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NEGATIVE LABOUR VALUES AND THE PRODUCTION POSSIBILITY FRONTIER

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ABSTRACT

Steedman's theoretical finding of negative labour values associated with positive equilibrium prices has been criticised on the grounds that this situation obtains only in inefficient economies. A recent paper by Hosoda claims that this criticism is valid only in two-dimensional joint-product systems. It is argued here that the dimensionality of the system is of no relevance to the "inefficiency critique" of Steedman. Rather, the validity of the critique turns on matters relating to the growth rate and the rate of profit. The argument that processes inefficient in a static context may be viable in the context of von Neumann growth is considered, and the implications for the labour theory of value are assessed. Marx's critique of capitalist economic calculation is supported by reference to the divergence of Sraffian prices and Samuelsonian values when the rate of profit is in excess of the rate of growth.

1. INTRODUCTION

In a recent contribution to this journal Eiji Hosoda (1993) argues that Steedman's (1975, 1977) derivation of negative labour values in the context of joint production has been "misunderstood a little" (p. 36). Among the criticisms that have been made of Steedman, Hosoda focuses on the argument that negative labour values—and the associated result of positive profit alongside negative surplus value—can obtain only in an inefficient economy, that is, a system employing a technologically inferior process of production (see for instance Wolfstetter, 1976; Farjoun, 1984). I shall refer to this argument as the "inefficiency critique". Hosoda replies that while this critique holds good for the simple two-commodity case, it does not generalize to higher dimensions. In the higher-dimensional case, "Steed-

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man's conclusion still holds even when there is no inferiority among processes" (Hosoda, 1993, p. 30).

I wish to argue that while Hosoda's algebra is sound, there are two problems with the substantive interpretation he places on his results. (1) *If* the inefficiency critique is granted in the two-commodity case, it generalizes. Hosoda's claim to the contrary depends on the use of a restricted definition of "inferiority" which is not a valid generalization of the problem diagnosed by Wolfstetter in the two-commodity case. On the other hand (2) Hosoda is too quick to concede the inefficiency critique for the two-commodity case. An answer is available—which I shall call the "growth argument"—namely that a system that appears to be inefficient in a *static* context (and that generates negative labour values) may nonetheless be viable if the economy is on a von Neumann growth ray. And there is a further point to be made: even if a system *is* in fact inefficient, it may nonetheless represent a possible equilibrium for a capitalist economy, provided that the rate of profit exceeds the rate of growth.[1]

2. WHAT IS AGREED?

Consider the simple 2×2 case (two processes, two commodities) as presented in Steedman (1977). Let us define a "dominated" process as one that produces smaller net output of at least one of the goods, per unit labour input, and does not produce larger net output of either of the goods—where "net output" is understood as gross output minus the input requirements for simple reproduction of the productive system. It is quite clear that a negative labour value for one of the goods can be obtained only if one of the processes is dominated by the other in the above sense (Wolfstetter, 1976). *Prima facie*, this suggests that Steedman's negative labour values are an uninteresting curiosity, arising only in an inefficient economy (i.e. one operating inside its production possibility frontier). Farjoun, for instance, contends that "since the net output is the aim of the production process", examples of systems exhibiting negative labour values "cannot be regarded as economically reasonable" (Farjoun, 1984, p. 19). As mentioned above, this interpretation is vulnerable to the growth argument (a "dominated" process may actually be employed in an efficient

[1] While the latter points vindicate Steedman's argument at the theoretical level, I do *not* see them as validating his general assault on the Marxian labour theory of value, which has received striking empirical support over recent years. See for instance Shaikh (1984), Petrovic (1987), Ochoa (1989), Cockshott, Cottrell and Michaelson (1995).

growth equilibrium), a point to which I return in section 6. For the moment, however, let us grant the inefficiency critique for the 2×2 case, in order to focus on Hosoda's claim that this critique fails to generalize to higher-dimensional systems. Since granting the critique means abstracting from the growth argument, I shall assume until further notice that we are dealing with an economy undergoing simple reproduction, and when I speak of "efficiency", and of the "production possibility frontier", these terms should be understood as relative to the simple reproduction path. While this may seem artificial, I think it will be useful for sorting out the issues at play in Hosoda's paper.

When we move beyond the 2×2 case, the notion of a dominated production process must be extended somewhat. We shall say that a given process is dominated if and only if there exists a zero-sum reallocation of labour away from this process in favour of some other process *or combination of other processes*, such that the vector of resulting changes in net outputs is semipositive, i.e., at least one element is positive and no element is negative. Note particularly that the property of being dominated, as defined here, is a property of an *individual* process—in the context, of course, of a set of other processes. (This is very close to Hosoda's first definition of an "inferior" process (1993, p. 32), although his is a slightly stronger condition: a process is "inferior" if and only if there exists a zero-sum reallocation of labour in favour of other processes, which produces a *strictly positive* vector of changes in net output.)

Hosoda then argues, quite correctly, that in systems larger than 2×2 one can obtain negative labour values *even if the system contains no dominated process*. In his Appendix he gives an example of a 3×3 system with no dominated process, where there is a unique solution for labour values featuring one positive and two negative values.

3. INEFFICIENCY IN HIGHER-ORDER SYSTEMS

The next step is to recognize that a further, and more general, conception of inefficiency is appropriate for higher-dimensional systems. Even if a system contains no dominated process as such, it may be the case that there exists a zero-sum reallocation of labour away from some *combination* of processes in favour of another or others, which produces a non-zero non-negative vector of changes in net outputs. At this point some formalism may be useful. Let $\boldsymbol{Q}$ be a $m \times n$ matrix of net outputs, where q_{ij} represents the output of commodity j ($j = 1, \ldots, n$) produced by process i ($i = 1, \ldots, m$) when operated at an intensity of one unit of labour. I refer to this matrix as the production table. The conception of inefficiency noted above corresponds to what Farjoun (1984) calls the "reducibility" of the $\boldsymbol{Q}$ matrix, which is defined formally below.

Definition 1 (reducibility of the production table)
$\boldsymbol{Q}$ is reducible if and only if there exists a m-vector $\boldsymbol{w} \equiv (w_1 \ w_2 \ \cdots \ w_m)$ such that

(a) $w_1 + w_2 + \cdots + w_m = 0$; and
(b) $0 \neq \boldsymbol{wQ} \geqslant 0$

The notation at point (b) says that $\boldsymbol{wQ}$ is semipositive.

We now note that there exists no strictly positive vector of labour values if and only if the $\boldsymbol{Q}$ matrix is reducible (see Filippini and Filippini, 1982; Farjoun, 1984; and for the basic theorems necessary to support this result, Gale, 1960). The example given by Hosoda in his Appendix is clearly reducible. $\boldsymbol{Q}$ in my notation is equivalent to $(\boldsymbol{B} - \boldsymbol{A})$ transpose in Hosoda's notation. The Appendix example is then

$$\boldsymbol{Q} = \begin{pmatrix} 2 & 2 & 1 \\ 3 & 2 & \frac{1}{3} \\ -10 & 0 & \frac{3}{2} \end{pmatrix},$$

which is reducible since $\boldsymbol{wQ} = (\frac{10}{3} \ \frac{2}{3} \ \frac{5}{18}) > 0$ for $\boldsymbol{w} = (1 \ -\frac{2}{3} \ -\frac{1}{3})$.

But this means—remembering that we have set aside the growth argument for the moment—that an economy operating all of the processes in Hosoda's table cannot be on its production possibility frontier.[2] Specifically, so long as processes II and III (corresponding to $i = 2$, 3 respectively) are *both* in use, it will always be possible to augment the net output of *all* goods (actually a stronger condition than is required for reducibility) by withdrawing labour from these processes in the right ratio—for instance 2:1, as shown—and applying it to process I. This is despite the fact that neither process II nor process III is dominated.

Hosoda's case is a counter-example to the proposition p_1 that in higher-dimensional systems negative labour values arise only if the production table contains a *dominated* process, but it is not a counter-example to the proposition p_2 that negative labour values arise only if the production table is *reducible*. I am not sure who has asserted p_1; but even if any of

[2] This is not, however, to say that such an economy could not be in a capitalist equilibrium. See section 7.

Steedman's critics were unguarded enough to do so,[3] it should be clear that p_2 is the appropriate generalization of the inefficiency critique as applied to the 2×2 case. It is quite explicitly p_2 that is asserted by Farjoun (1984). What "matters" from this point of view is that the economy in question can achieve a net output that is greater in some dimensions and no smaller in any dimension, via a reallocation of labour among processes. In the 2×2 case this implies the presence of a dominated process; in higher-order systems it does not.

4. HOSODA ON "F-INFERIORITY" AND "H-INFERIORITY"

Contrary to the above, Hosoda claims, or at least implies, that proposition p_1 is the important one. To understand how he reaches this view, we need first to draw a correspondence between the language used above and that used by Hosoda. Besides the notion of "inferiority" mentioned above, Hosoda talks in terms of "F-inferiority", his label for the conception of inferiority among sets of production processes set out by Fujimori (1982, pp. 49–50), and "H-inferiority", a definition of his own. A proper subset J of the available processes is said to be F-inferior to another non-intersecting subset I if and only if labour can be reallocated from members of J to members of I in such a way as to (a) produce a strictly positive vector of changes in net outputs, while (b) leaving some remainder of unallocated labour. "H-inferiority" is a slight variant of the same idea, in which condition (b) is replaced by the requirement that the reallocation of labour be zero-sum. Hosoda shows that for any feasible economy (capable of producing a strictly positive net output vector), F-inferiority is present if and only if H-inferiority is present. Given this result, it is unnecessary to maintain the distinction between F- and H-inferiority for my purposes here, and I shall take F-inferiority as representative of Hosoda's conception of inferiority in higher-dimensional systems.[4]

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[3] Wolfstetter (1976) asserts p_1 for the 2×2 case (where it is correct), but does not explicitly discuss the question of higher-dimensional systems.

[4] Hosoda's definitions may be translated into the notation employed in my Definition 1 as follows.

F-inferiority is present $\Leftrightarrow \exists \boldsymbol{w}(w_1 + w_2 + \cdots + w_m < 0 \wedge \boldsymbol{wQ} > 0)$; and

H-inferiority is present $\Leftrightarrow \exists \boldsymbol{w}(w_1 + w_2 + \cdots + w_m = 0 \wedge \boldsymbol{wQ} > 0)$.

It will be seen that the presence of H-inferiority is a special case of reducibility of $\boldsymbol{Q}$.

Clearly, the presence of Hosoda's F-inferiority implies that the production table is reducible. And in the example cited above processes II and III form an F-inferior set, as Hosoda himself points out.

We have to be careful with the economic interpretation here. If a given process is *dominated* then (in the context of simple reproduction at any rate) we can safely strike it from the production table. But from the fact that processes II and III in the example above are jointly F-inferior to process I, it does *not* follow that we can delete II and III from the table. In fact, it is clear from an inspection of $\boldsymbol{Q}$ that in order to maximize output of commodity 2 (respectively, 3) process II (respectively, III) must be used. What *does* follow, however, is that if the system is to be irreducible, one must not employ *all* the members of any given F-inferior set. In the example, one may use process II or process III, but not both, in conjunction with process I. In other words, the irreducible production tables derivable from the example are: process I plus process II, and process I plus process III. And of course each of these (2×3) tables has a strictly positive solution for the vector of labour values.[5]

The trouble with Hosoda's argument may be traced to his practice of referring to *particular processes* as F-inferior, when he means that they belong to F-inferior sets. Suppose some process P is F-inferior in itself, i.e., there exists an F-inferior set J which has P as its sole member. Then it follows that P is dominated, and should be struck from the table. But if P is not dominated it is potentially misleading to say *it* (singular) is F-inferior. If one does label all processes that feature in F-inferior *sets* as themselves "F-inferior", then the temptation may arise to reason thus: we clearly do not want to strike all F-inferior processes from the table; therefore the fact that a certain process is F-inferior is not sufficient reason to strike it out; therefore there is no need to strike any such processes from the table. The third clause is a non-sequitur.

Matters are clearer, I believe, if one uses Farjoun's terminology of reducibility, which refers to the production table as a whole. Once again, in order to attain the production possibility frontier (for an economy undergoing simple reproduction) one must ensure that the production table is irreducible. Starting from a reducible table, one has to eliminate *some* members of the F-inferior sets that are present, to the point at which there

[5] Labour values are given by the n-vector v that solves the system $\boldsymbol{u} = \boldsymbol{Q}\boldsymbol{v}$, where $\boldsymbol{u}$ is the m-dimensional unity vector (i.e. all elements equal to unity). When $\boldsymbol{Q}$ is composed of processes I and II there is a (non-unique) positive solution $\boldsymbol{v} = (\frac{1}{9}\ \frac{2}{9}\ \frac{1}{3})'$; and when $\boldsymbol{Q}$ is composed of processes I and III there is a positive solution (again non-unique) of $\boldsymbol{v} = (\frac{1}{40}\ \frac{7}{120}\ \frac{5}{6})'$.

are no such sets left. With the exception of the striking-out of *dominated* processes, however, the particular eliminations carried out will depend on the final demand proportions.

Consider in this light Hosoda's example (1993, p. 36n) of a 4×4 production table in which, in his terminology, *all four* processes are "F-inferior". That is because of the following two facts: (a) processes I and III form an F-inferior set in relation to process II; and (b) processes II and IV form an F-inferior set in relation to process III. Having pointed this out, Hosoda stops short. He leaves the impression that since we obviously do not want to eliminate *all* the processes on the grounds of their "F-inferiority", there is no call to eliminate any of them. But that would be wrong. From (a) it follows that we should not use both process I and process III when II is available; and from (b) it follows that we should not use both II and IV when III is available.[6]

One implication of the above argument may be noted in passing. In the examples discussed, the choice of the processes to be activated (and those to be eliminated) to ensure that the system is irreducible depends in part on the final demand proportions. Since the labour values of the commodities depend, in general, on the choice of processes, this means that labour values themselves depend on the demand proportions. This seems an unavoidable consequence, within the context of multiple joint-production processes, of the basic Marxian definition of values in terms of *socially necessary* labour time. It comes as no surprise, of course, being a corollary of Mirrlees' point (1969) that the nonsubstitution theorem, according to which relative prices are independent of demand, holds only on condition that "joint production of an essential kind" is excluded.

5. INFERIORITY AND INEFFICIENCY

As noted above, Hosoda seems to regard the question of whether or not a *dominated* process is employed (i.e., one he calls "inferior" without any prefix) as the only legitimate issue at play in the inefficiency critique of Steedman's negative labour values. Of the case where there is no

[6] In fact, the largest irreducible tables derivable from this example feature only two processes. They are, in a fairly obvious notation, (I, II), (II, III) and (III, IV). Given the pattern of F-inferiority indicated in the text, if one starts from any of these reduced tables the activation of an additional process will always produce a strictly smaller net output vector than a suitable re-scaling of the processes already in use.

dominated process, but yet "F-inferior" sets of processes are present, he says this "*merely* implies an inefficient combination of processes and does not fit our common usage of inferiority of techniques" (p. 30, emphasis added). Definitions of inferiority or inefficiency that go beyond the presence of individually dominated processes "*just* require that a certain type of combination of processes be inefficient, though each process may not be inefficient on its own. They *just* characterize the combination of processes" (p. 35, emphasis added). This, he says, "seems quite different from the normal usage of inferiority of processes" (p. 36). And the upshot is that "it seems quite inappropriate to criticize [Steedman's negative labour values] from a technological viewpoint" (p. 37).

Whether or not it is deliberate, the repetition of "merely" and "just" in the statements above creates the impression that the inefficiency of the *system as a whole* is somehow of no concern. But surely the question of whether or not individually dominated processes are in use is secondary at best. What matters for the inefficiency critique is that being inside the production possibility frontier, however it comes about, is "inferior" to being on it. If Steedman's negative values arise only in systems operating inside the PPF, then the inefficiency critique appears to strike home. But at this point we must face the second problem mentioned in the Introduction.

6. THE GROWTH ARGUMENT

In this section we revisit the contention that production processes that are dominated in the context of simple reproduction may nonetheless be activated on a von Neumann growth path.[7] This point generalizes: production tables that are reducible in the context of simple reproduction may be irreducible in a growth context. The definitions of domination and reducibility given above were framed in terms of net outputs, defined in the usual way as gross outputs minus ("historic") input requirements. But in a steady-growth context, we must shift our focus to the net output available for use after the *augmented* input requirements for the next period's production are met.

[7] It should be noted that Steedman made this point—or something very close to it—in the brief and somewhat elliptical section titled "An Implicit Assumption" of his original paper (1975, pp. 120–121). Steedman's version of the argument, however, claims greater generality and is also more contentious, relying as it does on the assumption that the slightest excess supply of a commodity is sufficient to send its price to zero. This assumption has been challenged by Morishima (1976, esp. p. 600) and Farjoun (1984).

If $\boldsymbol{B}$ denotes the gross output matrix (as before, normalized via a unit labour input for each process), and $\boldsymbol{A}$ denotes the input matrix, then the "production table", $\boldsymbol{Q}$, of which we spoke above is just $(\boldsymbol{B}-\boldsymbol{A})$. The modified production table for a system growing at a uniform rate g is given by

$$\boldsymbol{Q}_g = [\boldsymbol{B} - (1 + g)\boldsymbol{A}] = \boldsymbol{Q} - g\boldsymbol{A}.$$

The point then is that $\boldsymbol{Q}_g$ may be irreducible, for $g > 0$, even though $\boldsymbol{Q}$ is reducible. This can be illustrated with a simple 2×2 case. Suppose we have

$$\boldsymbol{B} = \begin{pmatrix} 2 & 2 \\ 10 & 33 \end{pmatrix} \quad \text{and} \quad \boldsymbol{A} = \begin{pmatrix} 1 & 1 \\ 8 & 30 \end{pmatrix}.$$

This gives us

$$\boldsymbol{Q} = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$$

while for $g = .10$,

$$\boldsymbol{Q}_g = \begin{pmatrix} .9 & .9 \\ 1.2 & 0 \end{pmatrix}.$$

$\boldsymbol{Q}$ is clearly reducible (process I is dominated), but $\boldsymbol{Q}_g$ is irreducible: process I must be operated if the system is to produce any output of commodity 2 over and above the growth-augmented input requirements.

I hope the reader will excuse the trivial example, but I cite it to underline the point that *the growth argument has nothing to do with the dimensions of the system.* (I also wish to have the example on hand for further discussion in section 7 below.) As we saw above, Hosoda produces an example of a 3×3 system in which no individual process is dominated, yet labour values are negative. We also saw that the negative values arose due to the reducibility of the production table. Yet in his Appendix, Hosoda is able to show an equilibrium in which all three processes are in use. What he does not make clear is that the equilibrium solution given in the Appendix is possible only because he assumes a positive growth rate and rate of profit (of 10 per cent, as it happens). As Voltaire said, one can kill a sheep with incantations ... and a little poison: in this case the "incantations" are the argument concerning dimensionality, while the "poison" is the positive growth rate. (The sheep, of course, is the inefficiency critique of Steedman.)[8]

7. CONCLUDING REMARKS

The main point of this note—that the dimensions of the productive system are quite immaterial for the "inefficiency critique" of Steedman's negative labour values—has now been made, but since we have raised the growth argument, a comment on its implications may be in order. That is, just how potent is the growth-rate "poison"? The point that the table $\boldsymbol{Q}_g$ may be irreducible, for positive growth, while the table $\boldsymbol{Q}$ is reducible would seem to invalidate the inefficiency critique of Steedman, *even for the two-dimensional system*, and restore the theoretical embarrassment for the labour theory of value under joint production. If $\boldsymbol{Q}$ is reducible then there will be no strictly positive solution for labour values as conventionally defined, yet all the processes contained in $\boldsymbol{Q}$ may nonetheless be in use in an efficient growth equilibrium (and all the commodities have positive prices).

To examine this point further we need to distinguish the rate of profit r and the growth rate g. It is common in the literature on this topic to assume that $r = g$, but of course the actual situation in capitalist economies is typically that r is well in excess of g, due to substantial consumption on the part of the capitalist class. The measurement of the rate of profit is, of course, subject to a number of theoretical and empirical problems, but in the course of a comprehensive review of empirical measures of profitability in the USA Duménil, Glick and Rangel (1987, p. 338) offer average figures of 16.2 percent and 11.4 percent for the periods 1948–1967 and 1970–1984 respectively.[9] This contrasts with average growth rates of real GNP of 3.9 percent and 2.5 percent over the same two periods.

Let $\boldsymbol{Q}_r$ denote the table formed by subtracting $(1 + r)\boldsymbol{A}$ from $\boldsymbol{B}$. Now consider the vector $\boldsymbol{v}$ that solves the system $\boldsymbol{u} = \boldsymbol{C}\boldsymbol{v}$, where $\boldsymbol{u}$ is the m-dimensional unity vector (all elements equal to unity). The vector $\boldsymbol{v}$ has the following interpretations:

1. Vector of Marxian values, if $\boldsymbol{C} = \boldsymbol{Q}$.

[8] I do not claim originality for the use of this *bon mot* in the context of economic argumentation—see Sraffa (1932) on Hayek.

[9] These figures are for the ratio of profit to the gross replacement cost of capital for the whole of the US economy.

2. Vector of "Samuelsonian values", if $\boldsymbol{C} = \boldsymbol{Q}_g$.
3. Vector of Sraffian prices, if $\boldsymbol{C} = \boldsymbol{Q}_r$.

I use the term "Samuelsonian values" at point 2 following the suggestion of Samuelson and von Weiszäcker (1972) that such modified values be used in the context of socialist planning in place of the standard Marxian values.

Clearly, the situation *may* arise where $\boldsymbol{Q}$ is reducible (therefore some labour values are negative) while $\boldsymbol{Q}_r$ is irreducible (all Sraffian prices are positive). And this may be regarded as an embarrassment in principle for the labour theory of value—although the empirical relevance of this possibility remains to be demonstrated, and it is a big leap from the point that some labour values might be negative under joint production to the idea that aggregate surplus value might be negative in any realistic case. But if we have $r > g > 0$ in a capitalist economy, a further possibility arises: it may be that $\boldsymbol{Q}_g$ is reducible while $\boldsymbol{Q}_r$ is irreducible.

To illustrate this, return to the 2×2 example discussed in section 6. We showed that while the $\boldsymbol{Q}$ matrix is reducible, $\boldsymbol{Q}_g$ is irreducible for $g = .10$. Suppose now that we have a rate of *profit* of 10 per cent, while $g = .03$, roughly plausible figures for a capitalist economy. In that case, $\boldsymbol{Q}_r$ will be irreducible, while

$$\boldsymbol{Q}_g = \boldsymbol{B} - (1.03)\boldsymbol{A} = \begin{pmatrix} .97 & .97 \\ 1.76 & 2.1 \end{pmatrix}$$

is clearly reducible. Not only are positive equilibrium prices associated with negative Marxian values: they are also associated with negative Samuelsonian values.

The implications of this possibility are two-edged. First, it means that one cannot assume, as Farjoun does in the context of his critique of Steedman, that the inefficiency of a production system is sufficient grounds for concluding that it will not be employed in a capitalist equilibrium.[10] To that extent, Steedman is vindicated. But, on the other hand, this possibility also vindicates Marx's critique of the irrationality of capitalist economic calculation (Marx, 1976, pp. 515–517)—a critique developed from the standpoint of the labour theory of value. For it says that in a capitalist equilibrium, with prices all positive (and the rate of profit equalized across all processes), a combination of processes may be in use that is inefficient

[10] I should point out that many interesting aspects of Farjoun's (1984) argument do not depend on this particular claim.

relative to the economy's growth path. Labour is being wasted, and a socialist planner using Samuelsonian values could do better. Indeed, if g is small and r is large, a planner using straightforward labour values might do better.[11]

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[11] For some new proposals on the use of a calculus of labour time in planning, see Cockshott and Cottrell (1993).