Metroeconomica 44:1 (1993) pp. 029–042

NEGATIVE SURPLUS VALUE AND INFERIOR PROCESSES*

Eiji Hosoda
Keio University

(October 1991: revised June 1992)

ABSTRACT

One of the most interesting results in value theory is that positive profits are consistent with negative surplus value. This result is obtained, using a two-commodity linear model with joint production. Since existence of an inferior process is always implied in a two-commodity model, the result has been supposed to be applicable to a limited type of technology. The purpose of this paper is to show that positive profits with negative surplus value do not necessarily imply the existence of such a process in higher dimensions. Although a different type of inferiority is implied, such a definition of inferiority is quite different from what is normally understood.

1. INTRODUCTION

In 1975 Steedman published an interesting paper on the relationship between surplus value and profits, which has attracted many economists' attention since then. His main result is that positive surplus value is neither a necessary nor a sufficient condition for positive profits in a joint-production economy.

Hot controversies broke out, and Steedman's conclusion was critically examined by many economists. One big issue was on the definition of value. In Steedman's example, the labour value of a commodity is calculated to be negative, which is crucial for his conclusion. Morishima (1976) strongly opposed the idea of negativity of labour value, and insisted that an inequality system be used instead of an equation system in determining labour value.

* The author is very grateful to the referees of this journal for their suggestions. He is also grateful to Professors I. Steedman and M. Terade for their comments on the earlier version of this paper.

Another problem, which was not mentioned as often as the former one, is the technological setting in Steedman's example. He considered the following technology: one process is inferior to the other, in the sense that the former process produces less net output than the latter, although both processes are activated.

Okishio (1977) questioned the validity of such a supposition on technology. Itoh (1981) also criticized Steedman on this point, saying "As far as circumstances permit, the more effective process 2 must increasingly be selected as a common rule of economic life". (p. 169).

Since the inferiority of one process turned out to be a necessary condition for Steedman's conclusion by Wolfstetter's theorem 3 (Wolfstetter [1975]), it may be said that his conclusion is derived from a limited type of technology.

The purpose of this paper is to show that Steedman's insight is immune to such criticism. Surely Wolfstetter's theorem 3 guarantees that inferiority of one process to the other is necessary for Steedman's conclusion in a two-commodity model. Yet, what is true in two dimensions is not necessarily true in higher dimensions. We will show that Steedman's conclusion still holds even when there is no inferiority among processes.

We will also examine the meaning of inferior processes in higher dimensions. A certain type of technique which brings about negative labour value has often been called inferior by some authors. We will demonstrate that this definition merely implies an inefficient combination of processes and does not fit our common usage of inferiority of techniques.

2. THE MODEL

In this section we briefly explain our model, and review how price, quantity and value are determined.

Let us consider a Sraffa–von Neumann type of economy with joint production, where constant returns to scale prevail. It is assumed that capitalists save all their profit income for accumulation, and workers consume all their wage income. Moreover, workers' consumption proportion is assumed to be given, although this assumption is not essential to our conclusion.

Then, a competitive equilibrium can be described as follows:

$$\mathbf{pB} \leqq (1 + r)\mathbf{pA} + \mathbf{L} \tag{1}$$

$$\mathbf{Bx} \geqq (1 + g)\mathbf{Ax} + \frac{1}{\mathbf{pd}}\mathbf{d} \tag{2}$$

$$\mathbf{pBx} = (1 + r)\mathbf{pAx} + \mathbf{Lx} \tag{3}$$

$$w = 1, \mathbf{Lx} = 1 \tag{4}$$

$$r = g \tag{5}$$

$$\mathbf{p} \geqq 0, \mathbf{x} \geqq 0.^{1} \tag{6}$$

Notation is as follows:
B an output matrix, **A** an input matrix, **L** a labour input vector (row), **p** a price vector (row), **x** an activity vector (column), r a rate of profit, w a wage rate, g a growth rate and **d** workers' consumption proportion vector. We assume **A** and **B** to be square.

Inequality (1) expresses a cost-price relationship, and (2) is nothing but a supply-demand inequality (i.e. no excess demand) with a balanced growth rate g. Equation (3) implies that a non-profitable process is not activated, and (4) means normalization for price and quantity systems respectively. Equation (5) is a result of the saving assumption, and (6) is a non-negativity condition. It is clear that (3), also implies zero price for an overproduced commodity. Although Steedman uses an equality system for price and quantity instead of an inequality system, there is no essential difference, as will be seen later.

Construct matrices $\mathbf{A}^-$ and $\mathbf{B}^-$, and vector $\mathbf{L}^-$, such that columns of $\mathbf{A}^-$ and $\mathbf{B}^-$ and components of $\mathbf{L}^-$ are exactly the same as those of **A**, **B** and **L**, whose corresponding activities in (2) are strictly positive. Then, Steedman's value system can be described as follows:

$$\mathbf{vB}^- = \mathbf{vA}^- + \mathbf{L}^-, \tag{7}$$

where **v** is a labour value vector.[2] Equation (7) means that the value defined above satisfied *additivity* and *actuality*, although it may not satisfy *non-negativity*. (See Steedman [1976].)

In the following, we adopt labour input as the unit of intensity, i.e. $\mathbf{L} = (1, \ldots, 1)$. Although this normalization excludes automated processes, it is not essential to our main results. Clearly, $[\mathbf{B} - \mathbf{A}]$ means a net

[1] We adopt the following convention: suppose **x** and **y** denote row vectors with n components. $\mathbf{x} > \mathbf{y}$ if $\mathbf{x}_i > \mathbf{y}_i$ for all $i = 1, \ldots, n$. $\mathbf{x} \geqq \mathbf{y}$ if $\mathbf{x}_i \geqq \mathbf{y}_i$ for all $i = 1, \ldots, n$. $\mathbf{x} \geqslant \mathbf{y}$ if $\mathbf{x} \geqq \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$.

[2] This value is called M_1-value in Fujimori (1982, p. 48).

output matrix at unit intensity level. We designate $[b_j - a_j]$ as the j-th column vector of $[\mathbf{B} - \mathbf{A}]$, i.e. a net output vector of the j-th process. We adopt the following definition for inferiority of process.

Definition 1 (Inferior process)
If there exists positive scalar α_i's $(i \neq j)$ such that $\sum_{i \neq j} \alpha_i = 1$ and

$$\sum_{i \neq j} \alpha_i (b_i - a_i) > b_j - a_j,$$

then $b_j - a_j$ is called an *inferior process*, and $(b_i - a_i)$'s are considered to be superior to $b_j - a_j$.

Definition 2 (Non-inferiority)
If there is no inferior process, technology is said to satisfy *non-inferiority*.

Definition 3 (Feasibility)
If there exists $\mathbf{x} \geqslant 0$ such that $(\mathbf{B} - \mathbf{A})\mathbf{x} > 0$, then an economy is called *feasible*.

Let us summarize one of Steedman's main results in brief. Using a similar model to our (1)–(7), he showed that positive profits coexist with negative surplus value in a two-commodity economy. The technology which he adopted in his example, however, does not satisfy the non-inferiority condition.[3]

As shown in Wolfstetter's Theorem 3, the existence of an inferior process is necessary and sufficient for the negativity of labour value of one commodity. Since negativity of labour value is a necessary condition for the existence of positive profits with negative surplus value (which we call PPNSV from now on), it may be said that Steedman's conclusion crucially depends upon the existence of an inferior process, *as far as a two-commodity economy is concerned*.

Yet, we have to be very careful on this point: Wolfstetter's Theorem

[3] In Steedman's example [1975], a net output matrix is

$$\begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}.$$

Therefore, this technology does not satisfy the non-inferiority condition, since the first process is inferior to the second.

3 holds valid only in a two-commodity world, unless the definition of inferior processes is altered. What holds true in two dimensions cannot always be applied to higher dimensions.

3. PPNSV IN HIGHER DIMENSIONS

Now we contemplate an economy where there are three or more commodities. If there is an inferior process as defined earlier, it is dominated by a convex combination of two (or more) other processes. Intuitively, we can tell that the labour value of, at least, one commodity is negative, since negative labour must be assigned to the inferior process as far as the process is activated.[4]

How about the case where the non-inferiority condition is satisfied? We must not jump to the conclusion that non-inferiority implies non-negative labour value and that PPNSV does not hold.

We can give an example where positive profits coexist with negative surplus value even when the non-inferiority condition is satisfied. (See Appendix.) In the example, we show that the equilibrium solution is unique and positive, and that labour values are also uniquely determined.

Even though the technology satisfies non-inferiority, the labour values of the two commodities are calculated as negative. Furthermore, a negative surplus value is obtained in the example. This result implies that PPNSV is not limited to a certain type of technology.

4. DISCUSSION

As we have just shown in the previous section, the relationship between technology and value determination in a three- or more commodity economy is quite different from that in a two-commodity economy. We will explore further the meaning of inferior processes in this section.

For this purpose it is quite convenient for us to utilize results obtained by Filippini and Filippini (1982) and Fujimori (1982, Chap. III). First of all, we define a different type of inferiority of process as elaborated by them.

---

[4] Indeed, this conjecture is correct. See the next secton.

Definition 4 (F-inferiority)
Suppose I and J are subsets of $\{1, 2, \ldots, n\}$ such that $I \cap J = \phi$. If there exist α_i's and β_j's such that

$$\sum_{i \in I} \alpha_i \begin{pmatrix} b_i - a_i \\ -1 \end{pmatrix} > \sum_{j \in J} \beta_j \begin{pmatrix} B_i - a_i \\ -1 \end{pmatrix},$$

then processes $b_j - a_j$ which belong to J are called F-inferior, where $\alpha_i \geqq 0$ and $\beta_j \geqq 0$.[5]

We have named these processes "F-inferior" to distinguish them from our definition of inferiority. The two types of inferiority are quite different. But before we explain the difference, we would like to refer to a helpful theorem which was proved by Filippini and Filippini (and Fujimori).

They proved that Steedman's values are semi-positive, if, and only if, there is no F-inferiority among processes. (Filippini and Filippini (1982), Theorem 1.)[6] In other words, positive profits with negative surplus value (PPNSV) hold true only when some processes are F-inferior.

In order to relate this theorem with the implication of the result in the previous section, we define another type of inferior process as follows:

Definition 5 (H-inferiority)
Suppose I and J are subsets of $\{1, 2, \ldots, n\}$ such that $I \cap J = \phi$. If there exist α_i's and β_j's such that

$$\sum_{i \in I} \alpha_i (b_i - a_i) > \sum_{i \in J} \beta_j (b_j - a_j),$$

[5] In Filippini and Filippini's (and Fujimori's) definition, a labour input vector is not normalized like our **L**. They define inferiority processes as follows: suppose I and J are subsets of $\{1, 2, \ldots, n\}$ such that $I \cap J = \phi$. If there exist i's and j's such that

$$\sum_{i \in I} \alpha_i \begin{pmatrix} b_i - a_i \\ -l_i \end{pmatrix} > \sum_{j \in J} \beta_j \begin{pmatrix} b_j - a_j \\ -l_j \end{pmatrix},$$

then processes $b_j - a_j$ which belong to J are called inferior, where $\alpha_i \geqq 0$ and $\beta_j \geqq 0$. The above inequality is equivalent to

$$\sum_{i \in I} \alpha_i l_i \begin{pmatrix} (b_i - a_i)/l_i \\ -1 \end{pmatrix} > \sum_{j \in J} \beta_j l_j \begin{pmatrix} (b_i - a_i)/l_i \\ -1 \end{pmatrix}.$$

This is equivalent to Def. 4.

[6] Exactly speaking, Fujimori gives a necessary and sufficient condition for *positiveness* of value. But there is no fundamental difference. See Fujimori (1982, p. 50.)

then processes $b_j - a_j$ which belong to J are called H-inferior processes, where $\alpha_i \geqq 0$, $\beta_i \geqq 0$, $\sum_{i \in I} \alpha_i = 1$ and $\sum_{j \in J} \beta_j = 1$.

At first glance, H-inferiority might seem different from F-inferiority. Labour coefficients do not appear in the former definition, while they appear in the latter definition. The two definitions are, however, very close to each other in the following sense: there are H-inferior processes if, and only if, there are F-inferior processes as far as an economy is feasible. (See Appendix for the proof.)[7] Therefore, if PPNSV holds, then the technology must satisfy H-inferiority.

Even at this stage, the circumstances may not be clear, since the example given in the Appendix might seem to contradict the above results. Actually, there is no contradiction. The definition of "inferior processes" is quite different from those of "F-inferior processes" and "H-inferior processes".

The latter definitions of inferiority just require that a certain type of combination of processes be inefficient, though each process may not be inefficient on its own. They just characterize the combination of processes.

This can be understood by the example in the Appendix. In the technology no process is inferior, but the first and the second processes are H-inferior (and F-inferior) since the combination of two-thirds intensity of the second process and one-third intensity of the third process gives less net product than unit intensity of the first process.

It is clear that the three definitions are different in three or higher dimensions, although they coincide in a two-commodity economy.

The difference is highlighted when we consider the ordering of inferiority. As far as "inferiority" is concerned, when some processes are inferior to other processes, there always exists at least one process which is never inferior to other processes. In other words, there is at least one process which is never dominated by any other combination of processes.

As for F- or H-inferiority, the above does not hold. That is, processes can be H-inferior to each other. Even every process can be H-inferior! In other words, any process may, *if wrongly combined*, produce less output than another process. We can easily make an example which

[7] It must be noticed that F-inferior processes may not coincide with H-inferior processes. Namely, F-inferior processes may not be H-inferior, though H-inferior processes are always F-inferior. See the remark in the Proposition in the Appendix.

justifies our argument, since H-inferiority actually refers to a net output matrix, which is square.[8]

Now, the difference between the definition of inferior processes and that of H-inferior (or F-inferior) processes is clear. According to the former definition, "inferiority" implies inefficiency of a process. On the other hand, the latter definition refers to the *inefficient combination* of processes, which seems quite different from the normal usage of inferiority of processes.

Incidentally, it must be emphasized that a combination of processes actually adopted in an equilibrium may not be inefficient, even if the technology satisfies H-inferiority. This can be confirmed with the example in the Appendix. Consequently, there is no *a priori* reason for getting rid of H-inferior processes in terms of efficiency.

5. CONCLUDING REMARKS

When Steedman revealed PPNSV, he was misunderstood a little. Since he gave an example of a two-commodity economy, his result necessarily implied the existence of an inferior process as normally understood. Some authors, like Itoh, pointed out seemingly unnatural settings of Steedman's example and questioned the applicability of PPNSV.[9]

Surely, PPNSV can only be applied to a limited type of technology as far as we concentrate on a two-commodity economy. But if we increase dimensions, PPNSV does not always imply the existence of inferior processes. Although it implies a different type of inferiority among processes, the new definition does not match our common usage, since,

---

[8] Suppose a net output matrix as follows:

$$\mathbf{B} - \mathbf{A} = \begin{array}{c} \begin{array}{cccc} \text{(I)} & \text{(II)} & \text{(III)} & \text{(IV)} \end{array} \\ \begin{pmatrix} 2 & \frac{3}{2} & 1 & 0 \\ 0 & 1 & \frac{3}{2} & 2 \\ 2 & \frac{3}{2} & 1 & 0 \\ 0 & 1 & \frac{3}{2} & 2 \end{pmatrix} \end{array}$$

This technology satisfies non-inferiority. Any convex combination of processes cannot dominate another process. Yet all processes are H-inferior (and so F-inferior), since the following holds:

$$\text{(I)} \times \tfrac{2}{5} + \text{(III)} \times \tfrac{3}{5} < \text{(II)}$$

$$\text{(II)} \times \tfrac{3}{5} + \text{(IV)} \times \tfrac{2}{5} < \text{(III)}$$

[9] Steedman gives the reason why an inferior process is activated in a steady-growth equilibrium (Steedman (1975, pp. 120–121)).

for instance, all processes can be inferior. It is clear that when PPNSV was critically examined by some authors, the former type of inferiority was envisaged, but not the latter.

Taking all these points into account, we can conclude that Steedman's result (PPNSV) does not depend upon inferiority of processes, and it has applicability to a wider range of technology than widely believed. It seems quite inappropriate to criticize PPNSV from a technological viewpoint.

APPENDIX

An example which shows the PPNSV holds even if the non-inferiority condition is satisfied.

Let us consider the following example:

$$\mathbf{A} = \begin{pmatrix} 10 & 0 & 10 \\ 0 & 10 & 0 \\ 10 & \frac{10}{3} & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 12 & 3 & 0 \\ 2 & 12 & 0 \\ 11 & \frac{11}{3} & \frac{3}{2} \end{pmatrix}$$

$$\mathbf{L} = (1\,1\,1)$$

$$\mathbf{d} = \begin{pmatrix} 5 \\ 8 \\ \frac{1}{20} \end{pmatrix}$$

A net output matrix is

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 & 3 & -10 \\ 2 & 2 & 0 \\ 1 & \frac{1}{3} & \frac{3}{2} \end{pmatrix}.$$

As easily seen from the net output matrix, each process has an advantage relative to other processes. Furthermore, any convex combination of two processes cannot produce more net output than another; that is, our technology satisfies non-inferiority. Notice that the technology also satisfies feasibility.

We assume that the rate of profit is 10%, i.e. $r = 0.1$. First, suppose that (1) and (2) hold with a strict equality. Then we can calculate equilibrium price and activity vectors from (1) and (2):

$$\mathbf{p} = (\tfrac{1}{5}\ \tfrac{2}{5}\ \tfrac{32}{15})$$

$$\mathbf{x} = \begin{pmatrix} \frac{559}{646} \\ \frac{41}{323} \\ \frac{5}{646} \end{pmatrix}.$$

(We will show that the solution is unique later.) That is, every process is activated and every commodity has a positive price in equilibrium.

Steedman's labour values are calculated as follows:

$$\mathbf{v} = (-\tfrac{4}{31}\ \tfrac{45}{62}\ -\tfrac{6}{31}).$$

Even though the technology satisfies non-inferiority, the labour values of the two commodities are calculated as negative. Since surplus value (S) equals unity minus variable capital (V), we can obtain S as

$$\begin{aligned} S &= 1 - V \\ &= 1 - \mathbf{v}\frac{1}{\mathbf{pd}}\mathbf{d} \\ &\cong -0.2. \end{aligned}$$

This calculation implies that positive profits exist with negative surplus value, *under the assumption that there is no inferior (and thus no superior) process in an economy*.

We can show the uniqueness of the solution and labour value as follows:

since (2) in the main text holds, we have

$$\begin{aligned} \mathbf{x}_1 + 3\mathbf{x}_2 - 11\mathbf{x}_3 &\geqq 5/\mathbf{pd} \\ 2\mathbf{x}_1 + \mathbf{x}_2 &\geqq 8/\mathbf{pd} \\ (3/2)\mathbf{x}_3 &\geqq 1/(20\mathbf{pd}), \end{aligned}$$

which implies that $\mathbf{x}_3$ is positive and at least one of $\mathbf{x}_1$ and $\mathbf{x}_2$ is positive. Suppose $\mathbf{x}_2 = 0$. Taking $\mathbf{Lx} = 1$ into account and coupling the first and the third inequalities of the above, we can deduce $(1/\mathbf{pd}) \leqq (5/27)$.

On the other hand, from the cost-price inequality we have

$$\begin{aligned} \mathbf{p}_1 + 2\mathbf{p}_2 &\leqq 1 \\ 3\mathbf{p}_1 + \mathbf{p}_2 &\leqq 1 \end{aligned}$$

$$-11\mathbf{p}_1 + (3/2)\mathbf{p}_3 = 1$$

The third holds with equality since $\mathbf{x}_3$ is positive. Utilizing these (in)equalities, we can calculate as follows:

$$\begin{aligned}\mathbf{pd} &= 5\mathbf{p}_1 + 8\mathbf{p}_2 + (1/20)(2/3)(11\mathbf{p}_1 + 1)\\ &\leqq (161/30)\mathbf{p}_1 + (1/30) + 4(1 - \mathbf{p}_1)\\ &\leqq 404/90,\end{aligned}$$

which implies $(1/\mathbf{pd}) \geqq 90/404$. A contradiction.

We can also show that $\mathbf{x}_1$ must be positive in the same way. All processes are activated, and (1) must hold with equality. Then, all prices are positive, and therefore, (2) must hold with equality.

Proposition

Suppose an economy is feasible. Then, some processes are H-inferior if, and only if, there is F-inferiority.

Proof. Suppose I and J belong to $\{1, 2, \ldots, n\}$ and $I \cap J = \phi$. If processes belonging to J are H-inferior,

$$\sum_{i\in I}\alpha_i(b_i - a_i) > \sum_{j\in J}\beta_j(b_j - a_j) \tag{A1}$$

holds for α_i and β_j such that $\alpha_i \geqq 0$, $\beta_i \geqq 0$, $\sum_{i\in I}\alpha_i = 1$ and $\sum_{j\in J}\beta_j = 1$. Since $\alpha_i > 0$ for some i, there exists positive t such that $t < 1$ and

$$\sum_{i\in I}t\alpha_i(b_i - a_i) > \sum_{j\in J}\beta_j(b_j - a_j)$$

with $\sum_{i\in I}t\alpha_i < 1 = \sum_{j\in J}\beta_j$. This means that processes which belong to J are F-inferior.

Conversely, suppose that some processes are F-inferior. Then, there exist non-negative α_i's and β_j's such that (A1) holds with

$$\sum_{i\in I}\alpha_i < \sum_{j\in J}\beta_j,$$

where I and J belong to $\{1, 2, \ldots, n\}$, and $I \cap J = \phi$.

Since the economy is feasible, there exists non-negative q such that

$$[\mathbf{B} - \mathbf{A}]q > 0.$$

Therefore, for sufficiently small positive t, the following holds:

$$\sum_{i \in I} t\alpha_i(b_i - a_i) + \sum_{i=1}^{n} q_i(b_i - a_i) > 0. \tag{A3}$$

Clearly, we have

$$\sum_{i \in I} t\alpha_i(b_i - a_i) + \sum_{i=1}^{n} q_i(b_i - a_i) > \sum_{j \in J} t\beta_j(b_j - a_j) + \sum_{i=1}^{n} q_i(b_i - a_i) \tag{A4}$$

and

$$\sum_{i \in I} t\alpha_i + \sum_{i=1}^{n} q_i < \sum_{j \in J} t\beta_j + \sum_{i=1}^{n} q_i. \tag{A5}$$

Then, there exists k such that $k > 1$ and

$$k\left(\sum_{i \in I} t\alpha_i + \sum_{i=1}^{n} q_i\right) = \sum_{j \in J} t\beta_j + \sum_{i=1}^{n} q_i. \tag{A6}$$

Considering (A3), (A4) and $k > 1$, we know

$$k\left[\sum_{i \in I} t\alpha_i(b_i - a_i) + \sum_{i=1}^{n} q_i(b_i - a_i)\right] > \sum_{j \in J} t\beta_j(_i b_j - a_j) + \sum_{i=1}^{n} q_i(b_i - a_i)$$

holds. Define $\mathbf{x}_1$ and $\mathbf{x}_2$, by $k(t\alpha_i + q_i)$ and $t\beta_j + q_j$ respectively. Then

$$[\mathbf{B} - \mathbf{A}]\mathbf{x}_1 > [\mathbf{B} - \mathbf{A}]\mathbf{x}_2 \text{ and } \mathbf{L}\mathbf{x}_1 = \mathbf{L}\mathbf{x}_2 \tag{A7}$$

hold. There exist non-negative w_1 and w_2 such that

$$\mathbf{x}_1 - \mathbf{x}_2 = \mathbf{w}_1 - \mathbf{w}_2 \text{ and } \{i | w_{1i} > 0\} \cap \{j | w_{2j} > 0\} = \phi$$

hold, where $\mathbf{w}_{k1}$ denotes the i-th component of w_k $(k = 1, 2)$. Then clearly,

$$[\mathbf{B} - \mathbf{A}](\mathbf{w}_1 - \mathbf{w}_2) = [\mathbf{B} - \mathbf{A}](\mathbf{x}_1 - \mathbf{x}_2) > 0$$

and

$$\mathbf{L}(\mathbf{w}_1 - \mathbf{w}_2) = \mathbf{L}(\mathbf{x}_1 - \mathbf{x}_2) = 0$$

hold. They imply that

$$[\mathbf{B} - \mathbf{A}]\mathbf{w}_1 > [\mathbf{B} - \mathbf{A}]\mathbf{w}_2, \quad \mathbf{L}\mathbf{w}_1 = \mathbf{L}\mathbf{w}_2$$

and

$$\{i|w_{1i} > 0\} \cap \{j|w_{2j} > 0\} = \phi$$

hold, which mean the existence of H-inferiority. Q.E.D.

Remark. As is clear from the proof, H-inferior processes are always F-inferior. Yet, the converse does not hold. We give an example:

$$\mathbf{B} - \mathbf{A} = \begin{array}{c} \begin{array}{ccc} (\mathrm{I}) & (\mathrm{II}) & (\mathrm{III}) \end{array} \\ \begin{pmatrix} \frac{6}{5} & 10 & \frac{2}{5} \\ \frac{2}{5} & 10 & \frac{6}{5} \\ -\frac{4}{5} & -2 & \frac{1}{5} \end{pmatrix} \end{array} \qquad \mathbf{L} = (1, 1, 1).$$

Consider an activity vector $(1/2, 1/10, 1/2)'$. Then, we have the following:

$$(1/2)(\mathrm{I}) + (1/2)(\mathrm{III}) < (1/10)(\mathrm{II})$$

and

$$(1/2)1 + (1/2)1 > 1/10.$$

Therefore, the first and the third processes are F-inferior. But the third process cannot be H-inferior, since any convex combination of (I) and (II) does not dominate (III). (Notice that the first process is not only H-inferior but also inferior.)

REFERENCES

Duménil, G. and D. Lévy (1984) "The Unifying Formalism of Domination: Value, Price, Distribution and Growth in Joint Production", *Zeitschrift für Nationalökonomie*, Vol. 44, No. 4, pp. 349–371.

Filippini, C. and L. Filippini (1982) "Two Theorems on Joint Production", *Economic Journal*, Vol. 92, pp. 386–390.
Fujimori, Y. (1982) *Modern Analysis of Value Theory*, Springer-Verlag.
Fujimoto, T. and U. Krause (1988) "More Theorems on Joint Production", *Zeitschrift für Nationalökonomie*, Vol. 48, No. 2, pp. 189–196.
Itoh, M. (1981) "Joint Production: The Issues After Steedman", in Steedman et. al. (1981).
Morishima, M. (1976) "Positive Profits with Negative Surplus Value – A Comment", *Economic Journal*, Vol. 86, pp. 599–603.
Morshima, M. and G. Catephores (1978) *Value, Exploitation and Growth*, McGraw-Hill, London.
Okishio, N. (1977) *Marxian Economics* (in Japanese) Chikuma Shobo, Tokyo.
Sraffa, P. (1960) *Production of Commodities by Means of Commodities*, Cambridge University Press, Cambridge.
Steedman, I. (1975) "Positive Profits with Negative Surplus Value", *Economic Journal*, Vol. 85, pp. 114–123.
Steedman, I. (1976) "Positive Profits with Negative Surplus Value: A Reply", *Economic Journal*, Vol. 86, pp. 604–608.
Steedman, I. (1976) "Positive Profits with Negative Surplus Value: A Reply to Wolfstetter", *Economic Journal*, Vol. 86, pp. 873–876.
Steedman, I. (1977) *Marx after Sraffa*, NLB, London.
Steedman, I. et. al. (1981) *The Value Controversy*, Verso, London.
Wolfstetter, E. (1976) "Positive Profits with Negative Surplus Value: A Comment", *Economic Journal*, Vol. 86, pp. 864–872.

Keio University 2-15-45 Mita,
Minato-ku
Tokyo 108,
JAPAN