

Toward a theory of planning

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1 Introduction

Planning refers to the elaboration of an explicit set of decisions concerning the present and future values of certain choice variables by a decision maker (planner) in order to achieve certain goals. Planning, therefore, involves the determination of a strategy that, in turn, involves decisions on both actions and their timing or pattern of implementation.¹ Planning in this sense is pervasive in the economy, and it is exemplified by national economic planning, corporate capital planning, inventory planning, household expenditure planning, investor portfolio planning, and planning in many other areas, for example, defense planning, development planning, environmental planning, energy planning, and educational planning. Most previous studies have treated the problem of planning in such a particular institutional or sectoral context. The purpose of this chapter is to contribute a general theory of planning that treats certain common features of all these particular forms of planning. These features involve certain basic choices concerning timing and implementation and typically must be chosen in advance of any specific plan. First is the choice of a *horizon*, that is, the time interval covered by the plan. Second is the choice of a *period*, that is, the time interval during which the plan remains in effect before it is revised. There is also a choice between *time planning*, in which the horizon and period are fixed time intervals, and *event planning*, in which the period may be influenced by the state of the system, with particular events triggering a revision of the plan.

Section 2 presents the basic concepts of horizon and period, and Section 3 develops the distinction between time planning and event planning.

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Section 4 presents a formal framework for the analysis of these choices. This framework leads in Section 5 to five theorems on planning. The theory is illustrated in Section 6 by the problem of a monopolistic firm adjusting prices in a situation of inflation. Conclusions are summarized in Section 7.

Before turning to the specific aspects of planning, it is important to consider both the nature of planning in several different organizations and its usefulness.

In many organizations, both public and private, planning is embodied in a budget or, more typically, in an interrelated set of budgets. There are often many overlapping plans at various levels, such as the operating and capital budgets in a corporation. There can be severe problems in obtaining consistency between these plans and in integrating short- and long-term plans, for example, annual budgets and five-year plans.

In other institutional settings, planning can take other forms. An example is household expenditure planning, which can involve purchases at fixed times (e.g., buy groceries on Monday), purchases when certain levels of goods fall below predetermined critical levels (e.g., purchase gasoline when the tank is one-quarter full), or purchases at random times (e.g., impulse purchases). A related example is inventory planning. Yet another example is the problem of a monopolistic firm revising its prices in a period of inflation (as discussed in Section 6) where there are costs associated with this action (e.g., printing new catalogs); the firm must determine both when to revise prices and what the new prices will be. Another example is a change in government, involving political rather than economic decisions. One alternative is the presidential system, in which a new administration is elected every certain number of years (e.g., four years in the United States and six years in Mexico). An alternative is the parliamentary system, such as that found in Western Europe and Canada, in which a new government is elected when there is a vote of no confidence. A third alternative is the quasipermanent system, such as that found in Eastern Europe and in most developing nations, in which a government lasts until a leader dies or the next coup replaces it. On the basis of these examples, it is clear that planning includes many different situations and a great variety of institutional contexts.

2 Horizon and period

Two fundamental decisions that must be made in advance of any specific plan and in any specific institutional setting concern the horizon and the period of the plan. The horizon of the plan is the time period covered by

the plan, whereas the period of the plan is the time period during which the plan remains in effect. These decisions are often made on the basis of traditional choices or rules of thumb rather than on the basis of conscious choices. Perhaps the most widely used rule of thumb is the annual budget, for which a plan covering one year (the horizon) is revised annually (the period). These traditional choices of horizon and period are based on seasonal factors, especially the harvest in agricultural planning. In other settings, they have been replaced by more deliberately chosen alternatives that are more appropriate to the particular set of decisions to be made. For example, many major corporations use capital budgets that extend over three years and are revised every six months, involving a three-year horizon and a half-year period. Another example is a consumer who plans purchases over horizons longer than one year, as recognized in the permanent income hypothesis, but who may revise plans more frequently than once a year, for example, a four-year horizon and a four-month period.

In most traditional approaches to planning, the horizon decision is linked to the period decision by the requirement that the time interval covered by the plan be the same as the interval between successive plans, so that the horizon and the period are equal. This requirement is clearly not necessary. It is possible (and generally desirable) to have plans overlap by choosing a horizon that is longer than the period. Such a choice implies that the decisions can be made over a long enough horizon to take account of their long-range impacts, although at the same time these decisions can be revised in the light of new information. In practice, this process of having a horizon longer than the period is frequently achieved by having both short- and long-term plans, the former referring to the period of the plan and the latter referring to the horizon of the plan.

The choice of a particular horizon and period depends on the degree of uncertainty concerning the decisions to be made.² Substantial uncertainty, particularly over the very long term, would imply that there is little value in planning over a very long horizon. An example is the use of relatively short horizons in the development of high-technology projects, such as advanced energy and weapons systems, because of their high degree of uncertainty. In such projects, short horizons of sequential decisions and prototypes result in more informed decisions than does integrated development, since each stage in the sequence can use information obtained during a previous stage.

However, whereas an infinite horizon is generally not optimal because of substantial uncertainty, a zero horizon is also generally not optimal since there is usually some information about the near future, and decisions

as to this near future are closely interrelated to those pertaining to the present. Thus, as stated in Theorem 1 (see Sec. 5), a positive but finite horizon is generally optimal.

Cost factors, in particular the costs of planning and of processing new information, are also important considerations in the choice of a particular horizon and period. Theorem 3 (see Sec. 5) states that if there were no costs of planning, then it would be desirable to plan over the indefinite future and to revise plans each instant, thus resulting in a *rolling plan* with a zero period and an infinite horizon.³ All planning entails some costs, however, so it is generally desirable to choose a finite horizon and a non-zero period for a plan.

Political, social, legal, and other constraints often set limits on the horizon and period of a plan. For example, the requirement that Congress must approve the U.S. federal budget implies that the period cannot exceed two years. In general, the horizon and period for many organizations, particularly for governments, are closely related to the decision makers' length of tenure.

3 Time planning and event planning

Decisions concerning the period of the plan can be made in several different ways. The traditional approach is time planning, in which the plan is revised after a fixed time interval has elapsed. Examples include the one-year period for annual budgets, the six-month period frequently used for corporate capital budgets, and the four-year U.S. presidential term. An alternative approach is event planning, in which the plan is revised after a certain event or set of events occurs. The event that triggers the drawing up of a new plan is usually closely related to the goals of the decision maker. Frequently, it is defined as the time that one or a set of the state variables describing the system under consideration reaches a particular value or values.

An example of event planning is a portfolio manager who reviews the portfolio when certain market measures reach predetermined values (e.g., the Dow Jones average passes above a particular ceiling or below a particular floor). By contrast, the portfolio manager who reviews the portfolio at fixed dates (e.g., the first of each month) would be an example of time planning. A second example of event planning is national economic planning in which a new fiscal plan is drawn up in the event of war or if inflation or unemployment rises above particular critical levels. A similar example is household expenditure planning in which the plan is revised in the event of a major illness or a loss of a job. A fourth example

is inventory planning in which a reorder decision may be triggered when inventory falls below a certain level. A specific illustration of this last example of event planning is the (s, S) inventory policy in which new inventory is ordered when the level of inventory falls below s and enough is ordered to bring the level up to S .⁴ This inventory policy may be contrasted to the (t, T) policy of time planning in which new inventory is ordered at fixed times t , with enough ordered to carry (expected) positive levels of inventory up to time T .⁵ This optimality of the (s, S) policy is an example of the general optimality of event planning in situations of uncertainty, formalized below as Theorem 5. A fifth example is investment for capacity expansion, where the time phasing of new investment depends on a projection of future demand.⁶

Hybrid planning represents a combination of time planning and event planning. In this case, either time or some event(s) can trigger a new plan being developed. Typically, a new plan is formulated if either a particular event occurs or a certain time interval has passed since the last plan revision. An example is the formulation of a new fiscal plan if either one year has elapsed since the last plan or if the inflation or unemployment rate exceeds 10 percent. Another example is a parliamentary system in which a new government is elected if there is a vote of no confidence or if five years has elapsed since the last election. This approach to planning has the desirable properties of both pure types of planning. It recognizes the existence of uncertainty by allowing events to trigger action. At the same time, it recognizes that a particular event or small set of events cannot embody all relevant information concerning a system.

Theorem 5 states that in any system involving uncertainty, revising plans on the basis of events is preferable to revising plans only on the basis of time. Thus, if the results of the plan or the state of the world are uncertain, then events should influence the period of the plan. A major problem of event (or hybrid) planning, however, is identifying the particular event(s) that would trigger the new plan. The event(s) should summarize relevant information available concerning the state of the system, involving certain significant changes in some fundamental variables of the system. Given a relevant index or set of indexes, the particular level(s) that would trigger a new plan would generally depend on the cost of planning and the opportunity cost of not revising the plan.⁷

4 A formal framework for the analysis of planning decisions

The concepts introduced so far, those of horizon, period, and time versus event planning, can be defined and analyzed in a formal framework.

This framework is introduced and used here to characterize optimal choices of horizon and period and in the next section to develop five theorems on planning. The development of the formal framework proceeds in four steps: from the overall planner to a sequence of subproblems to the individual subproblem planner to the overall results.

4.1 Overall planner

In the first step, the overall planner must select a trajectory over the entire time interval from time t_0 for a set of control variables. Letting $\alpha(t)$ be the vector of control variables at time t , the overall planner, starting at time t_0 , solves the problem of maximizing expected net benefit

$$\max_{\{\alpha(t)\}} V_0 = E \left\{ \int_{t_0}^{\infty} B(x(t), \alpha(t)) e^{-rt} dt - C e^{-rt_0} \right\}. \quad (1)$$

Here $\{\alpha(t)\}$ refers to the entire trajectory for $\alpha(t)$ from t_0 to ∞ ; V_0 is expected net benefit, to be maximized by the choice of trajectories for the control variables; $B(x(t), \alpha(t))$ is the benefit derived from the vector of state variables $x(t)$ and the vector of control variables $\alpha(t)$, discounted at the rate r , which is assumed given; and C is the cost of planning the trajectory $\{\alpha(t)\}$, which is incurred at time t_0 . Benefits are discounted continuously, and the costs of planning are discounted from the beginning of the period when they are incurred.

The system dynamics are given by the equations of motion

$$\dot{x}(t) = f(x(t), \alpha(t), u(t), t), \quad (2)$$

$$x(t_0) = x_0, \quad (3)$$

where $f(x(t), \alpha(t), u(t), t)$ gives the time rate of change of each of the state variables in x as functions of their levels $x(t)$; the control variables $\alpha(t)$; a stochastic disturbance term $u(t)$, which provides the underlying uncertainty in the problem and is assumed independent of $x(t)$ and $\alpha(t)$; and possibly time itself.⁸ The initial state of the system x_0 and initial time t_0 are given in (3).

4.2 Sequence of subproblems

In the second step, the overall planner is assumed to treat this problem by breaking it into a sequence of subproblems due to the complexity of formulating a trajectory over the entire interval $[t_0, \infty)$. The overall planner chooses a sequence of decisions times $t_0, t_1, t_2, \dots, t_\tau, t_{\tau+1}, \dots$ and dele-

gates the problem of choosing the trajectory to a sequence of planners. The zeroth planner is responsible for the interval $[t_0, t_1)$, the first planner is responsible for the interval $[t_1, t_2)$, ..., and the t th planner is responsible for the interval $[t_\tau, t_{\tau+1})$. Each planner is replaced by the next planner at the end of the interval, with the τ th planner replaced at time $t_{\tau+1}$ by the $(\tau+1)$ st planner. The τ th plan is the trajectory $\{\alpha_\tau(t)\}$ for $\alpha(t)$ over the period from t_τ to $t_{\tau+1}$, representing a set of decisions concerning present and future values of certain choice variables.⁹ The period of the plan is then simply the time interval between successive decisions times, period τ being

$$P_\tau = t_{\tau+1} - t_\tau.$$

The overall planner chooses the decision times t_τ as the solution to the problem

$$\max_{\{t_\tau\}} V = E \left\{ \sum_{\tau=0}^{\infty} \int_{t_\tau}^{t_{\tau+1}} B(\hat{x}(t), \hat{\alpha}(t)) e^{-rt} dt - C_\tau e^{-rt_\tau} \right\}, \quad (4)$$

where $\{t_\tau\}$ is the sequence of decision times.¹⁰ Here V is the expected net benefit, the expectation of a sum, the sum covering all plans, indexed by τ and ranging from the initial plan at t_0 , corresponding to $\tau = 0$, through all future plans, and C_τ is the cost at time t_τ of planning for period τ . As before, benefits are discounted continuously, and the costs of planning are discounted from the beginning of each period, when they are incurred. To solve (4), the control variables are set at expected values $\hat{\alpha}(t)$, which may, for example, reflect past trends or extrapolations. The state variables are also set at expected values $\hat{x}(t)$, which satisfy the equations of motion for expected values,

$$\dot{\hat{x}}(t) = f(\hat{x}(t), \hat{\alpha}(t), \hat{u}(t), t), \quad (5)$$

$$\hat{x}(t_0) = x_0, \quad (6)$$

where $\hat{u}(t)$ is the expected stochastic disturbance at time t . It is assumed that the sequence $\{t_\tau\}$ solves (4) subject to (5) and (6) for given expected values of control and state variables.

4.3 Individual subproblem planner

In the third step the τ th individual subproblem planner is assumed to solve a subproblem involving not just the interval from t_τ to $t_{\tau+1}$ but beyond this time to $t_\tau + H_\tau$, where H_τ is the horizon chosen by the τ th planner. The horizon is chosen subject to the condition that it be at least as long as the period

$$H_\tau \geq t_{\tau+1} - t_\tau = P_\tau \quad (7)$$

so that there are not “gaps,” that is, times for which decisions regarding actions to be taken have not been made. It is advantageous for the τ th planner to formulate plans over the interval $[t_\tau, t_\tau + H_\tau)$, even though only the portion from t_τ to $t_{\tau+1}$ is put into effect since considering the state and controls beyond $t_{\tau+1}$ can possibly improve decisions made for the interval up to $t_{\tau+1}$.¹¹ To the extent that the horizon exceeds the period, there is an interval in which actions planned at a particular time are superceded by actions planned at the next decision time, that is, by the $(\tau + 1)$ st planner.

The τ th planner solves

$$\max_{\{\alpha_\tau(t)\}, H_\tau} V_\tau = E \left\{ \int_{t_\tau}^{t_\tau + H_\tau} B(x(t), \alpha(t)) e^{-rt} dt - C_\tau(H_\tau) e^{-rt_\tau} \right\} \quad (8)$$

subject to (2) and (3). The τ th planner chooses the trajectory $\{\alpha_\tau(t)\}$ for $\alpha(t)$ over the interval up to the horizon, from t_τ to $t_\tau + H_\tau$, where the horizon H_τ is also chosen by this planner subject to (7). Note that the integral ranges from t_τ to $t_\tau + H_\tau$ and that H_τ influences the cost of planning since planning over a longer horizon generally entails higher costs of planning. In general, the τ th planner first chooses an optimal horizon H_τ and then chooses an optimal trajectory for the control variables $\{\alpha_\tau(t)\}$ within an admissible class of such trajectories A_τ , where $\{\alpha_\tau(t)\} \in A_\tau$ summarizes all actions at time t for $t_\tau \leq t < t_\tau + H_\tau$. The admissible class A_τ is chosen by the overall planner, and it embodies all of the technical constraints of the problem.

The equations of motion are given for the τ th planner by

$$\begin{aligned} \dot{x}(t) &= f(x(t), \alpha_\tau(t), u(t), t) \quad \text{for } t \in [t_\tau, t_\tau + H_\tau), \\ x(t_\tau) &= \lim_{t \uparrow t_\tau} x(t), \end{aligned}$$

where $x(t_\tau)$ is determined, in part, on the basis of actions taken by past planners.

Figure 1 illustrates this formulation of horizon and period for the case of a single control variable α in which the admissible set A_τ is the real line, shown as the vertical axis. The decision times t_τ appear on the time axis, the horizon H_τ gives the span over which actions are planned, and the period P_τ gives the time until the plan is revised. Any one curve is a plan $\alpha_\tau(t)$ chosen at time t_τ . Overlapping curves indicate plans that are superceded by later plans. In such cases, the horizon exceeds the period, so the time interval from $t_{\tau+1}$ to H_τ is one in which the initial plans are

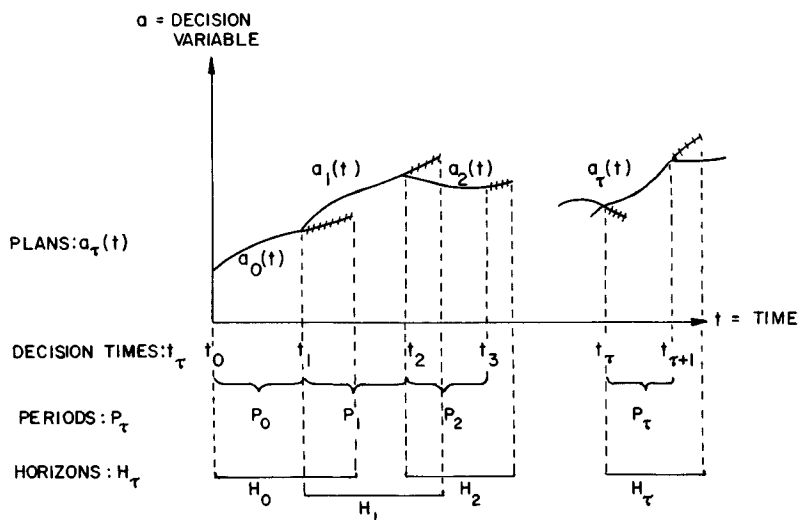


Figure 1. Plan $a_\tau(t)$, extending from t_τ to $t_\tau + H_\tau$, is put into effect at decision time t_τ , where the horizon is H_τ and the period is $P_\tau = t_{\tau+1} - t_\tau$.

revised and replaced by new plans. These portions of the plans are shown as the shaded segments of Figure 1. They can be interpreted as decisions concerning the future levels of the control variable that are superceded on the basis of information available at a later point in time. Consider, for example, the case of corporate capital budgeting introduced in Section 2, for which the horizon is three years and the period is six months. In this case, the last two and one-half years of plans are superceded and can be revised. In such a case, it is desirable to plan over a three-year period, but it is also desirable to reconsider and revise the plan every six months on the basis of the most recently available information. As another example, a U.S. president might commit his or her administration to a series of decisions extending over perhaps ten to fifteen years or even longer, but the latter portion of such a plan can possibly be revised by a new administration.

4.4 Overall results

The fourth and last step is that of overall results for the overall planner choosing decision times $\{t_\tau\}$ and delegating the problem of planning to a sequence of planners, planner τ choosing horizon H_τ and the trajectory

$\{\alpha_\tau(t)\}$ over $[t_\tau, t_\tau + H_\tau)$ and being replaced at the end of the period $t_{\tau+1}$ by planner $\tau + 1$. The resulting expected net benefit is

$$V = E \left\{ \sum_{\tau=0}^{\infty} \int_{t_\tau}^{t_{\tau+1}} B(x(t), \alpha_\tau(t)) e^{-rt} dt - C_\tau(H_\tau) e^{-rt_\tau} \right\}, \quad (9)$$

where $\alpha_\tau(t)$ depends on the horizon H_τ . Note that V is not the sum of the V_τ in (8). The system dynamics are then summarized by

$$\dot{x}(t) = f(x(t), \alpha_\tau(t), u(t), t) \quad \text{for } t \in [t_\tau, t_{\tau+1}), \quad (10)$$

$$x(t_\tau) = \lim_{t \uparrow t_\tau} x(t). \quad (11)$$

Traditional approaches to planning utilize a fixed time interval for both the horizon and period and set them equal to one another

$$H_\tau = H = P = P_\tau,$$

as in annual budgets, where $H = P = 1$ year. More generally, *autonomous* approaches to planning utilize fixed time intervals for both the horizon and period, which need not be equal,

$$H_\tau = H \geq P = P_\tau, \quad (12)$$

such as the corporate capital budgets that cover three years and are revised every six months, where $H = 3$ years $> P = \frac{1}{2}$ year. In *general* approaches to planning, only condition (7) is imposed, allowing both the horizon and the period to change over time and also allowing for the period to be determined possibly by events as well as by the time interval, as in the (s, S) inventory policy, parliamentary systems of government, and investment for capacity expansion. Thus, the autonomous approach is a special case of the general approach, whereas the traditional approach is a special case of the autonomous approach. When $u(t)$ is a stationary stochastic process, the solutions for the horizon and period involve the autonomous approach. If $u(t)$ is a nonstationary stochastic process, however, the optimal horizons and periods are generally not autonomous.

The formal framework thus far has involved time planning, in which the decision times t_τ are determined in (4) as certain specific times. In event planning, the overall planner determines not the specific t_τ but a decision rule under which the plan is revised when the state variables attain certain values in a given *revision manifold* R in E^{n+1} as

$$t_\tau \quad \text{defined by } (x(t_\tau), t_\tau) \in R \quad (13)$$

The overall planner solves (4) for R rather than for t_τ . An example is the (s, S) inventory plan where the manifold is defined by the two parameters s and S .

5 Five theorems on planning

The optimal planning framework introduced in the last section leads to five theorems on planning.

Theorem 1. In general, a positive horizon is better than a zero horizon.

Here, a zero horizon refers to the extreme case, in which decisions are made at each instant with no account taken of future decisions over later periods. This extreme case generally leads to a lower value of the expected net benefit of the plan than the case of an optimal horizon, which is generally not zero and, at time t , is generally determined from the first-order condition for problem (7):

$$E[B(x(t_\tau + H_\tau), \alpha_\tau(t_\tau + H_\tau))]e^{-(r(t_\tau + H_\tau))} - \frac{\partial C}{\partial H_\tau}e^{-rt_\tau} = 0. \quad (14)$$

The theorem holds other than in certain extreme cases, where the difference in (14) is negative when computed at $H_\tau = 0$. In such cases, $H_\tau^* = 0$ is optimal. For example, if the discount rate is infinite, $r = \infty$, so that no account is taken of the future, then the optimal horizon is zero. Another extreme case is one in which the cost function is discontinuous in H_τ , rising from a small (e.g., zero) value to a large (e.g., infinite) value in moving from $H_\tau = 0$ to $H_\tau > 0$. In this case, it is not worthwhile to take account of the future due to the extreme cost of doing so. In such extreme cases with $H_\tau^* = 0$, the expected net benefit function reduces to the discounted sum of benefits less costs, as in benefit-cost analysis, where $\alpha_\tau(t)$ summarizes the decisions made at time t_τ . Leaving aside such extreme cases, however, this type of analysis, in which no account is taken of the future values of the state variables, in particular, the effects of current choices on future states, is, by the theorem, generally worse than an analysis in which such effects are taken specifically into account. For example, weapons systems planning that takes into account only the first year's cost, as was done by the Department of Defense in the 1950s, is inferior to planning that takes into account the later years' costs as well, for example, planning over five years or over the lifetime of the weapons system.

Theorem 2. A permanent plan is optimal when there are no costs of planning and there is no uncertainty.

Here, a permanent plan is one with an infinite period, for which $P_0^* = \infty$, so $H_0^* = \infty$. In this case, all decisions are made at the outset, at $\tau = 0$,

covering all decisions to be made in the infinite future. Once this plan is put into effect, there is no revision of the plan. Furthermore, the horizon is infinite, since it must cover all future time. In this case, the expected net benefit function in (9) reduces to

$$V = \int_{t_0}^{\infty} B(x_t, \alpha_0(t)) e^{-rt} dt - C_0 e^{-rt_0}, \quad (15)$$

as in the usual *optimal control problem* where $\alpha_0(t)$ represents the time paths of the control variables chosen at t_0 and covering all future time.

To demonstrate this theorem, note that since there is no uncertainty, the equations of motion (10) can be integrated forward from the given x_0, t_0 values to yield values of the state variables at any time as functions of the present and all past (optimal) plans,

$$x(t) = F_t[\alpha_\tau(t), \alpha_{t-1}(t), \dots, \alpha_0(t)] \quad \text{for } t \in [t_\tau, t_{\tau+1}). \quad (16)$$

Thus, the expected net benefit in (9) can be expressed in this case as

$$\max V = \sum_{\tau=0}^{\infty} \int_{t_\tau}^{t_{\tau+1}} B(F_t[\alpha_\tau(t), \alpha_{t-1}(t), \dots, \alpha_0(t)], \alpha_\tau(t)) e^{-rt} dt, \quad (17)$$

the expectation sign being dropped because of the fact that there is no uncertainty and the cost function being dropped because of the assumption of no costs of planning. Consider an increase in $P_0 = t_1 - t_0$, or, equivalently, an increase in t_1 since t_0 is fixed. All of the later periods P_τ can be adjusted in such a way that there is no change in each of the integrals in (7) starting from $\tau = 1$. Thus, the effect of the increase in t_1 , which would reduce the second integral from t_1 to t_2 , can be offset entirely by an increase in t_2 . The effect of the increase in t_2 can, in turn, be offset by an increase in t_3 , and so on. Then the effect of an increase in t_1 is

$$\frac{\partial V}{\partial t_1} = B(F_{t_1}[\alpha_0(t)], \alpha_0(t)) e^{-rt_1} = \frac{\partial V}{\partial P_0} > 0.$$

Since this partial is positive for all t_1 , the optimal t_1 is infinite, so $P_0^* = \infty$, involving a permanent plan. Since $H_0 \geq P_0$, it follows that $H_0^* = \infty$. Thus, in this case, a permanent plan, with $P_0^* = H_0^* = \infty$, is optimal.

Theorem 3. A rolling plan with an infinite horizon is optimal when there are no costs of planning and there is uncertainty.

Here a rolling plan is one with a zero period, $P_\tau^* = 0$, so decisions are being continuously revised in the light of new information. In this case,

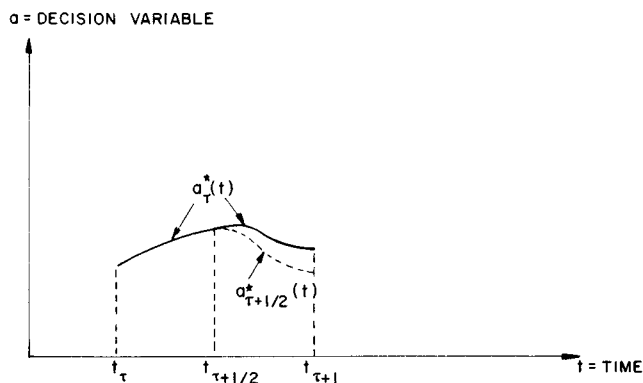


Figure 2. The plan $\alpha_t^*(t)$ is optimal conditional on information available up to t_τ , specifically all x_t for $t \leq t_\tau$. The plan $\alpha_{\tau+1/2}^*(t)$ is optimal conditional up to $t_{\tau+1/2}$.

the benefit and cost functions take explicit account of the effects of current decisions on all future values of the state variables.

This theorem can be illustrated using a geometric argument. Consider the optimal plan $\alpha_t^*(t)$ at time t_τ conditional on information available up to time t_τ , specifically the set of all x_t for $t \leq t_\tau$. In the case of a single decision variable, this optimal plan is shown as the solid curve in Figure 2, and it remains in effect from t_τ to $t_{\tau+1}$. Consider now dividing the interval in half, at $t_{\tau+1/2}$, allowing for a revision of the plan at this halfway point. The plan $\alpha_t^*(t)$ is now in effect from t_τ to $t_{\tau+1/2}$. A new plan $\alpha_{\tau+1/2}^*(t)$ starts at $t_{\tau+1/2}$, shown as the dotted curve Figure 2. This new plan, which remains in effect from $t_{\tau+1/2}$ to $t_{\tau+1}$, uses information available from t_τ to $t_{\tau+1/2}$, specifically the values assumed by x_t for $t_\tau < t < t_{\tau+1/2}$. This information was not available at time t_τ since x_t is affected by the stochastic term u_t . This added information may be of use in planning over the period from $t_{\tau+1/2}$ to $t_{\tau+1}$. Thus, the value of the expected net benefit function must be greater or at least no worse for the shorter interval between planning times. The same division-in-half argument can then apply for $t_{\tau+1/4}$. The argument can continue to be applied for any finite interval between successive plans so long as the two assumptions – that x_t is affected by the stochastic term (so there is a value in using the information in the later part of the interval between plans) – and that there is no cost of planning (so there is no cost in using a shorter interval between plans) are maintained. The result of this process is an optimal

period $P_\tau^* = 0$. As to the horizon, as long as there are no costs of planning, there is no reason to select any horizon short of an infinite one, since there may be a benefit and since there is no cost in selecting as long a horizon as possible. Thus, in this case, a rolling plan with an infinite horizon for which $P_\tau^* = H_\tau^* = \infty$ is optimal.

Theorem 4. In the absence of uncertainty, time planning and event planning are equivalent in having the same net payoff.

Here time planning refers to a choice of horizon and period based on specific time intervals, and event planning, as discussed in Section 3, refers to planning in which these choices are triggered by events. Specifically, in event planning, the plan is revised when the state variables attain values in a given revision manifold R as in (13).¹² Time planning, then, is the special case for which the revision manifold does not depend on $x(t_\tau)$, with it depending only on the t_τ . In the case of certainty, where the future of the system can be determined solely on the basis of decisions made by the planner, events can be predicted exactly, so there is no difference between time planning and event planning.

To demonstrate this theorem, note that here, as in Theorem 2, no uncertainty makes it possible to integrate the equations of motion forward to obtain the values of the state variables as functions of all past plans, as in (16). Time planning and event planning are then equivalent. Knowing the times t_τ^* at which plans are optimally revised, it is possible to define the revision manifold in terms of these t_τ^* , independent of $x(t_\tau)$. Conversely, given the optimal revision manifold R^* , knowledge of the time paths of the state variables implies that it is possible to determine explicitly those times t_τ^* such that $(x(t_\tau^*), t_\tau^*) \in R^*$.

Theorem 5. In the presence of uncertainty, event planning has a higher expected net payoff than time planning.

With uncertainty, events cannot be predicted exactly, so taking account of new information conveyed by the state of the system, which responds to uncertain “outside” influences as well as to the decisions of the planner, can improve performance as measured by the expected net benefit. An exceptional case is that for which $P_\tau^* = 0$, as in this continuous revision case there can be no difference between time planning and event planning.

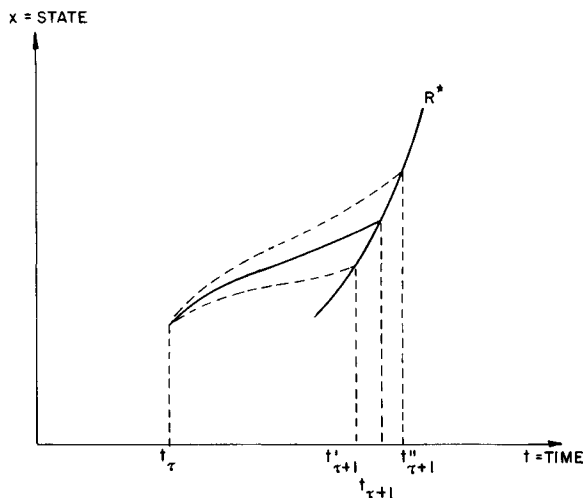


Figure 3. The anticipated optimal path for the state variable is the solid curve. Alternative possible paths are dotted. With event planning using the revision manifold R^* , the plan is revised at $t'_{\tau+1}$, $t_{\tau+1}$, or $t''_{\tau+1}$, depending on which path is taken by the state variable.

This theorem can be illustrated using a geometric argument, as in Theorem 3. Figure 3 shows, in the case of a single state variable, its anticipated optimal path from t_τ to $t_{\tau+1}$ as the solid curve. The plan is revised at time $t_{\tau+1}$. Because of the uncertainty that is present, however, the state variable may take a different path, and two alternative paths are shown as dotted lines. The revision manifold is given as R^* . Under time planning, the plan is revised at time $t_{\tau+1}$ (which can be considered the special case of a vertical revision manifold at $t_{\tau+1}$). Under event planning, the plan is revised at time $t_{\tau+1}$ if the actual path is the same as the anticipated optimal path or at time $t'_{\tau+1}$ or $t''_{\tau+1}$ if the actual path veers away from the anticipated optimal path. The decision to revise the plan is thus based on information on the state, which can increase the value of the expected net benefit of the plan.

An example of Theorem 5 on the optimality of event planning is the optimality of the (s, S) inventory policy. The (s, S) policy is one of reordering at times when the inventory level falls below a certain critical level s , which is precisely event planning, decisions being determined not a priori but in response to the (uncertain) evolution of the system. This policy is

superior to the alternative policy of time planning, namely reordering at certain fixed intervals.¹³ A fundamental theorem in inventory theory states that (s, S) is an optimal inventory policy. Theorem 5 broadens and extends this theorem to a wider class of problems, stating that event planning is an optimal planning policy. The next section presents an application of event planning to yet another area, that of price adjustment in inflation.

6 Example: price adjustment in inflation

An example of the concepts and theorems presented thus far is the planning used by a monopolistic firm in adjusting prices in a period of inflation.¹⁴ Such a firm must choose a horizon for its prices, specifically over how long a time interval it will honor the prices it announces. It must also choose a period for its prices, specifically the length of the time interval before it establishes new prices, and this time interval could be determined via time or event planning. A traditional plan would, for example, be an annual catalog, where the published prices are honored for a year and where at the end of the year a new catalog is published, entailing both a one-year horizon and a one-year period. A less traditional plan would, by contrast, involve a catalog published every six months where the published prices are honored for a year, entailing a one-year horizon and a six-month period. Such a firm must also choose between time planning, as in these examples, or event planning, where the decision to revise prices is influenced by the state of the system, such as the level of the aggregate price level. For example, in event planning, prices might be revised if the aggregate price level has risen by more than 50 percent.

All five theorems discussed earlier are relevant to this example. As in Theorem 1, a positive horizon is better (i.e., more profitable to the firm) than a zero horizon, with the firm honoring price commitments over the positive horizon. As in Theorem 2, a permanent plan is optimal if there are no costs of planning and there is no uncertainty. In this case, the firm might, for example, simply announce once and for all the rate at which its prices are rising. This fixed rate might be related (e.g., equal to) the (known) inflation rate. As in Theorem 3, if there are no costs of planning (i.e., costs of revising prices) and uncertainty, then a rolling plan is optimal. In this case, the firm continually revises its prices, based on the realized aggregate rate of inflation. As in Theorem 4, if there is no uncertainty (i.e., inflation is perfectly anticipated), then time planning and event planning are equivalent. Both will lead to the same level of profits,

and the firm could revise its plan at either fixed intervals or in response to a particular realized level of inflation. Finally, as in Theorem 5, event planning is superior to time planning in the case of uncertainty. In this case, where the inflation rate is not known or fully anticipated, the firm that revises its prices based on the realized rate of inflation will have a higher expected profit than an identical firm that revises its prices at fixed time intervals. The remainder of this section will summarize some results concerning the last two cases, involving the firm adjusting prices both where inflation occurs at a known constant rate and where inflation occurs at an uncertain rate.

The case of inflation at a known constant rate illustrates Theorem 4 on the optimality of time planning in the absence of uncertainty. Letting $p(t)$ be the nominal price charged by the firm at time t , if the general price level $\bar{p}(t)$ is rising exponentially at the known constant rate of inflation g , then the real price charged by the firm is

$$z(t) = p(t)/\bar{p}(t) = p(t)e^{-gt},$$

where, by normalization, $\bar{p}(0) = 1$. The firm revises its nominal prices at the times t_τ , starting from $t_0 = 0$, so that over the period from t_τ to $t_{\tau+1}$, the real price is given as

$$z_\tau(t) = p_\tau e^{-gt}, \quad t \in [t_\tau, t_{\tau+1}), \quad (18)$$

where p_τ is the fixed nominal price during the period $[t_\tau, t_{\tau+1})$. The firm chooses a set of decision times t_τ and a set of nominal prices p_τ so as to maximize the total discounted profits, given, as in (9), as

$$V_0 = \sum_{\tau=0}^{\infty} \int_{t_\tau}^{t_{\tau+1}} F(z_\tau(t)) e^{-rt} dt - \beta e^{-rt_{\tau+1}}.$$

Here, $F(z_\tau(t))$ is the real profit function of the firm, which depends on the real price level in (18) and is comparable to the benefit function $B(\cdot)$ in (9), and β is the real cost of nominal price adjustment assumed to be incurred at the end of the period, $t_{\tau+1}$. In this case of no uncertainty, the firm can, according to Theorem 4, optimally use time planning, and She-shinski and Weiss (1977) proved that such planning is indeed optimal. In particular, they showed that the optimal policy is one in which the periods $P_\tau = t_{\tau+1} - t_\tau$ are all equal,

$$P_\tau = \epsilon, \quad \text{for all } \tau, \quad \text{i.e., } t_{\tau+1} = t_\tau + \epsilon, \quad \text{for all } \tau, \quad (19)$$

and the nominal prices are chosen so that the real prices are the same at the beginning of each period

$$z_\tau(t_\tau) = S, \quad \text{for all } \tau, \quad (20)$$

$$\text{i.e., } z_\tau(t) = Se^{-g(t-t_\tau)}, \quad t \in [t_\tau, t_{\tau+1}).$$

Since the periods are all the same, the real prices are the same at the beginning of each period, and since inflation occurs at a constant rate, the real prices are also the same at the end of each period

$$z_\tau(t_{\tau+1}) = s, \quad \text{for all } \tau \quad \text{where } s = Se^{-g\epsilon}.$$

The real price thus moves between S and s in each period, declining, given the constant nominal price, at the inflation rate g . The firm thus starts at time $t_0 = 0$ by setting a nominal price $P_0 = S$, and it retains this nominal price up to time $t_1 = \epsilon$, at which time it revises the nominal price $p_1 = Se^{g\epsilon}$, so the real price is S . It retains this new nominal price up to time $t_2 = 2\epsilon$, at which time it revises the nominal price to $p_2 = Se^{2g\epsilon}$, and so on.

The interval between decision times is constant (ϵ), and the real price at each of the decision times is also constant (S). These two constants, determined on the basis of the rate of inflation (g), the cost of price adjustment (β), and the parameters characterizing the net revenue function ($F(\cdot)$), completely characterize the optimal pricing plan of the monopolistic firm in a situation of inflation in which there is no uncertainty. This solution is one of time planning, as in Theorem 4.

The case of inflation at an unknown and stochastically determined rate illustrates Theorem 5 on the optimality of event planning in the presence of uncertainty.¹⁵ In this case, if the general price level $\bar{p}(t)$ responds to random shocks instead of rising exponentially, then, at time t ,

$$\log \bar{p}(t) = \sum_{i=1}^{N_t} y_i. \quad (21)$$

Here the y_i are assumed independently and are identically distributed as $h(y)$, and the number of random shocks influencing the system at time t , given as N_t in (21), is determined by the Poisson process, where

$$\text{Prob}\{N_t = n\} = e^{-\lambda t} \frac{(\lambda t)^n}{n!}$$

so that

$$E(\log \bar{p}(t)) = \lambda t E(y).$$

Under these assumptions, Sheshinski and Weiss (1983) proved the optimality of event planning, where the firm always sets a nominal price p_τ at the decision times t_τ so that the real price is always S at these times, as in the case of a certain inflation rate (20). By contrast to the certainty case,

however, in this case of uncertainty, the periods are not constant. Rather, the real price floats down, depending on the random shocks in (21), and the optimal policy is one of event planning where the next decision time is determined as that time when the real price assumes the value s , as in (s, S) inventory planning. Because of the uncertainty in the system, the lengths of the periods are not constant, in contrast to (19), and they are not predictable a priori. This type of event planning, where the period is determined endogenously by the evolution of the uncertain system, yields a higher value for expected profits than does time planning, where the period is determined as a fixed time interval. It thus illustrates Theorem 5.

An extension of this case of price adjustment by a monopolistic firm in the case of stochastic inflation would treat the case in which the general price level is not known but must be determined, where its determination entails certain costs. For example, the general price level relevant to a particular firm may be obtained only via subscribing to trade journals or by sampling prices of firms in related markets. In such a situation, the optimal policy is one in which the firm determines the general price level at certain fixed time intervals but then adjusts its own price relative to the general price level as determined at these fixed times, according to event planning, as described above. Thus, at the time the general price level is determined, the nominal price would be left unchanged or revised upward according to whether the real price is above or below s . This case of stochastic inflation (in which the price level must be determined) therefore leads to hybrid planning, combining time planning for the determination of the general price level with event planning for the revision of the firm's nominal price.

This example of price adjustment in inflation illustrates the nature of time and event planning, the theorem on event planning, and also the nature of hybrid planning. It is, in fact, one of two problems discussed in the literature where the concepts of planning (as developed here) have been treated, the other being inventory planning, particularly the (s, S) inventory policy. An important implication of this chapter is that these concepts apply not just to these problems but, in fact, to a much wider range of planning problems for not only firms but also for all planners, whether firms, individuals, government agencies, or nations.

7 Conclusions

This chapter has identified two decisions that are part of any planning process, namely the horizon of the plan and the period of the plan. A for-

Table 1. *Optimal planning depends on whether or not there are costs of planning and whether or not there is uncertainty*

Cost of planning	Uncertainty	
	No uncertainty	Uncertainty
No cost of planning	<i>Permanent plan:</i> an initial plan covering all future time adopted at t_0 and never revised ($P_0^* = \infty$, $H_0^* = \infty$) (Theorem 2)	<i>Rolling plan with an infinite horizon:</i> continuous revision of the plan at each point in time, with planning over all future time ($P_\tau^* = 0$, $H_\tau^* = \infty$) (Theorem 3)
Costs of planning	<i>Time planning or event planning:</i> period chosen either as a specific time interval or dependent on certain events (P_τ^* and H_τ^* can be chosen independently of the state of the system) (Theorem 4)	<i>Event planning:</i> period dependent on events, with the revision of the plan possibly triggered by events rather than time (P_τ^* determined as $t_{\tau+1}^* - t_\tau^*$, where $(x(t_\tau^*), t_\tau^*) \in R_\tau^*$, a revision manifold at time t_τ) (Theorem 5)

mal framework for planning has been introduced in which the first stage is that of choosing the horizon and period, and the second stage is that of choosing a specific plan. This framework has led to five theorems on planning, of which the first states that a positive horizon is generally preferable to a zero horizon. The other four theorems are summarized in Table 1, showing the optimality of different types of planning in different situations, depending on the presence or absence both of costs of planning and of uncertainty. The problem of price adjustment in inflation illustrates the nature of these theorems. This problem and that of inventory planning by the firm (in particular, the (s, S) policy) are the two cases in which the issues treated here – in particular the concepts of horizon and period and the contrast between time planning and event planning – have been discussed in the literature. A principal conclusion of this chapter is that these concepts can be applied to a much wider class of planning problems.

It is hoped that this chapter will contribute toward more deliberately set and more informed choices about planning, which can significantly

improve performance in a wide variety of settings. It is also hoped that models of specific planning processes, such as capacity expansion and portfolio selection, will be expanded to include specific consideration of the choices of horizon and period and to allow for the possibility of event as well as time planning. Finally, it is hoped that this essay will stimulate the further development of a pure theory of planning.

NOTES

- 1 For discussions of planning and its different aspects, typically in the context of national economic policy, see Tinbergen (1952, 1954, 1956, 1964), Theil (1961, 1964), Hickman (1965), Kornai (1967), Hansen (1967), Chakravarty (1969), Lange (1971), Heal (1973), Czerwinski and Porwit (1977), Deleau and Malgrange (1977), and Intriligator (1978).
- 2 Using a numerical linear-quadratic formulation that allows for both the cost of uncertainty and progressively acquired information on uncontrolled factors, Deleau and Malgrange (1974) found that for a five-year horizon, revision near the end of the period (years 3, 4, and 5) is favored. Their numerical results in fact suggest that a four-year period is optimal, with the plan being revised in the fourth year. See Intriligator (1971) for a discussion of linear-quadratic control problems.
- 3 For a discussion of a rolling plan in the context of an optimal growth model, see Goldman (1968). For a discussion of a shifting finite time horizon, see Inagaki (1970).
- 4 For a discussion of the (s, S) inventory policy, see Scarf (1960).
- 5 For a discussion of the (t, S) policy and its relation to the (s, S) policy, see Naddor (1962).
- 6 See Srinivasan (1967) and Manne (1967). For related models dealing with the timing of investments, see Flemming (1969), Weitzman (1970), and Dixit, Mirrlees, and Stern (1975).
- 7 If there are several indicators of the state of the system, then the event triggering the need for a new plan should be based on all these indicators. Radner (1964) investigated this issue with reference to command and control decisions. He considered the case of two sensors, such as lights on a panel, that, when lit indicate the need for action; when not lit, involve no need for action; and which could be accidentally turned on, involving a known probability distribution. He showed that a desirable rule for action in such a case is the likelihood ratio rule of acting if $w_1 > \bar{w}_1$ or $w_2 > \bar{w}_2$ or $\min(w_1, w_2) > \bar{w}$, where w_i is the length of elapsed time during which the i th sensor is turned on ($i = 1, 2$) and $\bar{w}_1$, $\bar{w}_2$, and $\bar{w}$ are parameters obtained from the probability distribution and the disutility for a type I error (do not act when should act) as compared to that for a type II error (act when should not act). The examination of this rule and its comparison to other rival rules is instructive, but the major problem is probably that of identifying and obtaining the sensors in the first place or recognizing critical events, rather than in adjudicating between them.

- 8 As formulated here, the stochastic disturbance term applies only to the equations of motion (2). More generally, the benefit–cost functions in (1) can also be influenced, in part, by stochastic disturbance terms.
- 9 Note that a *plan* is a set of decisions concerning present and future values of certain choice variables. In particular, it is not the same as a *strategy*, which relates the choice variables to the state variables of the system.
- 10 For a similar formulation of a criterion function for a related problem, see Sheshinski and Weiss (1977, 1978). See also Chakravarty (1969) for a related approach.
- 11 As an example, weapons procurement decisions that treat only costs of the first few years can lead to commitments with ballooning costs in later years – the “camel’s nose under the tent” approach, avoided by using longer horizons.
- 12 The state variables may cross these values several times. To determine at which of these times the plan should be revised, it is necessary to discriminate among these several crossings. One such way is to introduce as added state variables the first, second, and higher time derivatives of the original state variables and to discriminate on the basis of these higher derivatives.
- 13 For the optimality of the (s, S) policy in inventory planning, see Scarf (1960). For the result that states that in the case of a separable cost function, the (s, S) policy of reordering at times when the level of inventory falls below a certain critical level (event planning) is superior to the (t, S) policy of reordering at fixed time intervals (time planning), see Naddor (1962).
- 14 See Sheshinski and Weiss (1977, 1978) for a treatment of price adjustment in the presence of a perfectly anticipated aggregate rate of inflation. For extensions to the case in which the aggregate rate of inflation is stochastic, see Padon (1981), Sheshinski and Weiss (1983), and Danziger (1984). For an empirical analysis of price adjustment in inflation, see Sheshinski, Tishler, and Weiss (1981). For related work on price adjustment by a monopolist firm, see Arrow (1962) and Barro (1972).
- 15 The developments reported here are based on Sheshinski and Weiss (1983), who developed event planning for the case of uncertain inflation. Danziger (1984) also treated uncertain inflation but developed only time planning for this case.

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