to the psychological experience of the observer looks very like a reversion to the Machist positivism that had been so influential in German physics back when he had been trained.

If we move from the perspective of the 1930s physics lab to that of modern communications technology, the special role of the observer loses all credibility. When Liao et al.[63] set up their lasers and telescopes to send polarized photons to the unmanned satellite *Micius*, where was the observer?

The automatic equipment in *Micius* recorded the polarized photons and the on-board processor used them to store in RAM the encryption key that it used to send subsequent radio messages to the ground. There was certainly no person up there. If wave packets collapsed, then the equipment up there did it unaided. Are we to say computers are subjects?

Or should we just recognize that the philosophical subject[64] is irrelevant to understanding this sort of engineering?

The latent subjectivism of the Heisenberg school gave rise to opposition, initially from[65] and later from the alternative realist interpretations of Bohm[66,67] and Everett.[68,69]

The two alternative realist approaches prioritize either the particle interpretation for Bohm or the wave interpretation for Everett.

Bohm argued that the existence of apparent indeterminism at a microscopic level and statistical predictability at a macro level should not prevent us from looking for deeper deterministic mechanisms.

> lawlessness of individual behaviour in the context of a given statistical law is, in general, consistent with more detailed individual laws operating in a broader context.[70]

He gave as an example the operation of a life insurance company that can predict the likelihood of a man dying in a particular year. They cannot predict with certainty if he will die, but they can make an accurate prediction over the ensemble of millions of customers they assure. The fact that the death of one man was only statistically predictable does not mean that there was not a definite material cause of this individual death. Perhaps it was an airliner crash due to a faulty flight control meachanism or perhaps lung cancer. Were we to just satisfy ourselves with statistical death rates we would not research into causes of cancer, the reliability of 737MAX flight computers, etc. Bohm took up the early work of De Broglie,[71] who was the first to show that material particles, like light, also show wave-like properties.

> THE quantum relation, energy = h × frequency, leads one to associate a periodical phenomenon with any isolated portion of matter or energy. An observer bound to the portion of matter will associate with it a frequency determined by its internal energy, namely, by its 'mass at rest.' An observer for whom a portion of matter is in steady motion with velocity βc, will see this frequency lower in

> consequence of the Lorentz-Einstein time transformation. I have been able to show (Comptes rendus, September 10 and 24, of the Paris Academy of Sciences) that the fixed observer will constantly see the internal periodical phenomenon in phase with a wave the frequency of which is determined by the quantum relation using the whole energy of the moving body provided it is assumed that the wave spreads with the velocity c / β. This wave, the velocity of which is greater than c, cannot carry energy.[72]

This wave property of matter, following on from Einstein's photo-electric effect and relativity theories, would entail a rethinking of the laws of dynamics when applied to small moving particles.

> The 'phase wave' has a very great importance in determining the motion of any moving body, and I have been able to show that the stability conditions of the trajectories in Bohr's atom express that the wave is tuned with the length of the closed path.
>
> The path of a luminous atom is no longer straight when this atom crosses a narrow opening; that is, diffraction. It is then necessary to give up the inertia principle, and we must suppose that any moving body follows always the ray of its 'phase wave'; its path will then bend by passing through a.sufficiently small aperture. Dynamics must undergo the same evolution that optics has undergone when undulations took the place of purely geometrical optics. Hypotheses based upon those of the wave theory allowed us to explain interferences and diffraction fringes. By means of these new ideas, it will probably be possible to reconcile also diffusion. and dispersion with the discontinuity of light, and to solve almost all the problems brought up by quanta.[73]

In the De Broglie/Bohm theory the wave equation is seen as establishing a potential, the spatial derivative of which imposes a guiding force on particles. The resultant trajectories may be curved in rather complicated ways. The mathematical structure of the theory is very classical, a generalization of Hamiltonian trajectories. It is also deterministic. Randomness of final results arises from the random spread of initial particle positions. In the two slit example, the final ending position of a particle is very sensitive to just where it passes through the aperture. The guiding wave potential steers particles to widely different final positions depending on their starting points. So, in Bohm's account, quantum randomness is no different from the randomness that was already present in Boltzmann's statistical mechanics.[74] In both cases it is assumed that the initial positions of particles in ensembles are random.

> Objective probability in SQM[75] implies that the correct frequencies of experimental outcomes are observed. However, the reverse is obviously not true. A statistical spread of measurement outcomes does not imply the existence of objective probability. For example, these frequencies also follow from the right

> statistical spread in the initial state across the ensemble of systems together with the right kind of laws of evolution. The fact that we cannot predict with certainty where a particular photon will hit the screen is only due to our ignorance of the initial state of the individual system. This is the approach taken by PWT[76,77]

Bohm took the particle view as fundamental; the Everett or 'many worlds' form of realism takes the wave function as fundamental. Everything is treated in terms of superposed waves with amplitudes, or in a finite model state vectors with amplitudes for every possible orthogonal state. There is no reduction of the wave function because everything is always in a superposition of states. The evolution of the universe is a series of unitary, that is to say, length preserving, rotations of this state vector. Observations yield definite answers; polarizations of photons go left or right, because the sub-manifolds of the multiverse associated with recording left and right polarizations are orthogonal to one another.

Whether any experiments will, in the future, be able to resolve whether Everett or Bohm was right we cannot know. At present, the accounts of Bohr, Heisenberg, Bohm and Everett are all equally compatible with the evidence. But the fact that Bohm and Everett have produced theories that explain things just as well as Heisenberg's positivism shows that science does not demand a subjectivist theory. Bohr's own account was, as we said earlier, in principle also compatible with strict materialism.

Chapter 7

Logic and Materialism

Introduction

> Clearly then it is the function of the philosopher, ie the student of the whole of reality in its essential nature, to investigate also the principles of syllogistic reasoning.[1]

Since antiquity, many long-lived transnational state systems have been explicitly founded on some organized religion as their explicit dominant ideology. In the current era, examples include Roman Catholicism for the Holy Roman, Spanish, Portuguese and French Empires; Islam for the Ottoman Empire; and Anglicanism for the British Empire. With one major exception, there have been no such systems based on secular philosophies; even the United States empire is openly deist,[2] if not outright Christian.

That one exception is *dialectical materialism*, the philosophy of the world communist movement since the late nineteenth century. Just as billions of people were taught the dominant religions in the empires that embraced them, so billions of people have been taught dialectical materialism in the former USSR and European socialist states until 1989, and in China, and extant socialist states, to the present. The reach of dialectical materialism should not be underestimated. Thus, in 2021, the Anglican Church had 85 million communicants[3]: markedly less than the Chinese Communist Party with 95 million members.[4]

In its materialist component, dialectical materialism is profoundly progressive, rejecting all forms of idealism and actively promoting science and rationality as the means for humanity to understand and transform the world. However, as we shall explore, the dialectical component is much more problematic, philosophically and ideologically.

In particular, in the USSR, dialectical materialism was deployed for partisan purposes, both to suppress opposition, and to constrain what were deemed appropriate areas for scientific investigation. The effects on Soviet genetics, with what was effectively Lamarckianism promoted over Mendelism, are well documented.[5] Here, we will focus on the less well-known impact on the study of formal logic, which, nonetheless, had profound implications for the development of Soviet mathematics, in particular computer science and practical computing.

Logic overview

A logic[6] is a formalized system for reasoning. It is important to note that logic is about *truth values* (i.e. true and false) rather than *the truth*. The things we reason about are *assumed* to be true or false independently of logic. They may be factual, or hypothetical, or speculations or beliefs. Logic cannot in itself establish whether or not something is truthful; rather, it is concerned with *correct reasoning*.

When we say logic is *formalized*, we mean that there are rules for:

- constructing statements to reason about, i.e.grammar or syntax;
- giving meaning to statements, i.e. semantics;
- manipulating statements to establish their properties, commonly whether they are always, or partially, true or false, i.e. through proof or evaluation.

We will further explore these below, but we won't exhaustively or formally treat them here.

A logical argument proceeds from *premises*, also known as *assumptions*, to *conclusions*. Reasoning steps are through *rules of inference*, a modern form of the more convoluted *syllogism* of antiquity, discussed in subsequent sections.

A syllogism has the general three term form:

$premise_1$
$premise_2$
conclusion

and reads as: given $premise_1$ and $premise_2$, we can conclude *conclusion*.

For example, suppose A and B are statements, and we accept that if A is the case then B is the case. Then, if we take it that A is the case, we will reasonably infer that B is the case as well, that is B logically follows from A. In syllogistic form, this is

1. A
2. if A then B
3. B

This fundamental rule of contemporary logic is known as *Modus Ponems*, or the method of affirming. For example, consider:

1. Bastet is a cat
2. if Bastet is a cat then Bastet likes fish
3. so Bastet likes fish

This seems entirely reasonable.

In arguments, people may vociferously question whether or not A is the case, or whether or not B actually follows from A. For example, consider:

1. Bastet is a warrior goddess
2. if Bastet is a warrior goddess then Bastet defends the king
3. so Bastet defends the king

We might argue:

- there are no goddesses;
- there are goddesses but no warrior goddesses;
- there are warrior goddesses but Bastet isn't one;
- Bastet is a warrior goddess but there is no king;
- Bastet is a warrior goddess but she doesn't defend the king;
- etc.

But if we accept the truth of 'Bastet is a warrior goddess', and of 'if Bastet is a warrior goddess then she defends the king', then clearly 'Bastet defends the king' is an unimpeachable conclusion.

That is, in the general case, nobody questions the deduction of B, assuming that both A, and A implies B, are the case. Any argument is about the premises, whose truth or falsity is ultimately determined outside of logic,[7] not the deduction.

In contemporary logic, we now distinguish *propositional* from *predicate* logic. It is common to refer to both as *calculi*, after George Boole, who called his pioneering system, discussed below, the *Calculus of Logic*.[8]

Propositional logic is to do with propositions (i.e. simple statements) being true or false. Propositions are built from truth values, and variables that abstract over them. Propositions may be *negated* (not) and combined through *disjunction* (or), *conjunction* (and), and *implication* (if . . . then . . .).

Predicate logic is then to do with propositions about collections of things, and their members, being true or false. Predicate logic extends propositional logic truth values and variables with *predicates*, which are functions that return truth values. Further, in predicate logic, propositions may be *quantified* to express *universal* properties, that is, all things having some property, and *existential* properties, that is, some things (i.e. at least one) having some property.

We also distinguish *pure* logics, which are not about anything in particular, from *applied* logics, which are about specific domains of things. As we shall discuss, from antiquity until the late nineteenth century, all of these aspects of logic were conflated.

A key philosophical question concerns the status of truth values and rules of inference, as part of a wider question about the status of mathematical entities like numbers and functions. Are they just marks on paper? Do they have some deeper ideal reality? Or are they, as we shall argue, components of materialized mathematical systems, abstracted from and with strong correspondences, to material reality?

Logic and dialectics

Logic and dialectics are core components of the *materialist dialectic.*[9] However, untangling logic and dialectics is a curiously difficult business.

Aristotle actually refers to dialectics rather than logic. In *The 'Art' of Rhetoric*,[10] he distinguishes rhetoric, concerned with informal persuasion, from dialectics, concerned with formal reasoning through the syllogism (p13). This implies that logic is part of dialectics.

Aristotle notes that both rhetoric and dialectics employ syllogistic and inductive reasoning. For reasoning based on the dialectical syllogism, all steps must be made explicit. However, for the rhetorical syllogism, the *enthymeme*, steps may be elided. Further, dialectical induction is based on finding patterns, whereas rhetorical induction is based on concrete examples. Overall:

> The function of Rhetoric, then, is to deal with things about which we deliberate, but for which we have no systematic rules; (p23).

The implication is that dialectics is systematic.

Aristotle also distinguishes sciences, concerned with particular domains, from both rhetoric and dialectic as universally applicable modes of discourse. This is reflected in the subsequent Trivium/Quadrivium distinction, with the Trivium providing the pure tools for reasoning and arguing about the Quadrivium applied domains.

He further says that, as someone develops richer understandings of a domain:

> the more he will unconsciously produce a science quite different from Dialectic and Rhetoric. For if once he hits upon first principles, it will no longer be Dialectic or Rhetoric, but that science whose principles he has arrived at. (p31)

Clearly, this applies to the dialectics/logic dynamic itself. As we shall see, as logic became more mathematically grounded, so the space for dialectics shrank, much as wider scientific advances shrank the space for both religion and philosophy, as discussed in earlier chapters.

In *The Organon (Prior Analytics)*,[11] Aristotle further abstracts logic from both dialectics and science in discussing types of premises for syllogisms (p200 & 202). A demonstrative (i.e. scientific) premise is true and based on 'fundamental postulates', whereas, for a dialectical premise, a choice may be made between two 'contradictory statements'. Then, for a syllogism, a premise is simply true or false, regardless of origin. That is, for both science and dialectics, once some premise is established, a syllogism may be applied to draw a conclusion.

Mathematical forms

We saw above that Logic in the Trivium was distinguished from the mathematical sciences of Arithmetic, Geometry and Astronomy in the Quadrivium. Further, in *The Metaphysics*,[12] Aristotle says that there is hierarchy in mathematics:

> mathematics too has divisions, – there is a primary and a secondary science, and others successively, in the realm of mathematics. (p151)

and, in considering how philosophy is layered, distinguishes universal (i.e. pure) mathematics, from specific (i.e. applied) mathematical sciences:

> One might indeed raise the question whether the primary philosophy is universal or deals with some one genus or entity; because even the mathematical sciences differ in this respect – geometry and astronomy deal with a particular kind of entity, whereas universal mathematics applies to all kinds alike. (p297)

Aristotle, a Platonist by training, nonetheless appears uncommitted as to the metaphysical status of mathematics, but notes that, for his master, mathematics lies between material reality and pure idea:

> Further, he [Plato] states that besides sensible things and the Forms there exists an intermediate class, *the objects of mathematics* [footnote: ie arithmetical numbers and geometric figures], which differ from sensible things in being eternal and immutable, and from the the Forms in that there are many similar objects of mathematics, whereas each Form is itself unique. (p45)

For Platonists, the *forms* are abstract ideals which nonetheless are real.[13] We argue that mathematical objects have material beings in their physical representations within symbol systems. So, for example, the ideal cube is no more than the mathematical cube, itself a materialized construct that characterizes physical cubes.

Syllogisms

Aristotle's formulations of syllogisms are key to pre-modern logic. We will now look in slightly more depth at these, but we won't give a formal treatment. For a succinct account, see Smith.[14]

Aristotle explores syllogisms in *The Organon*, considering reasoning about properties of individuals and collections of things, and about things from particular and general domains. He makes considerable use of concrete examples, which, as we saw above, he called rhetorical induction.

First of all, in *On Interpretation*,[15] Aristotle defines the syntax of propositions, but without using any notation. Single sentences are composed of nouns, and verbs that act on them, and may be further conjoined. Propositions are then sentences which are affirmations or denials, and subjects may be universal or singular. *On Interpretation* also introduces the key notion of contradictory propositions.

Thus, Aristotle enunciates four fundamental schemes for propositions, commonly expressed as:

- A: universal affirmative, for example, all X are Y;
- I: particular affirmative, for example, some X is Y;
- E: universal negative, for example, no X is Y;
- O: particular negative, for example, some X is not Y;

These schemes all have the structure: *subject* is/are *predicate*. Here, X, Y and Z may be replaced consistently by concrete nouns. The initial letters A, I, E and O are the classical identifiers.

Then, the three syllogistic *figures* explored in *Prior Analytics*[16] may be expressed as:

1. P is Q; Q is R; P is R
2. P is Q; P is R; Q is R
3. P is R; Q is R; P is Q

Note that these are second level schema, where P, Q and R may be replaced by one of the four proposition forms A, I, E and O, to give a first level schema.

The validity of these figures then depends on whether the terms we substitute for P, Q and R are universal or particular, and which subjects and predicates are common to which terms.

Aristotle exhaustively considers all the universal/particular and subject/predicate possibilities for the three figures, dismissing many by deriving contradictions. He also shows how the second and third figures may be reduced to the first figure.

Overall, Aristotle developed a framework both for constructing and analysing arguments that was in widespread use until the revolutions in logic and mathematics in the mid-nineteenth century.

After Aristotle

Aristotle's work was translated into Latin by Boethius,[17] who lived around 475 to 526 CE. Subsequently, it was effectively lost to European thought for several hundred years, until it was 'rediscovered' by Arab scholars. Aristotelian philosophy then gained steady traction among a significant movement of medieval Catholic scholars, now known as Scholasticism. The articulation of the Seven Liberal Arts, and their division into the Trivium and Quadrivium, was core to European education, especially after the establishment of the first universities from the late eleventh century onwards under the imprimatur of the Vatican.

France was a centre of syllogistic reasoning, particularly at the University of Paris. In the mid-seventeenth century, the *Port-Royal Logic*[18] was developed by Arnauld and Nicole, who had studied at the Sorbonne. Though they were clerics, they adhered to Jansenism, a sect in doctrinal conflict with mainstream Catholicism, and were heavily influenced by Descartes.

The *Port-Royal Logic*, published in 1662, retained the A, I, E and O syllogistic forms, but it had a more sophisticated term structure that allowed subordinate propositions. This made both semantics and reasoning more complex. The book became a key text on logic until well into the nineteenth century. It went into sixty-three editions, including ten in English, and was in use at Oxford and Cambridge Universities.

Nonetheless, Aritotelian logic fared less well in England after the Reformation, where the state ideology of Catholicism was displaced by Anglicanism, the English compromise with Protestantism. Francis Bacon (1561–1626), progenitor of what many view as the doleful British tradition of empiricism, rejected formal syllogism entirely. In *Novum Organon*,[19,20] from 1620, he argues that logic is a tool of persuasion, not reason, and promotes induction from observation as the means to understanding. Bacon's critique rested on his requirement for a unitary system of reasoning. He argued that logic, in which he conflates syllogistics and dialectics, cannot furnish this as it is unable to correct for unfounded premises.

Thomas Hobbes (1588–1679), whose philosophy may be characterized as materialist, was more measured than Bacon. In 1655, he gave a succinct account of Aristotle's logic in *De Corpore*.[21] While he rejected Aristotelian metaphysics, he viewed syllogistic reasoning as computational, by strong analogy with arithmetic. Thereafter, while logic continued to be taught at Oxford and Cambridge, there was little academic interest in syllogistics until the early nineteenth century, when its revival was wholly separated from dialectics.

The work of Richard Whately (1787–1863), the Archbishop of Dublin in the Anglican Church of Ireland, was particularly influential. In *Elements of Logic*,[22] written in 1826, he robustly defended logic, emphasizing its universal applicability, and the need to separate it from its subject matter. In an engaging analogy with arithmetic, he asserted that the strength of logic lay precisely in abstraction.

The laws of thought

The same period saw steady progress in the mathematization of logic, culminating in George Boole's seminal algebraic treatment in *The Laws of Thought*[23] in 1854. The book's title derived from the notion that thinking is underpinned by immutable laws. These go to the heart of the fundamental properties of reality and, hence, why the syllogism can capture unimpeachable reasoning.

Most accounts of the laws of thought describe what Stanley Jevons (1835–82) termed the *three primary laws:*[24]

1. The Law of Identity. **Whatever is, is.**
2. The Law of Contradiction. **Nothing can both be and not be.**
3. The Law of Excluded Middle. **Everything must either be or not be.** (p117)

The Scot William Hamilton[25] (1788–1856), a contemporary of Whately and Boole, discussed in detail the laws' origins in Aristotle's writings. In his posthumous *Lectures on Metaphysics and Logic*,[26] published in 1860, Hamilton reiterated the separation of logic from metaphysics, and logic's universal applicability. For Hamilton, there was something essential about the laws of thought, circumscribing even the deity.

There has been considerable disputation over the status of each law, especially the Law of the Excluded Middle (LEM), and whether or not there are further laws.

Logical operations and truth tables

Aristotle, and his successors, focused on grammar as central to constructing correct arguments, and depended on meanings and reasoning expressed in everyday language. Thus, much discussion of logic prior to, and indeed after, Boole was about what exactly logical operations meant.

Today, we use *truth tables* to give logical operators precise meanings, as promoted by Ludwig Wittgenstein (1889–1951).[27] We assume the basic truth values of true and false. Core operations are: negation (not) as reversing true and false premises; conjunction (and) as requiring both premises to be true; and disjunction (or) as requiring either or both premises to be true. See Table 7.1.

Note that this form of disjunction is called *inclusive*. We can also define *exclusive* disjunction (xor), which is true if either premise is true, but not both.

In logical reasoning, *material implication*, that is, 'implies' or 'if . . . then . . .', is central to forming rules of inference. For:

$$X \text{ implies } Y$$

we wouldn't like X to be true at the same time that Y is false. That is, we require the effect of:

$$\text{not } (X \text{ and not } Y)$$

Table 7.2 shows the corresponding truth table.

This feels counterintuitive, as 'X implies Y' is true whenever X is false. Here the distinction between logical and real-world notions of truth is stark: a logical implication which evaluates to true certainly does not permit us to deduce anything about the real world unless we know that the first premise is true. Indeed, a true implication resting on a false first premise is termed *vacuously* true.

Table 7.1 Logical Operations: Negation, Conjunction and Inclusive Disjunction

X	not X	X	Y	X and Y	X	Y	X or Y
true	false	false	false	false	false	false	false
false	true	false	true	false	false	true	true
		true	false	false	true	false	true
		true	true	true	true	true	true

Table 7.2 Implication

X	Y	not Y	X and not Y	not (X and not Y)
false	false	true	false	true
false	true	false	false	true
true	false	true	true	false
true	true	false	false	true

Table 7.3 Logical Operations as Arithmetic: Negation, Conjunction and Inclusive Disjunction

x	1 − x	x	y	x × y	x	y	x + y
0	1	0	0	0	0	0	0
1	0	0	1	0	0	1	1
		1	0	0	1	0	1
		1	1	1	1	1	1

Mathematizing logic

While Leibniz (1646–1716) had explored the formalization of logic well before Boole, his work was lost until the early twentieth century.[28] Thus, Boole (1815–64) is now recognized as the progenitor of modern logic, particularly propositional calculus. In *The Laws of Thought*,[29] Boole's intention was

> to investigate the fundamental laws of those operations of the mind by which reasoning is performed; to give expression to them in the symbolical language of a Calculus, and upon this foundation to establish the science of Logic and construct its method; (p1)

Boole thought that the laws of human thought were quite literally mathematical:

> There is not only a close analogy between the operations of the mind in general reasoning and its operations in the particular science of Algebra, but there is to a considerable extent an exact agreement in the laws by which the two classes of operations are conducted. (p6)

Boole's key insight was that if a Universe of discourse is represented as '1', and Nothing as '0', then logical operations on classes of things have arithmetical equivalents. Hence, algebraic techniques may be deployed to manipulate them.

It is straightforward to read off propositional logic from Boole's system, where 1 is true and 0 is false. See Table 7.3, which may be contrasted with Table 7.1.[30]

The information theorist Claude Shannon noted in 1938 that the correspondence between Boole's calculus and logic could be applied to switching circuits.[31]

Boole developed his logic with examples drawn from a range of domains, including contemporary economics and theology. He stated premises baldly, without discussion, and focused on what might be logically concluded from them. In so doing, he showed how logical arguments in everyday language might be formalized. He further demonstrated that all the Aristotelian syllogistic figures were subsumed by his approach.

Despite his book's title, Boole barely discussed the three laws of thought considered above. However, he called $x \text{ and } x = x$ the 'fundamental law of thought' (p49), and used it to develop the *principle of contradiction*, that is, nothing can be both x and not x.

While Boole was critical of scholastic logic, he acknowledged its importance. His conceptualization still conflated reasoning about things and about collections of things, and his notation, though cunning, is clumsy. Still, he enabled arithmetic certainty in chains of reasoning: that is, with Boole, logic truly became a matter of computation, as Hobbes had sought.

Boole's novel approach met with opposition, and the Aristotelian separation of logic based on syllogistic figures from mathematics persisted well into the twentieth century. Nonetheless, despite infelicities in Boole's system, his work has proved foundational, and marked a fundamental break with the longstanding Aristotelian tradition.

Frege and the foundations of mathematics

The German matematician Gottlob Frege (1848–1925) turned Boole's work on its head, and sought to found arithmetic, and then mathematics, on logic. The mathematical logic that underpins computing ultimately flows from Frege's.

Frege's *Begriffsschrift*,[32,33] published in 1879, was subtitled:

> a formula language, modeled upon that of arithmetic, for pure thought

The emphasis on 'pure thought' is very much in the Aristotelian tradition of separating reasoning from that which is reasoned about.

Here, Frege introduced a number of fundamental innovations. While, like Boole, he used variables and arithmetic operators, he replaced the Aristotelian notions of subject and predicate with those of *argument* and *function*. Further, he based all rules on material implication and negation, showing how to derive the other logical operators, and all syllogistic forms, from them. Finally, he introduced an operator for universal quantification, that is, for talking about all members of a class having some property.

In *The Foundations of Arithmetic*,[34] from 1884, Frege elaborated his *logicist* philosophy of mathematics. He explicitly sought to disassociate logic from subjective philosophy, which he called psychology, and asserted a strong connection with mathematics. However, he also criticized mathematics for accepting incomplete proofs, requiring that every step should be made explicit. Today, this is a characteristic of computer-based proofs.

In *Basic Laws of Arithmetic*,[35] published in 1893, Frege was frank that logic could not be justified by external appeal:

> The question why and with what right we acknowledge a logical law to be true, logic can answer only by reducing it to another law of logic. Where that is not possible, logic can give no answer. (p15)

Numbers and induction

Aristotle distinguished syllogistic, or *deductive* reasoning from *inductive* reasoning, that is, identifying patterns in collections of things. This was placed on a formal footing by Giuseppe Peano (1858–1932), who unified notions of number, set and induction.

Peano's approach,[36] from 1889, was based on a very simple conception of *successive* numbers, starting with one[37] and repeatedly adding one. Peano next presented having a property as akin to being in a collection with well-defined characteristics, much like Boole and Frege. He then defined induction over collections of numbers by considering common properties of their members regarded as a sequence.

In general, for an inductive proof, we assume that some property holds for 1. Then, suppose N is an arbitrary number. If assuming that the property holds for N, we can prove that it holds for $N+1$, then the property must hold for all numbers, as we can work our way forward from 1 to any number. That is, the property is an inductive pattern for the number sequence. Inductive proof is central to many areas of mathematics, complementing the traditional proof by contradiction. It is also the basis of the fundamental programming technique of *recursion*, where computations over collections are defined in terms of computations over sub-collections, down to an empty collection.

As with deduction, induction is a process of formalizing a property once we have one to reason about. Coming up with an inductive property in the first place, that is, identifying hypothetical patterns, is to do with the practice of mathematics.

Infinity and infinitesimals

Induction appears to justify counting without limit. How then might we characterize such seeming infinity? In the *Metaphysics*,[38] Aristotle distinguishes between *potential* and *actualized* infinities:

> Infinity and void and other concepts of this kind are said to 'be' potentially or actually in a different sense from the majority of existing things. e.g. that which sees, or walks, or is seen. For in these latter cases the predication may

> sometimes be truly made have without qualification, since 'that which is seen is so called sometimes because it is seen and sometimes because it is capable of being seen: but the Infinite does not exist potentially in the sense that it will ever exist separately in actuality it is separable only in knowledge.' (Book IX, Part 6, p447)

Zeno of Elea (495–430 BC) had constructed paradoxes that revolve around being able to divide things indefinitely and hence into an infinite number of components.[39] Aristotle dealt with such paradoxes at length in *Physics*. In *Metaphysics*, he deployed the distinction between potential and actual infinities to contest indefinite division:

> For the fact that the process of division never ceases makes this actuality exist potentially, but not separately. (Book IX, Part 6, p447)

The calculus of Newton and Leibniz raised acute problems of divisabilty.[40] In particular, integration involves summing the values of a function for successive values of vanishingly small differences. Consider Figure 7.1, which shows the curve for some function y = f(x).

We can approximate the area under the curve by dividing it into rectangles of width dx and height $f(x_i)$, and summing the areas:

$$f(0) * dx + f(x_1) * dx + f(x_2) * dx + \ldots + f(x_i) * dx \ldots$$

Of course, for each rectangle, there remains a vaguely triangular shape between it and the curve, which is not accounted for. These accumulate as an error value.

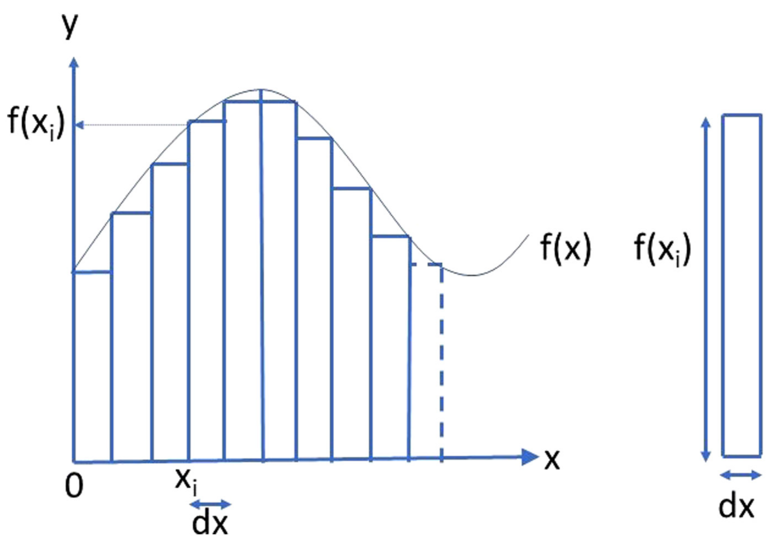


Figure 7.1 Integration by summing areas. Figure by authors.

If we can make dx smaller, then the error becomes smaller. How small can dx become?

This boils down to how many values there are in between the start and end values of the integration. If there are an infinite number, then the difference between them is zero. But then we have a paradox of summing an infinity of zeros giving a non-zero result: equivalent to the paradoxes of repeatedly dividing time and space that Zeno explored.

The practical solution is to sum the function for smaller and smaller values of dx until the difference between successive sums is small enough not to be concerned about. That is, the integration converges towards some acceptable value. There are precise analytic solutions for some classes of function, and the resulting values are explained as 'at the limit' of the equivalent sums of differences. However, the limit is often treated as if it is at infinity, that is, as if the range of integration is an actualized infinity.

To infinity, and beyond

The characterization of infinitesimals and infinity was central to the work of Georg Cantor (1845–1918), which underpinned the crisis in mathematical logic in the late nineteenth and early twentieth centuries. Cantor's work was refined over several decades.Tiles provides a thorough account.[41]

First of all, we will distinguish the integers, which have finite representations as sequences of digits, from the *rational* numbers, which have finite representations as the ratios of two integers, but may have infinite expansions if an attempt is made to divide the numerator by the denominator. For example:

10 / 3 ==> 3.3333333333...

Those numbers which cannot be represented as the ratio of two integers are called the *real* numbers. Examples are π and e. Such numbers also have infinite expansions, but are expressed as formulae for calculating them, for example, by summing series of rational numbers.

Now, if unbounded division is acceptable, then there appears to be an infinity of real numbers between two rational numbers. Cantor sought to characterize this *continuum* using set theory.

We will write sets as between the braces { and }, with elements separated by commas, for example, a set of even numbers:

{2,4,6,8,10}

The empty set is {}.

We distinguish *ordinal* and *cardinal* numbers. Ordinal numbers may be used as indices in ordered sets. So, for the set of even numbers, the first element is 2, the second element is 4, the third element is 6 and so on. Here we use the ordinals 1, 2, 3 . . . to select elements of the set in order. Cardinal numbers are used for sizes of set. So the cardinality of the set of even numbers above is 5.

We further distinguish between *finite* and *infinite* sets. For finite sets, like that of even numbers above, the cardinality is also the ordinal number for the last element. But, for an infinite set, the cardinality can't be an ordinal number, as there is no last element by definition.

While an infinite set clearly cannot be constructed in finite time, some infinite sets can be given finite characterizations. In particular, we can specify *algorithms* to *enumerate* some infinite sets, that is, to generate successive members. And an algorithm is a finite materialized description.

For example, the infinite set of numbers can be enumerated by starting at 1 and repeatedly adding 1. Note that the previous sentences is a finite description. However, it specifies a potentially infinite computation, which cannot be completed by a materialized system, which must necessarily be finite.

A set is said to be *countable*, or *denumerable*, if its members can be put into one-to-one correspondence with some set of integers. For example, for our set of five even numbers:

$$1 \leftrightarrow 2$$
$$2 \leftrightarrow 4$$
$$3 \leftrightarrow 6$$
$$4 \leftrightarrow 8$$
$$5 \leftrightarrow 10$$

Thus, an enumerated set is in this sense countable, even though it may be infinite, as its members can be put into one-to-one correspondence with the infinite set of integers. For example, to generate the set of even numbers, successively double the integers. So the set of all even numbers can be put into one to one correspondence with the set of all integers. This may seem counterintuitive, as only half of a finite set of integers are even.

Many other infinite sets derived from the integers can be put into one-to-one correspondence with them. This includes the rational numbers, which we can systematically enumerate as successive ratios of integers.

In order to compare properties of infinite sets, Cantor used the Hebrew symbol $\aleph$ (*aleph*) with successive subscripts to denote their cardinalities as *transfinite* numbers. Thus, the set of integers has cardinality $\aleph_0$ (*aleph nought*), as do all countable sets.

However, using a *diagonal* argument, Cantor sought to demonstrate that the real numbers are not enumerable and, hence, not countable. First of all, a real number can't be expressed as a ratio of two integers, so it must have an infinity of decimal digits. Each of those decimal digits can be put into one-to-one correspondence with the integers, so we can index them with ordinal numbers. Suppose we could enumerate the real numbers, so the digits of the ith real number d_i were d_i^1, d_i^2, d_i^3 and so on, as in Table 7.4.

We can make a new real number $0.n^1n^2n^3n^4$... where the first digit n^1 is different from the first digit of the first real d_1^1, the second digit n^2 is different from the second digit of the second real d_2^2, the third digit n^3 is different from the third digit

Table 7.4 Diagonalization

1.	0.	$\mathbf{d}_1^1$	d_1^2	d_1^3	d_1^4	...
2.	0.	d_2^1	$\mathbf{d}_2^2$	d_2^3	d_2^4	...
3.	0.	d_3^1	d_3^2	$\mathbf{d}_3^4$	d_3^4	...
4.	0.	d_4^1	d_4^2	d_4^3	$\mathbf{d}_4^4$	...
...						

Table 7.5 Power Sets

set	ordinality	power set	cardinality
{1}	1	{{},{1}}	2
{1,2}	2	{{},{1},{2},{1,2}}	4
{1,2,3}	3	{{},{1},{2},{3},{1,2},{1,3},{2,3},{1,2,3}}	8
...			

of the third real d_3^3 and so on. This new real can't be in the enumerated sequence, so there must be more real numbers than $\aleph_0$, so the reals are not enumerable.

Now, as well as sets of integers, it seems legitimate to make sets of sets. In particular, given a set, we can construct a set of all of its subsets, known as the *power set*. We start with the empty set {}, and then add all the single elements, pairs of elements, triples of elements and so on, ending with the whole set. See Table 7.5.

A power set always has greater cardinality, that is, more members, than the original set: for a set of N elements, the power set has 2^N elements. In particular, if the set of integers has cardinality $\aleph_0$, then the power set of integers has cardinality $2^{\aleph_0}$, denoted $\aleph_1$ (*aleph one*). Thus, the powerset of integers is uncountable, as $\aleph_1$ is necessarily bigger than $\aleph_0$. Cantor then demonstrated that the set of real numbers also has cardinality $\aleph_1$.

From an Aristotelian perspective, Cantor's diagonalization argument is illegitimate because it presupposes an actualized infinity of real numbers to arbitrary precision. For a finite set of 'real' numbers, represented to fixed precision, diagonalization produces a number which is already present.

Chapter 8

Logic and Dialectical Materialism

Hegel and logic

> . . . to regard the syllogism as merely consisting *of three judgments* is a formalistic view that ignores the relation of the determinations which alone is at issue in the inference. It is altogether a merely subjective reflection that splits the connection of the terms into isolated premises and a conclusion distinct from them:
>
> All humans are mortal.
> Gaius is a human.
> Therefore Gaius is mortal.
>
> One is immediately seized by boredom the moment one hears this inference being trotted out, a boredom brought on by the futility of a form that, by means of separate propositions, gives the illusion of a diversity which is immediately dissolved in the fact itself. G. Hegel, *The Science of Logic*[1]

The idealism of Georg Hegel (1770–1831) is central to European philosophy. Unlike the British tradition, which broke with Aristotle before reviving logic, dialectics is at the heart of Hegel's epistemology.

The title of Hegel's book *The Science of Logic*[2] seems strange from an Aristotelian perspective. Aristotle counterpoises dialectics to science, as a form of persuasion rather than an autonomous way of understanding how the world works, and to syllogistic logic, as a source of premises rather than a method for reasoning about them. Still, it is salutary that Hegel does view logic as being amenable to scientific exegesis.

Hegel was highly sceptical about the value of separating the form of a syllogism from its context of application. That is, the syllogism itself is of no interest. Rather, its importance lies in how it connects particular premises (determinations) to particular conclusions, which in turn depend on prior reasoning to establish relationships among premises. As we shall see, this attitude is a recurrent feature of dialectical materialism.

Nonetheless, Hegel does systematically explore the syllogistic forms,[3] and considers how they relate to each other using dialectical transformations. But he deploys no formal notation other than denoting the terms in syllogisms as universal (U), singular (S) or particular (P), a simplification of the earlier practice. Like his predecessors, Hegel uses terse arguments and examples.

Hegel's attitude towards syllogistic logic is further illuminated in the *Prologue* to *The Science of Logic*. First of all, he noted that science and commonsense had displaced metaphysics, as well as the notion that logic taught one how to think. Nonetheless, logic was retained among the sciences 'probably for the sake of a certain formal utility', though 'its shape and content have remained the same throughout a long inherited tradition' (p8).

For Hegel, thought is held in language and the form of language determines thought, for which German is clearly superior(!) (*Preface to Second Edition*, p12). And logic is central to the human condition:

> So much is logic natural to the human being, is indeed his very *nature*. If we however contrast nature as such, as the realm of the physical, with the realm of the spiritual, then we must say that logic is the supernatural element that permeates all his natural behavior, his ways of sensing, intuiting, desiring, his needs and impulses; and it thereby makes them into something truly human, even though only formally human – makes them into representations and purposes. (p12)

Here, he explicitly disagrees with Aristotle's position:

> 'In so many respects', says Aristotle in the same context, 'is human nature in bondage; but this science, which is not pursued for any utility, is alone free in and for itself, and for this reason it appears not to be a human possession'. (p14, citing Aristotle, *Metaphysics*, 982b)

Hegel argues that, on the contrary, logic is about abstract thought which is why it is taught to young people, with concrete matters coming later. Here, quite apart from Hegel's unawareness of how privileged such education was, he reflects the distinction between the Trivium and Quadrivium, which must have still been current.

For Hegel, a major benefit of logic lies in its utility as an abbreviation because of its universality. However, because logic shorn of content cannot attain truth, it is hardly surprising that it has been rejected by common sense as barren:

> Regarding the formulas that define the rules of inference which in fact is a principal function of the understanding, however mistaken healthy common sense might be in ignoring that they have their place in cognition where they must be obeyed, and also that they are essential material for rational thought, it has nonetheless come to the equally correct realisation that such formulas are indifferently at the service just as much of error as of sophistry, and that, however truth may be defined, so far as higher truth is concerned, for instance religious truth, they are useless – that in general they have to do only with the correctness of knowledge, not its truth. (p18)

Here, Hegel's position is reminiscent of Hobbes'.

Hegel argues that the separation of form and content is illusory. He goes on to considerably expand on this position in the *Introduction*. Herein lie the roots of the rejection of formalism, that is, the study of logic independently of content, in dialectical materialism.

Engels and logic

The revolutionary world view of Karl Marx (1818–83) and Frederic Engels (1820–95) is premised on scientific materialism. Marx wrote practically nothing about dialectics or syllogistic logic. Engels, in contrast, made a number of references to logic in his later work. In *Anti-Dühring*,[4] from 1877, he strongly asserted the subordination of logic in its union with dialectics. First of all, he reasserts Aristotle's position that, once a science is sufficiently specialized, it becomes independent of any wider epistemology. Syllogistic logic, now termed formal, and dialectics, as the laws of thought, are independent of philosophy:

> That which still survives, independently, of all earlier philosophy is the science of thought and its laws — formal logic and dialectics. Everything else is subsumed in the positive science of nature and history. (p40)

This formulation was repeated in the 1880 *Socialism: Utopian and Scientific*,[5] and reiterated in 1886 in *Ludwig Feuerbach and the End of Classical German Philosophy.*[6]

Returning to *Anti-Dühring*, Engels argues that mathematics has to borrow axioms from logic, which are 'expressions of the scantiest thought-content' (p60). Following Hegel, he says that logical axioms alone 'do not cut much ice', and to make progress it is necessary to draw on geometry (p61). His main plaint is that Dühring emphasizes the independence of pure mathematics from experience rather than acknowledging its abstraction from reality (p61).

Engels says that dialectics is more powerful than formal logic:

> Even formal logic is primarily a method of arriving at new results, of advancing from the known to the unknown – and dialectics is the same, only much more eminently so; moreover, since it forces its way beyond the narrow horizon of formal logic, it contains the germ of a more comprehensive view of the world. (p186).

Further, formal logic is only significant for elementary mathematics. In higher mathematics, indeed, in all new science, dialectics is needed to advance:

> Elementary mathematics, the mathematics of constant quantities, moves within the confines of formal logic, at any rate on the whole; the mathematics of variables, whose most important part is the infinitesimal calculus, is in essence nothing other than the application of dialectics to mathematical relations. In it, the simple question of proof is definitely pushed into the background,

> as compared with the manifold application of the method to new spheres of research. But almost all the proofs of higher mathematics, from the first proofs of the differential calculus on, are from the standpoint of elementary mathematics strictly speaking, wrong. And this is necessarily so, when, as happens in this case, an attempt is made to prove by formal logic results obtained in the field of dialectics. (p186–7)

Nonetheless, his characterization of formal logic involving constants rather than variables suggests a fundamental misunderstanding. This was also Marx's view of mathematics, as shall see below. And Engels betrayed a curious contempt for mathematics:

> The abstract requirement of a mathematician is, however, far from being a compulsory law for the world of reality. (p75)

Finally, in the posthumously published *Dialectics of Nature*,[7] Engels revisited these themes. Once again, dialectical logic is superior to formal logic, because it integrates both analysis and reasoning:

> Dialectical logic, in contrast to the old, merely formal logic, is not, like the latter, content with enumerating the forms of motion of thought, ie the various forms of judgment and conclusion, and placing them side by side without any connection. On the contrary, it derives these forms out of one another, it makes one subordinate to another instead of putting them on an equal level, it develops the higher forms out of the lower. (p296)

Engels acknowledges that, despite developments in natural and historical sciences, formal logic has an essential and knowable quality. Nonetheless, once again, Engels firmly rejects mathematical abstraction:

> What they [mathematicians and natural scientists] charge Hegel with doing, viz., pushing abstractions to the extreme limit, they do themselves on a far greater scale. They forget that the whole of so-called pure mathematics is concerned with abstractions, that all its magnitudes, strictly speaking, are imaginary, and that all abstractions when pushed to extremes are transformed into nonsense or into their opposite. (p359)

In his notes for *Dialectics of Nature*, Engels distinguishes between mathematical operations, which could be proved by 'material contemplation', and logical deductions, which could only be proved by deduction:

> Calculative reason – calculating machine! – Curious confusion of mathematical operations, which are capable of material demonstration, of proof because they are based on direct, even if abstract, material contemplation, with purely logical ones, which are capable only of proof

> by deduction, hence are incapable of the positive certainty possessed by mathematical operations—and how many of them wrong! Machine for integration; cf. Andrews' speech, Nature, Sept. 7, 76.317[8],[9]

The nature of infinity had also concerned both Engels and Dühring. Dühring's writings about infinity, were rather tetchily dismissed by Cantor.[10] Engels, in turn, criticized at length Dühring's notion of infinity. However, Engels confuses countabilty with counting:

> But what of the contradiction of 'the counted infinite numerical series'? We shall be in a position to examine this more closely as soon as Herr Dühring has performed for us the clever trick of counting it.[11]

Dietzgen, dialectical materialism and logic

Joseph Dietzgen (1828–88) met Marx during the 1848 German Revolution, and subsequently became his friend. Self-educated, Engels credited him, in *Ludwig Fuerbach and the end of Classical German Philosophy*, with independently discovering the materialist dialectic. Note the formulation 'materialist dialectic'. This sounds like a form of dialectic grounded in materialism. But this does not exclude materialism grounding other modes of analysis. So, we may admit a non-dialectical materialist science.

Burns[12] argues that Dietzgen coined the expression 'dialectical materialism', having read Engels on Feurbach in his *Excursions of a Socialist into the Domain of Epistemology*[13] from 1887.[14] This apparent point of trivia highlights how slippery these notions are.

'Dialectical materialism' sounds like a variety of materialism that is grounded in dialectics, that is, one which admits no other forms of analysis. As we shall see, dialectics and science are increasingly conflated in subsequent Marxist philosophy, and it was disputed that one could be separated from the other. Scientific materialism that was not avowedly dialectical was termed *mechanical*.

In his most mature work, *The Positive Outcome of Philosophy*,[15] Dietzgen conflated the premises of formal logic with the deductions made from them. He further confuses critiquing premises with critiquing the deduction:

> The premise from which he deducted the watchfulness of dogs in general, was handed down by tradition and had been accepted on faith. But was it founded on fact? Could there not be some dogs who lacked the quality of watchfulness, and might not our pug-dog be very unreliable, in spite of all exact deductions? (XIII)

Dietzgen goes on to cite Bacon's (and Descartes') rejection of syllogistic logic. As with Hegel, what is needed is a new synthesis of logic and dialectics.

Dietzgen's great strength lies in making explicit the class perspective of dialectics, in particular in an earlier collection of letters to his son, *Letters on Logic Especially Democratic-Proletarian Logic.*[16] Here, he criticizes class (i.e. bourgeoisie) logic for rationalizing exploitation as natural, by emphasizing difference. This leads Dietzgen to associate formal logic with class logic because it is based on reasoning from premises as separate aspects of things. However, he rejects the second law of thought, that contradictions demonstrate invalid premises, in an argument that is explicitly transcendental.

Echoing Hegel, Dietzgen has no time for logicians, because they separate logic from its content:

> The formal logicians are as ignorant as they are roguish, when they persist in discussing the intellect or thought in the traditional manner as if they were isolated things, while ignoring the necessary connection of the object of the logical study with the world of experiences. This interconnection leads to an explanation of truth and error, of sense and nonsense, of god and idols, and this is very inopportune for the professors. For this reason this unwelcome problem is handed over to the mystical departments, to metaphysics and religion, so that these venerable pillars of official wisdom may continue their services to the ruling classes. (Fourth Letter)

Early on in *Letters*, Dietzgen deploys an analogy of a potter to reject formal logic, saying that thoughts cannot be separated from actions, nor form from content. Here, Dietzgen seems to forget Marx's famous observation in *Capital* Volume 1.[17]

> But what distinguishes the worst architect from the best of bees is this, that the architect raises his structure in imagination before he erects it in reality. (p174)

Familiar with Aristotelian syllogistic logic, Dietzgen makes the reasonable criticism that it could not adequately capture multifaceted properties of things. However, invoking his revolutionary principles, he says in the later *The Positive Outcome of Philosophy*

> The philosophers should abandon their old hobby of trying to prove anything by syllogisms. Nowadays, a case is not substantiated by words, but by facts, by deeds. The sciences are sufficiently equipped, and thus the 'possibility of understanding' is demonstrated beyond a doubt. (XIII)

While Dietzgen is no longer part of the mainstream Marxist canon, he was highly influential before the Bolshevik revolution. In the introduction to the 1902 combined edition of *The Nature of Brain Work*, *Letters on Logic* and *Positive Outcome of Philosophy*, Anton Pannekoek wrote:

> a thorough study of Dietzgen's philosophical writings is an important and indispensable auxiliary for the understanding of the fundamental works of Marx and Engels.[18]

We will explore Marxist responses to logic after the Bolshevik Revolution, but first, we shall backtrack to key developments in formal logic at the start of the twentieth century.

Russell's paradox and Principia Mathematica

At the end of the nineteenth century, Frege's system was seen as the pinnacle of formal logic, on the high road to formalizing mathematics. Alas, this was short lived. In 1902, Bertrand Russell (1872–1970) wrote to Frege,[19] identifying a fundamental contradiction at the heart of his system.

In Frege's system, predicates are characterized by sets of items that satisfy some property. Russell's paradox involves observing that predicates may or may not apply to themselves. Equivalently, sets may or may not have themselves as members.

Now, a predicate applying to itself, represented as a set containing itself, sounds like it should lead to an infinite expansion. However, this may be avoided through the use of its name instead of its contents. That is, there are finite representations of apparently infinite constructs.

Consider sets that don't contain themselves. We might make a set of sets that do not include themselves. In turn, this set does not include itself, so maybe it should. But if it is included in itself, then it does include itself and so it shouldn't.

This is not merely an academic exercise; we can demonstrate this on any computer with a folder system. It is commonplace to make links from folders to other folders. Then, one way to make it easier to get from the bottom of a large folder back to the top is to make the last entry a link to the folder itself. Thus, we can make a new folder with links to folders that don't have links to themselves. Should that folder link to itself or not?

Russell's paradox drew into question the whole prospect of formalizing mathematics in logic, as Frege acknowledged.[20] Nonetheless, Russell, and his former teacher Alfred North Whitehead (1861–1947), embarked on trying to systematically ground mathematics in a formal logic that was broadly equivalent to Frege's or Cantor's. The three volumes of *Principia Mathematica (PM)*[21] appeared between 1910 and 1913 and reconstructed a significant portion of mathematics. While *PM* is now little read, not least because of its non-standard notation, it has long been heralded as foundational.

Hilbert's programme

Hilbert's programme, formulated by David Hilbert (1862–1943), framed the conduct of formal logic in the twentieth century, and to this day. The 'programme' was not a settled statement of purpose like a manifesto: rather, it was codified from Hilbert's evolving conceptions.

We can see the roots of Hilbert's programme in his 1904 response to Russell's paradox, *On the foundations of logic and arithmetic.*[22] He began by characterizing leading mathematicians' views on the foundations of mathematics. Leopold Kronecker (1823–91) was called a *dogmatist* for accepting integers as implicitly existent without recourse to foundations. Herman van Helmholtz (1821–94) was termed an *empiricist* for only accepting existence derived from experience, thus ruling out thought experiments as the basis for theories. Elwin Christoffel (1829–1900) was termed an *opportunist*. While he had opposed Kronecker's rejection of irrational numbers, he sought positive reasons for accepting them. Frege's work was acknowledged as foundational, but his system was criticized for a lack of rigour, giving rise to paradoxes of self-reference. Here, Hilbert writes:

> *Rather, from the very beginning a major goal of the investigations into the notion of number should be to avoid such contradictions and to clarify these paradoxes.*[23] (p130)

Hilbert went on to characterize Richard Dedekind (1831–1916) as following a *transcendental* method, as he assumed actualized infinities of objects. We will return to this position, which Aristotle criticized. Finally, Hilbert said that Cantor, while distinguishing consistent and inconsistent sets, gave no criteria for distinguishing them, necessitating *subjectivist* assumptions.[24]

Hilbert concludes these remarks by saying that all these difficulties could be overcome by what he called the *axiomatic method*. Further, the paradoxes might be avoided by acknowledging the co-dependence of logic and arithmetic and concurrently developing their laws.

Formal systems are based on *axioms*, elementary formulas which are true for all arguments, and *rules of inference*, for constructing or proving *theorems*, additional true formula, from axioms and other theorems. The Laws of Thought, derived from Aristotle, might be seen as progenitors of axioms. Then, the axiomatic method involves finding sets of independent axioms that, together with appropriate rules, are adequate for elaborating all of some domain.

Hilbert placed great stress on the *consistency* of axiomatic systems, that is, that it should not be possible to use them to derive contradictions. The impossibility of establishing consistency was to prove key to the later crisis in mathematical logic.

In a subsequent paper, *Axiomatic Thought*,[25] Hilbert first referred to a 'programme',[26] developing his objectives in greater detail. As well as consistency, he wished to determine the solvability in principle of arbitrary mathematical questions, to be able to check the results of mathematical activity, and to determine whether or not there might be simpler proofs. Here, Hilbert also returned to the old Aristotelian problem that so exercised Marxists, that of 'the relationship between *content* and *formalism* in mathematics' (p1113).

Notably, the ability to decide whether or not a mathematical question was solvable in a finite number of steps

> . . . goes to the essence of mathematical thought. (p1113)

This is still commonly referred to as the *Entscheidungsproblem* – the decision problem. There are lots of other decision problems, for example, whether or not two formulae are equivalent, but proof of properties in a finite number of steps was seen as key.

Meta-theory and logical schools

Hilbert thought that mathematical abstractions should, and could, be explored independently of any content of application. But Hilbert's programme required that mathematics itself should be subject to mathematical reasoning. That is, there should be *metamathematics*, with mathematics as its contents, but only to establish the consistency of axioms.[27]

Now, mathematics is expressed in a language,[28] with symbols, syntax and semantics. Thus, mathematics was to become its own *metalanguage*, a language for talking about language.

In the decades following Russell's paradox, three schools of formal logic emerged, reflecting different responses. We may characterize them according to what they accepted as admissible mathematical entities, and what forms of mathematical reasoning about them were admissible, that is, how meta-mathematics might be conducted.[29] This boiled down to their attitude to infinity.

The *logicists*, exemplified by Frege and Russell, accepted both infinite mathematical entities and infinitory reasoning, like Cantor's. They were Platonists, in that they accorded existence to ideal mathematical entities. Hence, they were idealists.

Next, the *formalists*, like Hilbert, accepted infinite mathematical entities as objects of study, but sought to only use finite reasoning. The existential status of mathematical entities was not of concern.

Finally, the *intuitionists* took the most radical stance. Building on Kronicker, they would only accept finite constructions in both mathematics and meta-mathematics, appealing to mathematical intuition. They also rejected the Law of the Excluded Middle (LEM), that is, that it is not possible for something and its negation to be simultaneously true.

There were profound, and sometimes vituperative, disagreements between proponents of these different schools. Nonetheless, their formal systems were actually very closely related.

Intuitionism

One response to Cantor was to entirely reject non-finitary methods, and, in effect, real numbers. Thus, in 1886, Kronecker argued that particular results found by manipulating infinite sequences were only admissible if they could be reconstructed without going 'beyond the concept of a *finite* series', using arithmetic over integers.[30]

Subsequently, Luitzen Brouwer's (1881–1966) intuitionism was concerned with the reconstruction of formal logic from a small number of 'intuitive' concepts, using constructive, finite techniques. In 1908, Brouwer published a critique of classical logic, whose themes recur in all his later writing.[31] First of all, he thought that logic separate from mathematics led to unfounded conclusions. Against the formalists, Brouwer thought that logic should be grounded in mathemetics, itself grounded in both observation and primordial intuitions of basic truth. Nonetheless, he suggests that, independently of mathematics, argument by syllogism and contradiction are both acceptable (p27,29).

Brouwer rejects the LEM,[32] that it is not possible for both something and its negation to be true (p29ff). This implies that one or other must be true, not allowing for the status of either to be uncertain. Similarly, Brouwer rejects double negation as cancelling, because something not being not true may still leave its status indeterminate.

Brouwer further argues that LEM

> demands that every supposition is either correct or incorrect, mathematically: that of every supposed fitting in a certain way of systems in one another, either the termination or the blockage by impossibility, can be constructed. The question of the validity of the principium tertii exclusi [ie LEM] is thus equivalent to the question concerning the *possibility of unsolvable mathematical problems*. (p42)

He was happy with LEM in finite cases, as it may be checked exhaustively. He was also happy with its application to infinite cases, so long as they may be constructed by induction. However, Brouwer objected to arguments from entities which are assumed to exist if their existence cannot be demonstrated. In particular, he rejected assumptions of completed infinities.

Bouwer's notion of 'ur-intuition' is plainly idealist, and his subjectivism is made explicit in his later work, for example, in *Mathematics, Science and Language*[33] from 1929.

From the Bolshevik revolution to Menshevizing Idealism

It is not possible to do justice to the catastrophic events of the first quarter of the twenteith century, killing millions of people and devastating the lives of millions more, without appearing to trivialize them. Nonetheless, from our current perspective, the key outcome of the 1914 to 1919 world war was the establishment of the Soviet Union,[34] the first state governed by a mass party explicitly committed to materialism.

Following the Civil War (1917–23), the Soviet focus was on reconstruction, and immediately bettering people's lives. Once it was clear that wider international revolution had stalled, the overwhelming priority was to stabilize and strengthen the Union. For this, and for building towards a Communist future, science was deemed central.

With a shortage of experts, lost through war or emigration, the state could not initially afford to place too much premium on the ideological rectitude of the remaining non-Communist intelligentsia, provided their loyalty was assured. At the same time, the expanding education system, under Communist direction, steadily produced a new generation of 'Red expert' scientists and engineers, who were explicitly committed to Soviet objectives, but who necessarily worked alongside the pre-Revolutionary cohorts.

Mathematics had a central role, and this period saw the growth of two world class mathematical centres, in Moscow and Leningrad. While pure mathematics research continued, the emphasis was on applied mathematics. Overall, formal logic was not prominent.

Nonetheless, the status of formal logic was still contested, even among polarizations within the Bolsheviks over the direction that the Soviet Union should take. For example, in 1921, Lenin,[35] in discussing Trotsky's Trade Unions proposal, attacked Bukharin for his neglect of dialectical logic.[36]

After Lenin's death in 1924, the struggles between different Bolshevik factions became acute. These took place against the background of the New Economic Policy (NEP), which, contrary to socialist aspirations, had introduced a substantial market component to try to accelerate recovery following the exigences of the pragmatic command economy of War Communism. To over simplify, the 'left' faction promoted a speedy transition from the NEP to a planned economy, whereas the 'right' faction sought a slower change. And these struggles were deeply entangled with jockeying for position and settling of scores.

As Helena Sheehan[37] systematically explores, the relationship of science to philosophy became an important component of these disputes, at both ideological and practical levels. Two positions developed. On the one side, the relevance of dialectics to science was questioned. This was in keeping with the Aristotelian tradition that mature sciences developed autonomously of their dialectical roots. This position was characterized by opponents as mechanist, descended from the mechanical materialism that Marx and Engels had opposed. On the other side, was a renewed emphasis on dialectics. This appeared to be in keeping with the mainstream Marxist tradition, drawing on Hegel.

Initially, ideological struggles within scientific discourse were against the 'right' tendency, characterized rhetorically as Menshevik. Here, the dialecticians under Deborin gained the upper hand against the mechanists. Within mathematics, this led to an increased repudiation of formal logic. The dialecticians' ascendancy was short lived. During subsequent struggles against the 'left' tendency, an over dependence on dialectics was characterized as idealist. As Friedmann subsequently wrote in the telling titled *Revolt Against Formalism in the Soviet Union*[38] :

> But the critics of mechanism, carried away by their zeal, fell into the opposite extreme of idealism. This required a 'struggle on two fronts' as the theoreticians of the Party call it. Apparently it was faults in practice which here too called attention to the theoretical problems. (p307)

However, the defeat of this tendency did not result in the rehabilitation of logic. Rather, both positions were conflated as *Menshevizing idealism*, and formal logic in the Soviet Union increasingly stalled until the early 1950s.

Menshevizing idealism and logic

The outstanding mathematician Andrei Kolmogorov (1903–87) exemplified the new generation of Red experts. His 1925 paper *On the principle of the excluded middle*[39] offered a critique of formalism and intuitionism, but also showed how classical and intuitionist logic might be reconciled, within an intuitionist framework.

Brouwer had argued that it was illegitimate to use both the LEM and transfinite premises to establish finitary results. Kolmogorov demonstrated that such finitary results still stood without recourse to either.

With the intuitionists, Kolmogorov accepted that, in the absence of other evidence, contradictory terms should be regarded as indeterminate. Further, with the intuitionists, he questioned whether transfinite premises had any meaning, even if they might be used to reach finitary results.

Kolmogorov's characterization of formalism, as uncommitted to choice of premises, sounds akin to Aristotelian dialectics. Noting that a contradiction may be resolved by adding one of the opposed terms as an axiom, he observed that, from a formalist perspective:

> The selection of the formula taken as the new axiom, from each pair of contradictory formulas, is thus subject only to considerations of convenience. (p417)

Further, while formalist logic attributed no meaning to axioms, intutionism was based on axioms that 'express facts given to us' (p417). This is the Hegelian, and also dialectical materialist, position of an integrated, content-full logic. However, Kolmogorov still identified 'mathematical logic' as a distinct component of mathematics. Here, Kolmogorov did not take an explicitly partisan philosophical, as opposed to mathematical, stance.

However, as Vucinich[40] explores, in that same year of 1925, Soviet ideologues explicitly attacked the idealism underpinning Cantor's results in the philosophical journal *Under the Banner of Marxism*. Vucinich notes that no attempt was made to find a materialist alternative to Cantor (p117–8), and that, while less attention was paid to Hilbert, dialectical logic was counterpoised to formal logic (p118–9). Nonetheless, the attacks were against Cantor's set theory rather than Soviet mathematicians (p122). The former Leningrad School mathematician G. G. Lorentz[41] notes that, in the same period, an algebraic school in Kiev was closed, under the direction of the Ukrainian Communist Party, and its scholars dispersed to other centres.

The Red experts Ernst Kolman (1892–1979) and Sofya Yanovskaya (1896–1966) provided a clear statement of the dialectical materialist position in 1931, in *Hegel and Mathematics*, also published in *Under the Banner of Marxism*.[42] Repeatedly citing *Anti During*, and the recently available *Dialectics of Nature*, they emphasized the roots of dialectical materialism in Hegel, while criticizing his dialectics.

Kolman and Yanovskaya summarized and rejected the schools of logic we identified above: intuitionism for depending on *a priori* assumptions; logicism for unifying logic and mathematics, and identifying the laws of reason with axioms and theorems; and formalism for treating logic independetly of content. They further reject 'mechanistic empiricists' who see mathematics as part of physics, and Mach for psychologizing. Overall:

> Thus none of these philosophical schools, which all grasp one and only one side of reality, is in a position to understand the link between mathematics and practice and its laws of development. Hegel alone gave mathematics a definition such as grasped the essence of the matter, a definition which, quite independently of Hegel's views, is actually profoundly materialist.

Nonetheless, taking a class standpoint on mathematics does not involve rejecting it:

> on the contrary it [bourgeois mathematics] must be subjected to a reconstruction, since it represents the material world, albeit one-sidedly and distortedly, nevertheless objectively.

Kolman and Yanovskaya identify attempts to reduce analysis to arithmetic as ultimately leading to

> the well-known paradoxes of set theory which destroyed the whole structure, not only of mathematical but also logical (sic), which had been specially erected for that purpose.

Thus, though for very different reasons, they shared the intuitionist suspicion of Cantor's infinities.

At the time, Yanovskaya was translating and editing Marx's recently rediscovered manuscripts on mathematics, which appeared in 1933.[43] In discussing Taylor's theorem, Marx wrote:

> This leap from *ordinary algebra*, and besides *by means of ordinary algebra*, into the *algebra of variables* is assumed as *un fait accompli*, it is not proved and is prima facie *in contradiction to all the laws* of conventional algebra. . . . In other words, the starting equation . . . is not only *not proved* but indeed knowingly or unknowingly assumes a substitution of *variables* for *constants*, which flies in the face of all the laws of algebra – for algebra, and thus the algebraic binomial, only admits of constants, indeed only two sorts of constants, *known* and *unknown*.

> The derivation of this equation from algebra therefore appears to rest on a deception. (p117)

It is as if Marx can only accept variables as place holders for actually existing values, rather than for values in general. This muddleheadedness gets to the heart of the Hegelian critique of formalism in logic: that it is illegitimate to remove content from logic, here constants, by abstraction to variables, which may in turn be replaced by arbitrary values. Engels' assertions about variables and abstraction, noted above, may well derive from discussions with Marx.

Of course it is perfectly legitimate to abstract over any constants, or indeed formulae, in an equation, provided such abstraction is made explicit.[44] However, Kolman and Yanovskaya quote Marx without further comment.

In 1932, Kolmogorov explored an approach to intuitionism that was acceptable to dialectical materialism. In *On the Interpretation of Intuitionist Logic*,[45] he reformulated intuitionistic logic as a *calculus of problems* and showed that it is formally equivalent to Heyting's formalization of Brouwer's logic. Cunningly, Kolmogorov avoided the critique of form without content and that of variables generalizing constants, by focusing on problems, which are grounded in reality:

> intuitionistic logic should be replaced by the calculus of problems, for its objects are in reality not theoretical propositions but rather problems. (p328)

That is:

> the concepts 'problem' and 'solution of a problem' can be employed without misunderstanding in all cases that occur in the concrete areas of mathematics. (p329)

This slight of hand substitutes abstraction over concrete problems for abstraction over premises.

Interestingly, Kolmogorov refers to the rules of his logic as 'computational' (p334). The idea of mathematics as computation is central to our conception of materialism.

Thus, intuitionism was the foundation of the Soviet school of constructive logic. Ironically, constructivism is now a core approach for the theory and practice of programming languages.

Menshevizing Idealism and British Marxism

The Communist Party of Great Britain (CPGB) was formed in 1920. A Marxist-Leninist party, the CPGM was explicitly aligned with the Communist Party of the Soviet Union and promoted the dialectical materialist world view. While it never enjoyed the mass membership, or electoral success, of other Western European communist parties, it had considerable influence in the trades unions.

The CPGB is often seen as a relatively philistine organization, more focused on day to day struggles than theory. Nonetheless, a significant number of what are now known as public intellectuals were members. These regularly appeared on the radio, and in popular print media, and their books were produced by mainstream, non-aligned publishers, as well as the Party press. Werskey's *The Visible College*[46] provides a through account of this milieu in the 1930s. Of particular interest to us are the mathematicians Hyman Levy (1889–1975) and David Guest (1911–1938).

Guest, who died fighting with the International Brigades in Spain in 1938, seems to have been the British Marxist who most closely studied the debates around foundations of mathematics. In 1929, in *Mathematical Formalism*,[47] he observed that Polish logicians had found contradictions in Russell and Whiteheads's *Principia* Mathematica. He also suggested that Hilbert's desire to formally demonstrate the consistency of mathematics was floored because the only way to do so was to 'produce a set of mental objects satisfying them' (p210). Ultimately, this depended on being able to minimally characterize the finite integers, a deep problem for Hilbert and the intuitionists.

In *The 'Understanding' of the Propositions of Mathematics*,[48] from 1930, Guest critiqued the LEM on the familiar grounds that it is meaningless to simultaneously consider a proposition and its negation. Further, he said that mathematical propositions are like empirical propositions, in that they may be overturned by new evidence. In contrast, properties of concrete instances may be established by carrying out processes, giving as an example trying to establish whether or not a specific number is prime. This seems similar to Kolmogorov's view of logical rules as computational.

Shortly before his death, Guest appeared sympathetic to intuitionism. In a review of E. T. Bell's *Men of Mathematics*,[49] he contrasted the reformism of *Principia Mathenmatica* with the 'revolutionary challenge of the "Intuitionists"'[50]:

> But what is this but the spirit of dialectics breaking through the hard shell of formal logic, within which so much scientific thought has been imprisoned in the past! (p256)

At the International Congress of the History of Science and Technology, held in London in 1931, the dialectical materialist position on logic was presented by Kolman. In his paper *The Present Crisis in the Mathematical Sciences and General Outlines for Their Reconstruction*,[51] Kolman surveyed what he saw as the current contradictions of mathematics:

> All the profound contradictions of mathematics–the contradiction between the singular and the manifold, between the finite and the infinite, the discrete and the continuous, the accidental and the necessary, the abstract and the concrete, the historical and the logical, the contradiction between theory and practice, between mathematics itself and its logical foundation–all are in reality dialectical contradictions.

While acknowledging some value in Hilbert's approach, Kolman criticized formal logic for ignoring the historical necessity of the concept of number:

> As for Hilbert's axiomatics, it is true that it is of use in explaining the logical connections between individual mathematical concepts, but, since it represents a construction post factum it, too, is unable to give a correct picture of development.

And, as before, Kolman dismisses both logical atomism and intuitionism as idealist:

> It is a matter of indifference whether the world of mathematical concepts is regarded as a world of rigid immovable universals, as it is by the logists, or whether it is looked upon as the sphere of action of the free becoming as it is by the intuitionists. . . . The most refined finesses of finitism, of metalogic, of mathematical atomistics, merely express the anxiety of bourgeois mathematicians to separate themselves from matter and dialectics by the veil of formal logic, guiding them directly into the desert of scholasticism.

Levy and Guest were both present at the Soviet sessions. The dialectical materialist position on analysing science more widely was presented by Boris Hessen (1893–1936) in a paper on Newton. In his eulogy for Guest, *The Mathematician in the Struggle*,[52] Levy reported that the audience seemed nonplussed, but Guest spoke in support of Hessen. In the same article, Levy made explicit the link between the disputes in Marxism over politics and economics, and

> struggles and confusions of a highly theoretical and abstract nature. In this, David could bring to bear his very valuable knowledge of mathematical logic. (p157)

Subsequently, Guest seems to have largely abandoned mathematics research for wider educational and Marxist activity. His teaching notes on Marxist philosophy were published posthumously as *A Text Book On Dialectical Materialism*.[53] In a short section on *Dialectics and Formal Logic*, Guest reiterated the line that formal logic, which he characterized as the 'logic of common sense' (p68), was based on absolute abstractions and hence was

> unable to grasp the inner process of change, to show its dialectical character. (p68)

To go beyond this, dialectical logic was required: attempts to further develop formal logic lead to metaphysical thinking. Here, Guest cited Engels' association of formal logic with lower mathematics and dialectical logic with higher mathematics, from *Anti-Dühring*. As we shall see, Turing's work in this period laid the basis precisely for characterizing 'the inner process of change', further shrinking the space for dialectics.

In 1934, several prominent CPGB members contributed to the collection *Aspects of Dialectical Materialism*.[54] In his paper *A Scientific Worker Looks at Dialectical Materialism*,[55] Levy summarized the orthodox account but ended it with an Aristotelian circumscription of dialectics:

> the so-called laws of the dialectic . . . appear to add little or nothing to the detailed methods of analysis that scientific workers have produced. . . . In a sense, they cannot be expected to add anything to these, for they profess to stand above science. . . . For science, therefore, it [dialectical materialism] is an *interpretative* method rather than a method of investigation. (p30)

In the same collection, the X-ray chrystallographer J. D. Bernal (1901–71) also sought to distance dialectics from science, echoing Levy in his paper *Dialectical Materialism*:[56]

> It [dialectical materialism] is not a critique of science; it does not claim to be a substitute either for experimental method or for the logical proof of laws or theories, but it does in a very important way supplement science by providing a definite method of coordinating the larger groups of special sciences and in pointing the way to new experiment and discovery. (p98)

Chapter 9

THE CRISIS IN LOGIC AND THE APOTHEOSIS OF ANTI-FORMALISM

Introduction

Hilbert's programme may be conveniently summarized as seeking to establish whether or not the formalization of arithmetic used in Russell and Whitehead's *Principia Mathematica*:

- is *consistent*, i.e., it is not possible to prove that both a formula and its negation are theorems;
- is *complete*, i.e., there are no theorems which cannot be proved to be so;
- has a *decision procedure*, i.e., a terminating mechanical procedure or algorithm, to establish whether or not an arbitrary formula is a theorem.

Russell's paradox had already strongly suggested that mathematics could not be consistent. Subsequently, as we shall next explore, Gödel established that mathematics could not be complete, and Turing that mathematics could not have a decision procedure for theoremhood.

Gödel and completeness

The key to meta-mathematics, where mathematics is used to quite literally talk about itself, lies in finding a mathematical representation or *encoding*, of formulae, for manipulation by other formulae. In Chapter 7, we saw how logic may be defined in terms of its symbols and syntax. If we assign numbers to symbols, we can turn a formula into a composite number, and then decompose it back into its symbols, using arithmetic.

In a seminal 1931 paper, Kurt Gödel (1906–78) used an encoding of number theoretic predicate calculus to demonstrate that it was incomplete, that is, there are theorems which cannot be proved[1]. Kleene[2] gives a thorough account.

Rather than using a simple multiplication technique, Gödel assigned a prime number to each position in a symbol sequence. He then accumulated the codes for symbols by multiplying together the prime numbers, with each raised to the power of the code at its position.

For example, suppose we give symbols for the codes shown in Table 9.1.

Then the formula for the syllogism Modus Ponems is:

$$\left(a \wedge \left(a \Rightarrow b\right)\right) \Rightarrow b$$

that is 'a', and 'a' implies 'b', implies 'b', becomes:

```
8 10 4 8 10 7 11 9 9 7 11
```

This could be encoded as:

$$2^8 * 3^{10} * 5^4 * 7^8 * 11^{10} * 13^7 * 17^{11} * 19^9 * 23^9 * 29^7 * 31^{11}$$

This approach works for an arbitrary number of symbols and sequences of arbitrary lengths. Decoding then involves a technique called *prime factorization*, which enables the exponents of all the prime factors in a number to be found. Of course, this encoding gives unimaginably large numbers, but it was not Gödel's intention to work directly with them.

Gödel wanted to encode proofs, that is, sequences of formulae where each is an axiom or a theorem, or follows from an axiom or theorem by application of a rule of inference. He showed how to construct functions that would pull these *Gödel numbers* apart to check, not just that they were well-formed, but that they corresponded to formulae for valid proofs. This works for establishing whether a sequence of formulas is or is not a proof, but it is quite different from checking whether a proof exists for an arbitrary formula.

On the assumption that it was possible to write a function to tell whether or not a formula was a theorem, Gödel constructed a paradox using self-reference,

Table 9.1 Codes for Symbols

T (true)	1
F (false)	2
=	3
∧ (and)	4
∨ (or)	5
¬ (not)	6
⇒ (implies)	7
(	8
)	9
a	10
b	11
c	12
...	

reminiscent of Russell's paradox. He showed how to make a formula with, say, Gödel number *N* that said, in effect:

the formula with Gödel number N is not a theorem.

Now, if this formula is a theorem, then what it asserts is true, so it can't be a theorem. And if it isn't a theorem, then what it asserts is false, so it must be a theorem. That is, assuming that there could be a function to check whether or not a formula was a theorem rendered the system inconsistent. And omitting the assumption rendered it incomplete.

Logicians who accept the LEM are agreed that preferring consistency to completeness is the safest course. For an incomplete system, there will be arbitrary formula whose status as theorems we cannot guarantee to determine. If they are simply added to the system as axioms, then it might be possible to use them to prove contradictions. However, for a consistent system, once we prove that a formula is a theorem, we know that we cannot prove its contrary.

Turing and termination

Five years later, Alan Turing (1912–54) showed that the third requirement of Hilbert's programme could not be satisfied.[3] That is, it is not possible to construct a terminating mechanical procedure for deciding whether or not a formula is a theorem.

Turing's approach was very different from that of the mathematical logicians we have considered above. Rather than using a formal system derived, say, from predicate calculus applied to set or number theory, Turing considered how people solve problems by hand. He speculated about someone using a pencil and squared paper, writing down a problem in the squares, letter by letter, and then working backwards and forwards, changing squares and writing in new ones.

Turing generalized this conception to what is now known as a *Turing machine (TM)*. A TM has a finite linear tape of cells. Each cell may hold a symbol. There is a reading head that can inspect and change cells. The head is positioned over the 'current' cell. For example, Figure 9.1 shows a stylized TM with Modus Ponems on the tape, symbol by symbol.

The tape may be moved to the left or the right, one cell at a time. New empty cells are added when either end of the tape is reached. Thus, the tape may grow to be arbitrarily long, but, at any stage, it is always bounded, that is, it is always of finite length.

A TM executes by repeatedly inspecting and modifying the tape, one cell at a time. The propensity of the machine, that is, how the current symbol determines what it will do next, in the light of what it has done previously, is called the current *state*.

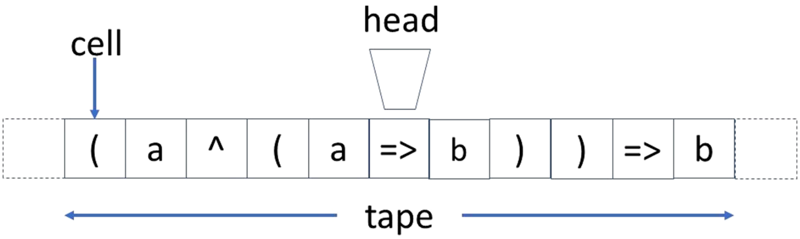


Figure 9.1 Turing machine. Figure by authors.

A TM is controlled by a set of instructions with five components, known as quintuplets. Each instruction says:

- $state_{old}$: if the machine is in this old state,
- $symbol_{old}$: and this old symbol is under the head,
- $state_{new}$: then change to this new state,
- $symbol_{new}$: change the cell under the head to this new symbol,
- *direction*: and move the tape one cell in this direction, i.e., left or right, or halt.

The machine is set up with the instructions in a control unit. This repeatedly looks for an instruction whose $state_{old}$ and $symbol_{old}$ match the current state and symbol under the head, which is then carried out. If there is no such instruction, the computation fails. Elaborate TMs may be constructed from the basic operations of searching for, changing and copying sequences of symbols.

Turing's key insight was that a TM tape could hold any symbol sequence, including that for a set of TM instructions. Indeed, it is possible to construct a *Universal* TM (UTM) that will execute an arbitrary TM, held on its tape, symbol by symbol, with appropriate data.

Asking whether or not an arbitrary TM will ever terminate, Turing constructed a paradox as follows: Suppose we have the symbols for a 'test' TM and its data on a tape. Let's assume that we can build a 'halting' TM that will inspect the tape and stop in one state if the 'test' TM halts on its data and in another state if the 'test' TM doesn't halt on its data.

It is easy to write a TM that doesn't halt. It starts on a blank cell, writes a 0 and moves one cell to the right. If there isn't a cell to the right, a new one is added. In effect, this TM will 'loop' forever, extending the tape to infinity, without ever reaching it.

We can modify the 'halting' TM to halt in one state if the 'test' TM doesn't halt on its data and to execute the loop instruction if the 'test' TM does halt on its data.

Suppose we apply the modified 'halting' TM to itself and some tape. If it doesn't halt on itself, then it halts, and if it does halt on itself, then it goes into the loop, and doesn't halt. Turing concluded that it wasn't possible to construct a TM to tell if an arbitrary TM halts; that is, the TM *halting problem* is *undecidable*.

This result falls out of Turing's more elaborate argument using Cantor diagonalization in considering TMs that generate real numbers. While above we critiqued the whole notion of diagonalization as involving completed infinities, this does not invalidate the paradox at the heart of the halting problem..

The Church-Turing thesis

Several other formalisms for meta-mathematics were explored in the same period that Gödel and Turing were working. Along traditional lines, Stephen Kleene (1909–94) developed recursive function theory, which inverted Peano's induction into a form for defining functions over number or sets.[4]

In contrast, Moses Schönfinkel (1888–1942) had developed combinatory logic in 1924.[5] This was refined by Haskell Curry (1900–82) in 1929,[6] and led to the highly influential work by Alonzo Church (1903–95) on λ calculus, from 1932.[7]

These systems seem particularly problematic for adherents of dialectical materialism. Schönfinkel's and Curry's combinatory logics are built from operators whose properties are defined by how they interact with other operators by eliminating or duplicating them. And Church's λ calculus is one of pure abstraction, with rules for combining abstractions through substitution. Neither makes any reference to concrete values.

Nonetheless, a key property of these systems is that they are all capable of representing logic and arithmetic, so they can all capture the notions of theorem and proof at the heart of Hilbert's programme. This is the root of the *Church-Turing thesis*: that all systems for performing calculations on numbers are equivalent in the sense that any may be translated into any other. That is, there is nothing that can be expressed in one system that cannot be expressed in any other system.

Recursive function theory, λ calculus and TMs were all demonstrated to be equivalent soon after they were developed.[8] Subsequently, they were shown to be equivalent to von Neuman machines, that is, generalized digital computers, and, hence, to arbitrary programming languages. Formalizations of analogue computing, and of quantum computing,[9] have also been shown to satisfy the Church-Turing thesis.

Note that this is a speculative *thesis* not a provable *theory*. We do not know how many different equivalent systems there might be. Nonetheless, it is a falsifiable thesis, as it may yet be proved that some new system has different properties to those that are currently known, and so there is no mutual translation. That is, the Church-Turing thesis is subject to experimental investigation as part of empirical normal science. Indeed, we will argue subsequently that it is central to a wider understanding of reality.

At heart, all these systems enable computation. This is explicit in TMs, where the machine manipulates the data on the tape. For combinators and λ calculus, where there is no obvious separation of instructions and data, rules are applied to formula until they cannot be simplified any further.

Herein lies the fundamental difference between TMs and other formalisms. A set of TM instructions is a blueprint for building an actual machine which, like a digital computer, will run autonomously on its data. In contrast, other formal systems require a human being, or an interpreting device, to carry out their rules. We think that this is a strong argument for the primacy of TMs in elaborating a materialism, and will return to this subsequently.

British dialectical materialist responses

For the dialectical materialists, Gödel's and Turing's results were further demonstrations of the limitations of logic without dialectics. Writing in 1938,[10] the Communist mathematician Alister Watson (1908–82)[11] thought that all the paradoxes

> *merely express in one way or another the well-known difficulties which arise when we attempt to treat an infinite process as completed.* (p450)

Watson is referring to Aristotle's distinction between potential and actualized infinity. We agree with Watson that the assumption of actualized infinities makes, for example, Cantor diagonalization problematic. However, Watson would be wrong to identify TM tapes as completed infinities. In any case, Turing's result of the undecidability of the halting problem may be established without diagonalization.

Watson's scepticism about meta-mathematics is clear:

> The attempts which have been made in the subject of the Foundations of Mathematics to justify or condemn mathematical arguments taken in the abstract, have given rise to a host of confusions, from which it has taken the most immense labour to escape. (p451)

He does not directly deploy the dialectical materialist critique of formal logic. However, in a footnote to this passage, he says that he was writing in opposition to Dedekind's claim that the foundations of mathematics did not require any mention of 'measurable quantities'. This is reminiscent of the Marxist objection to variables that are not derived from known quantities.

In 1938, in *The Marxist Philosophy and the Sciences*,[12] the evolutionary biologist J. B. S. Haldane (1892–1964) observed:

> On the whole we may take it that Marxists are rather sceptical of the more ambitious logical theories. For example, the system of Russell and Whitehead, in the *Principia Mathematica* is doubtless true, or largely true, if sufficiently sharp classification is possible.

That is, the truth or falsity of this logical system depends on the ability to elaborate concrete 'existents', 'relations' and 'propositions' arranged in classes, such that further classes may be abstracted. However, Haldane doubts the possibility of setting up such ontologies:

> it is probable that too great an emphasis has been attached to logical systems which will only work for material that has certain highly abstract properties, which are rather less frequently and much less completely exemplified in the real world than logicians would like to think.

In 1939, in the Preface to the English translation of *Dialectics of Nature*,[13] Haldane wrote of the mathematicians' claims to have removed contradictions in mathematics, noted by Engels sixty years earlier:

> Actually they have only pushed the contradictions into the background, where they remain in the field of mathematical logic. Not only has every effort to deduce all mathematics from a set of axioms, and rules for applying them, failed, but Gödel has proved that they must.

This is missing the key point that, while Gödel's results circumscribe what can be proved, they tell us nothing about establishing properties of individual formula. Indeed, automated techniques based on axiomatic systems are proving increasingly applicable to substantial real-world problems, like digital computer design.

Like Watson, Haldane does not use the language of dialectical materialism directly. Nonetheless, his implication, as for his comments on Gödel's results, is that logic is not adequate for concrete reality, for which dialectics is required.

Soviet logic after Menshevizing Idealism

The fortunes of Soviet mathematics, from the 1930s onwards, have been widely documented, though not always dispassionately, for example, in Vucinich,[14] Lorentz,[15] Seneta[16] and Kutateldze.[17] Mathias's egregiously titled 'Logic and Terror'[18] discusses Soviet logic to 1950. Anellis[19] criticizes Mathias for relying on the 'polemical and prejudiced account' in Philipov.[20] Nonetheless, Mathias contains telling, if poorly referenced, quotes from period publications:

> Formal logic is always a most trustworthy weapon in the hands of the predominant exploiting classes. a bastion of religion and obscurantism. (from a 1934 work on Dialectical and Historical Materialism)

> the laws of formal logic are opposed to the law of dialectical logic. formal logic is empty, poor, abstract, for the laws and categories which it sets up do not correspond to objective reality. (Concise philosophical Dictionary, 1940) (p7)

including the 1936 *Large Soviet Encyclopedia* (1936):

> formal logic is a metaphysical form of thinking . . . the lowest stage in the development of human knowledge, replaced by dialectic as the highest form of thinking.
>
> Formal logic, as we have seen, is not included in dialectic, but is displaced refuted and overcome by it.
>
> . . . the anti-Leninist deviations in the All-Union Communist Party (VKP(b)). Formal logic thinking is a characteristic trait of Menshevism frequently noted by Lenin who levelled devastating dialectical criticisms at the Menshevik formal-logical deductions of syllogisms and sophisms. (p7)

While the broad study of logic was curtailed, 'Red experts' like Kolmogorov and Yanovskaya were still able to explore and teach logic, and had access to western research. For example, Yanovskaya started to teach mathematical logic in 1936 at Moscow State University and, in 1943, was appointed Director of the Seminar in Mathematical Logic.[21]

The fortunes of logic in the Soviet Union were restored after World War Two, though not without continued fierce dispute over its relationship to dialectics, as we shall discuss below.

Digital computers

The key development of the twentieth century may well prove to have been that of digital computers. Certainly, early twenty-first-century life would be pretty well unthinkable without them. The history of computers is again thoroughly documented. Here, we will focus on salient aspects, albeit very briefly.

The Second World War, unprecedented in mass cruelty and immiseration, hastened the development of modern computers.[22] The Harvard Mark 1, built by IBM as the Automatic Sequence Controlled Calculator (ASCC), was designed by Howard Aitken (1900–73) in 1937, and first ran in 1944. It was electro-mechanical and could store data, but lacked the capability to store programs.[23] Instructions were encoded on punched paper tape to control a linear sequence of operations. Looping programs were accomplished by repeating the tape, and branching by changing tapes. The Mark 1 was used by John von Neumann (1903–57) to perform calculations for the Manhattan Project, developing the first atomic bombs.

The Electronic Numerical Integrator and Computer (ENIAC), among the first all electronic general-purpose computers, was completed in 1945. Much faster than the Harvard Mark 1, it was programmed by plugging components together in appropriate configurations. ENIAC was used, among other things, in the calculations for the first hydrogen bombs.

von Neumann had worked on set theory for his 1925 PhD, and had communicated with Gödel in the early 1930s about the incompleteness results. Strongly influenced by Turing's ideas about the UTM, where data and instructions share the same memory, von Neumann included this design, now known as the von Neumann architecture, in the highly influential *First Draft of a Report on the EDVAC.*[24][25] The report was circulated freely, influencing early digital computer development worldwide. Mid-century, there were numerous firms developing and selling von Neumann architecture computers, particularly in the USA and the UK.

Thus, while dialectical materialists, and Western European philosophers alike, saw the failure of Hilbert's programme as limiting the reach of mathematics, nonetheless it had profound and very long-lasting influences. Furthermore, the 1930s formalisms developed by mathematical logicians, especially recursive function theory and λ calculus, have long been the basis of the semantics, and the design, of practical programming languages.

Cybernetics

Cybernetics, the study of 'control and communication in the animal and the machine', was an area of major activity after WW2, alongside the development of computers. Norbert Wiener's (1894–1964) highly influential book[26] set out the key principles, drawing on information theory, statistical mechanics and Pavlovian behavioural psychology to elaborate on how machines might learn through feedback to optimize activities against observed outcomes.

Like Turing's, Wiener's work was based on abstracting from human behaviour, but at the level of the nervous system rather than higher cognition. Wiener observed the importance of feedback in governing activity that involved continuously predicting future behaviour, for example, in steering a craft, or following a moving target, and that this was carried out autonomously (pp6–8). Wiener also acknowledged the strong influence of mathematical logic on cybernetics. He observes that, for both formalists and intuitionists:

> the development of a mathematico-logical theory is subject to the same sort of restrictions as those that limit the performance of a computing machine. As we shall see later, it is even possible to interpret in this way the paradoxes of Cantor and of Russell. (p13)

That is, Wiener saw the paradoxes as limiting human reasoning in general, not just mathematics, because brains are machines. Citing Turing, he wrote:

> the study of logic must reduce to the study of the logical machine, whether nervous or mechanical, with all its non-removable limitations and imperfections. (p125)

Nonetheless, Wiener saw information as neither matter nor energy. Hence, we strongly contest his claim that:

> No materialism which does not admit this can survive at the present day. (p132)

because information must be embodied.

Wiener was well aware of the social implications of cybernetics and computing. He thought that:

> The modern industrial revolution is . . . bound to devalue the human brain, at least in its simpler and more routine decisions. (p27)

However, ideological objections to cybernetics proved a major barrier to the development of Soviet computers, as Gerovitch[27] recounts. To clarify this, we need to make an apparent segue sideways, and consider Soviet thinking about linguistics

Linguistics

For much of the nineteenth century, philology dominated linguistics. This sought to trace languages back to their origins by identifying common roots in words from different languages. Much of this work was distorted by concerns with establishing the historical primacy of contemporary national groupings.

Modern linguistics was founded by Ferdinand de Saussure (1857–1913), whose posthumous *Cours de linguistique générale*[28] was published in 1916. Saussure distinguished between language (*langue*) and speaking (*parole*):

> separating: (1) what is social from what is individual; and (2) what is essential from what is accessory and more or less accidental. (p14)

Saussure further distinguished the use of a relatively unchanging language by contemporary speakers from how language changes in time as users and usages changes:

> Everything that relates to the static side of our science is synchronic; everything that has to do with evolution is diachronic. Similarly, *synchrony* and *diachrony* designate respectively a language-state and an evolutionary phase. (p81)

Unlike the philologists, Saussure thought diachrony as of little use for understanding language. Rather, he saw linguistics as a component of a wider semiology:

> *A science that studies the life of signs within society* is conceivable; it would be a part of social psychology and consequently of general psychology; I shall call it *semiology* (from Greek *sēmeîon* 'sign'). (p16)

Sussure was particularly concerned with how *signs* combine a *signifier*, that which points, and the *signified*, that which is pointed at, i.e., a concept or idea. Saussure prefered the term 'sign' to 'symbol', as signs are arbitrary, where symbols are chosen for what they suggest (p68). Note that Saussure was primarily concerned with relations between signs in systems, not with semantics in itself, seeing changes in meaning as involving '*a shift in the relationship between the signified and the signifier*' (p75)

Finally, Saussure hinted at the relationship between linguistic activity and computation, discussed in Chapter 10:

> The mechanism of language, which consists of the interplay of successive terms, resembles the operation of a machine in which the parts have a reciprocating function even though they are arranged in a single dimension. (p128)

The Soviet linguist Valentin Vološinov (1895–1936) sought to develop a dialectical approach to language in *Marxism and the Philosophy of Language*[29] from 1929. Vološinov counterpoised the *individualistic subjectivism* of the Humboldt school,[30] to the *abstract objectivism* of Saussure (p48). Vološinov rejected Saussure's abstraction, identifying language as entirely sociological, that is produced by interacting speakers (p98). Thus, Vološinov saw *themes*, that is semantics, as central to understanding (p99ff).

For Vološinov, language was key to disentangling the Marxist problematic of the relationship between base, that is, material conditions, and superstructure, that is, social forms (pp18ff). Ideology is determined by the base, with the word mediating between base and superstructure.

Vološinov saw language materialized in speech as primary. Themes bear ideology, and signs have 'social *multiaccentuality*' (p23). Words have *evaluative accent* determined by *expressive intonation* (p103). The intonation, and how it is interpreted, also reflect the class orientations of speakers and listeners. Thus, 'Sign becomes an arena of class struggle' (p23), where:

> The ruling class strives to impart a supraclass, eternal character to the ideological sign, to extinguish or drive inward the struggle between social value judgements which occurs in it, to make the sign uniaccentual. (p23)

Vološinov approvingly cites his contemporary Nikolai Marr (1864–1934) in asserting that 'linguistics is the child of philology'. Both criticized traditional philology for focusing on utterances as monologues separated from dynamic verbal interactions (p72), but both thought it possible to derive the origins of languages in the contexts of material cultures.

However, unlike Vološinov, Marr thought that no language could be classless. Matejka[31] attributes Vološiinov's fall from favour to this difference (p173).

Marr also rejected formal logic as class based:

> Formal logic, a product of class thinking, together with the class that created it, is swept away by the dialectical materialist thinking of the proletariat in which thought gains ascendancy over language. (cited in Mathias[32])

Marr was the preferred Soviet linguistic until 1950.

The revival of Soviet logic

The detonation of three atomic bombs by the USA in 1945 radically changed Soviet priorities in science and weakened the dominance of ideology in policy, in particular anti-formalism.

Two Soviet agencies were set up in August 1945 to manage Soviet atomic bomb development[33] (p131). The first Soviet computer project began in Kiev in 1946, directed by Sergey Lebedev (1902–74), and the MESM became operational in 1950.

However, after the publication of Wiener's book,[34] a campaign against cybernetics was mounted on the grounds that it was a capitalist innovation to weaken working-class organization and, ultimately, entirely supplant workers who would be left destitute[35] (p128). This suggests that the ideologues had little grasp of Marxist economics, and the central role of living labour in the production of surplus value under capitalism. In Marx's scheme, profits derive from human activity, not machines. And, as Usdin[36] notes, the Soviet military was quick to deploy cybernetics and 'push ideological considerations aside' (p312).

Nonetheless, computers had to be presented as 'mathematical machines' to evade cybernetic scrutiny.[37] A clear distinction was made between the unacceptable use of cybernetics to model human behaviour and the use of computers for calculations and automation. Analogies between computers and human brains were deemed 'absurd'[38] (p142–3). This stance seems entirely retrogressive for materialism. If humans are more than machines, then their additional qualities must derive from non-corporeal properties.

Returning to logic, change came quickly in Soviet education. A 1946 CPSU(B) Central Committee directive,[39] cited by Campbell,[40] noted that it was:

> quite improper that logic and psychology are not taught in secondary schools. (p343)

and set out plans, with resources, for their introduction. Thus, Mathias[41] recounts how a 1918 edition of Chelapanov's *Textbook of Logic* was republished in 1946, followed by Strogovich's *Logic*, a new textbook by Asmus, and a further edition of Strogovich (p8).

Lorentz[42] observes that, following Lysenko's alleged achievements in genetics, there were moves in 1948 to systematically align wider Soviet science with the notion that science had a class character. However, the plans for mathematics and physics were halted by Beria, the chief of Atomic Missile Projects, after 'influential physicists explained to him that this may damage these projects' (p217–8).

In 1952, Campbell[43] noted the problems for Soviet logic education of squaring 'bourgoise' logic with dialectical logic. He summarizes a 1950 article by Osmakov[44,45] as arguing that:

> Unlike a world outlook, the logic of thinking is a *classless* phenomenon. (p281)

Further, in a formulation strongly reminiscent of Aristotle:

> The concepts on the basis of which the logic of thinking proceeds reflect objective reality well or badly according to the ideology or world view of the thinker. (p282)

Bazhanov[46] and Kilakos[47] recount how Yanovskya was central to the revival of Soviet formal logic. In 1947, she had translated *Grundzüge der theoretische Logik* by the formalists Hilbert and Ackermann and, in 1948, Tarski's *Introduction to logic and the methodology of deductive sciences.*[48] In 1947, Yanovskya was arguing that 'methodological formalism of mathematical logic' should be distinguished from the idealist philosophy underlying it, because[49] mathematical logic

> can be considered not only as logic of mathematics but also as mathematics of logic, for it is in large part the result of the application of mathematical methods to the problems of logic.[50]

This argument returns to Boole's mathematicization of logic, neatly inverting the stated objective of meta-mathematics.

Yanovskaya promoted formal logic throughout the rest of her life. Held in high esteem, she was the Chair of Mathematical Logic at Moscow State University from 1959 until her death in 1966.[51]

Stalin on linguistics

In 1950, the public reversal of anti-formalism was heralded by Stalin's repudiation of Marr. As Lorentz[52] observes, Stalin paid close attention to developments in exact science and, while endorsing Lysenko, expressed scepticism about science's general class character:[53]:

> Ha, ha, ha. And what about mathematics? And about Darwinism? (p217)

Marxism and Problems of Linguistics[54] was published in *Pravda* in 1950, followed by four further clarificatory exchanges. In the original article, Stalin roundly rejects two key tenets of Marr's linguistics, arguing that language is neither base nor superstructure, and is not class marked. These criticisms also apply to Vološinov.

In a subsequent reply to Krasheninnikova, Stalin chided Marr for overemphasizing semantics while acknowledging its importance for linguistics. From the context, Stalin appears to again be criticizing Marr for attributing different meanings to words and expressions, depending on a speaker's class. He suggests that such differences are very few, and lie in individual words, not in grammar, which is common to all speakers.

Most significantly, Stalin signalled the end of the attribution of formalism as a decisive critique:

> N. Y. Marr considered that grammar is an empty 'formality', and that people who regard the grammatical system as the foundation of language are formalists. This is altogether foolish.
>
> I think that 'formalism' was invented by the authors of the 'new doctrine' to facilitate their struggle against their opponents in linguistics.

Thereafter, Soviet logic flourished, though debate about the relationship between dialectics and logic continued.[55] Seventeen years later, in 1967, Kopnine[56] was still citing Engels[57] in defending the Hegelian verities.

Nonetheless, a 1959 report[58] by US computer scientists visiting the Soviet Union showed that they were very impressed by progress in logic and computing. They note that:

> There are obviously more and better logicians and mathematicians connected directly or indirectly with computers at Moscow, Kiev, and Leningrad Universities than at any universities in Western Europe or the United States. (p17)

They further note that, outside of MIT, the USA lacked programs in 'computer oriented' logic comparable to that at Moscow State University (p17), and warned that the production of:

> qualified computer oriented mathematicians, not just computer programmers - may, soon surpass that of the United States. (p17)

Constructivism

The Soviet school of *constructive* logic was founded by Andrey Markov Jr.[59] (1903–79).[60,61] Constructivism had its roots in intuitionism, but with a materialist orientation, as in Kolmogorov's approach.

Given the idealism underlying intuitionism, its embrace by Soviet logicians may seem surprising. However, Markov's collaborator Ngorny suggests that, before the 'thaw', set theory was seen as materialist rather than idealist, despite its Platonic roots.[62] Further, Vandoulakis[63] notes that Brouwer's intuitionism was seen as consistent with dialectical materialism, once its logical framework was separated from his philosophy. As Ŝanin wrote in 1962:[64]

> outside of the surface of intuitionistic philosophy one finds in many cases very valuable concrete observations and profound concrete analysis of the fundamental problems relating to the processes of forming mathematical abstractions and logical foundations of mathematics. (p7)

Markov's student Kushner identified four central characteristics of Markov's constructivism.[65] First of all, the objects under investigation are finite and generated by finite constructive processes from a finite alphabet according to definite rules of algorithm formation. Ŝanin rightly saw this as key to the success of constructivism:

> The enrichment of mathematics with the precise concept of arithmetic algorithm served as a starting point for fruitful investigation in a new direction by many authors. (p6)

Secondly, Kushner notes that, as with intuitionism, both the law of the excluded middle and double negation are rejected, and proofs of existence must be based on construction. Third, potential infinities are accepted, but not actualized infinities. And, finally, computability is associated with algorithms, and the Church-Turing thesis of the equivalence of all models of computability, demonstrated empirically, is accepted.

Like Turing, Markov started from mathematics as a material process using pen and paper. As an example, he considers drawing a row of vertical pen strokes on the paper forming his book manuscript. Discussing how this is subsequently reproduced by printing, he observes that:

> The constructive object [ie the original drawing] is a material body consisting of paper and dried ink, and the drawing given above is a copy of this constructive object, consisting of paper and dried typographic paint. It, too, is a constructive object, since the preparation of a copy may be considered a constructive act.[66]

Markov explicitly acknowledges the limits of material reality:

> Carrying out constructive processes, we often come up against obstacles connected with a lack of time, space and material. One usually succeeds in somehow by-passing these obstacles. However, our constructive possibilities really are limited, and there are no grounds for supposing that the obstacles caused by their restrictedness can always be obviated. Rather to the contrary,

> it seems that modern physics and cosmology testify to the impossibility in principle of surmounting such obstacles.[67]

Nonetheless, in an Aristotelian formulation, the *abstraction of potential feasibility*:

> allows us to consider arbitrarily long constructive processes and arbitrarily large constructive objects. Their feasibility is potential: they would be feasible in practice, had we available sufficient space, time and material.[68]

Markov explicitly contrasts this approach with classical mathematics based on set theory, which allows abstractions of actual infinity[69] and for which existence is a consequence of refuting non-existence, where 'a method for constructing the desired object may even be unknown'.[70]

Markov's notation was based on rules for rewriting symbol sequences, similar to Chomsky's subsequent characterization of classes of formal grammars. A *scheme* of such rules is called an *algorithm*, which Markov characterized as:

> a) the precision of the prescription, leaving no place to arbitrariness, and its universal comprehensibility – the definiteness of the algorithm;
>
> b) the possibility of starting out with initial data, which may vary within given limits – the generality of the algorithm;
>
> c) the orientation of the algorithm towards obtaining some desired result, which is indeed obtained in the end with proper initial data, the conclusivenes of the algorithm.[71]

Using this notation, Markov systematically reconstructs a considerable portion of Peano arithmetic using induction.

Markov suggests that his approach satisfies what he calls Church's Thesis, the equivalence of models of algorithms. He further argues that TMs are 'extremely convincing' as

> *in the essentials*, a Turing machine's performance adequately simulates the behaviour of a computing mathematician[72]

going from one state of mind to another.

The notation was intended for practical experimentation with formal systems. For example, the 'meta-algorithmic' programming language Refal[73] incorporated Markov rules within a more conventional framework as an aid to exploring formal semantics. However, Kushner[74] observes that, while Markov's work was known in the west, wider take-up was hampered by notational complexity.

As with intuitionism, accepting constructivist limitations on infinitary reasoning removed the foundations of much essential classical mathematics. Thus, as with intuitionism, considerable research was undertaken to refound such mathematics on a finitary base.

In particular, constructivism accepted integers and rational numbers, but not real numbers, which are necessarily infinitary. Hence, a vital step was the reformulation of real numbers as constructive functions. In the constructivist approach, drawing on work by both Turing and Weyl, real numbers are defined by an algorithm that generates rational numbers of increasing precision, determined by another algorithm. As Ŝanin put it:

> the actual use of concrete real numbers in the natural sciences and engineering is essentially based on the possibility of extracting from an individual representation of a real number an algorithm giving a sequence of rational approximating values for it.[75]

This recognized explicitly that, in practical applications, rational numbers are used in manipulating physical reality because measurement is bounded.

Despite Markov's materialist approach, he shared the view that the undecidability results were limitations on mathematics but not human beings:

> Therefore the conative, research enterprise in mathematics (as well as any other branch of learning) will never be transferred to machines, capable only of assisting man but not replacing him.[76]

Chapter 10

LANGUAGE, AUTOMATA AND MEANING

Language and meaning

There is a longstanding distinction between the meaning of a statement as its value compared with what it is about. These may be termed reference and sense, denotation and connotation and extension and intension.

Consider the descriptions of a number:

one more than two

The extensional meaning results from evaluating it to a value: three. Another way of expressing this is that the extensional meaning is all the things that can replace *the number that* in

the number that is one more than two

and retain the statement's truth. That is, we treat *the number that* as a variable whose sole valid value is 'three'.

Now consider the description of a number:

half of six

which also has the extensional meaning of three. These statements' intensional meanings are quite different. The first involves counting, and the second dividing. We could[1] use Peano arithmetic to formalize them, and prove that they are equivalent. They then have the property of substitutive synonymy. That is, they can be used interchangeably in formal expressions about numbers without changing the meaning of the expression that uses them.

Now, suppose that Chris can count but can't do division. Then:

Chris knows that three is one more than two.

is true. But we cannot replace *one more than two* with '*half of six*:

Chris knows that three is half of six.

and retain truth.

Bertrand Russell sought to analyse and reformulate intensional constructs to make them extensional.[2] In Russell's approach, we might write:

Chris knows that 'there is a number that is one more than two'.

to make the context of 'knows' clear. If we replace the quoted phrase with an apparent equivalent:

Chris knows that 'there is a number that is half of six'.

then we cannot replace *there is a number that* with *three* and retain truth.

Rudolf Carnap (1891–1970) was a logical positivist who sought fact in positivist science for manipulation by logic. Originally, Carnap considered all statements to be either extensional or meaningless. In his 1928 book *The logical structure of the world; pseudoproblems in philosophy*,[3] he enunciated a totalizing approach to denying meaning to any linguistic constructs that could not be grounded in fact. In particular, he disputed that either realism (i.e. materialism) or idealism was meaningful. That is, Carnap saw knowledge as grounded in verification via empiricism. Nonetheless, Carnap termed himself a 'physicalist' who saw physics as exemplifying how to establish facts.

Willard Quine (1908–2000), critiqued logical positivism in his 1953 *From a Logical Point of View.*[4] In the essay *Two Dogmas of Empiricism* (pp20–46), he first disputes that there is a deep distinction between analytic truths, based on meanings without considering facts, and synthetic truths, based on facts (p20). For example, we might contrast properties of the number three, which are internal to formalized arithmetic, with what someone knows about the number three, which is a reported fact. Secondly, Quine rejects the reductionism that only accepts as meaningful those statements that may be reconstituted as logical statements constructed from facts (p20). He is left with an explicit pragmatism.

Quine was particularly concerned with substitutivity as a criterion of synonymy of meaning, asking in what contexts substitution is legitimate and pointing out that substitutions necessarily change the form of statements (p56). His conclusion is again pragmatic:

> What matters rather is likeness in *relevant respects* . . . a problem typical of empirical science. (p60)

For Quine, given that formal systems are constructed with variables marking points of abstraction:

> To be assumed as an entity is, purely and simply, to be reckoned as the value of a variable. (p13)

Then:

> a theory is committed to those and only those entities to which the bound variables of the theory must be capable of referring in order that the affirmations made in the theory be made true. (pp13–14)

This is a pleasingly generous position. It admits not just of manipulating facts in logical systems, but of anything a system can legitimately manipulate.

Model theory

In the 1921 *Tractatus Logico-Philosophicus,*[5] Wittgenstein suggested that the world should be understood in terms of *states of affairs*, that is, configurations of things, which he termed objects. He asserts that:

> 4.1 Propositions represent the existence and non-existence of states of affairs.
>
> 4.11 The totality of true propositions is the whole of natural science (or the whole corpus of the natural sciences). (p25)

Wittgenstein further identifies the *sense* of a proposition with:

> 4.2 . . . agreement and disagreement with the possibilities of existence and non-existence of states of affairs. (p30)

He then proposed the use of truth tables to explore these possibilities (p30), having argued that:

> 4.25 If an elementary proposition is true, then the state of affairs exists: if an elementary proposition is false, the state of affairs does not exist.

In his 1947 book *Meaning and Necessity: A Study in Semantics and Modal Logic,*[6] Carnap, strongly influenced by Wittgenstein's states of affairs, systematically formalized the concept of extensional meaning through the idea of a *state description*:

> There is one and only one state-description which describes the actual state of the universe; it is that which contains all true atomic sentences and the negations of those which are false. Hence it contains only true sentences; therefore, we call it the true state-description. A sentence of any form is true if and only if it holds in the true state-description. (p10)

Note that, where Wittgenstein talks about determining the existence of a state of affairs, which is relative to what is known, Carnap asserts the possibility of describing 'the actual state of the universe'.

Carnap then systematically elaborated a formal *object language* for expressing extensional meanings through substitution of values from state descriptions. Intensional statements were made in a constrained natural *meta-language*[7] for translation into the object language for verification against state-descriptions.

By 1961, Carnap no longer thought that

> all statements about things can be translated into statements about sense data.[8]

Further, he was now cautious about how rigorously intensional statements might be converted to extensional:

> Hence I have later proposed a weaker version which claims that every nonextensional statement can be translated into a logically equivalent statement of an extensional language. It seems that this thesis holds for all hitherto known examples of nonextensional statements, but this has not yet been demonstrated; we can propose it only as a conjecture. (pix)

Nonetheless, the work of Wittgenstein and Carnap formed the basis of *model theory*, which now underpins the semantics of programming languages. And the development of Carnap's work continued, for example, in Marian Przelecki's *Logic of Empirical Theories*.[9]

For Przelecki, a *model* is state of affairs or a state description, formalized in set theory. Przelecki restricts models to what he calls *physical objects* with predicative properties. He suggests that this is insufficient as a basis for a science like physics, and it needs to be extended, though he doesn't do so, with real numbers, and higher order logic constructs like relations and functions (p104). Given that science continuously develops, Przelecki further suggests that the current state of a discipline might be characterized by taking a 'cross section' to determine the changing balance between determinate analytic and indeterminate synthetic knowledge (p105–6).

Badiou, model theory and materialism

Alain Badiou's *The Concept of Model*,[10] based on lectures given in 1968, sought to reconcile formal logic with the Marxist materialist tradition. Criticizing both Carnap and Quine, he sees the distinction between empirical 'fact' and logical form as common to both perspectives (p7), and claims that this actually serves to bind together formal and empirical science: that is, there is a

> dialectical complicity between logical neo-positivism and model theory. (p19)

The great strength of Badiou's analysis is that he characterizes mathematics as a unified material practice, linking formal systems of rules to practical calculation:

> The philosophical category of effective procedure – of that which is explicitly calculable by a series of unambiguous scriptural manipulations – is truly at the centre of every epistemology of mathematics. (p26)

Badiou emphasizes that while models are made to explore formal systems:

> *a model is the mathematically constructable concept of the differentiating power of a logico-mathematical system.*[11] (p40)

In turn, models are subject to experimental investigation:

> It is because it is itself a materialised theory, a mathematical result, that the formal apparatus can enter into the process of production of mathematical knowledge; and in this process, the concept of model does not designate an outside to be formalised, but a mathematical material to be tested. (p47)

Badiou further argues that a formal system is quite literally a 'machine for mathematical production', and notes that the

> increasingly evident kinship between the theory of these systems and the theory of automata, or of calculating machines, strikingly illustrates the experimental vocation of formalism. (p43)

As discussed below, we see automata themselves as not just experimental apparatuses but as providing a dynamic, physical basis for semantics that goes beyond mathematical constructs, no matter how materialized.

Badiou prioritizes set theory and integers for model making but does not mention the Church-Turing thesis. This implies that all Turing complete (TC) systems may serve as formal models for each other: demonstrating that a new system is TC requires precisely the ability to translate between it and some known TC system. Given that computers are physical TC systems, they ground formal models in actual reality.

Automata

In Chapter 9, we met the Turing machine and saw how it has a bounded but arbitrarily extensible tape. We also saw that, according to the Church-Turing thesis, the TM is a model for all possible computations.

We will now explore how restricting TMs' tape properties changes their computational properties. Hopcroft and Ullman[12] provide a succinct account.

We might consider a TM tape as a combined input and output, as well as a memory. The initial tape is the input, and the final tape is the output, and the TM can arbitrarily inspect, modify and extend the tape. We'll now consider three broad restrictions that we might place on TM behaviour by changing properties of the tape.

First of all, we might insist that the tape cannot be extended, that is, the output must be the same size as the input. This is called a *linear bounded automaton (LBA)*, and broadly corresponds to actual computers. Of course, computers may be given streamed input of arbitrary length, but only a finite amount can be held in memory at any moment. In practice, though, we treat computers as if they were Turing machines, and get grumpy when they run out of memory.

Secondly, we might give a FSA a tape which is extensible in one direction. The tape behaves like a *stack*, where items may be added to (*pushed*) or removed from (*popped*) the top, but only the top item can be inspected. This is known as a *push down automaton (PDA)*. PDAs are not a common form of practical machine but are central to software for initial syntax analysis of computer programmes.

Third, we might restrict access to a finite tape to only inspecting it from start to finish without changing the tape or reversing the direction. Thus, the machine can only chew through the tape, changing from state to state. If it is possible to enter a new state from more than one old state, then the machine has no memory of its computation history. This sort of machine is called a *finite state automaton (FSA)* or *Moore machine*.

A FSA, which can also emit outputs, is called a *Mealey machine*. This is equivalent to adding a single-cell writable tape to a Moore machine. Mealey machines are very widely used for controlling processes that go through fixed sequences of actions, for example, sets of traffic lights or different washing machine cycles.

Language games

We have elided the distinction between checking whether or not a symbol sequence has a required structure and wider computing. The connection is that, given a grammar of some type, there is an algorithm to generate its checking machine. However, it's not yet clear how affirming structure relates to the meanings of symbol sequences.

The key lies in what else the machine does as it checks the sequence. Once we locate meaning in state changes in materialized systems, we can clarify wider interaction in terms of linguistic exchanges.

Just like Carnap, Wittgenstein retreated from his totalizing vision of systematically elaborating states of affairs. He published little in his lifetime after *Tractatus*, and his later work is known through posthumous collections.

In *Philosophical Investigations*,[13] published in 1953, Wittgenstein explores the idea of a *language game* as a way of exploring how language functions. Like other games, language games involve players manipulating things according to rules.

They proceed by players taking it in turn to interact through talking and doing things, according to rules they share.

For example:

> Where is the connection effected between the sense of the expression 'Let's play a game of chess' and all the rules of the game? – Well, in the list of rules of the game, in the teaching of it, in the day-to-day practice of playing. (p80)

Wittgenstein counterposes this to the Augustinian idea of ostensive meanings based on the association of names with things, so children learn language by having things named as they're pointed at. Rather, Wittgenstein suggests, children learn language through training, which involves talking, that is, interacting (pp2–4).

He introduces a language game of a builder and assistant, where the builder calls out the name of a component and the assistant provides it (p3). He then considers the builder, asking the assistant how many slabs there are. Then, the assistant saying 'five slabs' in response is quite different from the builder saying 'five slabs' and expecting the assistant to hand over five slabs (p10). That is, rather than naming things, even broadly understood, the same linguistic constructs have different meanings in different contexts.

For Wittgenstein, we take part in a multiplicity of small language games in diverse concrete circumstances:

> We remain unconscious of the prodigious diversity of all the everyday language games because the clothing of our language makes everything alike. (p224)

However, language games are not fixed, but develop through interaction and may break down if players don't act appropriately or if actions have consequences that weren't foreseen. This may be resolved because we are immersed in language use:

> we lay down rules, a technique for a game, and then when we follow the rules, things do not turn out as we had assumed. That we are therefore as it were entangled in our own rule. (p50)

Grammars and automata

Noam Chomsky[14] explored different classes of well-formed symbol sequences without direct concern for their meanings. He identified four types of grammar that have been shown to correspond to the automata we discussed above.[15] That is, a Turing machine can check Type 0 grammars, an LBA Type 1 grammars, a PDA Type 2 grammars and a FSA Type 3. Thus, these types form a *hierarchy*, where each type can define more than the next, just as the corresponding machines are ordered by computational power.

The most powerful grammar, Type 0, can capture any TC computation. However, they are not used for practical computing because it is really hard to think through problems in terms of bounding symbol sequence contexts, which may be arbitrarily large and complex. Rather, programming languages are used. These are like formal systems in their rigour, and natural languages in their expressiveness.

Using a programming language involves thinking in terms of the abstractions that it supports, in particular what sorts of things variables may represent. Psychologically, this elides the physical grounding of computations in physical computers, manipulating physical instances of symbol encodings. Thus, what can be computed is determined, ultimately, by the physical nature of reality. This is particularly significant for computations that involve arbitrarily small constructs, like real numbers, or arbitrarily large ones, like databases that may grow open-endedly.

If reality is finite, then eventually we run out of stuff with which to do computations. We may think we're programming in a Type 0 language with potentially infinite resources, but we actually have bounded memory. And, even if reality is infinite, it makes no sense to talk about finite computations that deploy infinite resources in a finite time.[16]

Conclusion

Our account of the development of formal logic underpins our materialism. With Turing, we see mathematics as a mechanical activity grounded in physical machines. With Badiou, we see model-making and logic as complementary experimental activities. With Wittgenstein, we see meanings created in interactions that change reality. And, with Markov, we see meaning characterized by algorithmic processes modelled as automata.

From the Church-Turing thesis, our materialism is reductionist. If computability is key to understanding reality and all accounts may be demonstrated to have equivalent explanatory power, then their choice is a matter of pragmatism, not principle. In particular, we reject the Platonist primacy of pure mathematics but recognize its power in abstraction.

Our materialism is fundamentally scientific. We reject the primacy given to dialectics in the Hegelian and Marxist traditions, but recognize its power in exposing and resolving contradictions in interrogating reality.

And our materialism is finitist. With the constructivists, we reject arguments based on actualized infinities.

Chapter 11

Dialectical versus mechanist materialisms

Information theory and materialism

From the mid-twentieth century, our interaction with the world has been transformed by information *technology*, while information *theory* produced a radical shift in sciences and philosophy. These advances are striking illustrations of how the most radical changes in understanding can often arise from the demands of practice. In the nineteenth century, thermodynamics, which revolutionized our understanding of time, order and life, arose from the practical requirements of engineers working with steam engines. We have already mentioned how, in the late 1930s, Claude Shannon realized that Boolean algebra provided a formalism for the understanding of switching circuits. He was working for the US telegraph and telephone company Bell. His use of Boolean algebra stemmed from the need to rationalize the design of automatic telephone exchanges. From there, it spread out to become an enabling framework for digital circuitry employed by the nascent computer technology of the 1940s and 1950s.

Shannon's contribution did not stop there. He went on to found modern information theory. The notion that information is something that can be objectively measured in bits, something that is commonplace now, stems from a technical paper that he wrote for Bell.[1] Telegraph engineers had, as a practical matter, to estimate the carrying capacity of different ways of transmitting information: simple telegraph wires, cables with frequency division multiplexing, radio links etc. To do this, you needed a common unit of measure. This is analogous to the problem that Watt had faced about a century and a half earlier of quantifying the work provided by different forms of motive power. The solution then was to define the power of a standard horse. For a mid-twentieth-century engineer already used dealing with Boolean logic, which took on the values [0,1] the obvious unit of information was the bit.

Had he just stopped at a pragmatic adoption of the binary digits, something with which his engineering contemporaries were already familiar, his work would have had limited impact. What made it revolutionary was that he did not stop at Boole. He went beyond Boole to define bits in terms of probability theory.[2] Instead of just saying that a bit was either a 0 or a 1, he defined it as the quantity of information used to discriminate between two equally probable outcomes.

Suppose Alice in New York tosses a fair coin three times. Bob in Boston knows that on each toss, HEADS or TAILS are equally likely, but he cannot tell which falls or Alice's coin were HEADS and which were TAILS. If Alice transmits her sequence of Bob on a Telex machine, then whatever code she uses to transmit the falls, whether it be

```
HEADS
HEADS
TAILS
```

or `HHT` or `110` she has, according to Shannon, sent to Bob only 3 bits of information. If the information was sent using the 5-binary-digit Baudot Code in use in the 1940s, then the first alternative, spelling it out word at a time would have requires 105 binary digits.[3] If as `HHT` then 15 binary digits would have been sent. If raw binary data had been sent on a line then only 3 binary digits would have been used.

So for Shannon information and data were something distinct. Data might be very inefficient at encoding information. The first exampl of English text was thirty-five times less efficient than the most compact encoding.

So far, we have been considering only the paradigmatic fair coin. What if we have a very unfair coin?

What if your random number generator is biased?

What if the coin falls HEADS three quarters of the time?

Then it is clear that each transmission of a HEAD message must send less information than in the case of a fair coin. Having seen a long sequence of tosses, Bob can estimate the a priori probability of a HEAD. Were he to guess what the next message was going to be, he would guess HEADS each time. When he did so, he would be correct more than 50 per cent of the time. Consequently, each HEADS message now conveys less information than before. It conveys less than one bit.

Shannon came up with a formula for the information content of such biased streams of symbols. Suppose we have a long data stream with n possible distinct symbols, such that the ith symbol occurs in our stream with probability p_i, then the average information content per data symbol is given by

$$H = -\sum_{i=1}^{n} p_i \log_2 \left(p_i \right)$$

The minus sign is there to ensure we get a positive result since the probabilities are all < 1.and hence $\log_2 \left(p_i \right) < 0$.

This formula will be at a maximum when all the distinct symbols occur with equal probability. But in human languages, this is not the case. The frequency of occurrence of letters in written text is very non-uniform. In Shannon's terms, this means that written text is inefficient or redundant, even at the most basic character level.

His distinction between the information content of symbols and their raw codes has practical applications in cryptography[4] and data compression. Whenever you view a digital TV signal or listen to an MP3 track, you are implicitly relying on Shannon's information theory.

But the most striking feature of his formula, from the standpoint of materialism, is that his formula for information is functionally the same[5] as Boltzmann's formula for entropy.

It indicated that, at a deep level, information and entropy were the same thing. This was, from a philosophical standpoint, startling.

Physical meanings

Philosophy had not bothered to define information up to this point, but the normal usage of the word was so tied up with information understandable to humans that information was something physical and, moreover, something connected with the atomic level of physics, the idea that information could be analysed purely statistically, completely ignoring its meaning, was counterintuitive.

One might naively assume that data, for example, in the form of printed text only contained information because a reader was subjectively able to attach meaning to it. Conversely, this meaning, and thus the information, could be assumed to derive from the subjective intention of the original author.

That seems plausible when dealing with contemporary texts. But what about arcane information?

What about information in hieroglyphic texts prior to the discovery of the Rosetta stone?

Over centuries before the Rosetta's discovery, these Egyptian texts were meaningless to everyone. After it was found, they could be read. The inescapable conclusion is that the information objectively existed for centuries in the texts in the spatial configurations of ink on papyrus. The Egyptian texts could be understood after 1799 because they had objectively contained information, even though that information's code remained indecipherable. Shannon was adamant that information had to be considered distinct from its encoded representation.

> To define a concept of actual information, consider the following situation. Suppose a source is producing, say, English text. This may be translated or encoded into many other forms (eg Morse code) in such a way that it is possible to decode and recover the original. For most purposes of communication, any of these forms is equally good and may be considered to contain the same information. Given any particular encoded form, any of the others may be obtained, (although of course it may require an involved computation to do so). Thus we are led to define the actual information of a stochastic process as that which is common to all stochastic processes which may be obtained from the original by reversible encoding operations.[6]

That encoded information exists quite independently of any human subjective involvement, either as author or reader, became even clearer in the 1950s with the discovery of the genetic code. Here, information exists in DNA. With the development of DNA sequencing machines, this molecular code can be printed out in a form people can read: a sequence made up of the letters ATGC. Shannon's point is that the information content is the same whether it is efficiently represented in molecular form or much more redundantly as an ink-and-paper text. The information in molecular form can be 'read' either by a modern sequencing machine or by a ribosome. The fact that a trained person can read and, to some degree, interpret the printout is irrelevant to the information in the DNA.

It is not information because a person can read it.

A person can read it because it is information.

More generally, information content does not depend on human meanings. The opposite holds. Our ability to handle meaning depends on objectively existing information: both external to and internal to our nervous system.

Living organisms respond to their environment. Even single-celled organisms can respond in a purposeful way. Individuals of the unicellular aquatic photosynthetic species *Chlamydomonas* can swim towards a light source.[7] This purposeful behaviour has an obvious evolutionary advantage: access to light energy. But to achieve it, the cell must be able to store an internal state that is associated with the external state of its environment. The direction of illumination, which in geometrical terms is a vector in three-space.

Let us consider a possible internal encoding of such a vector. It would suffice to associate three states $(-1,0,+1)$ to each of the mutually perpendicular x,y,z axes, indicating whether the incident light vector is away from, at right angles to, or towards that axis. With a naive encoding, this would require the cell to be able to register $3\times\log_2(3)=4.755$ bits of information.[8]

What is the 'meaning' of these 4.755 bits?

Obviously, in one sense, it means that the light source is in a specified general direction. This specification can then steer the cell's motion. In terms of information theory, for the sensors to have meaning, there must be mutual information[9] between the states of the environment (directions of illumination) and internal states of the phototactic cell. The meaning then is the mutual information.

In this section, we have been examining one philosophically startling implication of information theory: that information is something objective and physical, something quite independent of human consciousness. It need hardly be emphasized that this had profound materialist implications. The formal definition of mutual information provides the scientific foundation for the representational model of knowledge defended in.[10]

Randomness

In this section, we look at a second surprising implication of what Shannon published.

Shannon showed that the formula for information is functionally identical to that of entropy. Indeed, the terms entropy and information have since then often been considered interchangeable.

But this equation of information with entropy appears to contradict popular understanding of entropy as disorder or randomness. Natural processes produce random results. A weathered rock face has a rough disordered appearance. A rock worked by human hand to bear an inscription is immediately distinguishable from something natural precisely because it is, in the main, relatively smooth and orderly. It would thus seem that recording human information goes hand in hand with an increase in order[11] and a reduction in randomness.

How are we to reconcile this with Shannon's identification of information with entropy, and thus with disorder?

This paradox arises when we neglect the redundancy of traditional human records. Monumental masons wanted their text to be clearly readable to stand out. To ensure this, they did two things:

1. They created flat surfaces. These contain no anthropic information but are low entropy and highly orderly.
2. They then incised letters, which were large and relatively deeply cut so that they would withstand weathering. These do contain information, but represent it very redundantly.

This process of first creating a low-entropy substrate on which information is then redundantly recorded is characteristic of lots of production processes[12]from printing to micro-chip manufacture.

We saw earlier that information about coin tosses could be recorded redundantly as text or efficiently as binary digits. The text is easily readable in the presence of errors. A one-bit error in the word HEADS could transform it to, for example, HECDS, which, in context, would still be understandable. A one-bit error in a pure binary stream encoding heads and tails, on the other hand, would pass unnoticed.

Practical and resilient ways of storing information use redundancy, order and as such are low entropy. A consequence of this is that the greatest possible information is encoded in a completely random bit stream in which 0 and 1 occur with equal frequency. This fact was of practical relevance in the cryptography on which Turing and Shannon had worked. Breaking codes was reliant on exploiting non-randomness in the clear-text, which somehow leaked through to non-randomness in the cipher text. Related to this, file compression techniques, like .zip files, produce bit strings which approximate to randomness.

But this raises a new question. What do we mean by randomness?

Audaces fortuna iuvat

Alea jacta est said Caesar; the die once thrown, he must wait and see how it lands. The result was unknowable: triumphs, victories or assassination.

Caesar forte regebatur. He was ruled by fortune. But how then does fortune differ from rules?

How can we, by observation, tell if something is just random or is following some pattern, which pattern may not be obvious to us?

Take the metaphor of a die. How can we tell, given a sequence of die casts, if the die is fair or if there is a bias on which we can soundly bet?

Suppose we have a ten-sided die labelled 0.9, and this gives us the following sequence:

4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3, 2, 3, 8, 4, 6, 2, 6, 4, 3, 3, 8, 3, 2, 7, 9, 5, 0, 2, 8, 8, 4, 1, 9, 7, 1, 6, 9, 3, 9, 9, 3, 7, 5, 1, 0, 5, 8, 2, 0, 9, 7, 4, 9, 4, 4, 5, 9, 2,

How do we know if this was random?

Well, we can look at how often each digit occurs.

Digit	Occurences
0	3
1	4
2	7
3	8
4	7
5	7
6	4
7	5
8	6
9	11

At this point, it looks as if our die is biased against zeros and towards nines. It would seem that were we to bet against zeros, we would, over the long run, win. Let's try generating more digits and see if that holds.

Figure 11.1 shows that as we get more and more digits, the percentage of zeros in the stream approaches 10 per cent, which is what we expect from an unbiased die.

If we wanted a more stringent test on randomness, we could look at pairs of digits. Perhaps zeros occur 10 per cent of the time, but they are still slightly more likely to occur after six than after a seven. If we find that, over a suitably large sample, the occurrence of each digit is unaffected by the preceding digit, then we could look at groups of three digits, groups of four digits, etc.

As we increase the length of groups we are examining, then the number of digits required in our sample will grow exponentially. If it took 44,000 digits for the share of zeros to converge to 10 per cent, then we could expect to look at over 400,000 digits before the percentage of sixes preceding these zeros would converge to 10%. For triples, we would need to check four million digits etc.

Proving randomness becomes increasingly difficult the longer are the patterns we are trying to detect. The subtler the pattern that might exist, the more impossible it is to find by exhaustive searching.

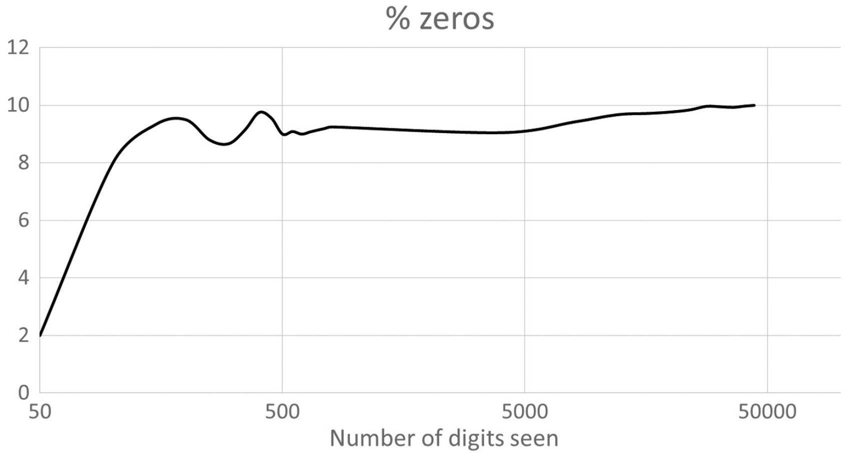


Figure 11.1 The initial bias against zero vanishes as we generate more and more digits. Figure by authors.

So the answer to our earlier question is **no**. We cannot in general prove by observation that a long sequence of numbers, or a long set of observations of nature, are random.

Obviously the digit sequence we have been using up until now was not actually generated by throwing a ten-sided die. Throwing it 44,000 times would have been impractical.

We could have used some electronic hardware to generate true random numbers.[13] In fact, we did use a computer-generated sequence, but it was not a truly random one. We used successive digits of π as the source.[14] Yet the successive digits of π seem to perfectly mimic randomness.

Here is a deep paradox of reality. *circulus regitur forte* The circle, the most paradigmatic of nature's orders, is ruled by what appears to be randomness. Where now lies the difference between rule and fortune?

Should we too consult the augurs and read the book of changes before our great decisions?

Is our edifice of scientific laws a mere illusion hiding chaos?

An answer to this was provided by the meta-mathematician Chaitin,[15] who gave a new definition of randomness based on computer programmes. According to him, a random sequence of digits is one which cannot be printed out by a shorter computer programme.

By this definition, π is not random. Here is a C programme by Dik T. Winter occupying 160 characters that will print out 800 characters of it:

```
int a=10000,b,c=2800,d,e,f[2801],g;main(){for(;b-c;)
  f[b++]=a/5;
for(;d=0,g=c*2;c-=14,printf("%.4d",e+d/a),e=d%a)fo
  r(b=c;d+=f[b]*a,
f[b]=d%--g,d/=g--,--b;d*=b);}
```

Other short programmes exist that will print π to millions of digits.

Soviet diamat

Dialectical Materialism was invented by Dietzgen, an intellectual influence on German Social Democracy, and became the official communist philosophy inherited from the German Social Democrats. However, the compatibility of this tradition with the stochastic basis of the material world is questionable. Nineteenth-century materialists like Boltzmann had previously struggled with the problem of directional change in a world governed by chance and the laws of thermodynamics. How should we understand the same paradox in the context of history?

To what extent is the Soviet doctrine of 'Diamat' compatible with the account of materialism we have been giving in this book?

Stalin, for long the orthodox authority, gave[16] the following four principles of materialist dialectics:

1. Nature is connected and determined
2. Nature is a state of continuous motion and change
3. Natural quantitative change leads to qualitative change
4. Contradictions are inherent in nature

We will consider these in turn.

Nature is connected and determined

> Contrary to metaphysics, dialectics does not regard nature as an accidental agglomeration of things, of phenomena, unconnected with, isolated from, and independent of, each other, but as a connected and integral whole, in which things, phenomena ,are organically connected with, dependent on, and determined by, each other.[17]

What strikes one immediately about this is that it is absolutely orthodox mechanics. Within the USSR in the interwar period, there had been a vigourous debate between the 'mechanists' and the Deborinists.[18] But Stalin's 1930s exposition on dialectical materialism states that nature's determination and connection via action at a distance forces are fundamental to both classical and quantum mechanics. In quantum mechanics, as our account of quantum cryptography in Section [sec:qcrypt] shows, the same applies. The physicist Bohm[19] called this the 'implicate order' of reality. So an assertion that nature is deterministic and connected is not something specifically dialectical. If dialectical materialism held this, it was because it was a faithful follower of mechanics.

Nature is a state of continuous motion and change

> Contrary to metaphysics, dialectics holds that nature is not a state of rest and immobility, stagnation and immutability, but a state of continuous movement and change, of continuous renewal and development, where something is always arising and developing, and something always disintegrating and dying away.[20]

Motion underlies any mechanical theory of dynamics. All statistical mechanics, too, are based on such a picture. With Smith, Engels and Darwin it gets extended to society and biology. Continuous motion and change in nature have indeed been asserted by all materialists since the ancient atomists.

> after the revolution of many of the sun's years a ring on the finger is thinned on the under side by wearing, the dripping from the eaves hollows a stone, the bent plowshare of iron imperceptibly decreases in the fields, and we behold the stone-paved streets worn down by the feet of the multitude; the brass statues too at the gates show their right hands to be wasted by the touch of the numerous passers by who greet them.
>
> These things then we see are lessened, since they have been thus worn down; but what bodies depart at any given time the nature of vision has jealously shut out our seeing.
>
> Lastly the bodies which time and nature add to things by little and little, constraining them to grow in due measure, no exertion of the eyesight can behold; and so too wherever things grow old by age and decay, and when rocks hanging over the sea are eaten away by the gnawing salt spray, you cannot see what they lose at any given moment Nature therefore works by unseen bodies.[21]

Natural quantitative change leads to qualitative change

The idea that quantitative change leads to qualitative change seems to have gained popularity from Engels' Dialectics of Nature, which argued that in physics, we work with bodies' molecular states and motion forms, which bring molecules into play. Every change is a transformation of quantity to quality,[22] resulting from a quantitative change of motion inherent in the body or communicated to it. Physical constants are designations of nodal points where addition/subtraction of motion alters the state of the body, transforming quantity into quality.

What Engels was describing here is basically what we now call phase-change phenomena. His examples of changes between liquid and gaseous states are valid phase changes. The terminology Engels used though, writing of quantity and quality is ambiguous compared to that used in physics. A physicist would define a phase change as occurring when the change in entropy becomes discontinuous for temperature, you require 1,000 calories per degree Celsius of heating until you reach 100 Celsius, then to go between water at 99 Celsius and steam at 100 Celsius you need 540,000 calories, a huge increase. Water evaporates from the surface using 540 calories of heat per gram. In the absence of external heat, water cools down by 0.5°C. This is realistic only for a closed container, and the evaporation rate below the boiling point is proportional to the temperature in Kelvin. For an open pan on the stove, the heat input required to raise a litre by 1 Celsius will be non-linear up to 100°C, at which point you still get a step jump.

The problem with the diamat language is that the terms quantity and quality are vague and ill-defined. In the melting/boiling example, we have a commonsense understanding of there being a qualitative difference between solid, liquid and gas phases. But if we have a super-critical boiler, one that operates above 3,200 psi, then liquid water and steam become indistinguishable as phases. One may dress this up in the language of quality by saying that above 3,200 psi, there is no qualitative difference between water and steam. Or one can be more precise and say that above 3,200 psi, there is no step jump in entropy as we heat the water up.

The vagueness of Engels's language leads him to make what are clear errors in his account. The example of the incandescent lamp is completely wrong.

If you alter the supply voltage to a lamp you get a gradual rise in the temperature of the filament. As the temperature rises, the intensity of light given off at different wavelengths gradually changes from red, orange and yellow to almost white. Anyone who has used a dimmer control for an incandescent bulb has seen this. There is no distinct phase change and no sharp threshold voltage. With a Light Emitting Diode, there is a distinct threshold voltage below which no light is admitted, and the colour of light emitted does not change above that threshold.

Only some systems undergo phase changes, others change continuously.

This confused light bulb example seems to be a consequence of Engels's ambiguous terminology.

What counts as a qualitative change?

What counts as a quantitative change?

For quantity, is he referring to whole numbers or to a continuum?

In the case of his correct example of temperature change associated with boiling, then the quantitative change is along a continuous scale. The same applies to his faulty example of the incandescent bulb.[23] Voltage is an analogue or continuous variable.

But in other places Engels wants quantity to be interpreted in terms of integer numbers.

> The sphere, however, in which the law of nature discovered by Hegel celebrates its most important triumphs is that of chemistry. Chemistry can be termed the science of the qualitative changes of bodies as a result of changed quantitative composition. That was already known to Hegel himself. As in the case of oxygen: if three atoms unite into a molecule, instead of the usual two, we get ozone, a body which is very considerably different from ordinary oxygen in its odour and reactions. Again, one can take the various proportions in which oxygen combines with nitrogen or sulphur, each of which produces a substance qualitatively different from any of the others! How different laughing gas (nitrogen monoxide N_2O) is from nitric anhydride (nitrogen pentoxide, N2O5) ! The first is a gas, the second at ordinary temperatures a solid crystalline substance. And yet the whole difference in composition is that the second contains five times as much oxygen as the first, and between the two of them are three more oxides of nitrogen (N0, N2O3, NO2), each of which is qualitatively different from the first two and from each other.[24]

So, we have a 'law of nature' discovered by Hegel!

Give us a break Fred. The name you are looking for is Dalton, John Dalton, not Hegel. Hegel's *Science of Logic*, cited as the source for this 'law' is dated 1812. Dalton's *A New System of Chemical Philosophy* came out four years earlier in 1808. At best we have a retrospective claim by Hegel to other's discoveries, as was the case when in 1801 Hegel claimed to have a superior proof of Newton's law of gravitation. We showed in Section 3.4 that those claims to originality by Hegel were incoherent nonsense. The same applies here. That chemical compounds must contain natural number ratios of atoms of their constituent elements was an integral part of Dalton's atomic theory. The analogy that Engels, with reference to Hegel, makes between phase change in boiling water and these numeric ratios is as incoherent as Hegel's arguments about gravity.

In the case of water boiling, you have one continuous analogue scale: temperature at certain points on which you get a phase change, marked by a discontinuity in another analogue scale: entropy. In the chemical case, there is no continuous analogue scale. In the chemical case, you do not get a sudden change in compound as you gradually add more Nitrogen. You cannot have a compound $N_{1.275}O$ between NO and N_2O. The number of each type of atom has to be an integer. This is a consequence of the (chemical) indivisibility of the atom. So instead of a gradual quantitative change leading to a sudden qualitative change, you have a discrete set of compounds, each containing discrete numbers of atoms.

The political appeal of the quantity into quality 'law' was strong. Statements about quantitative change leading to qualitative change could be taken as a metaphor for the increased quantitative exploitation of the working class leading to a sudden violent revolution. The metaphor of revolutions being the result of tensions boiling over was already in use by the nineteenth century.[25]

This alleged general law of quantity into quality is based, like much idealist philosophy, on superficial analogy and *double entendre*. As a general law, it is useless for making predictions. Indeed, even in the case of revolutions it is questionable whether the analogy makes sense. In Soviet revolutionary imagery, the masses unite into organized columns, marching towards the red future or gather in front of Lenin as he speaks. Revolution happens when entropy decreases, that is, when the masses gain order and direction instead of moving aimlessly.

It might be objected that the notion of phase change is also sometimes used in a rather metaphorical way. If we ridicule the Hegelian quantity in quality notion, why should we allow scientists to speak of other phase transitions?

No, because, in the scientific literature, when one uses the term phase change, it is expected that one will demonstrate that there is a discontinuity in a variable analogous to entropy while another variable undergoes continuous change.

Let's give a non-physics example where the term *phase transition* is used and show how the application of the term has to meet tight theoretical constraints.

In symbolic logic, there is something called the satisfaction problem (SAT). The problem takes some formula F in Boolean algebra defined over n Boolean variables $v_1..v_n$ and seeks to find an assignment of truth values to the variables such that F is true.

F is typically given in conjunctive normal form for example:

$$F(v_1,v_2,v_3,v_4) = (v_1 + v_2 + \neg v_3).(v_3 + \neg v_1 + v_4)$$

It has been observed that it is often very easy to find such an assignment of variables, but at other times it becomes very hard, requiring in the limiting case that one tries all 2^n possible distinct assignments of true or false to the n variables.

For other examples of F, it turns out to be very easy to prove that there is no possible assignment of variables that will make F true.

Further investigation has shown that the change from easy to satisfy, to hard to satisfy, to easy to prove unsatisfiable occurs at a critical point, which has been termed a phase transition.[26] The experimental work in this field justifies the term phase transition by showing that there is a constrainedness measure[27] m on F such that as m increases the hardness of the problem rises sharply and then falls. Why is it like a phase transition?

What does it have in common with water boiling?

They are similar because the constraint level m acts as an analogue for temperature T, and the hardness (measured as computational run time) is analogous to $\frac{dH}{dT}$ the derivative of entropy with respect to temperature. Obviously, in the water-to-steam phase, transition $\frac{dH}{dT}$ spikes analogous to the way that the hardness of F spikes as m increases.

Contradictions are inherent in nature

Supporters of dialectical materialism say that what they call 'contradictions' exist in the real world, not just in logical statements. It is clear that they are using the word in a somewhat different sense to what it usually means. Formal logical contradictions – not (not x and x); x or not x – depend on properties of truth values. But this is not what Stalin or Mao meant by contradictions. So we have Stalin writing:

> Contrary to metaphysics, dialectics holds that internal contradictions are inherent in all things and phenomena of nature, for they all have their negative and positive sides, a past and a future, something dying away and something developing; and that the struggle between these opposites, the struggle between the old and the new, between that which is dying away and that which is being born, between that which is disappearing and that which is developing, constitutes the internal content of the process of development, the internal content of the transformation of quantitative changes into qualitative changes.[28]

So, by contradiction, Stalin is talking metaphorically about the process of change in the world. But why is the recognition of change supposed to be something specific to dialectics?

That things have a past and a future, that things develop and die, is recognized by everyone. What special ingredient is added by using the term 'contradiction' to refer to this?

When Stalin elaborates on what he means, he has recourse not to dialectics but to what Marx had called the materialist theory of history. Stalin refers to it here as 'the historical approach to social phenomena'.

> The slave system would be senseless, stupid and unnatural under modern conditions. But under the conditions of a disintegrating primitive communal system, the slave system is a quite understandable and natural phenomenon, since it represents an advance on the primitive communal system
>
> . . .
>
> Everything depends on the conditions, time and place.
>
> It is clear that without such a historical approach to social phenomena, the existence and development of the science of history is impossible; for only such an approach saves the science of history from becoming a jumble of accidents and an agglomeration of most absurd mistakes.[29]

That there are successive social forms, successive relations of production and associated political superstructures was already evident to Adam Smith, as we have recounted in Section 5.1. Smith felt no need to invoke contradiction or dialectics to explain this. In their own first account of historical materialism, *The German Ideology*, Marx and Engels did often use the term contradiction even though they never referred to dialectics. Here are just a few examples from *The German Ideology* where Marx and Engels use the term contradiction in what later became the orthodox 'diamat' style:

> the division of labour implies the *contradiction* between the interest of the separate individual or the individual family and the communal interest of all individuals who have intercourse with one another.
>
> . . .
>
> The conditions of life of the individual burghers became, on account of their *contradiction* to the existing relationships and of the mode of labour determined by these, conditions which were common to them all and independent of each individual. The burghers had created the conditions insofar as they had torn themselves free from feudal ties, and were created by them insofar as they were determined by their antagonism to the feudal system which they found in existence. When the individual towns began to enter into associations, these common conditions developed into class conditions. The same conditions, the same *contradiction*, the same interests necessarily called forth on the whole similar customs everywhere.
>
> . . .
>
> This *contradiction* between the productive forces and the form of intercourse, which, as we saw, has occurred several times in past history, without, however,

> endangering the basis, necessarily on each occasion burst out in a revolution, taking on at the same time various subsidiary forms, such as all-embracing collisions, collisions of various classes, *contradiction* of consciousness, battle of ideas etc, political conflict etc.[30]

In 1932, *The German Ideology* was published and introduced the term 'contradiction'. Stalin would have been familiar with this term when he wrote his own text in 1938. The phrase 'class contradiction' is a relatively late addition to orthodox Marxism, first appearing in the English version of Bukharin's 1925 book *Historical Materialism*. In Maoist dialectics, this sense of contradiction became central.

Mao's account of dialectics

Mao's *On Contradiction*[31] was the most influential work of dialectics ever published. Its influence was twofold. Firstly, the great size of China means that it was printed in huge numbers and read by tens of millions. But that was only possible because the Chinese Communists, led by its author, put into practice a political strategy based on *On Contradiction* and emerged victorious to establish the People's Republic of China.

Coming out a year before Stalin's *Dialectical and Historical Materialism*, it has to be read in the context of both the ongoing debates in the Soviet Union and the Chinese Revolutionary War.

The preface positions the text as an attack on Hegel-influenced Deborin school:

> The criticism to which the idealism of the Deborin school has been subjected in Soviet philosophical circles in recent years has aroused great interest among us. Deborin's idealism has exerted a very bad influence in the Chinese Communist Party, and it cannot be said that the dogmatist thinking in our Party is unrelated to the approach of that school. Our present study of philosophy should therefore have the eradication of dogmatist thinking as its main objective.

Mao, like Stalin, sees the dialectical materialist world view as being tied to accounts of time and change. He contrasts what he calls metaphysical or vulgar evolutionist accounts of time with the view of Marxism. The vulgar evolutionists, he says, recognize change, but only in three forms: increase, decrease and cyclical repetition. From the context, it is clear that by vulgar evolutionism he does not mean the Darwinian theory, but simplified notions of social evolution:

> Metaphysicians hold that all the different kinds of things in the universe and all their characteristics have been the same ever since they first came into being. All subsequent changes have simply been increases or decreases in quantity. They contend that a thing can only keep on repeating itself as the same kind of thing and cannot change into anything different. In their opinion, capitalist exploitation, capitalist competition, the individualist ideology of capitalist

> society, and so on, can all be found in ancient slave society, or even in primitive society, and will exist forever unchanged. They ascribe the causes of social development to factors external to society, such as geography and climate.

An example of such a vulgar evolutionist view today might be Turchin's work,[32] which, despite its historical richness and recognition of class conflict, does formulate things in terms of what are basically unchanging dynamics over thousands of years. The key omission in the Turchin theory is the development of what Marx called the productive forces, and the way such developments condition social relations of production.

Mao believed that societies develop through internal conflicts, not external causes. He argued that China's development would be determined by the domestic class struggle between workers, peasants and patriotic capitalists versus landlords and comprador capitalists. This perspective proved correct in light of the subsequent economic and social trajectories of India and China.

He discusses the universality and particularity of contradiction. The former is a summary of concepts from Engels and Lenin, while the latter is the main focus of the pamphlet, analyzing specific contradictions in the Chinese Revolution. He criticizes dogmatism and highlights some examples of the universality of contradiction, which are no more convincing than those of Engels and Stalin. He did raise some examples that are worth greater attention. He quotes a list by Lenin of examples that are intended to illustrate the universal nature of contradictions.

> The correctness of this aspect of the content of dialectics must be tested by the history of science. This aspect of dialectics (eg in Plekhanov) usually receives inadequate attention: the identity of opposites is taken as the sum-total of examples ['for example, a seed,' 'for example, primitive communism.' The same is true of Engels. But it is 'in the interests of popularisation . . .'] and not as a law of cognition (and as a law of the objective world).
>
> 1. In mathematics: + and −. Differential and integral.
> 2. In mechanics: action and reaction.
> 3. In physics: positive and negative electricity.
> 4. In chemistry: the combination and dissociation of atoms.
> 5. In social science: the class struggle.
>
> The identity of opposites (it would be more correct, perhaps, to say their 'unity,' — although the difference between the terms identity and unity is not particularly important here. In a certain sense both are correct) is the recognition (discovery) of the contradictory, mutually exclusive, opposite tendencies in all phenomena and processes of nature (including mind and society). The condition for the knowledge of all processes of the world in their 'self-movement,' in their spontaneous development, in their real life, is the knowledge of them as a unity of opposites.[33,34]

We think that the gloss given by Lenin to these examples, implicitly accepted by Mao, is misleading. To show this, let's group the examples: (1) applies to arithmetic, a human art or technology; (2,3,4) apply to the natural sciences; (5) applies to social science.

Addition and subtraction, multiplication and division, differentiation and integration etc., are examples of inverse operators in maths. If α,β are inverse operators then $((A\alpha B)\beta B) = A$, and it is quite reasonable to call α,β *opposite* operations.

Arithmetic is a technical art. It involves physical objects such as tokens, beads and signs in ledgers. In the 1930s, commercial arithmetic in Russia or China was largely performed on an abacus. When adding 2, the operator slid 2 beads from right to left. When subtracting 2, the operator moved 2 beads left to right. An addition followed by a subtraction then left the beads in the same state as they started out. It is this return to the starting state that qualifies addition and subtraction as inverse operations on the state.

But if one wants to claim that the unity of opposites is universal, then there are problems. For a start, it is easy to come up with pairs of operators that are not inverse of one another. In logic, AND $(\wedge)$ is paired with OR $(\vee)$ as the two basic operators, but these are not inverses: $((A \wedge B) \vee B) \neq A$. So why claim that opposite operators are fundamental?

Is this not cherry-picking examples to support an argument?

Yes, it is arbitrary to claim that the unity of opposites is fundamental to mathematics. Yes, mathematics does have inverse operators, but it has other important ones as well.

If we go back to the idea that addition and subtraction return an abacus or other calculator to its start state, does this help?

Does returning something to its start state always require a pair of inverse operations?

No. Consider the operation of rotating a wheel by a half circle. If we perform this operation twice, the wheel returns to its starting state. An arithmetic analogue would be multiplication by −1. Apply that twice to a number, and you get back to your starting number. So the idea of a return to a starting point does not establish the necessity of a unity of opposites either.

The next three examples of action and reaction, positive and negative charge and combination/dissociation of atoms do have something in common, but reference to a unity of opposites gives only a partial account of what they have in common, missing out the most important thing.

All of these are ways of expressing, or consequences of, underlying conservation laws.

Equal and opposite forces result from the conservation of momentum. For instance, the Earth and the Moon exert equal and opposite gravitational forces on each other, resulting in no net force on the system. Without the Sun's external attraction, the Earth/Moon system would move in a straight line, conserving momentum.

Positive and negative electricity relate to another basic conservation law: the conservation of charge. When a carbon $_{14}$ nucleus with a charge of +6 undergoes β decay, it forms a nitrogen $_{14}$ nucleus with a charge of +7 along with a β particle, that is to say, a negatively charged electron. The net charge is still +6. The conservation of charge means that in all such nuclear transitions, the total net charge remains unchanged.

Association and disassociation of atoms in the chemical reactions also express a conservation relation: the conservation of the total number of atoms of each element in chemical processes.

Lenin's three examples are conservative processes and do not demonstrate the dialectical principle of the unity of opposites. The source of their shared property lies in Lucretius' maxim, 'nothing comes from nothing'. Despite Soviet diamat's criticism of mechanics, it is the mechanical world view that expresses the conservation laws behind Lenin's examples.

The last example given is class conflict. But this too relates to a conservation constraint since in every society, a finite labour force and limited working hours mean that an increase in surplus labour leads to a decrease in necessary labour. This creates irreconcilable class conflict between workers and employers, as any increase in labour appropriated by the capitalist class results in a corresponding decrease in the labour available to satisfy the needs of the working class.

It is from the despised mechanics that conservation laws originate. Forms of thought deriving from mechanics use conservation principles to explain how the world works. In Section 5.3, we argued that Marx's ambition to uncover the 'laws of motion' of capitalist society belonged within this line of thought.

When Mao was quoting of Engels and Lenin in *On Contradiction*, he was making perfunctory gestures towards orthodoxy. His innovation came in his treatment of the particularity of contradiction. Here, he deployed a set of entirely new conceptual terms and meanings, turning the language of dialectics into a tool of military and political strategy.

Mao claimed that it was the particularity of the contradictions that they studied that distinguished the different sciences. His description of the scientific method was:

> These are the two processes of cognition: one, from the particular to the general, and the other, from the general to the particular. Thus cognition always moves in cycles and (so long as scientific method is strictly adhered to) each cycle advances human knowledge a step higher and so makes it more and more profound.[35]

Allowing for a shift in vocabulary from forces to contradictions, this cyclical process exactly matches what Newton said in the introduction to the Principia: 'all the difficulty of philosophy seems to consist in this — from the phænomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phænomena'.[36] Mao was describing, in his terminology of contradictions, the Newtonian scientific method.

Mao contrasted the scientific approach to that of the Deborin influenced dogmatists:

> Our dogmatists are lazy-bones. They refuse to undertake any painstaking study of concrete things, they regard general truths as emerging out of the void, they turn them into purely abstract unfathomable formulas, and thereby completely deny and reverse the normal sequence by which man comes to know truth. Nor do they understand the interconnection of the two processes in cognition – from the particular to the general and then from the general to the particular. They understand nothing of the Marxist theory of knowledge.[37]

This need to base ones deeper understanding on concrete analysis of real historical conditions seems very plausible to the authors. Take the example of Marx's law of the falling rate of profit. It was our experience that only by means of a concrete statistical analysis of the accumulation process in the UK from the 1850s to the late twentieth century[38] were we able to deepen our understanding of its mechanism and come up with a precise mathematical formulation of its dynamical behaviour.[39] Had we just deduced things from the account given in Capital, without the guidance of actual historical statistics, it is unlikely we would have come up with the correct dynamic equations. Without that eq uation, that law of motion, you cannot grasp the limited and contradictory nature of the partial recovery in Western profitability that occurred in the last decade of the twentieth century, nor the contrasting trends of capital returns in China and Africa since 2000.

Mao wrote:

> There are many contradictions in the course of development of any major thing. For instance, in the course of China's bourgeois-democratic revolution, where the conditions are exceedingly complex, there exist the contradiction between all the oppressed classes in Chinese society and imperialism, the contradiction between the great masses of the people and feudalism, the contradiction between the proletariat and the bourgeoisie, the contradiction between the peasantry and the urban petty bourgeoisie on the one hand and the bourgeoisie on the other, the contradiction between the various reactionary ruling groups, and so on.[40]

These are very specific to China. But the same language of *many contradictions* provides a way of looking at the crisis of capitalism in the West where one has to consider:

- The 'contradiction' (class conflict) between labour and capital expressed in the rate of surplus value.
- The 'contradiction' between productive and unproductive labour. The higher the portion of surplus value used to pay wages for unproductive labour, the lower the flow surplus value that can be accumulated as a new means of production.

- The 'contradiction' between capital and landed property expressed as a rising share of surplus value going as rent.
- The contradictory effect of accumulation itself, which, by raising the capital/labour ratio, tends to depress the profit rate.

Mao's language of multiple contradictions in the process of the development of a society provides a framework for thinking through these processes. One can perhaps formulate these in mathematical terms so that the 'contradictions' become mathematical terms $s' = \frac{s}{v}$, $p = s - u - r$ (profit = surplus value - unproductive wages - rent).

In other cases, one can express contradictory relationships in the more familiar language of dynamics and calculus. Consider the contradictory relationship between accumulation and the rate of profit.

The rate of profit, let us use π, is given by $\pi = \frac{p}{K}$ where p as defined earlier, and K is capital stock.

The rate of change of the capital stock with respect to time would be written $\left(\frac{dK}{dt}\right)$ and this in turn is given by $\frac{dK}{dt} = \frac{A}{K}$ where A is annual accumulation. If we denote the share of profits going as net accumulation as $\alpha = \frac{A}{p}$. Now consider the rate of change of the rate of profit with respect to time $\pi' = \frac{d\pi}{dt}$ that is to say whether the rate of profit tends to rise or fall over time. It is clear from the previous equations that $\frac{d\pi'}{d\alpha} < 0$ that is to say the partial derivative of the rate of change of profit with respect to the rate of accumulation is negative. A high rate of accumulation leads to a decrease in the rate of profit. This is a negative feedback or contradictory relationship, described using the calculus of Newton and Leibniz in mathematics or different terminologies in other fields. Note that all of the contradictions in the initial list above, when analysed slightly more closely, involve negative (<0) partial derivatives. The derivative of surplus value with respect to wages is negative. The derivative of profit with respect to rent is negative, etc.

Indeed, if you mechanize logic with digital circuits, this relationship between logical negation and negative partial derivatives emerges.

It is absolutely standard practice in computer design to have wires representing both the true and the complement of a logic signal. If we make the kind of superficial analogies that Engels and Lenin use about materialist dialectics, then a circuit that contains the signals **A** and **not A** might be said to contain 'contradiction' or 'identity of opposites', and it turns out that these contradictory terms are related by negative partial derivatives. In Figure 11.2, the partial derivative of the voltage on the **not A** signal with respect to the voltage on the **A** signal will be negative. Negative partial derivatives are used to represent negation in physical systems. Logic is an abstraction used to describe these systems, whether they are electric circuits or people using notepads. As such, we can use the term "negation" metaphorically for any material process governed by negative derivatives.

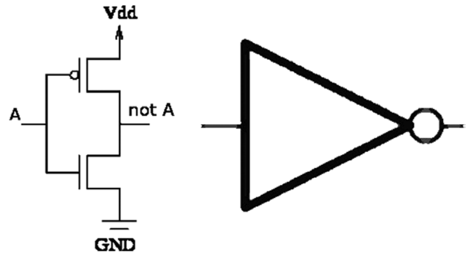


Figure 11.2 Left a standard component in computer chips is the CMOS inverter. Applying a positive voltage to input A switches the top transistor off and the bottom one on, resulting in the output voltage of not A being driven down to ground. **Right** the logic symbol for an inverter. A logic symbol allows you to abstract from the specific electrical circuit used to implement the logic. Figure by authors.

Describing a circuit containing both true and complement of a signal as a 'contradiction' is perhaps a rather free metaphor since, while signals **A** and **not A** can exist in a logic circuit, only one of them will be asserted.

Mao emphsisizes that different contradictions have their own specific forms of 'resolution'.

> Qualitatively different contradictions can only be resolved by qualitatively different methods. For instance, the contradiction between the proletariat and the bourgeoisie is resolved by the method of socialist revolution; the contradiction between the great masses of the people and the feudal system is resolved by the method of democratic revolution; the contradiction between the colonies and imperialism is resolved by the method of national revolutionary war; the contradiction between the working class and the peasant class in socialist society is resolved by the method of collectivisation and mechanisation in agriculture;[41]

He also introduces the notion of a 'principal contradiction', by which he means, roughly, the contradiction that is most important in determining the course of social development. This notion of a *principle contradiction* was essentially related to politics. He says that for capitalism, 'the two forces in contradiction, the proletariat and the bourgeoisie, form the principal contradiction'. That is because these opposing classes shape the structure of politics. He distinguishes this from what he calls a *basic* or *fundamental* contradiction, which operates at the level of production relations and will not disappear until a given stage in the development of society is completed.[42]

> When Marx and Engels applied the law of contradiction in things to the study of the socio-historical process, they discovered the contradiction between the productive forces and the relations of production, they discovered the contradiction between the exploiting and exploited classes and also the

> resultant contradiction between the economic base and its superstructure (politics, ideology etc), and they discovered how these contradictions inevitably lead to different kinds of social revolution in different kinds of class society.
>
> When Marx applied this law to the study of the economic structure of capitalist society, he discovered that the basic contradiction of this society is the contradiction between the social character of production and the private character of ownership. This contradiction manifests itself in the contradiction between the organized character of production in individual enterprises and the anarchic character of production in society as a whole. In terms of class relations, it manifests itself in the contradiction between the bourgeoisie and the proletariat.[43]

Applying this to modern capitalism, the contradiction between the forces and relations of production produces a basic economic contradiction in the tendency of the rate of profit to fall. At the class level, it is expressed in the economic class struggle between workers and employers over wages, the working day and conditions of work. It is this class contradiction that is the principle one, overruling the class conflict between capitalists and landed property. Understanding the relationship between the contradictions explains how the economic and political working out of a crisis can proceed.

So long as the principle contradiction at the political level between the working class and the capitalists is not resolved by a socialist revolution, the basic economic contradiction persists. The tendency of the rate of profit to fall is not removed!

Dialectics as problem solving

Let us now revisit the classic three laws of the dialectic in light of the foregoing discussion. We think that these are most charitably and fruitfully considered as methods of analysis rather than of explanation.

First of all, the transformation of quantity into quality is now popularly referred to as emergence. Observing that properties change on measureable boundaries and seeking to explain why is a core part of science. However, claiming that something changes through the transformation of quantity into quality is a way of invoking magic, rather than simply acknowledging that some observable phenomenum are not yet explicable in scientific terms. Appeals to emergence are most commonly found in discussions of artificial intelligence (AI), in particular in the notion of the singularity, when AIs will achieve competences greater than humans. How the singularity will come about is elided by appeal to increasing AI success, which somehow will gather pace and lead to qualitative change. While we are certainly open to the possibility of general AI, current evidence suggests that small, and indeed impressive, domain specific achievements do not scale, generalize or integrate well with other achievements.

Similarly, the negation of the negation is often presented as the abstraction of the trinity of thesis/antithesis/synthesis. Like the transformation of quantity into quality, it is typically invoked as a post-hoc explanation. In purely logical terms, the negation of a negation returns to the starting point.

Finally, the interpenetration of opposites is a way of expressing that processes often involve countervailing sub-processes, which act on each other. In itself, seeking interacting 'opposites' is one of many approaches to problem solving through decomposition. Thus, in contemporary computational thinking, the interpenetration of opposites is a form of divide and conquer,[44] where problem decomposition into two sub-problems is the simplest case. In solving arbitrary problems, such sub-problems are usually complementary, rather than contradictory. Nonetheless, understanding social processes by seeking contradictions between social classes is the key insight of historical materialism, so ably deployed by Marx, Engels, Lenin and Mao, to interpret the world in order to change it.

NOTES

Chapter 1

1 J. Boswell, *The Life of Samuel Johnson*, vol. I (Everyman's Libray, J,. M. Dent & Sons Ltd, 1906), 292.

Chapter 2

1 West is usually used as synonymous to European and is meant to cover its transatlantic extension. We choose to operate with 'European' as it is less exclusive towards the Southeast and East of Europe, in particular because its location has been relevant for far longer than its Westernized variant. Islamic philosophy and mathematics is of critical influence for its historical continuity and also scientific advancement. Also, we are equating here philosophy with European philosophy because we argue, concurring with François Laruelle, philosophy is a European phenomenon proper, notwithstanding the Arabic contribution to its preservation and development – in fact, including it.

2 Aristotle, 'Physics', in *The Complete Works of Aristotle*, trans. R. P. Hardie and R. K. Gaye, vol. 2 (Princeton University Press, 1984), 315–446 cf WilliamDavid Ross, ed., *Aristotelis Physica Clarendon Press* (Oxford University Press, 1950) , 200b33–201a3.

3 Please note that for the classical texts, we are using marginal pagination as it is universally legible regardless of the edition and language.

4 Joan Kung, 'Aristotle on "Being Is Said in Many Ways"', *History of Philosophy Quarterly* 3, no. 1 (1986): 3–18.

5 Walther Kranz and Hermann Diels, *Die Fragmente der Vorsokratiker*, vol. 3 (Weidmann, 1952), which in the inline citations here is abbreviated as DK and the marginal pagination used in the original publication (and in all future editions).

6 Plato, *Phaedo*, edited and with commentaries by C.J. Rowe, ed. Christopher J. Rowe (Cambridge University Press, 1992), The inline reference follows the internationally accepted pagination of Plato's originals.

7 Peter Wolfendale, *Object-oriented Philosophy: The Noumenon's New Clothes*, vol. 1 (MIT Press, 2019).

8 Wilfrid Sellars, *Science, Perception, and Reality* (Routledge & Kegan Paul, 1963), 1–40.

9 Georg Wilhelm Fredrich Hegel, *Georg Wilhelm Friedrich Hegel: The Science of Logic*, trans. A. V. Miller (Oxford University Press, 1977), 82.

10 The self-abstracted entity, fixed for itself, is man as abstract egoist—egoism raised in its pure abstraction to the level of thought' (Karl Marx and Friedrich Engels, *The Economic and Philosophic Manuscripts of 1844 and the Communist Manifesto* (Prometheus Books, 2009) 'Critique of Hegel's Philosophy in General').

11 Karl Marx, Friedrich Engels, and W. Lough, *Theses On Feuerbach Ludwig Feuerbach and the End of Classical German Philosophy* (1845) s.p.: 'The chief defect of all

hitherto existing materialism — that of Feuerbach included — is that the thing, reality, sensuousness, is conceived only in the form of the object or of contemplation, but not as a sensuous human activity, practice, not subjectively. Hence, in contradistinction to materialism, the active side was developed abstractly by idealism — which, of course, does not know real, sensuous activity as such.'

12 Marx and Engels, *The Economic and Philosophic Manuscripts of 1844 and the Communist Manifesto* : 'Critique of Hegel's Philosophy in General'.

13 Rosi Braidotti and Maria Hlavajova, *Posthuman Glossary* (Bloomsbury Publishing, 2018).

14 Aristotle, 'Physics'.

15 Aristotle.

16 Aristotle, *Aristotelis Opera*, ed. Immanuelis Bekkeri, Olof Gigon, and Hermann Bonitz (Walter de Gruyter, 1970).

17 Meta. ν.2 1089a15–19, ἔπειτα ἐκ ποίου μὴ ὄντος καὶ ὄντος τὰ ὄντα; πολλαχῶς γὰρ καὶ τὸ μὴ ὄν, ἐπειδὴ καὶ τὸ ὄν· καὶ τὸ μὲν μὴ ἄνθρωπον σημαίνει τὸ μὴ εἶναι τοδί, τὸ δὲ μὴ εὐθὺ τὸ μὴ εἶναι τοιονδί, τὸ δὲ μὴ τρίπηχυ τὸ μὴ εἶναι τοσονδί.

18 This and the following translation of the original are my own.

19 Aristotle, 'On Generation and Corruption', in *Complete Works of Aristotle*, trans. Harold H. Joachim (Princeton University Press, 2014).

20 Titus Lucretius Carus, *De rerum natura Volume primo (Libri I-III)*, ed. Marcus Deufert (de Gruyter GmbH, 2019).

21 Titus Lucretius Carus, *On the Nature of Things*, trans. By William Ellery Leonard, retrieved from Project Gutenberg Release Date: 31 July 2008 [EBook #785] Last Updated: 4 February 2013, accessed in April 2021. All citations of this title are taken from this translation. Pages are not cited but rather the original marginal pagination, in line with the classical canon, in order to ensure accurate location of the original as well as be able to compare other translations

22 As already noted, the translations are taken from Titus Lucretius Carus, *On the Nature of Things*, trans. By William Ellery Leonard, retrieved from Project Gutenberg Release Date: 31 July 2008 [EBook #785] Last Updated: 4 February 2013, accessed in April 2021. All citations of this title are taken from this translation. Pages are not cited but rather the original marginal pagination, in line with the classical canon, in order to ensure accurate location of the original as well as be able to compare other translations,

23 In the Latin original: Praeterea si iam finitum constituatur; omne quod est spatium, si quis procurrat ad oras; ultimus extremas iaciatque uolatile telum; (I, 970) id ualidis utrum contortum uribus ire; quo fuerit missum mauis longeque uolare; an prohibere aliquid censes obstareque posse?; alterutrum fatearis enim sumasque necessest.; quorum utrumque tibi effugium praecludit et omne; cogit ut exempta concedas fine patere; nam siue est aliquid quod probeat officiatque; quominus quo missum est ueniat finique locet se; siue foras fertur, non est a fine profectum. ; (I, 980) hoc pacto sequar atque, oras ubicumque locaris; extremas, quaeram: quid telo denique fiet?; fiet uti nusquam possit consistere finis; effugiumque fugae prolatet copia semper; Praeterea spatium summai totius omne; (I 984) undique si inclusum certis consisteret oris; finitumque foret, iam copia materiai; undique ponderibus solidis confluxet ad imum; nec res ulla geri sub caeli tegmine posset; nec foret omnino caelum neque lumina solis; quippe ubi materies omnis cumulata iaceret (I, 990); ex infinito iam tempore subsidendo. 991(Liber I, 970–90), or in English, let's, as an exception, use Martin Ferguson Smith's translation as it's easier to follow the alignment with the

Latin original: ‘Then again, just suppose that all the existing space were finite, and that someone ran forward to the edge of its farthest border and launched a spear into flight: do you favour the view that the spear, cast with virile vigor, would fly far and reach its target, or do you suppose that something could check it by obstructing its course? You must grant and adopt one or the other of these hypotheses, and yet both deny you a subterfuge and compel you to acknowledge that the expanse of the universe is infinite. For whether there is something to check the spear and prevent it from hitting its mark and lodging in its target, or whether it flies on, it did not start from the end of the universe. In this way I will dog you: wherever you locate the farthest border, I will ask about the ultimate fate of the spear. Our conclusion will be that nowhere can a boundary be fixed: no escape will ever be-found from the limitless possibility of flight (Liber I, 968–91), from Lucretius, *On the Nature of Things*, translated with Introduction and Notes by Martin Ferguson Smith (Hackett Publishing Company, 1969).

24 Translation is taken from Titus Lucretius Carus, *On the Nature of Things*, trans. By William Ellery Leonard, retrieved from Project Gutenberg Release Date: 31 July 2008 [EBook #785] Last Updated: 4 February 2013, accessed in April 2021. All citations of this title are taken from this translation. Pages are not cited but rather the original marginal pagination, in line with the classical canon, in order to ensure accurate location of the original as well as be able to compare other translations.

25 Same as in note 24, we use William Ellery Leonard and cite the original marginal pagination.

26 The same original source as in the citations above.

Chapter 3

1 Archimedes, *The Works of Archimedes: Edited in Modern Notation with Introductory Chapters*, ed. Thomas L. Heath, Cambridge Library Collection – Mathematics (Cambridge University Press, 2009), https: //doi.org/10.1017/CBO9780511695124.

2 Lucio Russo, *The Forgotten Revolution: How Science Was Born in 300 BC and Why it Had to be Reborn* (Springer Science & Business Media, 2013).

3 Rocco Leonardis, ‘The Use of Geometry by Ancient Greek Architects’, *A Companion to Greek Architecture* (2016): 92–104, https://doi.org/10.1002/9781118327586.

4 Galen and Arthur John Brock, *Galen On the Natural Faculties with an English Translation* (W. Heinemann, 1916), Book I, Chapter XIII.

5 Galen and Brock, *Galen On the Natural Faculties with an English Translation*, Book I, Chapter XIV.

6 The term pneuma was Greek for air.

7 C. E. Quin, ‘The Soul and the Pneuma in the Function of the Nervous System after Galen’, *Journal of the Royal Society of Medicine* 87, no. 7 (1994): 393–5, issn: 0141-0768, https://europepmc.org/articles/PMC1294649.

8 There is a similarity here with the nineteenth-century notion of Dr Frankenstein animating a corpse with the newly discovered electric energy.

9 Russo, *The Forgotten Revolution: How Science Was Born in 300 BC and Why It Had to Be Reborn*.

10 Isaac Newton, *The Mathematical Principles of Natural Philosophy . . . Translated . . . by Andrew Motte. To Which Are Added Newton’s System of the World; a Short Comment*

on, and Defence of the Principia, by W. Emerson. With the Laws of the Moon's Motion According to Gravity, by John Machin . . . (The Preface of Mr) (1848).

11 G. W. Leibniz, 'Discourse on Metaphysics', 1686, http://www.earlymoderntexts.com/assets/pdfs/leibniz1686d.pdf.

12 Russo, *The Forgotten Revolution: How Science Was Born in 300 BC and Why It Had to Be Reborn,* explains the similarity between Ptolemaic astronomy and Fourier decomposition.

13 We would say straight.

14 A sidereal day is measured in terms of the apparent rotation of the stars with respect to the Earth and is about 4 minutes shorter than a solar day.

15 'The spaces described by a body falling from rest with a uniformly accelerated motion are to each other as the squares of the time-intervals employed in traversing these distances', Theorem II, Proposition 2.

16 Newton, *The Mathematical Principles of Natural Philosophy . . . Translated . . . by Andrew Motte. To Which are Added Newton's System of the World; a Short Comment on, and Defence of the Principia, by W. Emerson. With the Laws of the Moon's Motion According to Gravity, by John Machin . . . (The Preface of Mr)*, Law I.

17 Newton, *The Mathematical Principles of Natural Philosophy . . . Translated . . . by Andrew Motte. To Which are Added Newton's System of the World; a Short Comment on, and Defence of the Principia, by W. Emerson. With the Laws of the Moon's Motion According to Gravity, by John Machin . . . (The Preface of Mr)*, Law II.

18 Newton, *The Mathematical Principles of Natural Philosophy . . . Translated . . . by Andrew Motte. To Which are Added Newton's System of the World; a Short Comment on, and Defence of the Principia, by W. Emerson. With the Laws of the Moon's Motion According to Gravity, by John Machin . . . (The Preface of Mr)*, Law III.

19 Newton, *The Mathematical Principles of Natural Philosophy . . . Translated . . . by Andrew Motte. To Which are Added Newton's System of the World; a Short Comment on, and Defence of the Principia, by W. Emerson. With the Laws of the Moon's Motion According to Gravity, by John Machin . . . (The Preface of Mr)*, Corrollary III.

20 Newton, *The Mathematical Principles of Natural Philosophy . . . Translated . . . by Andrew Motte. To Which are Added Newton's System of the World; a Short Comment on, and Defence of the Principia, by W. Emerson. With the Laws of the Moon's Motion According to Gravity, by John Machin . . . (The Preface of Mr)*, Corrollary V.

21 Relatively to things that lack it, and among things which all share equally in any motion, it does not act and is as if it did not exist. Thus the goods with which a ship is laden leaving Venice, pass by Corfu, by Crete, by Cyprus and go to Aleppo. Venice, Corfu, Crete etc., stand still and do not move with the ship; but as to the sacks, boxes and bundles with which the boat is laden and with respect to the ship itself, the motion from Venice to Syria is as nothing, and in no way alters their relations among themselves. This is so because it is common to all of them and all share equally in it. Galileo Galilei, *Dialogue Concerning the Two Chief World Systems, (1632)* (Robert C. Kuhmann, 2006), 41, http://www.kuhmann.com/starstuf f/DialogueConcerning theTwoChiefWorldSystems.pdf.

For a simple movable body, there can be only a single motion, and no more, which suits it naturally; any others it can possess only incidentally and by participation. Thus, when a man walks along the deck of a ship, his own motion is that of walking, while the motion which takes him to port is his participation; for he could never arrive there by walking if the ship did not take him there by means of its motion. Galilei, *Dialogue Concerning the Two Chief World Systems*, 45.

22 Newton, *The Mathematical Principles of Natural Philosophy . . . Translated . . . by Andrew Motte. To Which are Added Newton's System of the World; a Short Comment on, and Defence of the Principia, by W. Emerson. With the Laws of the Moon's Motion According to Gravity, by John Machin . . . (The Preface of Mr)*, 183.
23 Sesquiduplicate ratio means $r^{2.5}$
24 Th. is means in the ratio of the square roots, the subduplicate ratio of $r_1 : r_2 = \sqrt{r_1} : \sqrt{r_2}$ so if the ratio were 18:16, the. subduplicate ratio would be 9:4.
25 Newton, *The Mathematical Principles of Natural Philosophy . . . Translated . . . by Andrew Motte. To Which are Added Newton's System of the World; a Short Comment on, and Defence of the Principia, by W. Emerson. With the Laws of the Moon's Motion According to Gravity, by John Machin . . . (The Preface of Mr)*, PROPOSITION IV. THEOREM IV..
26 Georg Wilhelm Friedrich Hegel, 'Philosophical Dissertation on the Orbits of the Planets (1801), Preceded by the 12 Theses Defended on August 27, 1801', *Graduate Faculty Philosophy Journal* 12, no. 1/2 (1987): 282–3.
27 Hegel, 'Philosophical Dissertation on the Orbits of the Planets (1801), Preceded by the 12 Theses Defended on August 27, 1801', 285.
28 Hegel, 'Philosophical Dissertation on the Orbits of the Planets (1801), Preceded by the 12 Theses Defended on August 27, 1801', 285.
29 Hegel, 'Philosophical Dissertation on the Orbits of the Planets (1801), Preceded by the 12 Theses Defended on August 27, 1801', 290.
30 Hegel, 'Philosophical Dissertation on the Orbits of the Planets (1801), Preceded by the 12 Theses Defended on August 27, 1801', 296.
31 Vladimir Ilich Lenin, *The Three Sources and Three Component Parts of Marxism*, vol. 1 (Foreign Languages Press, 1977).
32 Hegel, 'Philosophical Dissertation on the Orbits of the Planets (1801), Preceded by the 12 Theses Defended on August 27, 1801', 294.
33 A. Emery, 'Dialectics versus Mechanics. A Communist Debate on Scientific Method', *Philosophy of Science* 2, no. 1 (1935): 9–38.
34 'An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line.'
35 Emery, 'Dialectics versus Mechanics. A Communist Debate on Scientific Method'.
36 Available in English in Emilie Du Châtelet, *Selected Philosophical and Scientific Writings* (University of Chicago Press, 2009).
37 Du Châtelet, *Selected Philosophical and Scientific Writings*, Foundations of Physics, Preface IV.
38 Ruth Hagengruber, *Emilie du Châtelet Between Leibniz and Newton* (Springer, 2012).
39 Du Châtelet, *Selected Philosophical and Scientific Writings*, 132, her argument against vegetative souls can clearly be applied to other variants of souls and similar religious explanations: the spirit, free will, the subject, gender identities etc.
40 Du Châtelet, *Selected Philosophical and Scientific Writings*, 149.
41 Du Châtelet, *Selected Philosophical and Scientific Writings*, 166.
42 Figure by authors.
43 Du Châtelet, *Selected Philosophical and Scientific Writings*, 188.
44 Du Châtelet, *Selected Philosophical and Scientific Writings*, 199.
45 Du Châtelet, *Selected Philosophical and Scientific Writings*, 199.
46 See the proof in John R. Taylor, et al., *Classical Mechanics* (University Science Books, 2005), .217.

47 Action is defined as $S = \int \mathcal{L}\,dt$ where $\mathcal{L}$. the Lagrangian, stands for the difference between kinetic and potential energy. For a detailed treatment, see see Taylor et al., *Classical Mechanics*, chap. 6, 7, 13.
48 See for instance *6 jun 2015 La ciudad como reivindicación identitaria: el caso de la New Town de Edimburgo.* at https://urban-networks.blogspot.com/2015/06/la-ciudad-como-reivindicacion.html.
49 Kord Ernstson et al., 'The Chiemgau Crater Strewn Field: Evidence of a Holocene Large Impact Event in Southeast Bavaria, Germany', *Journal of Siberian Federal University. Engineering & Technologies* 1, no. 3 (2010): 72–103; Barbara Rappenglück et al., 'The Fall of Phaethon: A Greco-Roman Geomyth Preserves the Memory of a Meteorite Impact in Bavaria (South-East Germany)', *Antiquity* 84, no. 324 (2010): 428–39.
50 These were named after al-Khwārizmī, whose method of arithmetic was introduced to Europe in Fibonnaci's *Liber Abaci*.

Chapter 4

1 Aristotle, *Ars Rhetorica,* ed. W. D. Ross (Clarendon Press, 1959), 1: 1.
2 Matthewe Duncombe and Catarina Dutilh Novaes, 'Dialectic and Logic in Aristotle and his Tradition', *History and Philosophy of Logic* 37, no. 1 (2015): 1–8.
3 Aristotle, *Rhetoric*, trans. W. Rhys Roberts (Bexar County, TX: Bibliotech Press, 2012), retrieved from *The Internet Classics Archive*, http://classics.mit.edu/Aristotle/rhetoric.1.i.html (accessed on 27 November 2021), Book 1: Part 1.
4 The distinction at issue is discussed in more detail below, where citations are provided, both in English translation and the original.
5 From Marx's dissertation on contradiction, objectification of form and objectification of relation: 'Through the qualities the atom acquires an existence which contradicts its concept; it is assumed as an externalised being different from its essence. It is this contradiction which mainly interests Epicurus. Hence, as soon as he posits a property and thus draws the consequence of the material nature of the atom, he counterposits at the same time determinations which again destroy this property in its own sphere and validate instead the concept of the atom. He therefore determines all properties in such a way that they contradict themselves. Democritus, on the other hand, nowhere considers the properties in relation to the atom itself, nor does he objectify the contradiction between concept and existence which is inherent in them'. Citation from Karl Marx, *The Difference Between the Democritean and Epicurean Philosophy of Nature*, Marx-Engels Collected Works Volume 1 (Progress Publishers, 1975), s.p., retrieved from the Karl Marx Internet Archive, https://www.marxists.org/archive/marx/works/1841/dr-theses/index.htm (accessed on 27 November 2021).
6 We will resort here to the method of unilateralization or dualysis elaborated by François Laruelle which is premised on dissembling philosophical 'worlds' – per definition postulating the real and treating that postulate as the real itself, the problem of amphibology and 'decisionism', Laruelle problematizes in many of his works, in particular more closely in the opening chapters of his *Philosophy and Non-philosophy*, trans. Taylor Adkins (University of Minnesota Press, 2013).
7 Ludwig Wittgenstein, *Tractatus Logico-Philosophicus* (Kegan Paul, Trench, Trubner & Co., Ltd., 1922), Gutenberg project (accessed on 27 November 2021): 2.150, 2.511, 2.1512, et passim.

8 Aristotle, *Ars Rhetorica*, 1:1, section 14, http://www.perseus.tufts.edu/hopper/text?doc=Perseus%3Atext%3A1999.01.0059%3Abook%3D1%3Achapter%3D1%3Asection%3D14 (accessed on 27 November, 2021).

9 W. Rhys Roberts' translation is retrieved from the Internet Classics Archive of MIT at http://classics.mit.edu/Aristotle/rhetoric.1.i.html (accessed on 27 November 2021).

10 The quote is from W. Rhys Roberts' translation; in its Greek original, it reads:ἐπεὶ δ᾽ αἱ πίστεις διὰ τούτων εἰσί, φανερὸν ὅτι ταύτας ἐστὶ λαβεῖν τοῦ συλλογίσασθαι δυναμένου καὶ τοῦ θεωρῆσαι περὶ τὰ ἤθη καὶ περὶ τὰς ἀρετὰς καὶ τρίτον τοῦ περὶ τὰ πάθη, τί τε ἕκαστόν ἐστιν τῶν παθῶν καὶ ποῖόν τι, καὶ ἐκ τίνων ἐγγίνεται καὶ πῶς, ὥστε συμβαίνει τὴν ῥητορικὴν οἷον παραφυές τι τῆς διαλεκτικῆς εἶναι καὶ τῆς περὶ τὰ ἤθη πραγματείας, ἣν δίκαιόν ἐστι προσαγορεύειν πολιτικήν. διὸ καὶ ὑποδύεται ὑπὸ τὸ σχῆμα τὸ τῆς πολιτικῆς ἡ ῥητορικὴ καὶ οἱ ἀντιποιούμενοι ταύτης τὰ μὲν δι᾽ ἀπαιδευσίαν, τὰ δὲ δι᾽ ἀλαζονείαν, τὰ δὲ καὶ δι᾽ ἄλλας αἰτίας ἀνθρωπικάς: ἔστι γὰρ μόριόν τι τῆς διαλεκτικῆς καὶ ὁμοίωμα, καθάπερ καὶ ἀρχόμενοι εἴπομεν: περὶ οὐδενὸς γὰρ ὡρισμένου οὐδετέρα αὐτῶν ἐστιν ἐπιστήμη πῶς ἔχει, ἀλλὰ δυνάμεις τινὲς τοῦ πορίσαι λόγους.The source in Greek is: Aristotle, *Ars Rhetorica,* and it is retrieved from the Perseus Tufts internet archive available at http://www.perseus.tufts.edu/hopper/text?doc=Perseus%3Atext%3A1999.01.0059 (accessed on 21 November 2021), 1: 2, section 7.

11 The quote is from W. Rhys Roberts' translation, retrieved from the Internet Classics Archive of MIT, available at http://classics.mit.edu/Aristotle/rhetoric.1.i.html (accessed on 21 November 2021); in its Greek original, it reads:ὅτι μὲν οὖν οὐκ ἔστιν οὐθενός τινος γένους ἀφωρισμένου ἡ ῥητορική, ἀλλὰ καθάπερ ἡ διαλεκτική, καὶ ὅτι χρήσιμος, φανερόν, καὶ ὅτι οὐ τὸ πεῖσαι ἔργον αὐτῆς, ἀλλὰ τὸ ἰδεῖν τὰ ὑπάρχοντα πιθανὰ περὶ ἕκαστον, καθάπερ καὶ ἐν ταῖς ἄλλαις τέχναις πάσαις (οὐδὲ γὰρ ἰατρικῆς τὸ ὑγιᾶ ποιῆσαι, ἀλλὰ μέχρι οὗ ἐνδέχεται, μέχρι τούτου προαγαγεῖν: ἔστιν γὰρ καὶ τοὺς ἀδυνάτους μεταλαβεῖν ὑγιείας ὅμως θεραπεῦσαι καλῶς): πρὸς δὲ τούτοις ὅτι τῆς αὐτῆς τό τε πιθανὸν καὶ τὸ φαινόμενον ἰδεῖν πιθανόν, ὥσπερ καὶ ἐπὶ τῆς διαλεκτικῆς συλλογισμόν τε καὶ φαινόμενον συλλογισμόν: ἡ γὰρ σοφιστικὴ οὐκ ἐν τῇ δυνάμει ἀλλ᾽ ἐν τῇ προαιρέσει: πλὴν ἐνταῦθα μὲν ἔσται ὁ μὲν κατὰ τὴν ἐπιστήμην ὁ δὲ κατὰ τὴν προαίρεσιν ῥήτωρ, ἐκεῖ δὲ σοφιστὴς μὲν κατὰ τὴν προαίρεσιν, διαλεκτικὸς δὲ οὐ κατὰ τὴν προαίρεσιν ἀλλὰ κατὰ τὴν δύναμιν. The source in Greek is: Aristotle, *Ars Rhetorica*, and it is retrieved from the Perseus Tufts internet archive available at http://www.perseus.tufts.edu/hopper/text?doc=Perseus%3Atext%3A1999.01.0059 (accessed on 21 November 2021), 1: 1, section 14.

12 Ferdinand de Saussure, *Course in General Linguistics*, trans. Wade Baskin, ed. Charles Bally and Albert Reidlinger (Philosophical Library, 1959), 53.

13 Karl Marx, *Grundrisse: Outlines of the Critique of Political Economy*, trans. Martin Nicolaus (Penguin Books, 1973), 41.

14 Laruelle, *Philosophy and Non-Philosophy*, 91–126.

15 Yuk Hui, *Recursivity and Contingency* (Rowman & Littlefield International, 2019), 41.

16 Paul Cockshott, 'Turing: The Irruption of Materialism into Thought', Oxford University Press's Blog: Academic Insights for the Thinking World (2012), https://blog.oup.com/2012/06/turing-the-irruption-of-materialism-into-thought/ (accessed on 27 November 2021).

17 Katerina Kolozova, *Toward a Radical Metaphysics of Socialism: Marx and Laruelle* (Punctum Books, 2015), 8–10.

18 Karl Marx, *Theses on Feuerbach*, translated from the German by W. Lough (Progress Publishers, 1969), s.p., https://www.marxists.org/archive/marx/works/1845/theses

/theses.htm (accessed on 22 March 2019). This citation is retrieved from an online source without pagination.

19 Thomas Nail, *Marx in Motion* (Oxford University Press. Kindle Edition, 2020), 20–1.

20 Alfred Sohn-Rethel, *Intellectual and Manual Labor: Critique of Epistemology* (London: Macmillan: 1978).

21 Karl Marx, 'The Difference Between the Democritean and Epicurean Philosophy of Nature', *Marx-Engels Collected Works* Vol. 1 (Moscow: Progress Publishers, 1975), s.p., retrieved from the Karl Marx Internet Archive, https://www.marxists.org/archive/marx/works/1841/dr-theses/index.htm, accessed on 21 November 2021. The online version is without pagination (s.p.); the quote is from Part 2 the Chapter 3 of the Dissertation.

22 Marx, *Dissertation*, s.p.: 'Difficulties Concerning the Identity of the Democritean and Epicurean Philosophy of Nature'.

23 Laruelle, *Philosophy and Non-Philosophy*, 51–3, 227, 228.

24 Laruelle, *Philosophy and Non-Philosophy*, 10.

25 Laruelle, *Philosophy and Non-Philosophy*, 12–13, 20, 24–29 et passim.

26 Sohn-Rethel, *Intellectual and Manual Labor*, 13–35.

27 We dare call it a dogma considering the fact that the USSR as a state, its institutions and the ruling party, and through that the Comintern, carefully curated a particular reading of Marx's and Engel's original writing, mainly based on Lenin's *Materialism and Empirio-criticism* (originally published in 1909).

28 Vladimir Ilych Lenin, *Collected Works Vol. 14*: *Materialism and Empirio-criticism* (Progress Publishers, 1977).

29 Lenin, *Materialism and Empirio-criticism*, 117–26.

30 Lenin, *Materialism and Empirio-criticism*, 341.

31 Evald Ilyenkov, *The Dialectics of the Abstract & the Concrete in Marx's Capital*, trans. Sergei Kuzyakov (Progress Publishers, 1982), https://www.marxists.org/archive/ilyenkov/works/abstract/index.htm (accessed on 27 February 2022), Ch. 1, s.p.

32 François Laruelle, *Theory of Identities*, trans. Alyosha Edlebi (Columbia University Press, 2016), 59, 120, 135–40.

33 Laruelle, *Philosophy and Non-Philosophy*, 22 et passim.

34 Wittgenstein, *Tractatus Logico-Philosophicus*, trans. Frank P. Ramsey and Charles Kay Ogden, 2.1512; or: 'scale applied to reality', in German: Es ist wie ein Maßstab an die Wirklichkeit angelegt (2.1512).

35 François Laruelle, *Introduction to Non-Marxism*, trans. Anthony Paul Smith (University of Minnesota Press-Univocal Publishing, 2014).

36 Tony Burns, 'Joseph Dietzgen and the History of Marxism', *Science & Society* 66, no. 2 (2002): 202–27.

37 Ilyenkov, *The Dialectics of the Abstract & the Concrete in Marx's Capital*, Ch. 1, s.p.

38 Lenin, *Materialism and Empirio-Criticism*, 45–6; cf.: 'The difference between materialism and "Machism" in this particular question thus consists in the following. Materialism, in full agreement with natural science, takes matter as primary and regards consciousness, thought, sensation as secondary, because in its well-defined form sensation is associated only with the higher forms of matter (organic matter), while "in the foundation of the structure of matter" one can only surmise the existence of a faculty akin to sensation' (46).

39 Alberto Toscano, 'The Open Secret of Real Abstraction', *Rethinking Marxism* 20, no. 2 (2008): 282–7.

40 Sohn-Rethel to Adorno cited in Toscano's 'The Open Secret of the Real Abstraction', 281.
41 Toscano, 'The Open Secret of Real Abstraction', quoting Sohn-Rethel to Adorno: Alfred Sohn-Rethel and Theodor Adorno, *Carteggio 1936/1969* (manifestolibri, 2000), 112.
42 Karl Marx, *Capital*, vol. I, Part 2: Chapter 4, trans. I. Lasker (Progress Publishers, 1956), https://www.marxists.org/archive/marx/works/1885-c2/ (accessed on 12 May 2022).

Chapter 5

1 Friedrich Engels and Tristram Hunt, *The Origin of the Family, Private Property and the State* (Penguin, 2010).
2 As the name implies, they were the lectures he gave a Professor of Philosophy at the University of Glasgow. In the mid-twentieth century, transcripts of the lectures were discovered in notebooks of John Anderson who had been one of Smith's students and later became a professor himself.
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11 It is interesting to note that this same process is occurring in the American Empire. Its citizens, having grown less fond of the hardships of war, from the 1980s, it increasingly resorted to hiring Afghans and Arabs to fight; bin Laden in due course turned his followers on his erstwhile paymasters. If history is any judge, that will not be the last time something similar occurs.
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25 'The difference between materialism and "Machism" in this particular question thus consists in the following.Materialism, in full agreement with natural science, takes matter as primary and regards consciousness, thought, sensation as secondary, because in its well-defined form sensationist associated only with the higher forms of matter (organic matter), while "in the foundation of the structure of matter" one can only surmise the existence of a faculty akin to sensation. Such, for example, is the supposition of the well-known German scientist Ernst Haeckel, the English biologist Lloyd Morgan and others, not to speak of Diderot's conjecture mentioned above"' (A quote from V. I. Lenin, *Collected Works: Materialism and Empirio-criticism,* vol. 14 [Progress Publishers, 1977], 46).
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Katerina Kolozova, *Toward Radical Metaphysics of Socialism: Marx and Laruelle* (Punctum Books, 2015).37 Diderot, 'Conversation Between D'Alembert and Diderot (1769)', fhttps://www.marxists.org/reference/archive/diderot/1769/conversation.htm (accessed on 12 December 2023).
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121 Thomson, 'The Kinetic Theory of the Dissipation of Energy'.

122 This state is a mirror image because every atom is now moving in the reverse of its original direction.

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Chapter 6

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22 Einstein, 'Concerning an Heuristic Point of View Toward the Emission and Transformation of Light'.
23 $S - S_o = k\ln\left(\frac{v}{v_o}\right)^n$ where $n$is the number of gas molecules. This was clearly the same functional form as the equation for change in electromagnetic entropy he gave earlier.
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25 Einstein, 'Does the Inertia of a Body Depend Upon its Energy-content'.
26 Readers who have not done so may well enjoy A. Einstein, *Relativity* (Methuen/ Company, 1920) where he gave an account for the non-specialist readership.
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35 Mach, 'Popular Scientific Lectures'.
36 Here Lenin is quoting the eighteenth-century materialist Diderot:

> Et pour donner à mon système toute force, remarquez encore qu'il est sujet à la même difficulté insurmontable que Berkeley a proposée contre l'existence des corps. Il y a un moment de délire où le clavecin sensible a pensé qu'il était le seul clavecin qu'il y eût au monde, et que toute l'harmonie de l'univers se passait en lui. Denis Diderot et al., *Entretien entre d'Alembert et Diderot* (Garnier-Flammarion, 1965).

37 Mach, 'Popular Scientific Lectures'.

38 According to Cercignani and Penrose, *Ludwig Boltzmann: The Man Who Trusted Atoms* Boltzmann took a very similar standpoint, seeing his opponents as Berkeleyian idealists:

> The central point of philosophy is, according to Boltzmann, the problem of the relationship between existence and knowledge, gnoseology (this term is usually rendered as epistemology in the English translation [2], although this somehow restricts its scope). To characterise his attitude in this respect, nothing serves better than a sentence thrown almost casually into the least philosophical and most entertaining of his essays, describing his trip to 'Eldorado' in 1905. This sentence is frequently quoted and follows his description of the campus of the University of California at Berkeley: 'The name of Berkeley is that of a highly esteemed English philosopher, who is even credited with being the inventor of the greatest folly ever hatched by a human brain, of philosophical idealism that denies the existence of the material world'. Cercignani and Penrose, *Ludwig Boltzmann: The Man Who Trusted Atoms*, 170

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52 Bohr had apparently not considered this sort of example until later discussions with Einstein, Jan Faye, 'Copenhagen Interpretation of Quantum Mechanics', in *The Stanford Encyclopedia of Philosophy*, Winter 2019, ed. Edward N. Zalta (Metaphysics Research Lab, Stanford University, 2019).
53 $\Delta t\Delta E = \Delta x\Delta I_x = \Delta y\Delta I_y = \Delta z\Delta I_z = h$

54 Bohr et al., *The Quantum Postulate and the Recent Development of Atomic Theory*.

55 Heisenberg had derived this the year earlier by a different method involving a thought experiment in which you attempt to fix the position of an electron by illuminating it with high-frequency radiation. The more precisely you want to fix its position, the higher the light frequency you need. This higher-frequency light perturbs the motion of the electron, giving the inverse relationship between positional and momentum uncertainty. Bohr's direct derivation from the properties of waves is more general.

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61 Howard ('Who invented the "Copenhagen Interpretation"? A Study in Mythology') claims that the term Copenhagen interpretation was invented by Heisenberg in a 1955 paper Werner Heisenberg, 'The Development of the Interpretation of the Quantum Theory', *Niels Bohr and the Development of Physics*, 1955, 12 in which he attempted to project his own opinions on to the Danish Bohr:

> What was new in 1955 was Heisenberg's dubbing his amalgam of ideas the 'Copenhagen Interpretation', but having so dubbed it, Heisenberg regularly reinforced the invention of a unitary Copenhagen point of view and posed as its chief spokesperson (see eg Heisenberg [1955]). Why? It helps to recall Heisenberg's situation in 1955, especially the fact that the person who was Bohr's favorite in the 1920s had become a moral exile from the Copenhagen inner circle in the postwar period, mainly because of the bitter rupture in Heisenberg's relationship with Bohr during his ill-fated visit to Copenhagen in September 1941 after taking over the leadership of the German atomic bomb project. What better way for a proud and once ambitious Heisenberg to reclaim membership in the Copenhagen family than by making himself the voice of the Copenhagen interpretation? Howard, 'Who Invented the "Copenhagen Interpretation"? A Study in Mythology'

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Chapter 9

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66 A. A. Markov Jr and N. M. Nagorny, *The Theory of Algorithms*, trans. M. Greendlinger (Kluwer Academic, 1988), 1.
67 Markov Jr and Nagorny, *The Theory of Algorithms*, 10.
68 Markov Jr and Nagorny, *The Theory of Algorithms*, 10.
69 Markov Jr and Nagorny, *The Theory of Algorithms*, 10.
70 Markov Jr and Nagorny, *The Theory of Algorithms*, 12.
71 Markov Jr, *The Theory of Algorithms*, 1.
72 Markov Jr and Nagorny, *The Theory of Algorithms*, 109.
73 V. F. Turchin, 'A Meta-algorithmic Language', *Kibernetika* 4, no. 4 (1967): 45–54.
74 B. A. Kushner, 'The Constructive Mathematics of A. A. Markov', *The American Mathematical Monthly* 113, no. 6 (2006): 562.
75 Sanin, 'Constructive Real Numbers and Constructive Number Spaces', 13.
76 Markov Jr, *The Theory of Algorithms*, 441.

Chapter 10

1 But won't.
2 B. Russell, 'On Denoting', *Mind* 14, no. 56 (1905): 479–93.
3 R. Carnap, *The Logical Structure of the World; Pseudoproblems in Philosophy*, trans. R. A. George (University of California Press, 1967).

4 W. V.-O. Quine, *From A Logical Point of View: Logico-philosophical Essays* (Harper Torchbook, 1963).
5 Wittgenstein, *Tractatus Logico-Philosophicus*.
6 R. Carnap, *Meaning and Necessity: A Study in Semantics and Modal Logic* (University of Chicago Press, 1947).
7 English rather than Hegel's preferred German.
8 Carnap, *The Logical Structure of the World; Pseudoproblems in Philosophy*, viii.
9 M. Przelecki, *The Logic of Empirical Theories* (Routledge and Kegan Paul, 1969).
10 A. Badiou, *The Concept of Model: An Introduction to the Materialist Epistemology of Mathematics*, trans. Z. L. Fraser and T. Tao (re.press, 2007).
11 Italics in original.
12 J. E. Hopcroft and J. E. Ullman, *Formal Languages and Their Relation to Automata* (Addison-Wesley, 1969).
13 L. Wittgenstein, *Philosophical Investigations*, trans. G. E. M. Anscombe (Blackwell, 1958).
14 N. Chomsky, 'On Certain Formal Properties of Grammars', *Information and Control* 2 (1959): 137–67.
15 Hopcroft and Ullman, *Formal Languages and Their Relation to Automata*.
16 Cockshott, Mackenzie, and Michaelson, *Computation and Its Limits*.

Chapter 11

1 Shannon, 'A Mathematical Theory of Communication'.
2 Attempts to define information in probability terms had been started by Fisher in the previous decade Ronald Aylmer Fisher, 'Probability Likelihood and Quantity of Information in the Logic of Uncertain Inference', *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character* 146, no. 856 (1934): 1–8, but Shannon's treatment was much more general and of more lasting influence.
3 Five characters per word, plus carriage return and line feed characters at the end of lines so $(5+2)\times 5 = 35$ binary digits per word, × three words.
4 The existence of slight deviations from uniformity in the random numbers used to encrypt Baudot code in the German 'Fish' encoding system, along with the unequal frequency of letters in the German language was what enabled the first digital computer Colossus to break the encoded messages B Jack Copeland, *Colossus: The secrets of Bletchley Park's Code-breaking Computers* (Oxford University Press, 2010). It is worth noting that Shannon had cooperated with Turing on cryptography during the war Andrew Hodges, *Alan Turing the Enigma* (Touchstone, 1983) and Turing was closely familiar with the Colossus approach.
5 Apart from the use of summation over a discrete space rather than integration over a continuous one.
6 Claude Shannon, 'The Lattice Theory of Information', *Transactions of the IRE Professional Group on Information Theory* 1, no. 1 (1953): 105–7.
7 Rachel R. Bennett and Ramin Golestanian, 'A Steering Mechanism for Phototaxis in Chlamydomonas', *Journal of The Royal Society Interface* 12, no. 104 (2015): 20141164.
8 In fact, as Bennett and Golestanian show, *Chlamydomonas* has a more efficient encoding. It has a single eye spot and swims in a spiral path spinning on its axis as it does so. This enables the one eye spot to perform time-multiplexed scanning of the three spatial axes with a sampling interval of about 60 ms. Assuming that the eye spot activates for $\frac{1}{6}$ of possible orientations and using Shannon's formula the mean

information content of a sample is 0.65 bits so the eye spot provides about 10 bits per second of information.

9 Mutual information between two random variables X, Y is denoted by $I(X;Y) = H(X) - H(X|Y)$ where $H(X)$ is the information entropy of X and $H(X|Y) = \sum_{x,y} p(x,y)\log_2 \frac{1}{p(x|y)}$ is the conditional entropy of X given Y.

10 Lenin, *Materialism and Empirio-criticism*.

11 This intuition is perhaps why some popular accounts refer to information as negative entropy (negentropy). This usage of the term stems from a misreading of Leon Brillouin, 'The Negentropy Principle of Information', *Journal of Applied Physics* 24, no. 9 (1953): 1152–63. What Brillouin did was introduce a new variable $N = -S$ as part of a generalization of the second law of thermodynamics:

> We may define the quantity N (negentropy)
>
> $$N = -S, \Delta(N + I) \leq O \qquad (7b)$$
>
> The sum of negentropy plus information must always decrease. Negentropy corresponds to the 'grade' of energy in Kelvin's principle of 'degradation of energy',
>
> Brillouin

From this he was able to draw conclusions about the entropic effect of observations that obtain information:

> Any experiment by which an information is obtained about a physical system corresponds in average to an increase of entropy in the system or in its surroundings. This average increase is always larger than (or equal to) the amount of information obtained.
>
> Brillouin

He was able to use this relation to demonstrate that Maxwell's daemon was unfeasible.

12 For an extended analysis of this alternation between entropy reduction and information addition in capitalist industry see Cockshott et al., *Classical Econophysics: Essays on Classical Political Economy, Thermodynamics and Information Theory*.

13 Since 2018 Intel processors have supported the RDRAND instruction which provides what are claimed to be true random numbers Intel, *Intel64 and IA-32 Architectures Software Developer's Manual*, vol. 2B (Intel, 2021).

14 We started at the third digit to prevent our source being obvious.

15 Gregory J. Chaitin, 'Randomness and Mathematical Proof', *Scientific American* 232, no. 5 (1975): 47–53.

16 Josif Vissarionovič Stalin, *Dialectical and Historical Materialism* (Lawrence & Wishart, 1943).

17 Stalin, *Dialectical and Historical Materialism*, 4.

18 See Emery, 'Dialectics versus Mechanics. A Communist Debate on Scientific Method', to whom we have refered in an earlier chapter.

19 Bohm, *Wholeness and the Implicate Order*.

20 Stalin, *Dialectical and Historical Materialism*, 5.

21 Titus Lucretius Carus, *The Complete Works of Lucretius* (Delphi Classics, 2015), H. A. J. Munro translation.

22 Friedrich Engels, *Dialectics of Nature* (Wellred Books, 1960), Chap 2.
23 The issue of continuous versus discrete energy levels associated with black body radiation like that from a bulb, was, as we have recounted earlier, fundamental to Einstein's validation of atomism against the Machian school.
24 Engels, *Dialectics of Nature*, Chap 2.
25 For instance in Charles Merivale, *A History of the Romans under the Empire* (Longmans, 1851) we have:

> But this was not a time in which such injustice could be perpetrated with impunity. The elements of war were fermenting throughout the peninsula; thousands of broken fortunes were suddenly thrown into the floating mass of discontent, which wanted only this ingredient to boil over in a new revolution. (223)

26 Ian P Gent et al., 'Scaling Effects in the CSP Phase Transition', in *International Conference on Principles and Practice of Constraint Programming* (Springer, 1995), 70–87; P. Prosser, 'An Empirical Study of Phase Transitions in Binary Constraint Satisfaction Problems', *Artificial Intelligence* 81, no. 1–2 (1996): 81–109.
27 Suppose each sum term in F contains 3 variables (3SAT) then a measure of the constrainedness might be the ratio of the number of sum terms in F to the number of variable *n*.
28 Stalin, *Dialectical and Historical Materialism*.
29 Stalin, *Dialectical and Historical Materialism*.
30 Karl Marx and Frederick Engels, *The German Ideology* (Progress Publishers, 1976).
31 Tse-Tung Mao, *On Contradiction, in Selected Works of Mao Tse-Tung*, vol. 1 (1937).
32 Peter Turchin and Sergey A Nefedov, 'Secular Cycles', in *Secular Cycles* (Princeton University Press, 2009).
33 Mao only quotes items 1 to 5, we include the surrounding paragraphs for context, since Mao appears elsewhere in the text to implicitly rely on this paragraph. The explicit numbering of items 1 to 5 is by us, in the Lenin text they are rendered simply as list.
34 V. I. Lenin, 'On the Question of Dialectics', in *Lenin Collected Works*, by Lenin, vol. 38 (Progress Publishers, 1915).
35 Mao, *On Contradiction*, 321.
36 'Omnis enim Philosophiæ difficultas in eo versari videtur, ut a Phænomenis motuum investigemus vires Naturæ, deinde ab his viribus demonstremus phænomena reliqua'.
37 Mao, *On Contradiction*, 321.
38 Paul Cockshott, Allin Cottrell, and Greg Michaelson, 'Testing Marx: Some New Results from UK Data', *Capital & Class* 19, no. 1 (1995): 103–30.
39 Cockshott et al., *Classical Econophysics: Essays on Classical Political Economy, thermodynamics and Information Theory*; Allin Cottrell and Paul Cockshott, 'Demography and the Falling Rate of Profit', *Indian Development Review* 4, no. 1 (2006): 39–59 .
40 Mao, *On Contradiction*, 322.
41 Mao, *On Contradiction*, 321.
42 Mao, *On Contradiction*, 325.
43 Mao, *On Contradiction*, 328–9.
44 G. Michaelson, 'Programming Paradigms, Turing Completeness and Computational Thinking', *The Art, Science, and Engineering of Programming* 4, no. 3 (2020): Article 4, https://arxiv.org/abs/2002.06178.

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