

# THE BEST USE OF ECONOMIC RESOURCES

*by*

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## EDITORIAL PREFACE

A CONSIDERABLE number of foreign works have recently appeared, concerned with problems of the organization and planning of production, and involving the practical use of methods of economic calculation—a new aspect of applied mathematics. This aspect, which has been developed abroad only in the last decade, is at present widely known as linear programming. In essence it deals with problems of the rational selection of variants in the solution of questions of economic planning so as to use a set of mutually related factors within a given complex in the best possible way.

The methods of linear programming can be utilized for the solution of the most diverse practical problems expressed in standard mathematical formulations.

Under capitalism these methods are employed with a view to the better utilization of resources (materials, equipment, areas, means of transport, capital investment, etc.) in order to achieve greater profit; these methods are mainly employed by firms internally. The sphere of application of the methods of linear programming is not restricted, however, to problems of production only; a number of government departments in the U.S.A. (the National Bureau of Standards, the Budget Bureau, the Bureau of Labour Statistics), are also very interested in this field of economic and mathematical research, and encourage the development of new methods which they utilize for their own purposes.

In a socialist economy quantitative methods of analysis and the solution of similar problems acquire an incomparably greater significance. The preparation of national economic plans embracing a vast number of mutually related and interdependent economic units and of factors of production, and the coordination of these plans—not only in space but also in time—in order to achieve the greatest possible economic or technical effect (at any given moment or in the long run) is a task of enormous magnitude. With the level of development of the national economy and the exceptional complexity of internal economic relations, the problem of finding the best

possible system for planning would become insurmountable without a fundamental appreciation of the quantitative methods of economic calculations and without the utilization of the latest computing techniques. The use of modern mathematical methods in the organization and planning of production provides a real and very efficient answer to the problem. It is, therefore, not surprising that linear programming, as an independent discipline, first emerged in the Soviet Union. Important results in this field were achieved in 1938-9 by the author of this book, L. V. Kantorovich, and published by him in a number of works beginning in 1939. The first of these contained fundamental advances and determined the content and further development of this discipline: it examined the mathematically new type of "extremal" problems; it evolved a universal method for their solution (method of solution multipliers) as well as various efficient numerical algorithms derived from it; it indicated the more important fields of technical-economic problems where these methods could be most usefully applied; and it brought out the economic significance of indicators resulting from an analysis of problems by this method which are quite essential in problems of a socialist economy.

It should be mentioned that in the period 1948-50, independent of the work referred to, an extensive programme of research on linear programming was begun in the U.S.A. (It was precisely during this period that the term "linear programming" appeared.) Eminent American mathematicians and economists (Koopmans, Dantzig, Tucker, Charnes, Dorfman and many others) have been engaged in various theoretical researches, and are showing the basic scope of the practical application of linear programming in a capitalist economy, in establishing its connection with the theory of matrix games, and in working out various numerical methods of solution. G. Dantzig, in particular, produced the "simplex method"‡ which is sufficiently universal and is now the most frequently used method. Up to the present time linear programming has been used abroad in various ways in the solution of the most diverse technical-economic problem. The literature on this subject runs into hundreds of titles.

† L. V. Kantorovich: *Matematicheskie metody organizatsii i planirovaniya proizvodstva* (Mathematical Methods in the Organization and Planning of Production), Leningrad University, 1939

‡ For a comparison of Dantzig's method and the methods of the author see Appendix II, p. 310.

Moreover, many of the findings of the work of L. V. Kantorovich (which were unknown abroad for a long time) were newly "discovered" in one way or another in American works in the period 1949–56. At the present time, the priority of Soviet science in developing the basic propositions of this new discipline is acknowledged even by the American scholars themselves.†

Almost at the same time as linear programming, a new trend in the utilization of mathematical methods in economics was formulated, i.e. "input-output analysis". The matrices of inter-sector activity developed by V. V. Leontief for the economy of the U.S.A.‡ were the principal element in this analysis. Soviet literature on economics shows that Leontief's method was considerably influenced by Soviet economic ideas of the 1920's and in particular by the first "Balance of the National Economy of the U.S.S.R. for 1923–4". Leontief completed his studies at Leningrad University in the 1920's and was familiar with Soviet balancing methods. Leontief's model is a very special case of the linear programming problem. "Input-output analysis" can be applied to some problems not connected with the finding of extremal solutions (analysis of the balance of consumption and production, clarification of the structure of inputs, study of inter-sectoral and inter-regional relations, etc.).

In this work offered to the reader by Corresponding Member of the Academy of Sciences of the U.S.S.R., L. V. Kantorovich, which sums up his extensive researches, methods of linear programming are further developed and used in a new field—planning and economic calculations. This study includes a number of aspects which bring the formulation of the problems under consideration much closer to concrete national economic conditions.

In conditions of socialist production, methods of linear programming can be of enormous benefit. They are the principal means of

† See, for example, p. 63 of T. C. Koopmans' book: *Three Essays of the State of Economic Science*, McGraw-Hill, New York, 1957.

‡ V. V. Leontief: *The Structure of American Economy, 1919–1939. An Empirical Application of Equilibrium Analysis*, Oxford University Press, New York, 1951. The first edition of this work, with calculations for 1919 and 1929 only, was published by Harvard University in 1941. A collection of articles by V. V. Leontief and other American authors has appeared in Russian under the general title *Studies in the Structure of the American Economy*, State Statistical Publishing House, Moscow, 1959 (a translation of the 1953 American edition).

analysing and accurately solving those problems which arise when separate economic questions are brought into a schematic and mathematical formulation. These methods have been tested in practice to a considerable degree in separate sectors of the national economy.

Proofs and illustrations of the possibility of their practical application in the solution of specific technical-economic and planning problems constitute the major part and the principal value of L. V. Kantorovich's book.

This book is not intended for the professional mathematician and therefore the purely mathematical examination of problems is kept separate from the main body of the work. The author expounds gradually the basic ideas and concepts connected with the proposed methods for the quantitative analysis of economic questions. In examining separate problems the author explains the principal methods of calculation, while the systematic presentation and mathematical principles are given in the appendices.

The author's exposition is based on convincingly selected examples, which are simplified in order to show the essence of the approach he is using and to give a step-by-step exposition of basic ideas. The analysis of each example is completed by the formulation of certain general principles (conclusions), and the role and significance of these principles in specific economic conditions are explained. The mathematical analysis of each specific problem shows the auxiliary criteria objectively determined by the problem itself, and evaluates materials, equipment and other factors of production involved. In relation to a specific problem, taken in isolation, the objectively determined valuations are a very important instrument for its solution: they represent the particular technical indicators which characterize a given problem.

The method of objectively determined valuations (solution multipliers) advanced by L. V. Kantorovich has been worked out for fully determined conditions when some of the resources among those being utilized are scarce. In the *present work the author broadens* the concept of scarce resources, and extends it to cover all resources available in a restricted quantity, including here both those already taken up fully (allocated) and those which are temporarily deficient, i.e. those resources in which temporary shortages have arisen. In the proposed system of economic calculations, scarce resources receive a

high valuation, while those available in excess receive zero valuation.

The system of economic calculations using objectively determined valuations makes it possible—on the basis of the valuations of shortages, scarcity and prior commitments of factors of production—to give a variant for their utilization which would ensure, with the given resources of these factors, the maximum fulfilment of the programme task (in terms of the given assortment of goods). In this lies the main interest of L. V. Kantorovich's work.

However, the author attaches to his valuations and to his system of economic calculations an interpretation of such amplitude and such a universal significance that it cannot be accepted. He has come to regard them as universal equivalents for the substitution of some resources by others. Arising from the condition that the sum of the valuation for production, on the basis of given resources, must be equal to the sum of the valuations of utilized resources, the author also begins to regard the valuations of various types of production in the same way as equivalents for the substitution of some products by others. Not only that—he attaches universal significance to these substitution equivalents, and demands that the utilization of factors of production according to objectively determined valuations should be included in costs of production. These claims by the author are completely unfounded.

It should be pointed out that objectively determined valuations can only play a subsidiary role of valuations of shortages and scarcities of resources. They can only be successfully employed in the solution of specific problems connected with the rational allocation of a production task under given concrete conditions.

The valuations *are for purposes of allocation*. They cannot be regarded as criteria for production and they must not be treated as costs. The author is right when he points out that rent from equipment can be calculated (hire valuation), although no money is paid (*see pp. 78–9*). But he is wrong when he begins to regard objectively determined valuations as cost elements. They only represent criteria of shortages or limiting factors, calculated for a given type of equipment only for fully determined purposes connected with such distribution of the production task which must take into consideration the factor of temporary scarcity of available resources.

Available resources of course characterize the conditions of the use of labour, but these conditions cannot be regarded as being

the same as labour outlays. The fact that the author ignores the given situation, leads him to include in costs not only *outlay* on the means of production, but also the costs of their scarcity (e.g. commitments of transport and of equipment). In the mathematical formulation of a problem, L. V. Kantorovich introduces the cost of factors which increase the productive power of labour (various forms of equipment, natural resources, etc.) as mathematical variables, equivalent to the expenditure of labour and outlays on the means of production (pp. 274, 291). In practice this leads the author to compare the real outlay on repairs (p. 82) with the "loss of hire valuation" (conditional expenditure). It is impossible to agree in any way with comparisons of this type.

The introduction to the problem of conditional valuations (multipliers), which characterize the shortage and scarcity of factors, only permits the solution of the question of choice of the most rational and expedient method of utilizing available resources, ensuring the most correct distribution of a given production task or programme.

The author lays claim to the universality of the proposed method of economic calculations based on objectively determined valuations. This leads him to a number of inconsistent and incorrect conclusions. On the one hand, the author stipulates that objectively determined valuations should not be directly linked with tariffs for electric power (p. 60), with wage rates (p. 65), or with prices (p. 135), while on the other hand, all his examples are drawn up in such a way that the existing system of national economic valuations (prices, tariffs) often leads to wrong conclusions, and only objectively determined valuations give "complete harmony" (pp. 11, 32, 134). He contrasts objectively determined valuations, which only reflect local conditions, with valuations emerging from the national economic objectives, valuations which he calls *a priori* (pp. 11, 32), rejecting them for local problems and imparting to them a subsidiary role (p. 235). The author's desire to underline (p. 213) the common factor which links objectively determined valuations with market prices—their divergence and not their coincidence with the necessary expenditure of labour is typical.

L. V. Kantorovich's conception of an optimal plan is also very weak. For him, an optimal plan is only one in which objectively determined valuations, for products as well as for factors, are

consistent. The existence of consistent objectively determined valuations is the main condition for an optimal plan (p. 222). The author considers that as a criterion of normal efficiency and of optimality of a plan all other national economic criteria are not very essential, and in particular he devotes quite insufficient space in his economic analysis (p. 228) to the index of the growth of labour productivity. He ignores such an important aspect as the optimal character of the relation between consumption and accumulation. The plan itself is treated by the author in a very narrow sense, as a "collection of numbers" (p. 267, Mathematical Appendix).

Of course, it is impossible to agree with the author's point of view; it must be rejected. He imparts to one of the methods of economic calculations, which is very useful in a strictly limited sphere, a general and universal character which is alien to it. His objectively determined valuations are only criteria which permit a numerical valuation of the scarcity of the conditions of production, the scarcity of resources, restrictions of equipment and the strain of the programme. These valuations do not characterize anything else. But this does not detract from their significance in a specific field. They permit, for example, the numerical determination of differential ground rent. Hire valuations for equipment are also, in their own way, valuations of rent. The author himself indicates the "rent" character of hire valuations (p. 79, footnote).

However, objectively determined valuations have significance only in the solution of problems where the important role is played by processes of distribution and re-distribution. These criteria are particularly important when, by the nature of the problem, it is necessary to take into account shortages and the scarcity of resources. But these are *not indirect costs*, as treated by the author (pp. 227, 259), but merely a constant element of economic calculations of a particular type, which take into consideration the scarcity of resources. These valuations cannot characterize the actual extent of costs nor, what is more, full national economic costs as the author asserts (pp. 92, 244) even if only because objectively determined valuations for surplus products and surplus resources are equal to zero (pp. 96, 264) and because the total profitability of the plan is also equal to zero (p. 121). They can take into account divergences from social costs only if the worst conditions for the application of labour (e.g. poor land) are operating, which only happens in special cases.

Differential ground rent cannot be regarded as part of the social expenditure of labour (p. 208), since the conditions for the application of labour do not create cost. Rent is only a part of the surplus product created by the socially necessary expenditure of labour. It can be isolated only in the process of the distribution of income, and in that case the method of economic calculations based on *objectively determined* valuations can be useful for the determination of rent as a separate part of the surplus product.

When the nature of objectively determined valuations is understood correctly and when the unfounded claims of the author to the universality of the method are withdrawn, then only will the method of economic calculations put forward by L. V. Kantorovich be used to advantage. In this field the author has done a not insignificant service in publishing his work, despite a number of erroneous principles and the controversial nature of a number of his conclusions.

Thus, while the use of the methods and the objectively determined valuations advanced by L. V. Kantorovich do not raise any serious objections in relation to the solution of specific problems of economic planning, the extension to cover the national economy as a whole in order to achieve an optimal economic plan is controversial and has been quite inadequately studied.

Of course the structure of an optimal plan is an extremal problem which assumes mathematical formulation and mathematical solution, but under conditions of accurate economic premises and formulations. Indeed, even bourgeois mathematical economists as represented by the pure mathematical (so-called Lausanne) school and its modified Anglo-American form also aim to find the maximum value of a certain "general utility function" or a "welfare function". This function, however, has nothing in common with the economic effect or with the development of the productive forces of a country and is in fact never realized. No less basic is the fact that Western mathematical economists arrived at this problem by rejecting an analysis of causal connections, by rejecting the structure of a unified theory of prices and of the categories of value, replacing the study of the latter—the basis of all value formations—by *formal mathematical* calculations to determine the interdependence among specific external manifestations of economic phenomena. As they understand it, the relation of supply and demand is the cornerstone of this interdependence.



L. V. Kantorovich is anxious to avoid such a usage of mathematics. He relates the objectively determined valuations not to the categories of demand but to the labour theory of value. He tries to give a real economic meaning to objectively determined valuations and tries to indicate a way of obtaining, by means of objectively determined valuations, the valuations of all goods on the basis of all the socially necessary labour expended on their production, in accordance with the Marxist concept of value. While disassociating himself, however, from the concepts of bourgeois economists he nevertheless introduces into his structure, to a certain degree, the dependence of objectively determined valuations on demand, though the role which the author assigns to this demand remains unclear. L. V. Kantorovich repeatedly points out that the methods of economic calculation and of objectively determined valuations put forward by him must be used within the framework—already predetermined—of leading directives and main proportions; i.e. basically he determines not what is to be produced but how it should be produced (pp. 138, 184). In other words, these valuations cannot serve as a regulator of the allocation of social labour between the main sectors of the national economy. But nevertheless the character of these objectively determined valuations as interpreted by the author, is such that they do appear to a certain extent to be such a regulator, whether the author subjectively desires this or not.

Of course, many of the proposals and conclusions of L. V. Kantorovich relating to the conditions of price formation are worthy of note. Attention should be drawn to the timeliness and accuracy of the author's formulation of the problem of the construction of a system of valuations—of prices, capable of serving as a basis for economic accounting and a means of finding the most expedient variant for the utilization of resources of the whole national economy. Naturally, in his research work the author could not finally elaborate theoretically all the numerous problems he touched upon which arise in the construction of an optimal plan for the national economy.

It must be hoped that the publication of this piece of research, despite the controversial nature and theoretical imperfection of a number of the author's principles, will influence the further fruitful development of the methods of planning and economic accounting. A number of the author's arguments and elements of his construction

can be utilized in the future, though in a different context, both in the practice of the construction of the national economic plan and in the solution of specific problems of the socialist economy.

Academician V. S. NEMCHINOV

*6 June, 1959*

## AUTHOR'S PREFACE

THE present author wrote a study on the *Mathematical Methods of Organizing and Planning* (Leningrad University, 1939) arising from advisory work on production problems in 1938–9. It set out a method for finding the solution of technical and economic problems, such as the least wasteful allocation of work to machines, the cutting of material with the minimum loss, and the distribution of loads over several different means of transport.

These investigations were continued in 1940–1 and 1948–50 at the Institute of Mathematics of the Academy of Sciences of the U.S.S.R. Some of the problems were further developed in special studies. Methods of cutting which resulted in the smallest waste were introduced into some Leningrad factories. Various computing techniques were generalized and simplified.

Soon after this work began it became clear that the methods being developed there could find a far larger field of application to general problems in economic accounting and planning. The results of some of the investigations in this field were submitted by the author in 1942 to the Institute of Mathematics of the Academy of Sciences of the U.S.S.R. (at that time in Kazan), and in 1943 to the Institute of Economics of the Academy of Sciences of the U.S.S.R. in Moscow.

This is a statement of the results of the investigations referred to above, supplemented by new calculations and examples to include economic data of recent years. It gives only the basic elements of a system of economic accounting.

The essential problem appears to be one of constructing an optimal production plan which would ensure the best results by the greatest use of available resources, and also the study of the economic indices of such a plan.

The work is in two sections. In the first, the optimal solutions of some specific economic planning problems (allocation of the programme, efficiency measures, utilization of equipment, effectiveness of capital investment) within a single factory, a group of factories, an economic region or a sector are analysed and solved. In the second,

some general economic accounting and planning principles in a socialist society are explained on this basis. The results may be applied to economic planning and in choosing economic indices.

The main conclusion is that a system of production valuations correctly constructed and conforming to real conditions is an effective means of analysing the best use of available resources. Under given conditions in an optimal plan these valuations fully agree with the accounting cost of social labour necessary for the production of a unit of output. To find such a system of valuation and an optimal plan, an effective approach and special accounting methods are proposed.

Such a method is superior to existing ones because many factors usually disregarded or only considered qualitatively are accounted for by quantity, as a result the choice of the solution conforms more fully to the national economic interest.

At present, calculations have not yet been made for the necessary indices consistent with the proposed methodology, but the author thinks that the results of this study, even at this stage, can be of real assistance in the solution of many practical economic problems.

The applicability and high efficiency of the methods of optimal planning for a production unit, a workshop, factory or group of enterprises are undoubted. They have been sufficiently tested in practice. However, experiments in the use of optimal planning methods on a scale covering the whole national economy have yet to be made. The place and value of these methods in national economic planning can therefore not be considered sufficiently clarified, and any categorical assertions in this respect would be premature.

*One thing is certain: in a socialist society where the whole economy is built on a scientifically planned basis the field of application of mathematical methods (especially of the extremal principle) is immeasurably wider; here, in contrast to capitalism, the possibility exists of applying mathematical methods in national economic planning.*

The preparation of such a plan and its indices is an extremely complex task. For this reason it would be particularly important to introduce more accurate quantitative methods.

The application of mathematical methods to the analysis of economic planning in a socialist economy raises a whole series of complex problems of method: the role and extent of the application

of these methods, the discovery of the economic meaning of the new indices resulting from the application of these methods, and their connection with the usual economic categories. In so far as these indices appear in an objective quantitative economic investigation, they should agree with and fit into the general assumption of the labour theory of value.

In this book some attempts have been made to establish such relations, but the final clarification of these problems should follow from further investigations, and the constructive criticism of such problems by a wide circle of specialists in the field of economic theory as well as by practical workers.

Many useful comments on this work were made by the editor in charge, V. S. Nemchinov, and the readers K. I. Klimenko, I. I. Lukomski, and A. L. Lure and also by V. V. Novozhilov, V. A. Zalgaller, A. L. Vainshtein, A. S. Konson, A. I. Katsenelinboigen and G. N. Soloveichik.

G. S. Rubinshtein was of great help to the author in compiling the appendices. L. I. Gorkov, A. A. Korbut, I. V. Romanovskii, L. S. Soboleva, V. N. Sokolova, and I. N. Sokolova checked the examples in the main text and worked out the examples in Appendix II. They also read through the final text.

To these and to others who gave helpful advice and criticism the author expresses his warmest thanks.

### PUBLISHERS' NOTE

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# INTRODUCTION

## How to Improve Planning and Economic Accounting

PLANNING of the national economy and of individual branches within the framework of the state is only possible when private ownership of the means of production is replaced by common socialist ownership. Such planning becomes possible only when capitalist relations of production have been eliminated and replaced by socialist ones.

V. I. Lenin inspired the first scientific plans in the history of the Soviet economy—plans for electrification and co-operation in agriculture. The genius of Lenin's ideas about planning formed the basis for all plans for the development of the national economy of the U.S.S.R. Lenin's ideas of building a communist society permeated the target figures of the Seven Year Plan for the development of the U.S.S.R. as confirmed by the XXI Congress of the Communist Party of the U.S.S.R. The party has frequently emphasized that only with detailed and correct scientific planning is it possible to achieve full and balanced use of all existing resources and to demonstrate the general superiority of the socialist method of production which will guarantee the victory of socialism in the world-wide contest with capitalism.

“In a socialist society there is no room for the contradiction which exists under capitalism between the social character of production and the individual manner of appropriation, there is no room for such phenomena as competition, anarchy in production, unemployment, economic crises. In the socialist society different economic laws have emerged: planned and proportional development of the national economy, and the continuous and rapid growth of production without depressions and crises. This makes possible the planning of the national economy, the direction of its development, the continuous increase of the volume of production, the efficient

distribution of the forces of production with large-scale specialization and co-operation on the road to socialism."†

Much experience has been obtained in the planning of an entire economy controlled by a socialist state. Socialist construction in the U.S.S.R. has fully confirmed the basic principles of planning.

In practice there still are substantial shortcomings. This applies above all to the techniques and methods of planning and to economic accounting generally. The Communist Party and the Soviet Government have frequently turned their attention to the imperfection of planning methods and construction of economic indices, and have persistently tried to improve these shortcomings.

Moreover, planning deficiencies exist as a direct result of economic science lagging behind the requirements needed in the building up of a communist state.‡

These questions have received much attention in recent years when a whole series of measures were taken; they were aimed at the removal of shortcomings in planning and in economic indices, especially in agriculture, and at improving the organization of management in industry.

The task of further improving the methods of solving these problems remains topical. Herein lie the essential reserves for the faster growth of the national economy.

No doubt, correct planning methods should lead to the attainment of the optimal plan, ensuring the best use of all the resources and yielding the maximum quantity of the required products. Are all the existing production plans efficient? There are no grounds for an affirmative answer to this question.

On the one hand, the work of our leading factories proves that great unused potentialities exist. As a result of better utilization of available resources, these factories reach outputs which greatly exceed the targets set by the plan.

On the other hand, considerable losses take place even now—

† Target figures for the development of the national economy of the U.S.S.R. for 1959-65 *Report of the XXI Congress of the Communist Party of the U.S.S.R.*, vol II, p 461

‡ See the paper at the XXI Session of the Communist Party of the Soviet Union by I I Kuzmin and A N Nesmerjanov (*Report*, vol. II, pp. 197-208).

Such backwardness and lack of solutions to a whole series of basic tasks in the field of economic science is admitted by the economists themselves. See the paper by K. V. Ostrovitianov (*ibid.*, p 372)



idleness of labour and equipment, and losses in raw materials and fuel, owing to unsuitable programmes, rush work towards the end of the plan period and delays in supply, the freezing of materials in surplus stocks and in protracted construction—these are also evidence of the lack of sufficiently rational planning.

No less significant are the indirect losses caused by the improper utilization of resources. As they are not recorded they are less noticeable. For example intricate equipment is used for simple work, with low efficiency, while in other places, where it could be most effective, the absence of this equipment causes delays or necessitates the use of primitive methods. This is also true of materials. Particularly frequent are the losses due to the lack of flexibility in allocation, resulting in the lack of small quantities of any necessary material becoming a hindrance to raising output.

All these losses are basically due to imperfect production planning and economic accounting, caused by inadequate methods.

If, with identical production capacity, one factory yields two to three times less output than another, blame is usually laid on inadequate production aims, or the difficulty in obtaining raw materials or tools.

Loss of production due to rush work alone was estimated at one time at about a quarter of possible output. The removal of these losses by improved planning techniques and better economic accounting would make it possible, within a short time, to raise output by 30 to 50 per cent, using available resources only in the most economic manner at all stages of production. That is why the task of working out and introducing such methods is both important and urgent.

It should be stressed that in order to solve the economic problems of industry, agriculture and construction, shortcomings in planning and in the preparation of economic indices have to be removed. The reorganization of management carried out by industry and construction makes it considerably easier to apply improved methods.

What is, therefore, the main task in solving economic and planning problems in a socialist society?

All the purely economic questions, i.e. the extent to which production requirements will be met and the speed with which output will grow, may be divided into two types.

(1) *The correct choice of method in the production of a given output or the completion of a given operation.* The type of resources (labour,

raw material and other materials, equipment, transport, power) and the quantities in which they will be used will depend primarily on the production method chosen. For instance, should aluminium or steel alloys be used for components, wood or cement in building, excavators or manual labour for foundation work, fuel brought in from a distance or local fuel? These are the questions which are decided every day in the *sovmarkhozy*, factories, *kolkhozy*, project bureaux, and in construction firms.

(2) *The allocation of the programme and of the resources available among individual enterprises and operations, etc.* This task must be solved so as to ensure the correct composition of final output, and not to disturb the general availability of resources, and to achieve balance between production and requirements of each type of final or intermediary products. Upon the basic solution of this task will depend the possibility of uninterrupted work of the factories and hence the quantity of output. These problems are solved by the organizations which perform the planning and exercise operational control at all stages. *Both these tasks are inseparably linked.*

Only in a few instances, where a given factory or operation can be considered in isolation is it possible to solve the first problem independently of the second. For instance, when a new operating method makes it possible to produce more than previously with the same equipment but with smaller expenditure on labour and materials, no economic problem arises. It is obvious that the improved method should be given preference. Such cases are analysed in many manuals of applied economics.

In practice the process is more complex. For instance, it may be possible to turn to local raw materials (but as a result the volume of production may decrease), to replace one material by another; to increase the volume of production by adding to the existing equipment, to reduce the consumption of fuel by spending substantial sums on boilers of a new type. The solution of similar problems will depend upon the operating conditions of many other enterprises and upon general economic conditions. If transport is overloaded, local raw materials should be used irrespective of the losses this may involve. If the means available for capital investment are very limited, plans for a new boiler must be abandoned in spite of its superiority. In deciding, for instance, whether to replace one ton of tin by three of aluminium, the fact that one material is available

at a given moment in large quantities should not be the deciding factor, but rather which material is of greater economic importance.

The second task should never be solved independently of the first. The general balance of production and productive resources consists of the relationship between individual enterprises and organizations. Changes in this balance are only possible as a result of changes in the programme and operating conditions of individual factories.

These two tasks cannot be solved independently. However, in practice, it is very difficult to solve both simultaneously. When solving production problems of a given factory which affect general relationships, an analysis of the latter could not be carried out on a national scale at the same time. But failure to consider such problems amounts to disregarding all the possibilities of improving the operation of the factory. Meanwhile, when solving planning problems for the economy as a whole, a branch of it, or for an economic region, it is not possible to consider simultaneously all the operating conditions and possibilities of numerous individual economic units.

To bring about an agreement between the general and the particular is the basis of the difficulty of planning and economic accounting. This difficulty could be overcome by creating a method enabling the solution of planning problems for individual sectors and factories to be obtained separately, but at the same time maintaining the consistency between one another and leading to an optimal (or near-optimal) system of planned results.

The first task referred to above may also be encountered in a capitalist society. A capitalist compares operating methods and chooses that which brings him the maximum profit. Of the two possible types of raw material the cheaper one is chosen; if economies in labour expenditure can be achieved by the increased use of electric power, the expediency of using one or the other is determined by a comparison of costs. The financier invests available capital where he can obtain the highest return. The decisive factors are the system of prices operating at the time, the system of tariffs, and the rate of interest on capital.

The second task—the preparation of a general consistent balance—cannot arise under a system of private ownership of the means of production. This balance is automatically achieved as a result of the competition on the market and is accompanied by great losses. A spontaneous “system” of economic solutions cannot ensure the

character. Frequently, the prevailing custom is important here, and so is the necessity of a solution which allows for the inflexibility of allocation when in spite of its undoubted superiority some method cannot be used owing to the lack of materials for its application.

Against the background of the impressive progress of our industrial and agricultural economy these difficulties and deficiencies are less noticeable. Nevertheless it is essential that they should be removed; it should help to increase further the pace of development of the socialist economy.

In the solution of the second task, a rational distribution of the programme and of resources, considerable difficulties arise. Frequently there appears to exist considerable disparity between the orders and requirements of individual firms for materials, equipment, electric power, transport, means of capital investment as against existing possibilities. Such disparity arises because orders fail to take into consideration the actual supply of a given factor of production and the requirements of other enterprises for such factors: this is due to the absence of sufficiently good methods of making such assessments.

In fact, objective indices, showing the actual degree of importance of each order, are not applied or the characteristics are purely qualitative ones, such as, "very necessary" or "absolutely indispensable". In view of this, orders are usually cut, partly for those firms which are considered less important, and partly automatically by some percentage without any objective analysis of the losses that may result. Moreover, such arbitrary and subjective factors as the opportune moment for an order and continuity in the requirements play an important part.

As a result of these deficiencies in distribution, relative proportions are frequently much disturbed (not only in the plan but also in actual realization); this leads to losses like under-employment of the labour force, rush work (due to delays in supply), idleness of means of production (for lack of certain parts), greater use of manual methods and the consequent lowering of labour productivity (due to lack of equipment and co-operation). Many of these losses could easily be avoided (for instance, in one firm wagons stay unloaded owing to the lack of motor transport, in another movements over distances of hundreds of kilometres are carried out by motor transport as a result of the non-arrival of railway wagons).

criteria for finding the best solution. For this reason defects in the plan are unavoidable. Even though, in the experience of the chief planners, the basic instructions are determined correctly, the absence of competent, objective computational methods in economic analysis when such instructions are put into practice by subordinate bodies leads in numerous cases to less successful results; and hence waste occurs. Thus industry is far from using its full capacity.

One is confronted with the task of establishing methods that would ensure an objective approach and the best solution of the problems of economic planning.

Such a solution, once it is required to be objective, must inescapably be quantitative, since one set of initial numerical data will furnish one solution while another solution will emerge from different data (to substitute a ton of aluminium for one ton of lead may be correct, but it might not be so for thirty tons and a qualitative approach is of no use here). It is, therefore, clear that such a method should represent some system of *economic accounting*. This book aims at clarifying some principles and procedures of such a method. A more detailed elaboration would require very much research, generalization and the analysis of an enormous amount of factual material together with long experience in planning. Such a task can only be solved by the efforts of a great number of scientists in various fields of specialization, and of practical workers. The object of this book is to bring out certain properties and possibilities of economic accounting in a socialist society which in our view are of fundamental importance, but which at present are neither utilized nor even duly considered.

To ascertain the basic economic categories, the author was guided on the one hand by the fundamental propositions and methods of analysis in the economic theory of Marx, on the other hand by the assumption of the objective nature of the economic laws of socialism. Of course, the author does not aim here at a theoretical analysis of the basic economic categories of a socialist society. The scope of this work is considerably narrower and of a more practical nature: a preliminary exploration of the method of economic accounting making possible a systematic approximation of the optimal plan; in addition it aims at elucidating the approach to the further development of a method of solving economic problems which would ensure

and government. In passing, we show the importance that the application of these results could have (*see* pp. 27, 43, 61, 73-85, 100, 116, 197).

A systematic analysis applied to problems of economic planning in an abstract and simplified form, consisting essentially in the substitution of some model scheme for the original problem, greatly facilitates analysis and makes it possible to carry it through quite fully, to apply objective computational methods and to arrive at exact quantitative solutions. In such cases, the results obtained refer only to the scheme under consideration. The incomplete agreement of this scheme with the complexity of real problems arising from necessary omissions makes it impossible to apply the results obtained directly to practical problems. At the same time we suggest that the more important basic economic factors can be calculated accurately enough with the aid of these schemes, and that the assumptions closely approach real conditions. For this reason, the results may still be of real advantage. This aspect which is common when using the abstract method in scientific investigation must be continuously borne in mind by the reader.

The book is divided into the following parts. In Chapter I we consider questions of programme allocations under special limiting conditions in the presence of which the solution of the problem, that is, the finding of the optimal plan, may be attained by using only a certain valuation of production. Here we introduce the basic concept of objectively determined valuations which are intimately and inseparably connected with the optimal plan. The ratio of these valuations for two types of operation (of production) represents the real equivalent by which one operation may be replaced by another in the optimal plan. This is also consistent with the ratio of costs necessary for the completion of one or the other operation in any given situation, provided this cost is calculated correctly and fully. Moreover, under given conditions, the relationships shown may be determined by correctly allocating costs among types of products manufactured simultaneously without analysing in detail the structure of this outlay. In passing, an account is given of a numerical method of finding an optimal plan and objectively determined valuations for simpler cases. This method is quite easy, while the numerical analysis quoted in the examples is of great importance. It is necessary to absorb it fully as otherwise it may be difficult to understand the basic meaning of the concepts introduced.

of certain propositions stated in the preceding chapters. In Appendix II we describe the procedures for solving these problems which are necessary for the utilization, in the more complex cases, of this method of economic accounting, as set out in the body of the text.

This work deals only with the range of problems relating to planning and economic evaluation of methods of production. Some other problems closely connected with these are not subjected to systematic investigation and are touched upon only in passing; for instance, the question of choosing an index of performance for a factory consistent with the interests of the national economy in such a manner that the improvement in the operation of the factory (from the point of view of the general plan) should be reflected in an improved index. For this and many other problems our method may prove useful.

In this book we have worked out some principles of the objective method of solving problems of economic accounting and planning. The details and technique of the use of this method in practice is not considered since these questions have not yet been sufficiently investigated and should be solved in the course of the practical realization of the work, depending upon actual conditions and on its progress.

Certain concrete solutions of individual problems, the application of which may be recommended, are quoted in the book; but the number of such problems could be multiplied.

The book touches upon a fairly wide range of economic problems connected with the task of optimal planning; nevertheless, many of these have only been partially solved and our conclusions and proposals are necessarily rather sketchy.

Familiarity with the proposed approach should promote discussion, further development and practical use which will open up new horizons.

## CHAPTER I

# PRODUCTION PROGRAMMING AND THE VALUATION OF PRODUCTION

### Section 1. The Problem of the Best Distribution of the Programme among Several Enterprises

#### *Statement of the Problem*

As a first problem of a technical and economic nature in the solution of which the application of correct economic accounting is of primary importance, we shall consider the question of the most suitable distribution of the production programme among several firms. We assume that there are many different ways of solving the problem—one and the same article may be put into production at several factories. The problem arises from the choice of an optimal distribution such that in the programme for each works are included only those articles for which it is best suited; as a result aggregate expenditure would be least.

Just as the production of various articles is to a certain extent interdependent, the costs of their production are also inter-related. For this reason, a solution is called for both for the distribution of costs amongst the articles and for an objective determination of the necessary expenditure on each article.

To explain the method of solution, we shall consider this problem with a practical example using a whole series of simplifying assumptions.

EXAMPLE. To give the best distribution of the programme among factories under the following conditions:

(1) Two articles have to be put into production: No. 1 and No. 2; the requirements for both are unlimited but it is necessary that they should be produced at a fixed ratio (problem of assortment), e.g. twice as much should be produced of article No. 1 as of article No. 2.

(2) Each of these goods may be put into production at factories of



types A, B, C, D and E. The number of factories of each type and the production capacity per month of parts No. 1 and No. 2 are given in Table 1. It is assumed that each enterprise will produce only one type of article.

TABLE 1. NUMBER OF FACTORIES AND MONTHLY PRODUCTION CAPACITY

Type of factory	Number of factories	Production capacity of each factory		Relative labour content in the manufactured product	
		For article No 1	For article No. 2	Article No. 2 compared with No. 1	Article No 1 compared with No. 2
A	5	100,000	15,000	6.7	0.15
B	3	400,000	200,000	2	0.5
C	40	20,000	2,500	8	0.125
D	9	200,000	50,000	4	0.25
E	2	600,000	250,000	2.4	0.41

Production costs of factories as a whole (in addition to raw materials and basic materials), namely wages (the number employed is constant), electric power, fuel, expenditure connected with equipment, for other workshops, and general factory expenditure and depreciation are approximately the same whichever article is put into production at the works.

(3) All the necessary materials are available in the required amounts. Expenditure on basic materials (and also power and fuel, if required) per unit of a given article is identical at all types of factory and constitutes, say, 10 roubles per unit of article No. 1 and 15 roubles per unit of article No. 2 (the latter figure, however, is not very important in the subsequent analysis).

(4) Transport problems are of no great importance; all the factories and workshops are situated in one town or in several towns close to each other.

Briefly, all the productive expenses may be divided into two groups: those which do not change at a given works, independent of the type and quantity of article of each type irrespective of where it is produced, and those which are proportional to output.

It is necessary: (1) to determine the possible volume of the production programme, (2) to distribute the programme among the firms in the best possible manner, (3) to carry out a scientific allocation of the costs incurred on the articles.

It is clear that an optimal plan is the one in which the proposed assortment in the programme is observed and in which production is at the highest volume. Such a plan would, of course, have its counterpart in the lowest costs of production as the cost of material (per article) is in all cases identical and the remaining sum of expenditure for the operation of the enterprise is constant; consequently, the cost for a specified number of parts (and for each part) will be the smaller, the larger the total output.

The general plan and the total output depend essentially upon the method adopted for the allocation among the factories. Let us quote an example (Table 2). The allocation was carried out in such a manner that in each type of firm parts No. 1 and No. 2 were produced approximately at the required ratio. In other words the programme is "allocated" to groups of factories.

TABLE 2. ALLOCATION OF THE PROGRAMME

Type of factory	Article No. 1		Article No. 2	
	Number of factories	Aggregate output	Number of factories	Aggregate output
A	1	100,000	4	60,000
B	2	800,000	1	200,000
C	10	200,000	30	75,000
D	2	400,000	7	350,000
E	1	600,000	1	250,000
Total		2,100,000		935,000

Table 3 shows another plan in which the output of both the first and the second article is smaller—this is an inferior plan.

TABLE 3. INFERIOR PLAN

Type of factory	Article No. 1		Article No. 2	
	Number of factories	Aggregate output	Number of factories	Aggregate output
A	—	—	5	75,000
B	3	1,200,000	—	—
C	—	—	40	100,000
D	—	—	9	450,000
E	1	600,000	1	250,000
Total		1,800,000		875,000

*The Optimal Plan*

A great number of various plans may be constructed. The optimal plan has to be chosen. How is this to be achieved? It is evident that this will be a plan in which each factory will produce, as far as possible, the type of output for which it is best suited. To arrive at such a plan, we reason in the following manner. If we turned all the factories to the production of article No. 1, we shall produce (see Table 1):

$$5 \times 100,000 + 3 \times 400,000 + 40 \times 20,000 + 9 \times 200,000 + 2 \times 600,000 \\ = 5,500,000 \text{ units of article No. 1.}$$

But we also require article No. 2; consequently, some of the factories must be turned to the production of article No. 2 and then we shall obtain a smaller quantity of article No. 1. How many less? In turning a factory of type A from production of article No. 1 to No. 2, instead of 100,000 articles No. 1 we obtain 15,000 articles No. 2 or instead of one article No. 1—0.15 article No. 2; similarly, for a factory of type B—0.5; a factory of type C—0.125; a factory of type D—0.25, of type E—0.41 article No. 2 instead of one article No. 1 (see Table 1, last column).

As we see, it is most advantageous to turn the three factories of type B to the production of article No. 2; but this is not enough: we shall obtain 600,000 articles No. 2 and 4,300,000 articles No. 1. After these, we shall turn both factories of type E to the production of article No. 1 but this too is insufficient as we shall obtain 600,000 + 500,000 = 1,100,000 articles No. 2 and 3,100,000 articles No. 1, i.e.

TABLE 4 THE OPTIMAL PLAN

Type of factory	Article No. 1		Article No. 2	
	Number of factories	Aggregate output	Number of factories	Aggregate output
A	5	500,000	—	—
B	—	—	3	600,000
C	40	800,000	—	—
D	6	1,200,000	3	150,000
E	—	—	2	500,000
Total		2,500,000		1,250,000

nearly three times as much. The next in importance is the ratio 0.25 which corresponds to factories of type D; however, if we converted all the nine factories of this type to the production of article No. 2 we should obtain too many of the latter. To arrive at the requisite ratio, six factories of type D need be used for article No. 1 and three for article No. 2. We come to the plan, given in Table 4—the optimal plan ensuring the highest production of the required composition.

This plan furnishes an appreciably higher output (24–40 per cent more) than the plans in Tables 2 and 3.

### *Objectively Determined Valuations*

The method whereby we arrived at the optimal plan merits attention. Let us take the problem of costs connected with the work for the production of articles No. 1 and No. 2.† Naturally, here the relative importance of labour in the production of both articles must be accounted for. This has its own value for each type of factory.

In the example under discussion, labour input for the production of either article at a given factory is inversely proportional to the productive capacity for that article. At a factory of type A, in the time spent on the production of one article No. 1, 0.15 unit of article No. 2 can be produced, i.e. in this factory labour input is 6.7 times higher for article No. 2 than for article No. 1. Similarly, it requires twice as much labour in a factory of type B, 8 times as much in type C, 4 times as much in type D, 2.4 times as much in type E (Table 1). Which figure should, therefore, be taken as relative labour input when taking the factories together? We must take into consideration that article No. 2 is in fact not produced at factories of type A (in accordance with the programme of Table 4); on the other hand, at the factory of type E article No. 1 is not produced. The only ratio of those mentioned which in fact is achieved in the optimal plan is the ratio of 4 for factories of type D.

For this reason, it is natural to apply this ratio to all the factories together. Indeed, the cost ratios for various products must be assessed

† It is important to turn attention to the fact that if, as in the given example, the production of the articles is interdependent (even if indirectly) then the question of necessary outlay for the production of each article may only be analysed simultaneously and for all the articles together. Shifting production of one article and reducing its production costs may change the cost of production of the other article.

on the basis of the necessary expenditure for their production, that is, ultimately according to the expenditure of labour. In so far as the plan obtained is optimal under given conditions, expenditure incurred on it may be considered necessary. The possibility of comparing directly the cost of production of articles No. 1 and No. 2 at factories of type D (and only if both articles are produced here simultaneously) enables the ratio of expenditure for these articles to be established, and correspondingly the ratio of their valuations under the given conditions. The valuations for outputs established in this manner we shall call "objectively determined valuations" (o.d. valuations). In the present case, we have established only the relationship of these valuations, i.e. 4:1, so that if, for instance, the valuation for article No. 1 equals  $a$ , for article No. 2 it equals  $4a$ . It is important to note that this ratio has not been chosen arbitrarily, but it is determined by the given conditions and is revealed in the course of analysis of the optimal plan.

Later on (Chapter II), when considering the question of finding absolute values for objectively determined valuations we shall establish that the o.d. valuations are arrived at by the total of the necessary costs of production which must be fully accounted for in the given conditions.

In these circumstances it seems justifiable to apply the term "valuation" and not "cost" or "price": the valuations here obtained are to some extent of a limited and local nature since the analysis of costs and the plan are not carried through for the economy as a whole but only for the group of factories under consideration. For this reason, such analysis is not complete enough to establish value relations. It should also be noted that we establish valuation not for an article as a whole but only for the operations necessary for its production and the application of the term "price" in such conditions cannot be generally accepted. Therefore, the term "necessary expenditure" (of labour) for production appears to us more appropriate in this case than "socially necessary" as the analysis of costs here does not relate to society as a whole but only to a section of a group of enterprises under given conditions. The introduction of such a special term would be of no advantage when considering the national economy as a whole.

Let us start from the ratio found—taking the valuation for article No. 1 as equal to  $a$  and of article No. 2 as equal to  $4a$ . More

precisely, these figures do not reflect the valuations of the articles themselves but the valuations of net production or the labour used on the production of these articles.† We shall calculate in these valuations the monthly production by the factory of each type if one or the other article has been produced here. The results are given in Table 5 where the figures relating to the method adopted in the optimal plan are set out.

TABLE 5. NET MONTHLY PRODUCTION BY FACTORIES OF ARTICLES NUMBERS 1 AND 2 EVALUATED AT  $a$  AND  $4a$  RESPECTIVELY

Type of factory	Valuation of production	
	Article No. 1	Article No. 2
A	100,000a	60,000a
B	400,000a	800,000a
C	20,000a	10,000a
D	200,000a	200,000a
E	600,000a	1,000,000a

As may be noticed, in the optimal plan each factory was used in such a manner that its net output has the highest valuation. In solving the problem of using factories and the allocation of the programme we observe (starting from the valuations mentioned above) the principle of the highest yield, i.e. we obtain the maximum (net) production in value terms for a given outlay yielding the highest profit. This corresponds also to minimum expenditure per unit output (expressed in value terms).

The results obtained in considering this question remain true and can be ascertained for other similar cases, in particular for any number of factories and types of articles.

CONCLUSION 1.‡ Of all the possible allocation programmes there is always one which is the best—the optimal plan. In this plan the ratio of the individual types of production satisfies the condition

† By net production we mean the production of a given enterprise, i.e. the cost of material factors required by the enterprise is not included in the total cost. In other words, in the production is included not the article itself, but its (costs of) processing. For instance, the net production of a clothes factory is not the overcoat but the operation of “sewing a coat”.

‡ All the conclusions obtained here and below presuppose that the conditions remain as set out in the statement of the problem.

given in the problem, output is larger (or equal) and the expenditure less than in any other plan that fulfils the same conditions. The cost of production in this plan is less than (or equal to) that of any other allocation plan.

CONCLUSION 2. With the optimal plan are associated determinate valuations for each type of product, more exactly for the operations in the manufacture of a unit of each type of product—an objectively determined valuation (o.d. valuation).

These valuations are such that, if they are taken as a basis, it is found that in the optimal plan the principle of profitability is observed, i.e. under this plan, each factory is assigned the production of that type of goods on which it has the highest net product.

The principle of profitability as stated here is applied in a somewhat wider sense than generally accepted. It is necessary to explain now the meaning of this principle of profitability, and its function and role.

By the principle of profitability we understand the choice of an economic, planned solution on the basis of the effect expressed by one value index; choice of technology affording the lowest cost; choice of a programme ensuring the maximum production in value or maximum accumulation for a given outlay; choice of the cheapest raw materials and other materials, etc.

The conclusions to which the principle of profitability may lead are essentially dependent upon the initial system of valuations.

This principle operates fully under capitalism where it rests on the current system of market prices. In the U.S.S.R. it is of limited significance even in the computation of official sales prices since for various reasons it is sometimes necessary to abandon it. What is decisive in such problems is not one or the other index for a given sector but the interest of the national economy as a whole and the calculation of the effects on it.

In a socialist society higher profitability should not be an aim in itself (as under capitalism) but a means of attaining the best result or the lowest expenditure for the whole society. In this connection, the order in which it is proposed to apply this principle—on the basis of the system of o.d. valuations, based on real conditions—is to subordinate it to the need of achieving the targets of the general plan.

After having first refused to be guided by profitability, the task of

constructing an optimal plan led us again to conclude that this principle should be applied to each sector (in order to reach the general optimum), but on condition that the indices of cost shall be calculated on the basis of the o.d. valuations, taken from the given conditions and problems.†

The question, considered in this section, of the correct allocation of the programme to enterprises is of great practical importance; as shown by experience, expenditure on the same article in one enterprise may in many instances be two to three times higher than in another, even with the same technical equipment. In addition to defects in the organization of production, the unsuitability of the enterprise for the production of a given type and the insufficient allowance for this circumstance in the allocation of the programme play undoubtedly an important part.

### *Use of Other (a priori) Valuations*

Objectively determined valuations have an inherent character: they are entirely determined by the conditions of the problem under consideration.

As against these, there exist other valuations of production prepared independently of a given problem arising from allocation which may in this context be called *a priori*, external valuations. Among this kind of valuation may be considered current prices or the cost of some products. Naturally the question arises: are the objectively determined valuations necessary and could not any other available *a priori* valuations have been used instead? We shall show that this does not always lead to satisfactory results.

Let us assume that the sale prices of articles No. 1 and No. 2 were previously determined on the basis of production conditions of factories of type B. It is clear that the cost of labour for the manufacture of article No. 2 at this type of factory is double the cost for article No. 1, for instance 20 roubles and 10 roubles. And the full cost, allowing also for materials, will be  $20+15=35$  roubles for article No. 2 and  $10+10=20$  roubles for article No. 1.

† The conclusion reached on the necessity of applying o.d. valuations refers to the particular economic problem under consideration. Some observations about the importance of the results obtained for the national economy as a whole, and in particular in relation to price formation, are given in Chapter II, Section 8.



Let us try to construct a programme starting from these prices. If we calculate the production of each factory for one or the other article (Table 6), we can see that the highest production is obtained from article No. 1 at all factories, i.e. for all the factories it is more profitable to include in the plan article No. 1 rather than article No. 2. If we followed here the principle of profitability, article No. 1 would be put into production at all enterprises. But then the required articles No. 2 would not be available at all, and the assortment task would not have been fulfilled. This forces us to depart from the principle of profitability and to put into production article No. 2 although this may not be financially advantageous to the

TABLE 6 OUTPUT OF ARTICLES NUMBERS 1 AND 2 IN ROUBLES  
(AT PRICES OF 20 AND 35 ROUBLES RESPECTIVELY)

Type of factory	Production of article No. 1		Production of article No. 2	
	Number of articles	Production (in roubles)	Number of articles	Production (in roubles)
A	100,000	2,000,000	15,000	525,000
B	400,000	8,000,000	200,000	7,000,000
C	20,000	400,000	2,500	87,500
D	200,000	4,000,000	50,000	1,750,000
E	600,000	12,000,000	250,000	8,750,000

factory. Here it is difficult to determine in which factories article No. 2 should be produced. If article No. 2 is necessary although its production is not profitable for any type of factory, the task of producing article No. 2 is assigned to various types of firms. In this manner any "random" plan of the kind quoted in Table 2 may emerge.

Let us note that such unsatisfactory results may also arise from valuations prepared on the basis of average costs at all firms where a given output is produced even if the optimal allocation plan is taken into consideration in the calculations (see Chapter II, Section 6, pp. 97-9).

Thus, when arbitrary *a priori* valuations different from the o.d. valuations are used, it is not possible to follow the principle of profitability and to satisfy at the same time the assortment condition (while in the case of the o.d. valuations full harmony was attained). Not only do these *a priori* estimates fail to assist in finding an optimal plan, but they confuse the issue. For instance, it seems fully

justified to produce article No. 1 at B-type factories when this task is considered in isolation, but should this be done it is most likely that the plan would be far from optimal.

CONCLUSION 3. Starting from any (*a priori*) valuations, which differ from the objectively determined ones, it is usually not possible to follow the principle of profitability and to ensure the output of the necessary products. The application of the principle of profitability on the basis of such valuations may result in looking in the wrong direction for the optimal planned solutions.

So we see that the comparison of valuations concerning the volume of production leads to the optimal plan when o.d. valuations are used. It is important to emphasize that to direct the allocation of the programme correctly the following two conditions were essential: that the comparison should be made on the basis of the valuation of the completed net production, and that for individual work correct valuations should be used. If either of these conditions were violated—if correct valuations for production were used but comparisons were made not for net output but for commodity production, or if in comparing net output incorrect valuations of production were used, then in either case we should be led astray in the drawing up of the optimal plan.

When the problem of determining the production programme in practice arises, prices are often used which have been fixed some years earlier under different conditions. Hence some articles appear “convenient” for the factory—their plans are easily fulfilled (in terms of gross production), others are “inconvenient”. Sharp differences in profitability lead managers to exceed the production targets of profitable outputs to the detriment of the production of articles important for the national economy but not advantageous to the factory. It is known, for instance, that owing to the unprofitableness of some articles of children’s wear, sufficient numbers were not manufactured.†

The misleading effect of the valuation of labour efficiency is magnified by calculations that are made on the basis of gross production or of commodity production rather than of net production. All hinders the efficient allocation of the programme and fails

† Paper of A. N. Kosygin at the XXI Congress of the Communist Party of the Soviet Union (*Report*, vol. I, p. 160).

to arouse the interest of the factory in the proper fulfilment of the assortment plan.†

In the allocation of the programme or in the placing of a given order direct comparisons of expenditure are also used in order to minimize costs, but this way does not ensure either that the optimal allocation will be secured. In fact, an optimal solution guarantees a minimum total expenditure of the whole production unit for all outputs (namely that which is of interest to society as a whole!). At the same time, if we consider costs of each individual product, the best location of its production does not, generally speaking, ensure a minimum of total costs. For this reason, the analysis of expenditure per unit product taken individually frequently does not lead to an optimal solution. To arrive at a correct solution it is necessary to analyse simultaneously the allocation of the whole programme over the entire production unit while bearing in mind the general objectives of a socialist society. On the basis of this analysis one should decide on the indices to be used for comparison. The assessment of particular solutions in accordance with these indices ensures then the choice of solutions in agreement with the general interest (of the whole unit). This furnishes a harmonious combination of general and local interests.‡

These principles are continuously being used in socialist construction. The method of obtaining an optimal plan and o.d. valuations furnishes means for a more precise and systematic construction of such types of indices, and thereby allows a fuller use of the possibilities and advantages of a socialist economic system.

† These considerations confirm the correctness of the statements in the press in favour of substituting net production for gross and commodity production when describing the volume of work carried out by a factory during a given time; that is, accounting only for the newly created value (and not the transferred one). This measure is of course advisable when at the same time the system of valuation of individual types of production is improved.

‡ Here we have a typical example of the contradictions that may arise in the planning of our economy (between the particular interests—improving the conditions of production for one product, and the general interest—the fulfilment of the whole plan). Like all such contradictions they have not, under socialism, an antagonistic character. This contradiction is resolved by determining the indices on the basis of which the comparison of particular solutions is made, starting from the tasks and interest of the society as a whole.

### *Criteria for an Optimal Plan*

We shall now consider the question as to how to ensure that a given plan is optimal.

A direct comparison with all other plans is, as a rule, not practicable, as the number of possible plans could be enormous. Let us quote a convenient method for this purpose using the example given above. That the plan given in Table 4 was optimal we already knew when it was set out; however, two features make it clear had we not known this. First, the set ratio between the two articles is observed in this plan (the assortment task); secondly, in it the principle of profitability is satisfied for certain valuations (Table 5) of the given types of production ( $a$  and  $4a$ ).

On the basis of these two features we can conclude that the plan is optimal. Let us assume the contrary. Let us suppose that there is some other allocation of the plan in which the requisite relationship is also observed, but the total output is still higher. In the case of such a typical plan a higher output is observed both for articles No. 1 and No. 2 than in the plan given in Table 4. Then, no matter what valuations be adopted for each article, the total valuation of net production in this hypothetical plan will be higher than for that of Table 4. This should in particular obtain if valuations  $a$  and  $4a$  are used. But the total valuation of the whole production is made up of valuations of the outputs of individual factories; for this reason the volume of production (according to the valuations  $a$  and  $4a$ ) would be higher for one factory in the case of the hypothetical plan than with the one given in Table 4. However, such an assumption is impossible for, as shown in Table 5, by allocating any factory differently from Table 4, no higher valuation could be obtained for its production because in the plan Table 4 each enterprise was utilized in such a manner that the valuation of the attained output was at a maximum. The argument used here proves that there could be no plan yielding a higher output and for this reason the one given in Table 4 is optimal. The reasoning we have applied here is general,† and this brings us to Conclusion 4 which is essentially supplementary to Conclusion 2.

**CONCLUSION 4.** If in a certain plan (a) the target ratio of assortment set by the plan is observed (by types of production); (b) the principle

† This reasoning is given in mathematical form in Appendix I.

of profitability for a certain set of valuations for different articles is satisfied then first, the given plan is optimal—there can be no other plan in which the assortment condition could be fulfilled and the volume of production increased or which would give a higher output of each type of production than the given one, and secondly, the above valuations are objectively determined for the given case.

It follows that if a non-optimal plan has a correct assortment ratio there can be no valuations in which the principle of profitability is observed (if such a valuation were found, the plan would be optimal). What would happen if such valuations were attempted for a plan which is not optimal? Let us try to do this in the plan quoted in Table 3. Let us assume that such a valuation will amount to  $m$  per article No. 1 and  $n$  per article No. 2. As one of the factories of type E is used for the production of article No. 1 and the second for article No. 2, then if the principle of profitability were observed, the valuation of net production in both cases would be identical, i.e.  $600,000 m = 250,000 n$ , from which  $n = 2.4 m$ .

Let us now try to verify whether the condition of profitability will be observed in a factory of type C. We find that the valuation of the output for the method used (in the production of article No. 2) equals  $2500 n$  or  $6000 m$ , as  $n = 2.4 m$ ; in the case of the method not used (in the production of article No. 1) it equals  $20,000 m$ , so the principle of profitability is clearly violated, for  $2500 n < 20,000 m$  (it would not be violated if  $2500 n \geq 20,000 m$ ). It is also immediately clear how a plan can be obtained giving a higher output of one or the other article. In fact, turning type E factories from production of article No. 1 to production of article No. 2, for each additional unit of article No. 2 we lose 2.4 units of article No. 1, but changing type C factories from production of article No. 2 to the production of article No. 1, by giving up one unit of article No. 2 we gain eight units of article No. 1. It is evident, that by combining two such changes, we shall obtain a plan giving a higher volume of production for both articles. Such an improved plan is given in Table 7. As may be seen, in this plan the volume of production of each article is 11 to 17 per cent higher than in that of Table 3.

Thus, the attempt to find valuations for a non-optimal plan failed—in arriving at such valuations we met with contradictory requirements.

Having revealed the impossibility of deriving such valuations, we have at the same time established what changes in the allocation could furnish a plan giving a higher production of each article and thereby revealed, or more exactly confirmed, in this case that the plan was not optimal. We thus reach:

CONCLUSION 5. If it is impossible for a certain plan to yield valuations in which the principle of profitability is observed (for such valuations contradictory conditions are obtained), then the given plan is not optimal, that is, there is a plan in which the volume of production for each type of article is higher than for the given one. Meanwhile, the analysis carried out points to the possible way of improving the plan.

TABLE 7. IMPROVED PLAN  
(AS COMPARED WITH THE PLAN IN TABLE 3)

Type of factory	Article No. 1		Article No. 2	
	Number of factories	Aggregate output	Number of factories	Aggregate output
A	—	—	5	75,000
B	3	1,200,000	—	—
C	40	800,000	—	—
D	—	—	9	450,000
E	—	—	2	500,000
Total		2,000,000		1,025,000

Thus, it becomes evident that the analysis of the valuations provides a very simple criterion as to whether any given plan is optimal or not, i.e. by comparing Conclusions 4 and 5 we obtain:

THE RULE. To decide whether a given plan is optimal, it is necessary to look for such valuations of production in which the condition of profitability would be fulfilled for the given plan. Then: (a) should such valuations be obtained the plan would be optimal—no other plan with that particular allocation would give a higher volume of production; (b) should it prove impossible to arrive at such valuations (contradictory conditions are found for them) then the plan is not optimal—there exists one giving a higher output of each article.

*Methods for Finding an Optimal Plan  
and O.D. Valuations†*

To find an optimal plan in the case of two articles the method we used above is sufficiently simple and convenient. However, in the case of a greater number of products and also for the task which we shall encounter further no similar method is available. In this case it is necessary to use some special methods based on the relationship between the optimal plan and the o.d. valuations corresponding to it. Although these examples worked out above are comparatively simple they have distinct peculiarities which merit some consideration. This is all the more useful as understanding the calculation makes it possible to penetrate more deeply into the meaning of the concept of the o.d. valuations. Therefore, without touching upon complicated cases, we shall give here the basic methods of calculation.

Although in the case of two products, mentioned above, the solution was obtained by the previous method, it is better to use the simpler case as an example. A solution of a more complicated example of that nature is given in Section 2.‡

*1. Choice of Valuations.* As shown above, for each type of factory a corresponding valuation of labour input (cost) is obtained for the manufacture of article No. 2 as compared with article No. 1, namely (see Table 1): 6·7 for A; 2 for B; 8 for C; 4 for D; 2·4 for E.

Which of these valuations is the most appropriate one?

Let us consider the valuations 2·4. Comparing the valuations of the production of the enterprises, we find the figures shown in Table 8.

From this table it may be seen that following the principle of profitability factories of types A, C, and D should be turned to the production of article No. 1, those of type B to the production of article No. 2 and those of type E to one or the other article as desired, upon which will also depend the quantity of each kind of article that will be obtained. But even in the most favourable case when both factories of type E are put to the production of article

† This part (pp 16-24) may be omitted at the first reading.

‡ A more general discussion of the computational methods of finding an optimal plan and o.d. valuations is given in Appendix II.

No. 2, 1,100,000 articles will be obtained as compared with 3,100,000 articles of No. 1, i.e. it is impossible to meet the necessary assortment ratio.† It is therefore necessary to increase the valuation for article

TABLE 8. CALCULATIONS OF NET PRODUCTION AND OF POSSIBLE OUTPUT OF TWO ARTICLES USING VALUATIONS 1 AND 2·4

Type of factory	Number of factories	Net production in the manufacture of		Number of articles in the profitability plan	
		Article No. 1	Article No. 2	No. 1	No. 2
A	5	100,000	36,000	500,000	—
B	3	400,000	480,000	—	600,000
C	40	20,000	6,000	800,000	—
D	9	200,000	120,000	1,800,000	—
E	2	600,000	600,000	(1,200,000)	(500,000)
Total		Minimum		3,100,000	600,000
		Maximum		4,300,000	1,100,000

No. 2. Taking valuation 4, we obtain Table 5 instead of Table 8; then as was seen it was possible to satisfy the assortment ratio (2:1) and we arrive at the optimal plan (Table 4).

2. *Another Method of Assigning a Valuation.* The difference consists in that a valuation is chosen starting not from special values but from arbitrary ones.

A first approximation for the valuations can be found as follows. Let us calculate the aggregate production of the factories when they all produce article No. 1 only or article No. 2 only. We shall obtain 5,500,000 and 1,725,000 respectively, i.e. three times more of article No. 1. This shows that labour consumption in the manufacture of article No. 2 is on the average approximately three times higher than that for article No. 1. For this reason we shall, for example, take valuations 1 and 3 and shall set out a table as above; in this case we

† Thus, the plan that can be obtained is "profitable"—it is set up in accordance with the principle of profitability starting from some system of valuations—yet it does not satisfy the assortment ratio and for this reason it is not an optimal plan.



select not only figures corresponding to the maximum valuation, but also those which approach them (Table 9). In the latter case, as a result of the analysis, one article may be chosen as well as the other. Therefore, the number of articles in this case is put for both variants. (Corresponding figures are in brackets.)

TABLE 9 CALCULATION OF NET PRODUCTION AND THE POSSIBLE OUTPUT OF ARTICLES, USING VALUATIONS 1 AND 3 RESPECTIVELY

Type of factory	Number of factories	Valuation of production		Number of articles	
		Article No. 1	Article No. 2	No. 1	No. 2
A	5	100,000	45,000	500,000	—
B	3	400,000	600,000	(1,200,000)	(600,000)
C	40	20,000	7,500	800,000	—
D	9	200,000	150,000	(1,800,000)	(45,000)
E	2	600,000	750,000	(1,200,000)	(500,000)

We shall now try to fulfil the prescribed assortment. First of all, production of article No. 2 would have to be carried out by those factories for which the aggregate production valuations of this article are much higher than for article No. 1, but none exist. In that case, those factories for which the aggregate production valuations of article No. 2 are only slightly higher—those are factories of type B and E. The volume of production for article No. 2 will amount in these to 1,100,000, which is insufficient (there will be 3,100,000 units of article No. 1). Therefore, factories of type D have to be used in part for which the total production valuation of article No. 2, although less, is near the total production valuation for article No. 1. Thus a plan is obtained as given in Table 4. Since factories of type D are used for the production of both articles, when we compare their profitability we obtain valuations 1 and 4. With the aid of these valuations, we establish (on the basis of Conclusion 5) that the plan given in Table 4 is optimal.

*3. Method of Successive Adjustment (Improvement) of the Plan.* We start from some plan which gives an approximately correct allocation. We then determine whether it is optimal. If it is not optimal, we can see how it can be changed to produce more of both articles. When checking the profitability of the plan, a comparison of the valuations for the methods used or otherwise leads to contra-

dictory inequalities. Considering several possible methods of providing the output, the comparison of which led to contradictory valuations, we see how to improve the plan by incorporating some method hitherto not used while maintaining strictly the prescribed ratio of the allocation of production. This change is carried on until the method being excluded ceases to be applied or until the method being increasingly utilized is used to its maximum. Thus we arrive at a new plan with an output of the required assortment at a higher volume. For this plan we shall repeat the check and should it not prove optimal, we can improve it further. Thus we reach an optimal plan and at the same time obtain the o.d. valuations.

We shall illustrate this method by taking the plan of Table 3. We have already improved it and obtained the plan in Table 7. We check it to see if it is optimal. We shall assume that the valuations for articles No. 1 and No. 2 will be  $m$  and  $n$ . Since article No. 2 was put into production at type A factories, it should be profitable—we should have:  $100,000 m \leq 15,000 n$ , or  $n \geq 6.7 m$ .

As article No. 1 was put into production at the B type of factory, we should obtain  $400,000 m \geq 200,000 n$ , or  $n \leq 2 m$ . These conditions are contradictory: in the first instance, one article No. 2 is preferred to 6.7 articles No. 1, in the second case, two articles No. 1 are preferred to one article No. 2. This points to the way of improving the plan. It is necessary to turn type A factories to the production of article No. 1 and in order to preserve approximately the assortment one factory of type B to the production of article No. 2. The corresponding plan is given in Table 10.

TABLE 10. PLAN OF CHANGING THE PRODUCTION OF FACTORIES TO OTHER ARTICLES, MAINTAINING THE ASSORTMENT

Type of factory	Article No. 1		Article No. 2	
	Number of factories	Output	Number of factories	output
A	5	500,000	—	—
B	2	800,000	1	200,000
C	40	800,000	—	—
D	—	—	9	450,000
E	—	—	2	500,000
Total		2,100,000		1,150,000

We check again whether the plan is optimal.

Since type B factories are used in the plan for articles No. 1 and No. 2, both should be equally profitable:  $400,000 m = 200,000 n$ ;  $n = 2m$ , e.g.  $m = 1$  and  $n = 2$ . Using these valuations, we check the profitability of the other types of factories—the principle is violated for type D factories. We readjust the plan and change type B factory to the production of article No. 2. This increases the output of article No. 2 by 200,000 and reduces article No. 1 by 400,000, but turning three factories of type D from the production of article No. 2 to the production of article No. 1, the output of this article is increased by 600,000 and that of No. 2 is reduced by 150,000. As a result, the number of the two articles is increased by 200,000 and 50,000 respectively, while at the same time the assortment is maintained. If we transfer two instead of one factory of type B to the production of article No. 2 and if production can be changed in six factories of type D, the effect is twice as great. As a result we arrive at the plan shown in Table 4.

As the factories of type D are used for the production of both articles, the valuations will be 1 and 4; having checked that according to these valuations the most profitable article was manufactured by the remaining types of works, we see that the plan is optimal. The process of improvement has been completed.

4. *Graphic Methods.* Instead of computational methods, simple graphic methods may be used in checking whether a given plan is optimal and also in finding the optimal plan and the o.d. valuations. Although these methods can in practice only be used for two or three

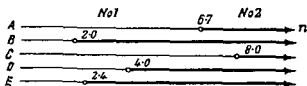


FIG. 1

types of product, we quote them as graphs to make the problem and the properties of the valuations clearer.

To explain the optimum of a plan we start from the following

considerations. The question as to whether it will be more profitable for a given factory to produce article No. 1 or article No. 2 will depend upon the valuations of these articles. Taking as a valuation of article No. 1  $m=1$ , a solution will be found with valuation  $n$  for article No. 2. Thus, for type A factories, if  $100,000 > 15,000n$ , that is when  $n < 6.7$ , the output of article No. 1 is more profitable, and when  $n > 6.7$ —that of article No. 2 is more profitable. This is also the case for the remaining types of factory. Thus, marking the corresponding figures on the drawing, the range of the values of  $n$

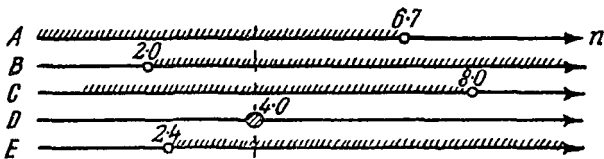


FIG. 2

that are favourable to article No. 1 and article No. 2 can be shown. In Fig. 1 this range is shown for all the types of factory (on the left of the dividing point the range is advantageous to produce article No. 1, on the right article No. 2).

Let us now consider some plan (for instance, that of Table 4). The choice of a given article for manufacture at a factory of a given type means that  $n$  must lie in the range favourable to this article. Let us mark for each type of factory the range corresponding to the article by hatching the production which is used by the factory of the type envisaged in the plan (Fig. 2).† If the given plan is optimal (and only in such a case), there must exist a valuation on the basis of which all choices of production utilized are profitable.

In Fig. 2 which corresponds to the plan of Table 4 a point common to the whole range ( $n=4$ ) exists (the dotted line corresponds to it); consequently the plan is optimal.

In Fig. 3 drawn in accordance with the plan of Table 3, it is not possible to draw such a line—the plan is not optimal.

† For instance, for type A factories engaged in the manufacture of article No. 1 the range is shown to the left of the dividing point where  $n=6.7$ . In the case of type D factories utilized in the manufacture of both types of articles, the dividing point is only noted since their production as shown is only economic when  $n=4$ .

In order to find the optimal plan, the following calculation may be made, repeating the original calculation by which this plan was found (p. 4). First of all (Fig. 4), for each type of factory the greater range of articles No. 1 and No. 2 is marked (the limits are already shown in Fig. 1). At first, all the factories are put to the production of article No. 2, and 1,725,000 of these articles are

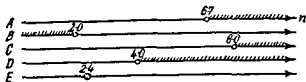


FIG. 3

manufactured. The corresponding point (I) is marked on Fig. 5. The highest corresponding valuation of article No. 2 is obtained (see Fig. 4) for the factories of type C. Therefore, if any No. 1 articles are required, factories C should be first to change their production. In transferring them to the production of article No. 1 a new position is obtained (II); continuing in this manner, points I, II, III, IV, V, VI are constructed. The coordinates of these points correspond to the volumes of productions, shown in Table 11.

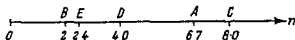


FIG. 4

Then a curve is drawn connecting all these points. Each section of the curve is divided into equal parts in accordance with the number of factories of a given type (except for the first section in which one division corresponds to four factories of type C). Now, whatever the assortment ratio for total output, the optimal plan may be found at once. Thus, in the case of the ratio 2:1 (in which the production of article No. 1 is twice that of article No. 2), drawing a straight line corresponding to this ratio, the point of intersection  $\alpha$  is obtained and corresponds to the optimal plan. In Fig. 5 it is apparent that the output of article No. 1 amounts to 2,500,000 and of article No. 2 to 1,250,000. Factories of types C and A and six factories of type D should produce article No. 1; the rest should produce No. 2. Thus the plan of Table 4 is obtained. Let us note that the slope (the

absolute magnitude of the tangent) of the segment III-IV equals  $1:4 = 0.25$ , and furnishes the value of the ratio of the o.d. valuations for articles No. 1 and No. 2.

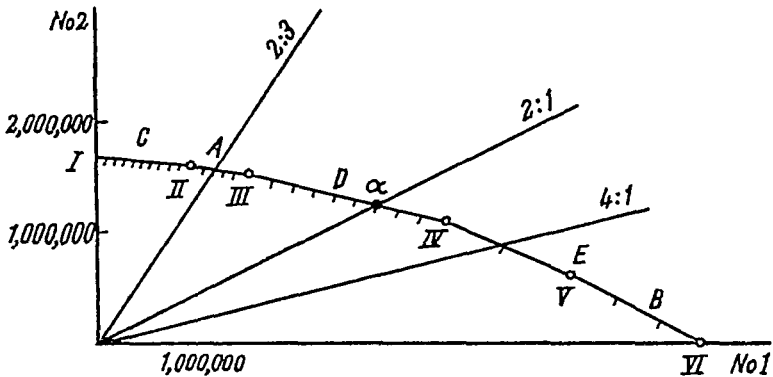


FIG. 5

The line I-II-III-IV-V-VI together with the axes forms a polygon in the plane. This is the polygon of feasible plans since the output under each plan achieved in the given conditions (for instance, the plans given in Tables 2 and 3) will be represented by some point of this polygon and conversely, each point of this figure corresponds to the output under some feasible plan.

This graphic solution may be arrived at in another way by using a linear instead of a plane projection. Namely, for points I-VI, instead of absolute production magnitudes for each article, we shall find their relative shares (Table 11).

TABLE 11. PRODUCTION UNDER SUCCESSIVE RATIONAL TRANSFERS OF WORKS TO THE MANUFACTURE OF ARTICLE No. 1

Points	Articles		Articles as % of their total number	
	No. 1	No. 2	No. 1	No. 2
I	0	1,725,000	0	100
II	800,000	1,625,000	33	67
III	1,300,000	1,550,000	46	54
IV	3,100,000	1,100,000	74	26
V	4,300,000	600,000	88	12
VI	5,500,000	0	100	0

Let us mark point I-VI on Fig. 6, starting with the percentage found for article No. 1. If we have to determine now the optimal plan for some assortment ratio, for instance 2:1, this relationship

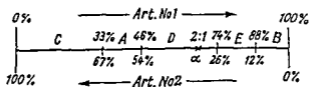


FIG. 6

is marked on the drawing (67 and 33 per cent—point  $\alpha$ ) and it may be seen at once that in the optimal plan factories C, A and a part of D (in Fig. 6 on the left of point  $\alpha$ ) should be used for article No. 1, i.e. the plan of Table 4.

#### *Properties of the O.D. Valuations*

Let us show some of the important properties of the o.d. valuations on which their application is based and which reveal their meaning and value. First of all, o.d. valuations are specific—they are determined by specific conditions and are dependent upon all the conditions of the problem: the prescribed assortment of output, the number of factories of each type, the planned production capacity of each article. A change in any of these conditions may lead to a change in the o.d. valuations. Let us, for instance, observe the change in the o.d. valuations as a result of a change in the assortment condition.

Let us consider this example when 1.5 times more is required of article No. 2 than of article No. 1, that is a ratio of 2:3, instead of 2:1. In this case, to arrive at the optimal plan, it is necessary to transfer the remaining factories of type D to the production of article No. 2; however, this is insufficient. Therefore two factories of type A must be added. Then the production of article No. 1 will amount to 1,100,000 and of article No. 2 1,580,000, i.e. the requisite ratio of 2:3 is almost attained (see also Fig. 5).

Further, since in this case type A factories are used for the manufacture of both articles, the o.d. valuations are determined on the basis of equal profitability for this type of factory on the two articles, and prove to be equal,  $n=6.7$  (keeping  $m=1$ ), that is they are deter-

mined on the basis of the ratio of expenditure at factories of type A. We see thus that the o.d. valuation has increased. This is natural: as the requirements of article No. 2 increased it became necessary to use for its manufacture factories less suited to the production of this article; consequently the relative cost of this article increased.

Assume now that the assortment ratio is set at 4:1, that is the requirements of article No. 2 are reduced. In this case, it is necessary to change to the production of article No. 1 all factories of type D and also one of the factories of type E. In the plan obtained (which is optimal for the given ratio) the output of article No. 1 will amount to 3,700,000 and of article No. 2 to 850,000. O.d. valuations are determined on the basis of equal profitability (for both articles) for E-type factories where  $n=2.4$  ( $m=1$ ). The reduction in the o.d. valuation is again natural as the production of article No. 2 was maintained only at factories best suited to it and where the relative labour input for its manufacture was lower. The results thus obtained can be formulated as follows.

**CONCLUSION 6.** Objectively determined valuations are concrete and dynamic; they are defined by all the conditions: the required assortment of products, the number of factories of each type, the planned output capacity; and they change when these conditions change. In particular, following an alteration in the assortment ratio, any increase in the requirements of some article entails a corresponding increase in costs and consequently in its o.d. valuation; a decrease in the requirements entails a reduction in its o.d. valuation.†

It should be noted that this proposition was, in fact, taken into consideration in various economic measures by the party and government on several occasions. Thus, in 1963, in connection with the tasks set for the further development of light industries it became necessary to increase the output of cotton. To achieve this, the price of cotton was raised sharply. In consequence, many areas which previously produced cereals found it more advantageous to grow cotton. This had a favourable effect on the increase of the area under cotton cultivation.

Other similar examples could be quoted. However, it should be pointed out that such changes by individual bodies were far from

† It should be pointed out that this latter statement is correct for the simplified problem considered. In real conditions, owing to a whole series of circumstances which we have not taken into account it does not always apply.



being carried out in every case where it would have been advisable; even the magnitude of the requisite change was determined to some extent subjectively.

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### *Stability of O.D. Valuations*

Let us introduce some slight change in the prescribed condition, for instance, changing the ratio 2:1 to 4:3. In this case, as may be easily seen, the optimal plan is obtained by changing another three of the six factories of type D to the production of article No. 2. The production of article No. 1 amounts to 1,900,000 and of article No. 2 to 1,400,000, thus the required ratio is almost exactly obtained. As both articles are again produced at type D factories, the o.d. valuations remain the same (1 and 4). It can be shown that slight changes in other conditions (the number of factories, production capacity) either do not affect the value of the o.d. valuations or else change them only a little.

It should be mentioned that if a more realistic example were chosen in which the number of types of factories is greater or in which the ratios of productive capacities for the articles are different for the same type of factory as well, then the number of possible values for valuations  $n$  would be much more than 5 (assuming  $m=1$ ) (as above: 8; 6-7; 4; 2-4; 2). In such a case (and also if productive capacities change) and with slight changes in other conditions, the valuations might change, but not greatly. Thus we obtain an important property of the o.d. valuations which shall be called the *stability* property.

CONCLUSION 7. O.d. valuations possess a certain stability; with slight changes in the conditions of the task (assortment quota, number of factories, production capacities), the ratio of o.d. valuations, as a rule, remains either unchanged or changes merely a little.

This feature is extremely useful, particularly in the following two cases. First of all, it makes it possible, in finding an optimal plan and o.d. valuations, to confine the task initially to the more important and most representative types of factory, and by way of solving it to determine the values of the o.d. valuations. Then the question of utilizing the remaining factories may be solved on the basis of valuations already found, since the calculation for these factories

cannot substantially change them. Thus, we shall arrive at an optimal plan or one only slightly different. Secondly, when ascertaining changes connected with the operation of any individual factory (for example, increase in its capacity, temporary stoppage, etc.), calculations may be carried out starting from existing o.d. valuations, disregarding those changes in them which may take place as a result of accounting for the transformations at the given factory.

It is appropriate in this connection to draw attention to the resolutions of the June 1958 Plenum of the Central Committee of the Russian Communist Party and the report by N. S. Khrushchev *On the abolition of compulsory deliveries and payments in kind for work by the machine and tractor stations, on the new order, prices and conditions of state purchases of agricultural products* which emphasize the importance of a scientifically established system of prices that reflects the dynamic changes in operating conditions and costs, while ensuring at the same time the necessary stability of prices.

### *The Realistic Nature of O.D. Valuations*

In our example, the ratio of valuations of the work in the manufacture of articles No. 1 and No. 2 was 1:4. This ratio is not fictitious and can actually be realized, i.e. instead of four units of article No. 1, one unit of article No. 2 may be manufactured and vice versa. In fact, only one factory of type D need be transferred from the production of article No. 1 to article No. 2 and instead of 200,000 articles No. 1, 50,000 articles No. 2 are obtained. In this manner with a transfer in the reverse direction, instead of article No. 2 we shall obtain the corresponding number of article No. 1 in the proportion (1:4).† Each of these transformations will change our optimal plan to another optimal plan, corresponding to a slightly different assortment quota. Thus we have:

CONCLUSION 8. The ratio of o.d. valuations is realistic, that is, on the basis of the equivalence determined by these valuations, some units of one type of product can be replaced by a corresponding number of units of another type of product and vice versa. More

† This realism in the ratios of the optimal o.d. valuations once more shows that they correctly reflect the ratio of costs in the manufacture of articles under the given conditions.

exactly, if for two articles the ratio of o.d. valuations of the work in their manufacture is  $m:n$ , then broadly speaking it is possible to carry out such changes in the programme that the quantity of the first article will be reduced by some number of  $m$  units and the quantity of the other article will be increased by  $rn$  units. With such changes the programme remains optimal (for the changed assortment requirements).

This property of the o.d. valuations is useful as it makes clear what sort of changes can be made in the plan. It should be pointed out that our usual prices are far from always having this property. If it was planned to acquire 1000 roubles'-worth of timber, it is certainly not always possible to exchange this for cement worth the same amount of money. In particular, rigid control of expenditure by commodities reflects the lack of realism about relative prices—the impossibility of exchanging certain materials and services for others at such prices.

The difficulty arises not only from such exchange not being permitted or because it is impossible for lack of the necessary materials but because of a certain belief that the two products are not equivalent as regards their national economic significance and the real magnitude of costs necessary for their production.†

Such an impossibility of establishing valid price relationships makes the calculations based on them frequently appear unreal and the results derived from them in practice incorrect or else impossible to attain.

### *Application of O.D. Valuations*

**CHANGES IN THE PROGRAMME.** As a result of their properties and their connection with the optimal plan o.d. valuations may be used with success in solving various problems of economic planning.

Let us assume that the previous programme (article No. 1—2,500,000 units, article No. 2—1,250,000 units) is changed and a new task is set, namely: article No. 1—3,000,000 units, article No. 2—1,000,000 units. Can this programme be fulfilled? As the change is

† An undisputed proof of such a situation is the (planned) losses of certain factories, in particular in the heavy industry, which persisted for a long time. Apparently it is thought justified for a factory manufacturing 1000 roubles'-worth of iron or steel to incur an expenditure of 1500 roubles (of other kinds).

slight, we shall use the previous valuations (1 and 4). Evaluating the previous task, we obtain:

$$2,500,000 \times 1 + 1,250,000 \times 4 = 7,500,000.$$

The new target:

$$3,000,000 \times 1 + 1,000,000 \times 4 = 7,000,000.$$

From the calculation it is apparent that the target is not merely fulfilled, but may also be exceeded by

$$(7,500,000 - 7,000,000) : 7,000,000 = 7 \text{ per cent.}$$

And in fact, transferring three factories of type D from the production of article No. 2 to the production of article No. 1, we obtain 3,100,000 of article No. 1 and 1,100,000 of article No. 2—a surplus over and above the set task.

*Second example.* Three factories of type A are taken off the production of the given articles. It is necessary to assess to what extent this will affect the fulfilment of the programme with the other previous conditions remaining the same, in particular as regards the assortment ratio. As the type A factories in the plan of Table 4 were used for the manufacture of article No. 1, the output of the three factories which went out of production is as follows:  $3 \times 100,000 \times 1 = 300,000$ . At an aggregate production valued at 7,500,000, it is apparent that the total output is reduced by 4 per cent—the number of article No. 1 must be reduced by 100,000 (4 per cent of 2,500,000), and the number of article No. 2 by 50,000. As a result of three factories having been removed from production, the number of article No. 1 is diminished by 300,000. To restore the allocation, one factory of type D is transferred from the production of article No. 2 to the production of article No. 1. We then obtain a plan (again optimal) in which the output of article No. 1 will amount to 2,400,000, and that of article No. 2 to 1,200,000—a 4 per cent reduction in production.

**CONCLUSION 9.** O.d. valuations may be used in the calculation of the possible fulfilment of a programme as a result of any small changes in the plan target or in the productive capacity. To arrive at a solution of the problem, it is necessary to evaluate the target or productive capacity using the existing magnitudes of the o.d. valuations, which satisfy the optimal plan.

*Comparison of Methods of Organizing Production*

O.d. valuations may also be used in the solution of another important problem, namely in the choice of one of several possible *methods of organizing production*, which should supply a varied output. Let us consider some examples.

(1) To the given group of factories a new type of factory F is added. If it is used for the production of article No. 1, it will supply 450,000 of them, and if for article No. 2, 150,000. Which is preferable?

Let us compare the net production of both kinds on the basis of the o.d. valuations. We obtain  $450,000 \times 1 = 450,000$  and  $150,000 \times 4 = 600,000$ . Therefore, it is more advantageous to produce article No. 2 at this factory. As there will be an increase of 150,000, one D-type factory is changed from the production of article No. 2 to that of article No. 1 in order not to disturb the allocation ratio. We then obtain:

$$\begin{aligned} 2,500,000 + 200,000 &= 2,700,000 \text{ of No. 1 and} \\ 1,250,000 + 150,000 - 50,000 &= 1,350,000 \text{ of No. 2.} \end{aligned}$$

The plan obtained is again optimal as it fulfils the prescribed ratio (2:1) and the principle of profitability is observed (at valuations 1 and 4).

(2) At one of the type E factories a new method of organizing production is proposed whereby the production of both articles is combined. The planned output is: 55,000 of No. 1 and 150,000 of No. 2. Is this method advantageous?

The net production of the enterprise, as previously used, was (for article No. 2)  $250,000 \times 4 = 1,000,000$ ; by the proposed method it is  $550,000 \times 1 + 150,000 \times 4 = 1,150,000$ . Thus, the comparison based on o.d. valuations shows that the proposed method is preferable. Having recourse to this method it is not difficult to construct a plan in which the output of the group of factories of both types of articles will be higher than the original one.

(3) It is proposed that A- and B-type factories co-operate so that the aggregate output shall amount to 250,000 of article No. 2. Is this advantageous?

Evaluating production, we obtain:

for the factories as previously utilized:

$$100,000 \times 1 + 200,000 \times 4 = 900,000;$$

as proposed:

$$250,000 \times 4 = 1,000,000.$$

Co-operation is advantageous. That the conclusion arrived at is correct is confirmed by the programme given in Table 12 in which such co-operation for the three pairs of factories A and B leads to an increase in the output of both articles.

TABLE 12. CO-OPERATION OF FACTORIES A AND B

Type of factory	Number of factories	Production of	
		Article No. 1	Article No. 2
A	2	200,000	—
A + B	3 + 3	—	750,000
C	40	800,000	—
D	8; 1	1,600,000	50,000
E	2	—	500,000
Total		2,600,000	1,300,000

It is interesting to see what we might obtain if we tried to solve these problems starting from *a priori* valuations (or prices) differing from the objectively determined ones. Let us, for instance, take valuations 20 and 35. Then, evaluating (in the first instance) the production of type F factory in the manufacture of article No. 1, we obtain  $450,000 \times 20 = 9,000,000$  roubles, and in the manufacture of article No. 2  $150,000 \times 35 = 5,250,000$  roubles. We reach the conclusion that article No. 1 should be put into production, contrary to the results obtained above. The conclusion is certainly incorrect. If we followed this conclusion, we should deliberately obtain a non-optimal plan. In the second instance, the use of these *a priori* valuations leads accidentally to a correct conclusion (the same as with o.d. valuations).

In Example 3, comparing the aggregate production of factories A and B, the following actual and planned production is obtained respectively:

$$100,000 \times 20 + 200,000 \times 35 = 9,000,000 \text{ roubles, and} \\ 250,000 \times 35 = 8,750,000 \text{ roubles,}$$

therefore the proposed method results in the lowering of the output and should be rejected. The conclusion is certainly incorrect as clearly shown in Table 12, as the use of co-operation gives an increase in the volume of the programme by 4 per cent. The incorrect guidance resulting from *a priori* valuations is due to the fact that, in distinction to the o.d. valuations, they are not concrete, they do not take account of all the circumstances (for instance, a sharp rise in the demand for a given article).†

Even a comparison of the cost of production at a given factory with its average values does not, in many instances, make it possible to arrive at a correct conclusion on similar problems.

CONCLUSION 10. O.d. valuations make it possible to compare two methods of organizing manufacture of different products, and in particular to solve the problem as to whether some newly proposed production methods will result, under the given conditions, in increased output.

For the purpose of such comparison it is necessary to use the o.d. valuations in order to calculate the resultant (*net*) production in both methods of production, and to choose the method for which the total valuation of output is higher (principle of profitability). The use of *a priori* valuations (or of prices) in the solution of such problems, as distinct from the objectively determined valuations, may lead to a wrong decision.

Of course, this conclusion should not be understood to mean that the existing methods of economic analysis must not be applied in such problems, but only that the method of o.d. valuations enables one to reach a closer approximation to an optimal solution under given conditions. The usual methods furnish results which are the better the more closely the valuations used in them (price, cost) approach

† The dependence of the o.d. valuations upon production requirements, reflected in the requisite allocation, derives from the influence of the allocation conditions on the objectively determined allocation of costs to articles the production of which is interdependent.

o.d. valuations. However, in so far as in the application of o.d. valuations only their relative values are of importance (the result of comparison will not differ if they are all changed in proportion), then *a priori* valuations may lead to correct results even if they do not coincide with the o.d. valuations, but are only proportional to them.

Let us note further that with the aid of o.d. valuations the problem of a reallocation of the task may be solved. Let us assume that in each of two different groups of factories the task is allocated in the best possible manner to individual factories. Then, if the ratios of o.d. valuations for both groups should differ, such as ratios of 1:4 and 1:3, this shows that it would be advantageous to carry out a reallocation of the task, that is, to transfer part of the output target of article No. 2 from the first group to the second, and of article No. 1 in the reverse direction. As a result of this, the total output at both groups will increase for each article.

### *More Complex Cases*

The example considered was particularly easy to solve as the problem was to allocate the programme for two articles only. It should be pointed out that with the more complicated task of allocating a programme for several articles, the method in which we first found the optimal plan is not applicable.

However, all the conclusions concerning o.d. valuations, and also other methods of arriving at an optimal plan based on them, retain their full validity with not only two but more articles as well.

A suitable example based on several types of production is given in the following section, in which the problem is essentially the same although formulated somewhat differently.

A systematic exposition of the computational methods of arriving at an optimal plan is given in Appendix II.

## Section 2. Allocation and Choice of Means for Work Performance

### *Statement of the Problem*

Let us now consider the question of constructing an optimal plan of allocating means for performing a total amount of work by using the same methods. In order to show the essence of the problem we consider it once again schematically.



Let us assume that it is necessary to complete some total quantity of work simultaneously (agricultural, excavation, work connected with transport). These projects may be split into several types according to their character and conditions (agricultural work into tillage, harrowing, sowing, harvesting the crop; excavation into planning of the plot, sinking of pits, ditches, culverts; transport into transportation of various kinds of load over various distances, etc.). Various means can be used in carrying out these operations (several types of tractors, combines, harvesting machines for agricultural work; excavators, graders, scrapers, spades for excavation; motor lorries, tipping lorries, conveyors, narrow-gauge railways, wheelbarrows for transport).

The majority of the means of production and transport may be used for several types of work and there exist standard indices of their efficiency in various operations. For each type of work one of the means will be the most effective (resulting in the lowest cost and a correspondingly high productivity). However, such means are not always available in the necessary quantities. In many cases, when one wishes to accelerate the work but is restricted to a certain set of machines, it becomes necessary to utilize fully all available means even if they are hardly suitable. Yet one must ensure that the given total amount of work is completed in the shortest possible time. So one has to abandon the idea of using each means exclusively on operations to which it is best suited.

We shall also deal with the question of allocation of means under prescribed conditions. Let us consider a practical numerical example of several types of operations and means (machines).

**EXAMPLE.** The daily standard output of each machine (means of production) on each type of operation for which it can be used is shown in Table 13. This gives the volume of operations (expressed in units of measure corresponding to the given type of work: hectare, cubic metre, ton-kilometres).

It is necessary to indicate the optimal allocation of means, i.e. an apportioning which would enable one to complete the given amount of work *within the shortest time*.

This example does not essentially differ from the example of Section 1, as the individual machines here take the place of factories, and the volume of work for each type corresponds to the number of articles.

TABLE 13. STANDARD OUTPUT

Machine	Operation	Type	I	II	III
		Volume	5000	10,000	10,000
Type	Number	Standard daily output			
A	20	4	10	11	
B	50	0.4	—	10	
C	30	—	4	6	
D	100	0.4	2.5	2.5	

SOLUTION. When the number of operations exceeds two, the first method cannot be used to establish an optimal plan, and it is necessary to apply the method based upon finding o.d. valuations. Let us use the second method (p. 17 and further). In the first place, it is necessary to determine these valuations if only roughly. For this purpose, we calculate the total daily output of all the machines on each type of operation. We obtain:

For type I  $20 \times 4 + 50 \times 0.4 + 100 \times 0.4 = 140$

For type II  $20 \times 10 + 30 \times 4 + 100 \times 2.5 = 570$

For type III  $20 \times 11 + 50 \times 10 + 30 \times 6 + 100 \times 2.5 = 1150.$

Since labour inputs in the operations are inversely proportional to the output and since the aggregate outputs are in an approximate relationship of 1:4:8, it is natural to adopt as rough approximations in the corresponding valuations of labour input the inverse ratios, i.e.  $1:\frac{1}{4}:\frac{1}{8}$  or 8:2:1.

Using these conventional valuations, we can calculate the daily production of each machine for each type of operation (Table 14).

TABLE 14. DAILY PRODUCTION OF MACHINES ON EACH OPERATION (ACCORDING TO VALUATIONS 8:2:1)

Machines	Operation		
	I	II	III
A	32	20	11
B	3.2	—	10
C	—	8	6
D	3.2	5	2.5

TABLE 15. DAILY PRODUCTION OF MACHINES ON EACH OPERATION (ON THE BASIS OF VALUATIONS 25:4:1)

Machines	Operation		
	I	II	III
A	100	40	11
B	10	—	10
C	—	16	6
D	10	10	2.5

Let us also calculate the aggregate volume of operations and the total capacity of the machines according to these valuations.

For the aggregate volume of operations we obtain

$$5000 \times 8 + 10,000 \times 2 + 10,000 \times 1 = 70,000 \text{ conventional units.}$$

In so far as the daily output of the machines on the various operations differs, we calculate it roughly, assuming that each machine is used in the best possible manner, i.e. the highest number is chosen in each series. Then, for the aggregate daily output we find

$$20 \times 32 + 50 \times 10 + 30 \times 8 + 100 \times 5 = 1880.$$

From this we may arrive tentatively at the time (an underestimate compared with time actually required) for the accomplishment of the total of operations as  $70,000/1880 = 37$  days. We now determine the means for the completion of each operation. For operation I we shall use mostly machine A because this operation is the most profitable for machine A (see Table 14; maximum outputs according to the given valuations are in bold type). However, the total output of these machines for a given operation in the course of 37 days will amount to only  $20 \times 37 \times 4 = 2960$  as against the necessary 5000. Consequently, it is necessary to put into operation I an additional machine, namely machine D provided that the output for operation I is near the maximum (3.2 as against 5). The production from machines of type D on operation I will amount to:

$$100 \times 37 \times 0.4 = 1480.$$

But this, too, is insufficient. It will, therefore, be necessary to utilize on the given operation some machines of type B as well. For operation II the most suitable are machines C and D of which there seems to be a sufficient number; and for operation III the B machines of which there also is a surplus. In Table 14 in each line the bold figures show which of the machines we intend to utilize in the optimal plan on the corresponding type of operations. From this it is easy to determine the o.d. valuations. Let us adopt, as we did before, for type I operations the valuation  $m = 8$ . As we intended to utilize machine D both for operations of type I and II, the valuation  $n$  of operation II should be such that an equal valuation of production is obtained (equal profitability) for both operations, i.e. (see Table 13)

we should have  $0.4 m = 2.5 n$ ;  $0.4 \times 8 = 2.5 n$ , or  $n = 1.28$ . Further, if it is intended to use machine B for operations I and III, the  $r$  valuations for operation III will follow from the condition  $0.4 \times 8 = 10 \times r$ ; from which  $r = 0.32$ .

We thus obtain for the valuations of operations the relationship  $8:1.28:0.32$  or  $25:4:1$ . We repeat the same calculations as above, starting from these new valuations. The valuation of production (with new conventional units) is given in Table 15.

$$5000 \times 25 + 10,000 \times 4 + 10,000 \times 1 = 175,000 \text{ conv. units.}$$

For the daily output of all the machines:

$$20 \times 100 + 50 \times 10 + 30 \times 16 + 100 \times 10 = 3980 \text{ conv. units.}$$

Hence the time necessary for the completion of the operations is determined as  $175,000/3980 = 44$  days.

Let us verify if in fact all the operations could be completed within that number of days. It is more convenient to start from operation III. It should be completed by machines of type B for which 1000 machine-days will be required or  $1000/44 = 23$  machines. The remaining machines must be used in operation I.

On operation II, machines C will produce  $30 \times 44 \times 4 = 5280$  units in 44 days. The remaining 4720 units must be provided by machines D which requires  $4720/44 \times 2.5 = 43$  machines. The remaining 57 machines of type D must be used on operation I. Then, for operation I, 20 A-machines, 27 B-machines and 57 D-machines will produce

$$44 \times (20 \times 4 + 27 \times 0.4 + 57 \times 0.4) = 4998,$$

i.e. indeed in the course of 44 days the whole set of operations will be completed. The corresponding plan is shown in Table 16. That it is optimal and the valuations objectively determined can be easily proved by verifying the following two circumstances: first of all, the completion of the ratio required by the plan. This is directly evident from Table 16.

Secondly, it is apparent from Table 15 that in the given plan the machines are utilized in the most profitable manner (they furnish the maximum output if the operations are valued in the proportions

25:4:1). These two circumstances, as already known (Conclusion 4), ensure that the plan is optimal and the valuations objectively determined.

TABLE 16. OPTIMAL PLAN

Machines	Operations				Total number of machines
	I	II	III		
A Number	20	—	—		20
Output	3520	—	—		
B Number	27	—	23		50
Output	475	—	10,120		
C Number	—	30	—		30
Output	—	5280	—		
D Number	57	43	—		100
Output	1003	4720	—		
Total output	4998	10,000	10,120		

#### *Valuations of Machine Production and Their Application*

In this example, as in the previous one, the features of the o.d. valuations could be demonstrated again (Conclusions 6, 7, 8) and the use of the o.d. valuations for the solution of various problems connected with changes in the plan could be quoted as analogous examples to the preceding ones. Let us quote only one example to explain some new aspect of the problem. Let us assume that it is necessary, in addition to the operations shown in the plan, to complete an operation of a new type IV to the amount of 2500 units and that this operation can only be carried out on machine C (with a norm of 8 units per day) or by machine D (with a norm of 3 units per day). Which of the machines should be used? As the volume of operations is small in comparison with the whole plan, it is possible (because of the stability of the valuations) to use in the solution the values found above. From Table 15, we see that the valuation (of daily production) for machine C equals 16 and as for the completion of a unit of operation IV with machine C  $\frac{1}{8}$  of a day is necessary, unit costs will amount to  $16 \times \frac{1}{8} = 2$  conv. units. From the valuation of 10 for machine D, it follows that when the latter is used on operation IV ( $\frac{1}{3}$  of a day per unit), costs amount to  $10/3 = 3.3$  conv. units. So we see it is best to carry out operation IV with machine C. From this it

is also apparent that the valuation of one unit of operation IV will equal 2. To what extent will the time for the completion of operations be increased by adding operation IV? The valuation of the whole volume of operations IV will amount to  $2500 \times 2 = 5000$  conv. units. As the daily total production of the machines constitutes 3980 conv. units, it will be necessary to spend a further  $5000/3980 = 1.26$  days, therefore, together with the previous operations, 45.26 days.

Let us note that in solving the given problem, in addition to obtaining unit valuations of each type of operation, valuations were also produced for each machine. And this is the daily production of a given machine expressed in conventional units, we find on those operations for which it is used in the optimal plan in our example (Table 15), 100 for A, 10 for B, 16 for C, and 10 for D. These valuations, as may be seen, were helpful in solving the problem just considered: the choice of a suitable machine for operation IV. We made the choice following the principle of profitability, viz. on the basis of these valuations. This may be formulated as follows:

**CONCLUSION 11.** In the allocation of operations, each machine has an o.d. valuation (or more correctly its productivity) as well as a unit for each type of operation. This o.d. valuation equals the daily production (in conv. units) of a given machine in those operations on which it is used in an optimal plan.

With the aid of these valuations the problem of suitable means for a given operation can be solved. One must be guided by the principle of least costs and the greatest profitability: costs must be evaluated from the machine time necessary for the completion of a unit operation and from the machine o.d. valuations, and of all the possible means the cheapest must be chosen.

Using valuations of production and of the productivity of the machines simultaneously, an optimal plan together with the methods used in it may also be shown differently. If we carried out for a given production method (the completion of a fixed operation by means of a given machine) a comparison of aggregate valuations of daily production with cost valuations (machine-time spent according to o.d. valuations), it would become evident that these valuations agree with the methods used in the optimal plan. For instance, for machine C on type II operation

$$4 \times 4 = 16 \times 1$$

while conversely, for the methods not used, the valuation of production is lower (more exactly  $\leq$ ); for instance, for operation II on machine A

$$4 \times 10 < 100 \times 1$$

In other words, in the optimal plan methods used are justified (no loss); the production justifies the costs incurred: conversely, the methods not used in the optimal plan are, as a rule, not justified: in every case, the valuation of production for these is lower than or equal to the valuation of costs incurred (all on the basis of o.d. valuations).

This situation has an understandable economic meaning: o.d. valuations determine the necessary expenditure on production, under given conditions, and the optimal plan may therefore be confined to those methods in which expenditure per unit of production is shown to be equal to necessary expenditure.

Let us analyse in detail an application of the o.d. valuations in a second problem which was not touched upon in the first example.

#### *Allocation of Means of Production to Production Units*

Let us assume that the type of operations referred to are carried out at three production units. The means of production are allocated to these units and the plan of operation for each given. The corre-

TABLE 17 ALLOCATION OF MEANS OF PRODUCTION BY PRODUCTION UNITS

Production unit	Means	Programme	Allocation of means by operation (in machine-days)	Time (in days)
1st	A—10 B—50 C—10	I—3000 II—4000	450(A) + 3050(B) 160(A) + 610(C)	61
2nd	D—100	I—2000 III—2000	5000(D) 800(D)	58
3rd	A—10 C—20	II—6000 III—8000	600(A) 30(A) + 1260(C)	63

sponding data are shown in Table 17. In the last column but one the operation of each type of machine in machine-days used for the given operation is shown, in the last column the time taken for the completion of the operation at a given production unit.

It is easy to verify that at each production unit available means are utilized in the best possible manner and the plan is completed in the shortest possible time. The allocation of operations in the first two sections is only practicable when the available machines are fully utilized.

Considering the totality of operations and means at the three production units, we see that of type I operation there are altogether  $3000 + 2000 = 5000$  units, of type II—10,000 units, of type III—10,000 units; there are  $10 + 10 = 20$  machines of type A, 50 machines of type B, 30 machines of type C, 100 machines of type D; indeed, just the target and means as in the problem considered above.

TABLE 18. REALLOCATION OF MEANS OF PRODUCTION BY PRODUCTION UNITS SO AS TO SHORTEN THE TIME NECESSARY FOR THE COMPLETION OF OPERATIONS

Production unit	Means	Pro-gramme	Allocation of means by operation (in machine-days)	Time (in days)
1st	A—17 C—23	I—3000 II—4000	748(A) 1012(C)	44
2nd	A—3 B—32 D—57	I—2000 III—2000	132(A), 1188(B) 2508(D) 220(B)	44
3rd	B—18 C—7 D—43	II—6000 III—8000	308(C), 1829(D) 792(D)	44

Why does the task now require an average of 61 days while before 44 days appeared sufficient? The means are improperly allocated among the production units. In order to complete the operation in 44 days they would have to be allocated differently, for example, at the first production unit 17 machines of type A and 23 machines of type C. Such an allocation is shown in Table 18. In this example the machines will be operating in accordance with the use adopted for the optimal plan of Table 16. In other cases it may prove more convenient to carry out a reallocation of the plan by production units or a partial reallocation of means of production and a partial reallocation of the plan.

Given some plan of allocation of means and operations by production units, it is clear at once whether such a plan is rational or irrational if we try to establish a system of o.d. valuations for the



various types of operations (or machines) in it. Forming the ratios of the valuations of operations (assuming that machines A and also machines D are utilized on operations of type I and III) under the plan given in Table 17, inconsistent ratios are obtained:

$$4 m = 11 n \quad 0.4 m = 2.5 n$$

Consequently, this plan is not rational (see Conclusion 5). On the other hand, the system of valuations for the plan of Table 18 conforms to the principle of profitability ( $m=25$ ;  $n=4$ ;  $r=1$ ) and thus confirms its rationality.

It is important to note that such improper allocation could pass unnoticed if no special analysis was carried out as to what constitutes the optimum of an aggregate plan since at first sight everything would appear to proceed properly: the machines used to full capacity, the targets fulfilled. The plan is not rational because one machine of high quality is used for comparatively ordinary operations, and a complex and labour consuming operation is carried out by hand or by simpler machines. But this may remain unnoticed for at the first production unit a complex machine might be used profitably for light operations (on the basis of the adopted calculations) and as the machine is available, it is justified to use it for this operation; at the second production unit the hand operation, or by a simpler machine, is also justified since the appropriate machine is not available at this unit. Such incorrect allocation of means is usually accompanied by unavoidable delay.

Losses of this nature from an inferior allocation of means or operations are unfortunately frequent in our country.

Thus, in the past, during the spring sowing period in one district tractors were sometimes used even for light operations (harrowing) if they were available in sufficient numbers and supplied with fuel; while in an adjacent district even ploughing had to be done with horses.

Excavators may frequently be used for the digging of small pits and other shallow work requiring frequent movement. This explains that its load coefficient is low. At other places where this excavator could be working at full load, important excavations are carried out by hand.

Similar situations also frequently arise in transport and other activities.

The schematic solution of the problem considered above was worked out on the basis of consolidated indices, and ignored a whole

series of specific features; it does not, of course, fully exhaust the question of the efficient organization of operations and the choice of means.

A practical detailed plan of operations must consider all the conditions accompanying each operation, the peculiarities of the operation of individual machines, costs of each type of operation and so forth. However, the preliminary schematic plan on a basis of consolidated data and the derived values of o.d. valuations can help substantially in the working out of such a concrete plan and provide a fundamental guide as to what means should be utilized for given types of operations, and how to allocate means to consolidated sections by considering the volume and type of operations. Thus, as a result of the construction of such an aggregate plan it becomes possible to analyse each concrete problem (such as the choice of means for a given operation) and to evaluate, even though only roughly but objectively and quantitatively the whole concrete situation (the prevalent types of operations in other places, the limited supply of means of one type or another). This is bound to furnish considerably better results than the usual way of solving problems in isolation and then later some mechanical linking of solutions. The use of the proposed system of allocation of means may prevent the considerable losses which occur at present.†

At each production unit taken individually, the conventional methods of economic analysis combined with experience use, in many instances, available resources efficiently enough to produce a plan that approximates optimal conditions without recourse to special calculating methods of finding an optimal plan and o.d. valuations. However, a simultaneous assessment of conditions and resources at a whole series of production units proves difficult to accomplish with conventional methods, and the possibilities of increasing efficiency as a result of reallocation remain often unrealized. Problems of such joint utilization of productive capacity and reallocation of the programme (specialization, co-operation) in the regions could be easier achieved with the creation of *sovnarkhozy* (economic councils). Doubtless, the use of methods of optimal planning should prove highly fruitful in the solution of these problems.

† We return to the analysis of this problem below (Chapter II, Section 5).

## CHAPTER II

# MAXIMUM FULFILMENT OF THE PROGRAMME WITH AVAILABLE RESOURCES. VALUATIONS OF FACTORS OF PRODUCTION

### Section 1. General Propositions

#### *Statement of the Problem*

Only in comparatively few cases do we find the conditions outlined in Chapter I, namely: all costs are divided into (1) fixed expenses, independent of the chosen method and type of production being carried out at a given factory or farm (in the examples—expenditure on labour of the factory and on machines), and (2) proportional-variable costs, representing a fixed value per unit of each type of output whatever method of production is chosen (in the examples—expenditure on materials is the same no matter at which factory a given type of article is produced).

Usually, a change in the method of manufacture of a given output entails a substantial change in costs both quantitatively and in composition: some items of expenditure increase, others become smaller. In such cases, the method of solving the problem quoted in Chapter I, based only on ascertaining objectively determined valuations of production without analysing individual costs, seems insufficient. In questions of this kind it becomes necessary to consider fully all basic factors of production.

The problem is formulated in the following manner. A given production unit of the national economy (a factory, sector, an economic district) has available for the planning period certain resources of basic factors of production (labour force, equipment, productive capacity, raw materials, other materials, electric power and natural

sources of energy). For certain types of input, instead of limiting their resources there may operate norms limiting their cost per unit output. A production unit is set a fixed task in terms of output of a given assortment, or a task of fulfilling some plan of different kinds of work, giving at the same time the order of priorities for their completion (if the plan is not fulfilled or exceeded). The given plan may be allocated in various ways among the productive units. For the same types of production various methods of manufacture may be applied (techniques, organization of production) using different kinds of resources.

The problem consists in the construction of an optimal plan—in such a work allocation and a choice of manufacturing methods that within the fixed time (and with the available resources) the highest possible fulfilment or the maximum over-fulfilment of the plan may be attained.

We encounter problems of this type in instances such as these.

If a factory with a fixed quantity of means of production is set a plan, its problem becomes the optimal over-fulfilment of the plan with the available means by organizing production in the best possible manner and by choosing the best possible technological processes. In particular, this type of problem arises when ensuring the fulfilment of commitments taken on in emulation. Similar problems are met in planning the work for groups of factories.

A like problem arises in carrying out a construction plan when as much of such a plan as possible is to be fulfilled by the most efficient allocation and use of available resources (the labour force, building materials, stocks of machines and fleets of vehicles).

For short-term planning, in so far as one has to start from existing resources, costs of production must be determined not by potentially better methods but by methods which can be used in the given situation and under actual conditions. In choosing likely technological processes and methods of organization of production it is necessary to take account of limiting conditions—such as the equipment available or output capacity. Here an analysis of factors used in production or which influence the latter plays an important role.

In this section, rather than following the method adopted in Chapter I, we shall find it more convenient to start by stating the basic conclusions to which the analysis of a given problem leads; we

shall then illustrate such an analysis on practical examples and explain at the same time the particular role of individual factors.

### *Classification of Factors of Production*

The allocation of the production plan and the choice of methods of production substantially influence the volume of the task that can be completed. Of all the methods of organizing production there is always one optimal method (plan) which ensures the highest *over-fulfilment of the production task*. To arrive at such an optimal plan it is of primary importance to have a correct allocation of the plan and of available resources.

These resources may usually be reduced to four groups according to their use.

I. *Factors used in fixed proportions* per unit of each type of output in an amount independent of the choice of methods of production and of the total output. For instance, in the motor car industry, for each motor vehicle a fixed number of tyres are used which come from another factory. It is presumed that the supply of such factors is adequate for the volume of output that could be achieved.

II. *Factors used in fixed amounts* include factors (types of input) used to the same amount independently of the planned volume of output and of the methods of production selected (for instance, management expenses, safety precautions, lighting and heating).

III. *Non-limiting (surplus) factors* which under given conditions remain in surplus whatever the choice of methods of production (for instance, some types of equipment, water), and the surplus of which cannot be utilized.

IV. *Substantially variable factors* which are available in limited amounts, of which the cost per unit production depends upon the method of production chosen. It is, however, presumed that this cost is independent of the volume of output produced.†

The problem of the best utilization of the factors of the last category must now be considered.

The most important among the factors of this group are: the

† The latter condition for factors of group IV is somewhat restrictive and not always sufficiently closely satisfied; but as far as known, it is indispensable for the present analysis.

labour force (of different grades), productive capacity of individual types of equipment, electric power, fuel, certain materials, production space and, in many instances as well, land, water and other natural wealth.

The number of consumed factors is quite large, but in the analysis of their costs it is usually possible to reduce this number by consolidating a whole series of costs into one index (for instance, the operating time of a machine and all the expenditure connected with it: labour time for operating and servicing the machine, time the machine tool is in use, lubrication, maintenance and other servicing, electrical energy).

We speak of the utilization of a given factor irrespective of whether it is actually consumed in the manufacture (materials), or merely occupied or hired for a given period (equipment, production space, land in agriculture).

Among all the productive factors labour occupies quite a special place since it is the only source of value produced. And besides direct expenses of live labour we must also take into account the expenditure of stored-up labour represented by such factors as materials, services, wear and tear of equipment. Moreover, in so far as the productive ability of live labour depends upon the conditions of its application, in particular upon the associated equipment, and in so far as productive capacity and space at our disposal are limited, one cannot omit from the valuation the use of such equipment in production as a particular type of cost, for the absence of such a valuation would render the plan unrealistic.

In analysing the problem of constructing an optimal plan, it is apparent that factors used in fixed proportions may be excluded (since their cost for a given plan does not in fact depend upon the choice of a particular plan),<sup>†</sup> as well as constant factors, and finally those which are not limiting in the sense that a reduction in their use is not essential. Thus, only the analysis of the substantially variable factors is of importance.

In a schematized form the task of constructing an optimal plan covering a certain period (the task of short-term planning) may be formulated in the following manner.

<sup>†</sup> This means, in essence, that in production one considers not the article in itself but its manufacture on the basis of expenditure of raw materials, semi-manufactures, etc., according to fixed norms.

Resources are available for the planning period, that is, various types of factors of production exist or become available in given amounts. It is necessary to produce certain types of goods in fixed proportions (assortment). For each type of product there are one or more technically feasible methods of manufacture, the structure of inputs of each of which is given—it is known in what quantities factors of production used in a given method are required (consumed or utilized) per unit output.

An optimal plan has to be constructed, therefore it is necessary to show what quantities of each type need be produced by each method, so that the aggregate requirement of factors of production does not exceed the given resources, and such that production is carried out in the specified quotas and in the highest possible volume. This task, representing an idealized and simplified model of the real task of short-term production planning, is considered further and the conclusions reached below relate directly to it. In the real task of economic planning this model represents only a certain approximation.† For this reason, the conclusions reached may also be applied only approximately. However, as usually happens in scientific analysis, these conclusions are of substantial importance both for the quantitative and the essentially qualitative analysis of the real planning tasks.

### *Objectively Determined Valuations of Factors of Production*

The question of the utilization of resources which are available in limited amounts, the cost of which is dependent upon the choice of production has been discussed already in Chapter I, Section 2. Let us turn back to the result obtained for this problem. We constructed an optimal plan and o.d. valuations for each type of operation. The feature of the optimal plan here was that each machine was used at the highest degree of efficiency (the valuation of output was the highest). In addition to this we obtained valuations of operations per day (daily productivity) for each machine or of the cost of factors of production (Conclusion 11). Here it became evident that the optimal plan may be characterized with the aid of these

† For instance, it is only possible to calculate approximately the increase in those costs (substantially variable factors) in production with a given method which are proportional to the volume of output (even if fixed costs are excluded)

valuations as satisfying the principle of profitability as in each type of operation those machines are used which are the cheapest, if the daily cost of operation of each machine is accounted for on the basis of the valuations mentioned.

As we shall see below, this proposition holds also for other types of factors of production, i.e. each factor has its valuation, and the optimal plan is constructed on the principle of minimum costs. For these valuations the propositions are similar to those of the valuations of production. We shall state these propositions in the form of the following conclusions.

**CONCLUSION 12.** A definite system of o.d. valuations of variable types of costs exists and is related to the optimal plan in such a manner that by using them the plan satisfies the principle of profitability (minimum costs). In carrying out each type of production (or operation) those methods are utilized for which the total expenditure of factors of production (based on the o.d. valuations) is smallest.

Conversely, if with a certain plan of construction (1) productive factors of group IV are fully used, (2) the required quota of each type of production is obtained, and (3) a system of valuations of basically variable factors exists such that for a given plan the principle of profitability is observed, then such a plan is optimal and the valuations are objectively determined.

Thus, as in the previous statement (and with the aid of similar methods), o.d. valuations may be used both to verify that a given plan is optimal and to find such an optimal plan.

Further, in the o.d. valuations of factors of production those features are preserved which were given above for o.d. valuations of production in Conclusions 6, 7, and 8.

**CONCLUSION 13.** Objectively determined valuations of factors of production are concrete; they are determined by allowing for the totality of cost conditions (resources of these factors, production programme, methods of production). In particular, a reduction in the resources of a given factor or an increase in the rate of production in which the latter is consumed is normally related to an increase in the o.d. valuation of such a factor. For slight changes in the conditions, o.d. valuations usually do not change or change only slightly. The ratio of the o.d. valuations of factors of production may be realized by using a certain amount of one of these factors. It is



possible to achieve a corresponding reduction in the quantity of another factor.

Objectively determined valuations of factors of production may be used in solving various problems arising from the plan. Examples will be given below where more practical problems are considered. The economic meaning of o.d. valuations of individual factors of production will then be analysed in more detail.

In a whole series of cases of planning one finds requirements other than those of achieving a maximum output of a required composition from given resources. For instance, we may be interested (for a given volume of output) in the maximum reduction of cost per unit output (where relative costs are given), in the maximum reduction of one type of cost (for instance, labour) while observing the limits set for the remaining types. It is nearer reality when only some main types of production and expenses may be planned in natural units and the remainder accounted for in a summary form at cost. Some of these tasks may be similar to those already considered (for instance, the first). However, it is essential that the analytical methods as well as the final conclusions regarding the existence of o.d. valuations characterizing an optimal plan apply to these tasks as well.

Let us consider yet another problem. We saw in the example of Chapter I, Section 2, that the valuations of factors of production were calculated from valuations of production. On the other hand, if o.d. valuations of factors of production are known, it is possible to determine with their aid production valuations by calculating the total cost per unit output from the method used in the optimal plan.

Thus, o.d. valuations are determined both for factors of production and for outputs.

However, as it was shown (Conclusions 2 and 12), one of these two systems of valuations often proves sufficient to characterize an optimal plan. In a more complex case, when several types of output are obtained simultaneously, and where it is not possible to show directly which of the costs should be related to which product, a single system of valuations is insufficient, and to attain the optimal plan both systems have to be used. The optimal plan in such a case is characterized in the following manner.

**CONCLUSION 14.** In a complex production plan, a fixed system of o.d. valuations of factors of production and of all types of production

is related to the optimal plan. Here the principle of profitability holds for the optimal plan, that is, for the methods of production used in the optimal plan, the valuation of total costs agrees with the valuation of total production,† while for those methods which are not used in the optimal plan, it is higher than or equal to the valuation of production.

Let us note that this general criterion of the optimal plan includes also the two special cases quoted when it is possible to limit the requirements to valuation of production, or of costs, alone. Its application to such a problem is illustrated by the example in Chapter I, Section 2 (p. 39).

## Section 2. Valuation of Factors of Production which Increase Labour Productivity

### *Statement of the Problem*

Let us consider the problem of the use of resources when there are only two variable limiting factors: labour, and some other factor which increases labour productivity. The amount of the latter factor is limited and the problem is how best to use it. Fairly often, electric power or fuel, etc., may be such a factor. We analyse the problem in its pure form when the scarcity cannot be changed by man and the factor must be used economically. The only other variable factor is labour; we shall assume that further factors (such as equipment) are available in sufficient quantity. Such a problem is fairly frequent in practice. For this reason it is interesting to find a solution, but the problem is also important in the general train of our argument. We shall demonstrate this method by a concrete example.

EXAMPLE 1. Let us consider the question of drawing up a monthly plan for some section of a factory which has to perform consecutively a whole series of production tasks: I, II, III, and so on. The labour force (of one type only) and electric energy are the only

† It should be borne in mind that the extent of using the methods is unlimited, or at present, not exhausted. Should this not be so, corrections are introduced for methods fully utilized: in such methods the valuation of production is not less than total cost (*see* Appendix I, p. 275).

variable factors. The number of operating hours per month is 5000, the given limit for electric energy is 21,000 kWh.

Requirements of labour and electric energy necessary for the completion of each of the tasks is given in Table 19. Because of the shortage of electric power, in addition to the basic (power-intensive) variant A for the completion of the task, a variant B (labour-intensive) is available which economizes power by increasing the use of labour.

TABLE 19. REQUIREMENTS OF LABOUR AND ELECTRIC ENERGY

Task	Variant A		Variant B	
	Requirement of		Requirement of	
	Labour force (hr)	Electric energy (kWh)	Labour force (hr)	Electric energy (kWh)
I	300	4000	500	2000
II	500	9000	1000	3000
III	500	8000	800	5000
IV	1000	6000	2000	2000
V	1000	6000	1500	3000
VI	200	2000	500	1000
VII	100	1500	200	500

Thus, for instance, electric welding and electric cutting of metals may be replaced by riveting and mechanical cutting; certain types of operations may be transferred to machines consuming less power although they may be less efficient for hardening; electrical may be replaced by thermal hardening.

An optimal operating plan has to be drawn up for a given productive unit and production methods have to be chosen which ensure the maximum fulfilment of the plan (the highest number of tasks) while not exceeding the limits of the given resources. In essence, productive methods have to be chosen correctly in order to achieve the requisite economy in electric power.

In such instances, current prices and tariffs frequently do not furnish the means of obtaining an optimal solution of the problem. For instance, let us assume that these prices constitute 2 roubles per hour of working time and 14 kopecks per kilowatt hour of electric

energy.† Then it will become evident that the high-energy variant A is cheaper for all tasks. Thus, for task I, we have

$$300 \times 2 + 4000 \times 0.14 = 1160 < 500 \times 2 + 2000 \times 0.14 = 1280, \text{ etc.}$$

Therefore, starting from the above prices it is difficult to determine how to achieve economy and the solution may prove to some extent arbitrary. For instance, for the first task the power-intensive variant will be chosen, ensuring the speedier completion of the operations

TABLE 20. ARBITRARY PLAN

Task	Chosen variant	Requirements	
		Labour force (hr)	Electric energy (kWh)
I	A	300	4000
II	A	500	9000
III	B	800	5000
IV	B	2000	2000
V	B	1500	3000
Total		5100	23,000

and for the subsequent ones the labour-intensive variant so as not to exceed the given limit of electric energy. Such a plan is shown in Table 20.

This plan ensures the fulfilment of five tasks only by slightly exceeding the operating time (overtime) and the limit for electric energy.

### *The Optimal Plan and O.D. Valuations*

Let us now take the problem of finding an optimal plan. For this purpose we shall use the first of the methods based on o.d. valuations. The valuation of the operating time will be expressed as one unit and we shall select an o.d. valuation for electric energy. For each task we

† As this and other illustrating examples are somewhat arbitrary and have for their purpose only the demonstration of the principles of the method, all prices, tariffs and other numerical data are taken approximately.

TABLE 21. EFFICIENCY OF SAVINGS

Task	Transfer from variant A to variant B		
	Savings of electric power (kWh)	Additional labour force (hr)	Hours of operating time (per kWh)
I	2000	200	0 10
II	6000	500	0 08
III	3000	300	0 10
IV	4000	1000	0 25
V	3000	500	0 17
VI	1000	300	0 30

shall show how many man hours are lost per kilowatt hour of electric energy saved in passing from method A to method B. The relevant calculation is given in Table 21. Using method A, say for the first six tasks, requires 35,000 kWh of electric energy. In accordance with the limit set (21,000) 14,000 kWh must be saved.

From Table 21 it is clear that a saving may be achieved with the least expenditure in labour by transferring to method B consecutively tasks II, I, III, and V. It is enough to convert these four tasks as the resulting saving will amount to

$$6000 + 2000 + 3000 + 3000 = 14,000 \text{ kWh.}$$

The plan thus obtained (Table 22) is optimal for this case. In this plan, as compared with the arbitrary one, the task is better fulfilled and there is no excess expenditure in electric energy or labour force.

TABLE 22 OPTIMAL PLAN

Task	Chosen variant	Expenditure	
		Labour force (hr)	Electric energy (kWh)
I	B	500	2000
II	B	1000	3000
III	B	800	5000
IV	A	1000	6000
V	B	1500	3000
VI	A	200	2000
Total		5000	21,000

It is also easy to understand that a plan arrived at on the basis of a maximum plan calls at the same time for the least labour outlays for the fulfilment of the projected number of tasks without exceeding the limit set for electric energy. At the same time, we have determined in essence the o.d. valuation  $m/\text{kWh}$  of electric energy (we shall assume the valuation of operating time  $n=1$ ).

To achieve a saving of one kilowatt hour of electric energy in task V we sacrificed 0.17 of a working hour, hence  $m > 0.17$ . On the other hand, we did not save energy when additional labour cost rose to 0.25 of a working hour (task IV), and for this reason  $m < 0.25$ . In the present example we do not obtain a unique value for  $m$  but only succeed in establishing that it is contained within the limits of 0.17 and 0.25. Any number within those limits could be adopted as the o.d. valuation per kilowatt hour. Let us assume that  $m=0.2$ . It is easy to verify that with a valuation productive methods as given in Table 22 are used with the minimum amount of expenditure. This again confirms that the given plan is optimal.

In this simple instance we could obtain an optimal plan by comparing directly expenditure on labour and electric energy for any possible choice of a production plan. There are 32 such plans for five and 64 for six tasks. But, of course, this method is impracticable in more complex cases in which millions of solutions have to be compared. The superiority of the methods based on o.d. valuations consists in making it possible to avoid direct comparisons of all the plans. It is also essential that they are equally successfully applicable when a whole series of factors is taken into account simultaneously.

A valuation of a given type of production or of services must correspond to the necessary expenditure (of labour in the final calculation) involved in its realization. What is the meaning of the o.d. valuation of electric energy as obtained here?

It should be pointed out that under short-term planning, when it becomes necessary to start from the available production base, it may not always be possible to produce additional output of a given type if manufacturing capacity is fully utilized. There is another practical way of achieving this, that is, by savings or by substituting for a given output some other one (or some other factor) in any of its uses. The expenses this may entail then determine the o.d. valuation; in the given instance, for example, the o.d. valuation is determined by the expenses necessary for the saving of electric energy on tasks IV

and  $V\ddagger$  (0.17—0.25). Nevertheless, as will be established below, this valuation must also agree with the total cost of electricity generation.

We could also illustrate on this example all the features of the o.d. valuations. Thus, the valuation of electric energy depends upon all the conditions of the task. It is not difficult to realize that if the limit for electric energy was, for instance, 26,000 instead of 21,000 kWh (in the given example this change is not trivial), then the o.d. valuation would equal 0.1 and not 0.2. Conversely, a lowering of the limit would increase its o.d. valuation (Conclusion 13).

### *Application of O.D. Valuations*

We shall show in this example the possible applications of o.d. valuations. Thus, if for some operation a new method is put forward, calling for other expenditure, it is easy to decide whether this is advisable. Supposing, for instance, that for task III method C is proposed which requires 1100 working hours and 1500 kWh of electric energy. Is its application advisable under the given conditions? Comparing cost on the basis of o.d. valuations, we realize that method C is cheaper than method B used in the plan (and even more so than method A). In fact:

$$1100 + 1500 \times 0.2 < 800 + 5000 \times 0.2 < 500 + 8000 \times 0.2$$

(1400)                      (1800)                      (2100)

And, in fact, the use of method C, as shown in the revised plan given in Table 23, makes it possible to add task VII to the other tasks. It is of interest to note that if *a priori* valuations were used (prices of 2 roubles and 14 kopecks) it would seem that method C entails higher expenditure than the two previous methods A and B:

$$1100 \times 2 + 1500 \times 0.14 > 800 \times 2 + 5000 \times 0.14 > 500 \times 2 + 8000 \times 0.14$$

(2410)                      (2300)                      (2120)

On this basis it should be rejected, which proves that in such problems valuations that do not take into consideration the actual conditions of the task are not admissible as they may lead to an incorrect result.

† Let us note that here we have two different values: 0.25 man-hour per kilowatt hour resulting from excess requirement of electric energy, and 0.17 man-hour per kilowatt hour increase arising from its actual saving. Such a case is not typical but may occur in real life (see Appendix I, p. 287).

For example, problems regarding measures aiming at economy and relating to changes in consumption of electric energy should be decided by means of the o.d. valuations of electric energy. Is it advisable, for instance, to stop a machine tool in order to overhaul its components even though this reduces the productivity of the worker?

TABLE 23. A PLAN USING METHOD C FOR TASK III

Task	Selected variant	Cost of	
		Labour (hr)	Electric energy (kWh)
I	B	500	2000
II	B	1000	3000
III	C	1100	1500
IV	A	1000	6000
V	A	1000	6000
VI	A	200	2000
VII	B	200	500
Total		5000	21,000

EXAMPLE 2. The introduction of an auxiliary worker leading to an increase in the productivity of an essential worker by 30 per cent with the same consumption of energy results in an increase in the cost of the given operation; however, if electric power is scarce this may prove advisable as it reduces expenditure on energy per unit operation. A correct solution of similar problems regarding the advisability of applying various methods of saving is only possible by reference to the o.d. valuations of electrical energy.

In particular, refraining from carrying out such measures because of the losses they may involve—increase in cost—can often not be justified since with a particularly tight energy balance this may lead a factory to a standstill owing to excess consumption of electrical energy and, consequently, to even higher losses.

CONCLUSION 15. Objectively determined valuations may be used in the comparison of methods of production and, in particular, when deciding whether a new method is advisable. For this purpose, the total expenditure (on the basis of the o.d. valuations) involved in the new method must be compared with a corresponding total in the methods previously used. If expenditure with the new method appears lower, its application is advisable, while in the opposite case



it is inadvisable. *A priori* valuations may lead to an incorrect solution of the problem.

Objectively determined valuations may also be used in the solution of various problems relating to more than one factory. Let us assume that particular valuations have been made for electric power for all the factories and that the o.d. valuation for factory N was 0.1, and 0.3 for factory K. As o.d. valuations are realistic, it means that by increasing the number of workers in factory N say, by 100 (20,000 working hours), 200,000 kWh of electric energy may be freed, while factory K with an additional 67,000 kWh of electric energy may save 20,000 working hours. In such a case, by moving 100 workers from factory K to N (for simplicity we shall assume that the workers in question are semi-skilled auxiliary labour and that therefore such transfer can be accomplished) and by passing 67,000 kWh from factory N to K, we shall be in a position where the two factories fulfil the previous plan while saving 133,000 kWh of electric energy. Thus, the o.d. valuations help in demonstrating the possibility of a better allocation of electric energy.

It should be mentioned that in spite of the unquestionable advantage of such a transfer, it may prove impossible to accomplish this with a low tariff for electric power (lower than the o.d. valuations for most factories). Thus, with a tariff of 14 kopecks/kWh and a wage rate of 2 roubles/hr, it will become apparent that factory N will not be interested in economies of electrical energy as this would entail an increase in costs (by economizing  $200,000 \times 0.14 = 28,000$  roubles in electrical energy, it loses  $2 \times 20,000 = 40,000$  roubles in payment of the labour force). Factory K, although interested in such a change, will not be in a position to fulfil it as it is not entitled to exceed the limit for electric power which it has been allowed. Therefore, such a reallocation of resources, irrespective of whether it is advantageous to the national economy as a whole, may pass unnoticed (if o.d. valuations are not taken into account) and will not be realized.

Let us consider another problem. Let us assume that the o.d. valuation of electrical energy for all the factories in a town is approximately 0.2. Some measure at the power station (for instance, grading of coal before feeding it into the combustion chamber of the boiler) helps to improve the operation of boilers and to produce more electric power with the same consumption of fuel. However,

the expenditure on labour for this work is considerable, and each additional kilowatt hour of electric energy will cost 24 kopecks (0.12 of a working hour at a rate of 2 roubles/hr). For the power station this measure is not advantageous as it increases the cost (it is assumed that the tariff is 14 kopecks/kWh). However, this measure is of course advantageous to the government as expenditure of labour incurred at the power station will be offset almost twice at those factories which receive additional electric energy (on the basis of the o.d. valuation of 1 kWh).

Let us now emphasize that while at the factories electric power represented a factor of production (a form of expenditure), at the power station it represented output. Nonetheless, in both cases it was correct to be guided by the same o.d. valuations.

The reasoning used in the discussion of these examples is of a general character and may therefore be taken as a basis for the following conclusion.

**CONCLUSION 16.** Objectively determined valuations may also be utilized in problems relating to several factories. The difference in the ratios of o.d. valuations of factors of production at some factories shows that it may be possible to reallocate such factors (assuming that this is permissible) in such a manner as to enable all the factories to increase their output. If some factors at one factory appear as productive outlays and at another factory as types of output and if the ratio of their o.d. valuation is different, then it is possible to introduce changes in the plan of both factories as a result of which the total output of both will increase. Therefore, in the general optimal plan (provided that the transfer of resources and the reallocation of the plans are feasible) there should be a common ratio for the valuation of various factors.

It should be mentioned that in the process of planning and operational control mistakes which are likely to arise in the usual calculation of profitability are rectified by allowing for actual conditions. Sometimes, by such accounting, even the sale prices are changed. Thus, the district tariff for electrical energy is fixed, to some extent, in relation to the tightness of the energy balance. If electrical energy is extremely scarce various measures are taken to reduce its consumption and increase its generation. However, the advisability of such measures is ascertained only qualitatively; they are not

adopted sufficiently systematically, and often only in cases of extreme necessity. *The application of o.d. valuations and of the conclusions derived from them would make possible an objective quantitative approach to a more precise solution of these problems and to ensure a better solution, thus preventing avoidable losses.*

The important fact is that as expressed by the tariff, electricity in territories with a tight energy balance is usually undervalued and the tariff is considerably lower than its o.d. valuation; it does not reflect the actual relationship between the requirements of electricity and the amount generated, and its real national economic cost. This makes all economic accounting involving the use of electric energy unrealistic in such conditions. For this reason, many measures calling for an increase in consumption of electric energy and clearly inadvisable with the available supply, produce, according to the calculations, a reduction in cost and seem advantageous. Conversely, other measures resulting in an economy in electric energy do not appear advantageous on the basis of such calculations or lead to quite insignificant reductions in cost (for in the majority of types of production, electric energy constitutes a negligible proportion of costs, of the order of 1-2 per cent).

Such a discrepancy between the tariff of electric energy and its real value is generally characteristic of the prices and tariffs of the majority of factors, the production involving the use of large-scale equipment (see Section 5, pp. 75-92); it is also partially due to the failure of accounting for the temporary strain on the supply of electricity at a given time or place.

Correct economic analysis and accounting may lead to contradictory conclusions for the same problem depending upon actual circumstances. Thus, in the given problem the tightness of the balance of energy is determined to a large extent by the shortage of fuel or shortage of generating capacity at the power station (in the calculation this will show for instance in the ratios of o.d. valuations of electric energy and fuel). During the war the electricity balance was often restricted more by lack of capacity at the power stations than by the shortage of fuel. Under such conditions, it was advisable from a national point of view to switch the supply of electrical energy at a factory from the main grid to its own generating unit, although this might have entailed an increase in cost and even an increase in consumption of fuel. On the other hand, if there had been spare

capacity and generating of electric energy had been limited by available fuel resources, this would have been inadvisable.

In this connection it is appropriate to note that economic accounting, if it is complete and scientifically established and allows for actual conditions, is more likely to lead to a correct conclusion than reliance on one or other partial index. One of the basic indices of the operation of a power station is electricity consumption by the station itself. In order to improve this index, power stations frequently use steam instead of electric energy. This leads to an increase in the consumption of fuel and is economically inadvisable.†

Using objectively determined valuations of electric power (and bringing tariffs into close approximation) is of great importance in other respects. Such a precise quantitative index of tightness in the energy balance would be a much more correct guide for the need and urgency of measures for expanding generating capacity in a given district or for connecting the given district with another network, rather than qualitative characteristics. Such an index would at the same time indicate those factories consuming electricity which it would be of advantage to contract or not to expand.

The problem of the economy of fuel and of its best utilization is similar to the problem of electric power. Here, analogous calculations and examples could be quoted and similar practical conclusions drawn, although the sharp rise in oil output has made this question less topical. When considering various measures relating to changes in the consumption of fuel in short supply, it is necessary to bear in mind that its o.d. valuation is also apparently higher (if only in relation to labour) than the ratio of money values. For this reason, calculations based on selling prices of fuel may lead to incorrect results.

The adoption of o.d. valuations for the calculation required in solving problems of allocation of fuel would ensure a more rational use. In such a case the calculation would in many cases reveal the advantage of fuel economy where this is not apparent. At the same time, it would make possible the supply of fuel to a whole series of operations where shortage of fuel leads to losses.

A correct valuation may be used in the solution of problems connected with oil output and gas production (*see* Conclusion 16). It would show the advisability of taking certain measures aimed at increasing output, even if the usual calculations show a rise in cost

† *See the Promyshlenno-ekonomicheskaya gazeta*, 1 September 1957.

(the selling price appears to reflect more correctly the importance of fuel from a national economic point of view). In individual cases such measures were actually taken. Thus, in 1936, in accordance with the instructions of the Central Committee of the Communist Party, oil factories were shown the necessity of exploiting wells of low output in spite of their apparent unprofitability. Correct accounting based on the o.d. valuation of fuel would confirm the unqualified truth of this conclusion. The present drive to reduce cost or to increase productivity per worker to the detriment of the volume of output is often a mistake.

### Section 3. Advisability of Using Qualified Labour and the Valuation of Labour Force

#### *Features of Labour as a Factor of Production*

Among the factors of production upon whose correct use output depends we list labour. However, labour occupies quite a special place among these factors which cannot be compared with any other.

First of all, in the final account labour is the only source of value created. The scheme of planning adopted above is fully consistent with this basic assumption of the labour theory of value. In fact, although apart from labour other factors of production were considered, some of them presented in themselves a materialized product of labour (materials, fuel, depreciation of equipment, the service of transport): they again are brought back to labour as the ultimate source. Other factors (productive capacity, land and other natural resources) are not independent sources of value and their use (allocation) only influences the productivity of labour which alone creates value.

The features of this factor are: extraordinary diversity and heterogeneity, and multiple possibilities of using human labour; the dependence of labour productivity not only on its position in the production process but also on many other conditions: qualification, conditions of work, form of payment, general education, various features and conditions of the life of the worker. Finally, the task of utilizing labour in a socialist society cannot be reduced solely to attaining the highest productive efficiency, but requires the protection of the physical health of the worker by continuously improving his

working conditions, and also by making him satisfied with his work. All these circumstances greatly complicate economic problems, the standardization of work and its rating, and the establishment of norms as a basis of calculation which are unavoidably approximate and conditional. At the same time, the role of labour as the decisive factor of production urgently calls for further scientific analysis of the problem of its utilization, especially as the scales system in operation at present and the practice of rate fixing are far from satisfactory.

The problems of using labour as a basic factor of production must also occupy an important place when we consider problems of constructing an optimal plan and its indicators. The study of these problems, in view of their nature, will doubtless present considerable difficulties and will necessitate special investigation. A schematic model of these problems will lead to the construction of new and more complex schemes, while the conclusions and results obtained in the course of such study will not allow for all the actual data and circumstances, and can for this reason only be applied with caution.

Notwithstanding the shortcomings of the investigation of this problem, it seems to us that here too the approach used in the present work will be fruitful (as one of the possible ways) and will enable us to obtain useful conclusions immediately. We only speak of one way since the comparison and valuation of labour consistent with its payment according to operating conditions and scale rates also involves some method of simplifying a complex task and thus of arriving at roughly approximate valuations of various categories of labour.

We shall consider now those cases in which the only substantially variable factor is labour.

If only one type of labour is available (for instance, unskilled labour or labour of one particular speciality), the problem is solved very easily. Of all the possible methods of manufacture for each type of product in an optimal plan, it is evident that the method requiring the least working time per unit production should be retained. The valuation of one hour of this labour, for instance, may be taken as the unit. As in this instance there is only one factor of production, the valuation of a unit of each type of product (more exactly, of the work for its manufacture) will equal the expenditure in the optimal plan, that is, the minimum time necessary for the production of such a unit. This corresponds to what Marx calls "socially necessary working

time" with this difference that it is calculated only for the production unit under consideration.† And in the case when the law of value operates in its simpler form relative o.d. valuations do not differ from relative values. We shall now consider this problem in a more complex case.

### *Valuation of Labour of Different Skill*

Let us assume that we have several categories of labour, varying in kind, specialization or skill. How do we obtain their comparative valuations? A certain basis of such comparison is always provided

TABLE 24. STANDARD TIME AND CATEGORIES OF LABOUR IN THE COMPLETION OF A SERIES OF OPERATIONS

Category of labour		A	B	C
Labour time (hr)		80,000	190,000	125,000
Type of work	Volume of work (in units)	Standard time per unit of operation (hr)		
I	10,000	10	20	—
II	2,000	50	—	40
III	50,000	—	2.5	1.5
IV	10,000	3	—	—
V	20,000	2	2	2

by the degree of interchangeability of various categories of labour. For instance, the work may be performed by a less skilled worker but with much lower productivity, or replaced by equivalent work of another specialization. The work of a welder may be replaced by the work of a riveter, the work of a blacksmith sometimes by the work of a welder, etc. Frequently, this change may be accomplished in a more complex manner, i.e. by substituting for one type of production another one which requires other types of labour. For certain types of operations a given specialization or skill may not be interchangeable.

† Thus, if one applicable method requires four hours of working time and another two hours and if in addition there are no restrictions as to the application of the latter method (other factors not being limiting), this method will be systematically used and in accordance with it the socially necessary working time will be determined.

Table 24 gives as an illustration a numerical example of three categories of labour with a whole series of operations to be performed. The standard time is given for each category of labour and for all the types of operations where it may be applied, together with the volume of operations of each type and the number of working hours for each category of labour.

The total amount of work needs to be completed in the shortest possible time. The optimal plan is given in Table 25. It is easy to

TABLE 25. THE OPTIMAL PLAN

Type of work	Units of measurement	Type of labour			Total number of units
		A	B	C	
I	Hours	50,000	100,000	—	10,000
	Physical units	5000	5000	—	
II	Hours	—	—	80,000	2000
	Physical units	—	—	2000	
III	Hours	—	50,000	45,000	50,000
	Physical units	—	20,000	30,000	
IV	Hours	30,000	—	—	10,000
	Physical units	10,000	—	—	
V	Hours	—	40,000	—	20,000
	Physical units	—	20,000	—	
Total number of hours		80,000	190,000	125,000	

establish that this plan is really optimal in the following way. Assuming valuations of 1 for one hour's work of category B labour, 2 for category A and 0.67 for category C, we find that expenditure of labour used for the methods of the plan shown is the lowest (while other expenditure is the same for all methods, as already stated) (see Conclusion 12).

Here the time for the completion is 125 working days (1000 hr of an eight-hour working day).

We shall not consider the derivation of the solution or the process of finding the quoted valuations as this does not differ essentially from the solution of the example in Chapter I, Section 2 (pp. 34–8).

What is the meaning of these valuations? They show that under the given conditions (valuations, as usual, are concrete), one working



hour of category A labour has the same value in production as two working hours of category B labour, i.e. one may be replaced by the other. This valuation should be used as a guide when evaluating expenditure and allocation of work. But on no account should these valuations be directly related to wage scales and payment of labour. Thus, with extreme shortage of welders, it may be found that the valuation of a welder's work and of an unqualified worker are in an 8:1 relationship. This does not mean that a welder should be paid eight times more, but that these relative valuations should be used as a guide in the organization of production. With such a relationship, for instance, even five to six hours of unqualified labour may be used in order to save a welder one hour's work. In such exceptional conditions it might be best to give the welder a permanent assistant even if this were to increase the productivity of his work by only 20 per cent.

#### *Valuation of Production through Outlay on Labour*

Valuations of various categories of labour enable one also to obtain valuations of production. For instance, per unit of job I, 10 hr of labour A are spent or 20 hr of labour B. Its valuation, therefore, is  $1 \times 20 = 2 \times 10 = 20$ . Similarly, job IV requires 3 hr of labour A which means in terms of labour B:  $3 \times 2 = 6$ . Consequently, in this case too, in order to obtain an o.d. valuation of production the time necessary must be calculated, but for qualified labour this must be converted to units of unskilled labour (or generally to one type of labour). This is a very well known theoretical principle applied in the calculation of value, supplemented here only by some specific quantitative method of converting skilled to unskilled labour, i.e. by deriving objectively determined coefficients of conversion applying to actual conditions.

We thus arrive at the following conclusion.

**CONCLUSION 17.** Like other factors of production, labour time of each category has a definite o.d. valuation. For each category of labour a fixed objectively determined coefficient for conversion to unqualified labour is obtained (the value of which will depend upon the actual conditions of the task). If the sole type of outlay is labour required for a given production (or if all other types of outlay are unrestrictive or superfluous), the ratio of o.d. valuations of various

types of production is determined by expenditure in labour per unit of each type of production provided all the categories of labour are converted to one of them by means of the coefficient described above.

We shall not deal in detail with the numerous properties and applications of o.d. valuations of labour. For these Conclusions 12-16 hold throughout. In particular, Conclusion 16 states that the ratio of o.d. valuations of the labour force may be a guide to the advisability of transferring certain categories of labour from one factory to another and also of training labour in one or another special skill.

In conclusion, even a rough estimate of o.d. valuations of skilled labour and the application of such valuations in problems of labour utilization might be of substantial benefit in such problems. At present one often finds highly qualified labour used at times in unskilled work where it could easily be replaced by much less skilled labour. On the other hand, workers with little training are entrusted with complicated unsuitable work which frequently results in considerable waste, extremely low productivity of labour and insufficient use of equipment.

The cause of this, besides the shortcomings of calendar planning and rush work, seems an insufficiently realistic valuation of labour. In fact, if with wage scales and standards used as a guide, a productive method is generally chosen according to its cost of production, actual conditions are not allowed for. They are particularly misleading when conditions and production tasks change rapidly, as happened for instance during World War II when the country was faced with radical changes in the composition of labour requirements of various professions and their social valuation.

It must be pointed out that such need for a correct valuation of labour from a national economic point of view arises not only from the diversity of skills, but also from the nature and location of its utilization, and the time period (male and female labour, distinction as to the physical composition, age, labour in individual districts, seasonal labour).†

† As has been already noted, the difference in the o.d. valuation of various types of labour in no way contradicts the fact that used on the same job, their payment would be identical, and does not infringe the principle of equal pay for equal work.

In the main, the calculation of such a national economic valuation for various categories of labour (by analysing the efficiency of their use), of requirements and of resources appears entirely feasible. The existence of valuations related to real conditions should prove very useful; they would promote a proper allocation and use of labour by categories. (The shortcomings in this respect were frequently reported in the press, for instance, the case of physically healthy men used on light work while female labour was being used on heavy work.) The use of such valuations is also essential in an economic analysis of the advisability of developing and expanding production at a given place when the sequence of measures regarding mechanization and automation of operating processes has to be determined.

Further, in order to encourage a proper allocation of labour, and to arouse the interest of the factories and the workers themselves in this, such economic valuations must be reflected in the payment of labour and in economic accounting, although as already mentioned, we do not consider that the earnings need correspond directly to the economic valuation of its efficiency. A further consideration is that when a certain type of labour is not used to its full value this is detrimental not only to labour itself but also to the state which does not receive a full return of the product for society. For this reason, for instance, an order might be adopted whereby economic units which use particularly scarce labour would have to pay a fixed supplement into a special fund. Such a measure would prevent the recurrence of cases where the use of such categories of labour was little justified and would, at the same time, promote the use of the reserves of those categories of labour which are not scarce.

#### Section 4. Measures for Economies of Scarce Materials and their Valuations

##### *Scarce Materials*

Let us consider the case when the only variable factors are materials. As was shown in the preceding section, in a simpler case where the sole type of expenditure in the production of materials (apart, possibly, from factors in excess supply) was labour,

o.d. valuations of materials are established on the basis of the quantity of labour (converted to basic labour) necessary for their production.

If, besides labour, some other factors enter into the production of materials (for instance, electrical energy), this expenditure too may be converted to labour by using its o.d. valuations.

The valuation of materials obtained should be the starting point. In other words, in tackling problems of the utilization of materials, of two possible types of material that give the same results when used in the manufacture of a given type of output, that material should be chosen the utilization of which would produce the lowest total valuation of expenditure. However, this is so only in the simpler case when there are no other limiting factors, and where therefore the possibility exists of producing the given material in the required amounts consistent with its o.d. valuation (that is, with the corresponding expenditure of labour). However, if such production or the quantity of a given material becoming available for use is limited for some reason (scarce sources of raw material, shortage of equipment, overloading of transport) and does not fully meet requirements, the problem of its most advantageous use should be solved in a different manner. Such kinds of material we shall call *scarce*. In view of the broad meaning given to this designation, it will embrace a fairly large number of important materials.

Apart from increasing production of a scarce material, there is usually another way of improvement—by an economy in the material, by reducing its consumption or replacing it by other materials or methods of production. This way of increasing the material is also tied to a definite objectively determined expenditure under given conditions.† If the production or use of scarce materials cannot be increased at a given moment, the second method of obtaining such result should also determine the o.d. valuation of the material under given actual conditions. In its calculation the

† The enormous and varied possibilities of economy and substitution of metals are shown in the speech of L. I. Brezhnev at the XXI Congress of the Communist Party of the U.S.S.R.: "It was calculated, for instance, that in order to construct containers for petrol and other products of a volume of 130–150 million cubic metres 3·5 million tons of sheet steel is required. By constructing these reservoirs from reinforced concrete, it is possible to save more than 1½ million tons of metal. Equally important are such possibilities in building, in metal manufacturing industries, in shipbuilding, in the oil and gas industries, in the municipal economy" (*Report*, vol. I, p. 425).

economic influence of other possible uses of a given material which remain unexploited can be evaluated.

We shall show in an example how the problem of the proper utilization of such a material and its valuation may be approached.

EXAMPLE. Material A is used in several types of product. Consumption of material and labour per unit of each type of product is given in Table 26. To achieve the required (say monthly) output, 350,000 kg of material A are needed. At present, only 200,000 kg of this material can be supplied. Suppose that the cost of material A equals 4 roubles per 1 kg, and that we assess the working hour at

TABLE 26 EXPENDITURE ON MATERIAL AND LABOUR PER UNIT OF EACH TYPE OF PRODUCT

Type of product	Number required	Expenditure per unit product of				Cost of one unit in roubles
		Material A		Labour		
		kg	rouble	hr	rouble	
I	6,000	5	20	20	40	60
II	1,200	100	400	100	200	600
III	2,000	25	100	10	20	120
IV	100	500	2000	1000	2000	4000
V	100,000	1	4	15	30	34

2 roubles. It is necessary to save 150,000 kg of material A. It is known that for all types of product with the exception of IV, it could be replaced by material B which is not scarce and the cost of which is 2 roubles/kg. It is true that this calls for greater expenditure on this material and what is more important, labour for its processing.

The necessary expenditure is given in Table 27. Let us assume that the products obtained are in both cases approximately the same. Sometimes, even when this is not the case, they may be converted to the same quality. For instance, if in the second case the product has one-third of the useful life of the first (here is meant a product the useful life of which cannot be extended), the cost of one unit in the first case must be compared with that of three units in the second case.

From Table 27 we can see that in all cases changing to substitutes entails an increase in costs. All the same, it is necessary to do so.

How does this lead to the smallest losses? At a first glance it would appear that the losses would be the smallest if the substitute was used in product V where the cost increases by only 41 per cent while for product I the increase is 133 per cent, for product II 100 per cent, and for product III 83 per cent. However, such a solution would be superficial.

TABLE 27. EXPENDITURE WITH MATERIALS IN ADEQUATE SUPPLY

Type of product	Number required	Expenditure per unit on				Cost (in roubles)	Increase in cost (in roubles per unit)	Increase in cost per 1 kg of A (in roubles)
		Material B		Labour				
		kg	rouble	hr	rouble			
I	6,000	20	40	50	100	140	80	16
II	1,200	100	200	500	1000	1200	600	6
III	2,000	50	100	60	120	220	100	4
V	100,000	4	8	20	40	48	14	14

### *Optimal Solution and Conclusions*

A correct solution is obtained in the following manner. For each product we calculate where a saving could be achieved and the expenditure involved in saving 1 kg of material A. For product I a saving of 5 kg entails additional expenditure of 80 roubles or 16 roubles/kg. Similarly, for product II, 6 roubles/kg, for III, 4 roubles/kg, for V, 14 roubles/kg. It is necessary to save 150,000 kg; this means, we carry out economies on products III and II for which these result in the lowest losses (for 1000 units of product II material B will be used and for 200 units material A). For products I, IV and V material A is retained. The total increase in cost will amount to:

$$1000 \times 600 + 2000 \times 100 = 800,000 \text{ roubles.}$$

If, in order to achieve this saving of material A (150,000 kg), for all the possible products (I, II, III and V) material A were replaced by material B, in half of the output ( $150,000 : 300 = 500$ ) the losses would amount to

$$\frac{1}{2} \times (6000 \times 80 + 1200 \times 600 + 2000 \times 100 + 100,000 \times 14) = 1,400,000 \text{ roubles.}$$

Thus, the optimal result produces a gain of 600,000 roubles or 300,000 working hours in comparison with the simpler mechanical method.

As was shown above, considerably higher losses would result than in the optimal solution if the analysis of the problem was approached superficially and material A was saved on product V.

The result obtained furnishes at the same time the o.d. valuation for material A. Once it has been decided to increase the cost by 6 roubles in order to reduce the consumption of material A by 1 kg, its valuation must be increased by this amount and will rise at most to  $4 + 6 = 10$  roubles/kg.

If we started from such an o.d. valuation of material A and used it in the re-evaluation of the cost per unit of each type of product (calculating expenditure from Table 26) the following o.d. valuations would be obtained: product I—90 roubles, II—1200 roubles, III—270 roubles, V—40 roubles. Comparing these data with the value of costs in Table 27 where it agrees with the o.d. valuations (since material B is not scarce), it is apparent that the use of material B is possible for products II and III, and is not advisable for I and V.

In the o.d. valuations of scarce materials all the characteristic features are maintained and they have the same application as in the preceding examples. The valuation of 10 roubles/kg of material A obtained is realistic; if the calculation with such a valuation should show the profitability of material A and not a substitute, production may be carried out with this material. Thus, particularly high losses resulting from a shortage of material A could be avoided.

The ratio of the o.d. valuation between a scarce product and its price (or cost) in the example is  $10 : 4 = 2.5$  which may be considered as a coefficient characterizing quantitatively the scarcity of a given material. Such coefficients of scarcity were sometimes applied in planning practice in order to rectify the lack of agreement of prices of some materials with their national economic value. However, their values were taken rather arbitrarily without reference to the method described here.

Further, o.d. valuations may be used in solving problems relating to the manufacture of material A where it appears itself as a product. Assume it is possible to produce material A from a local raw material at a cost of 8 roubles/kg. On the basis of current cost this should be rejected since it increases costs by 100 per cent. On the basis of the

o.d. valuation, however, such production will prove entirely profitable, and for this reason advisable. At the same time, if sufficient production from local raw materials can be provided, then it would be better not to use substitute B in the manufacture of product II and the o.d. valuation for A will have to be taken as equal to 8 roubles.

We shall give the results obtained as follows:

**CONCLUSION 18.** In the case of a scarce material, that is, a material the production possibilities of which are limited for a given period, the o.d. valuation is obtained both by the direct expenditure on labour per unit material and all the actual conditions relating to its production and use, and in particular by expenditure on labour necessary when recourse is had to substitutes or saving. The relationship between the requirements of a given material and available production is also relevant in its calculation.

If an o.d. valuation of a scarce material determined in this manner were taken as a starting point, the principle of profitability may be a suitable guide to its use: to select the materials which entail the lowest expenditure; but in the calculation of expenditure, allowance must be made for scarce materials on the basis of o.d. valuations.

It should be pointed out that such a departure from cost in determining prices of scarce materials occurred in the economic policy of the Soviet government as a means of encouraging economy in these materials. Thus, in 1940 (and on several occasions in subsequent years), the prices of non-ferrous metals, and above all copper, were considerably increased. What is new in the conclusion quoted above for o.d. valuations of scarce materials?

First, it shows that such a change in the calculated prices would be advisable for all scarce materials, and in particular for ferrous metals and cement; and it would help to use them in a more suitable manner.

Secondly, it shows that such a price must be ascertained objectively and at the same time it must be realistic, i.e. in all those cases where the use of a given material should prove profitable in spite of the rise in price, its supply will be fully ensured. This was not always accomplished in practice and resulted in innumerable cases where production was brought to a standstill, equipment lay idle, through the impossibility of obtaining even small quantities of certain materials the quality of production deteriorated, and high expenditure was incurred.



Meanwhile, these materials could have been saved in other lines of production or replaced by materials which were not scarce, harmlessly and with much less loss.

*This kind of loss relating to the unrealistic nature of prices is very frequent (in spite of the clear need for a scarce material at a given place and the profitability of its use at any possible price, no supply is forthcoming). Thus, at oil extracting factories the possibilities of extraction are not fully utilized and great losses of output occur because of insufficient equipment, often simple. Let us mention that a ton of metal used in the construction of oil tanks could save tens of tons of oil per month. At the same time, metallurgical and metal manufacturing works were sometimes at a standstill or did not work to full capacity owing to difficulties of supply of one type or another of raw material and other materials caused by fuel shortage. It is quite likely that an additional ton of oil might reduce losses by 10 tons in the output of metal and metal components. Meanwhile, at both places this state of affairs may be acceptable. At the same time, with a proper allocation and supply, tens of tons of oil and metal could be provided as well. Examples of this kind are all too frequent.*†

Using correct realistic valuations of scarce materials (even if they have been determined only roughly) and following them approximately, using a scarce material whenever this is advantageous despite the increased valuation of such a material, would lead to the elimination of the losses mentioned. The quantity of material necessary for this purpose could be obtained by carrying out economies at those points where they would prove advisable from the calculation of o.d. valuations, regardless of whether this might appear to be superfluous with the usual calculations, and whether the use of scarce materials for this purpose might have been hallowed by tradition.

In conclusion let us note that the concept of scarcity of materials and the degree of scarcity depend upon circumstances. From an

† Even more impressive losses of this type are noted in the speech of M. T. Efremov at the XXI Congress of the Communist Party of the Soviet Union: "In the oil industry of our region 2 million cubic metres of gas are daily burnt in torches. In 1957 664 million cubic metres of gas were burnt. This amount would have sufficed to heat the town of Kuibyshev for three years. The burning of gas in torches is current at many points of our oil industry. This is due chiefly to the lack of a grid and of installations for the transport and utilization of gas, and also because the problem of storing large volumes of gas has not yet been solved" (*Report*, vol. II, p. 28).

economic point of view, the field in which the use of a given material might be appropriate and the requirements for it depend upon current prices. Thus, if in the example under consideration the price of material A were 10 roubles and not 4, one would at once plan to use material B for product III and this product would not be included in the statement about material A. In this manner the determination of the volume of production of a given material that may be advisable from an economic point of view will depend upon the price fixed. Consequently, in order to arrive at an economically correct price of a material, one which approaches the o.d. valuation and encourages economy and substitution of materials on the one hand and the growth of their production on the other, it is necessary to balance in the plan requirements and production and thereby to end the shortage of a given material.

The above discussions of the determination of valuations at the lowest level of savings that may still be achieved or of the consistency of the valuation of a given material when it is listed both as a product and as a raw material may be superficially reminiscent of the constructions of the vulgar economic schools (the law of equilibrium between demand and supply) and of the subjective school of marginal utility. In fact, the analysis applied by us differs essentially from these theories by its scientifically objective approach. Neither the demand nor the "utility" is decisive, but perfectly realistic objective data of production: the degree of saving that may be attained, the extent of the requirements for a given material for production at a given production unit, the cost of a given complete production process. Finally, as explained in several places, o.d. valuations are determined quite consistently with the labour theory of value by the economic expenditure (of labour, in the final account) necessary to achieve production under given conditions (*see also* Sections 5 and 6).

### Section 5. Efficient Use of Equipment. Hire Valuation

#### *Statement of the Problem*

Up to now we considered only those planning tasks in which equipment was a surplus factor and the question of its utilization did not arise. However, the problem is one of vital importance: its

analysis has its own features and leads to important conclusions. If there is a shortage of equipment its correct allocation in the direction in which it may be best utilized consistent with its technical possibilities, and its most efficient use is a very important task. The existing methods of analysing these problems are not sufficiently developed to be satisfactory and frequently do not guarantee a correct solution. We shall demonstrate the proposed method of solving this problem by a practical example.

TABLE 28. PRODUCTIVITY OF OPERATIONS BY HAND AND BY MACHINES

Type of operation	Volume of operation	By hand		By machine		Number of machines necessary to carry out the total volume of operations
		Daily output (in units)	Cost per unit (in roubles)	Daily output (in units)	Cost per unit (in roubles)	
I	2,000,000	40	0.6	1000	0.2	20
II	1,500,000	10	3.0	500	1.2	30
III	200,000	4	7.0	50	1.0	40
IV	40,000,000	200	0.15	10,000	0.05	40
V	2,500,000	20	1.5	500	0.3	50

EXAMPLE. The efficacy of using different equipment and machines for excavations, freight traffic, loading and unloading and other work will depend upon various conditions: the volume and concentration of work, the distance of travel, etc. We assume that the work may be carried out manually (with simpler tools) or with one single type of machine available in limited quantity.

Let us suppose that the operations may be divided into five types. For each type Table 28 shows daily production by hand and by machines, as well as the volume of operations (in corresponding units) and the cost per unit operation of each type. In calculating cost, all the expenditure relating to the use and also that part of depreciation and repairs which is connected with productive work are included in both cases.†

† Some depreciation of equipment also takes place when it is not being used; it is preferable not to take this rate into account here as it remains the same for all operations even when the machines are idle, and consequently represents in itself a fixed cost element.

It is assumed here that outlay has been costed correctly, and if not then the requisite corrections are introduced such as a more precise conversion of qualified labour to unskilled labour; expenditure on electric power and fuel is accounted for on the basis of o.d. valuations, and scarce materials utilized are also included on the basis of o.d. valuations.

A hundred machines are available; the time for the completion of all the operations is 100 days. A method has to be chosen for the performance of each type of operation whether by hand or machine in such a manner that the total cost of all the operations be the lowest. The results would not substantially change if we set ourselves the task, all other conditions remaining the same, to use a minimum number of additional workers for operations by hand, besides those operating the machines.

In all cases I-V (Table 28) operations by hand involve higher costs, yet manual labour has to be used since in order to complete all the operations in the planned time by machines, 180 machines and not 100 would be required. On which operations should machines be used and on which manual labour?

At first sight it seems that machines should be utilized where they would result in the highest reduction in costs as compared with work carried out by hand, say for type III operation for which the cost is seven times lower. However, such a solution, as will be seen subsequently, is incorrect and superficial.

In order to arrive at an optimal solution we calculate for each type of operation by how much the use of machines will reduce total costs as compared with manual labour. We obtain per unit of type I operations:  $0.6 - 0.2 = 0.4$ , per machine-day:  $1000 \times 0.4 = 400$  roubles; similarly, for type II operations 900 roubles, for III, 300 roubles, for IV, 1000 roubles and for V, 600 roubles. From this it is clear that a machine should first of all be used on operation IV, and then on II, V, I and III. To complete type IV operations in 100 days, 40 machines are necessary, for II, 30 and for V, 50 machines. Thus all the machines are exhausted. A part of type V operations (and all of type I and III operations) has to be completed by hand.

In this manner we arrive at an optimal plan (Table 29).

In order to complete the work 1500 manual workers have to be engaged in addition to the workers operating the 100 machines. The total cost of the work, at its minimum under the given conditions,

amounts to 8,350,000 roubles. If, for instance, type IV operation were executed by hand and type III operation by machine, as appeared correct from a superficial analysis, 3000 workers and not 1500 would be required for all the operations, and the cost of the work would have increased (by 2,800,000 roubles).

TABLE 29. THE OPTIMAL PLAN

Type of operation	By hand			By machine		
	Number of workers	Volume of operations	Cost (in roubles)	Number of machines	Volume of operations	Cost (in roubles)
I	500	2,000,000	1,200,000	—	—	—
II	—	—	—	30	1,500,000	1,800,000
III	500	200,000	1,400,000	—	—	—
IV	—	—	—	40	40,000,000	2,000,000
V	500	1,000,000	1,500,000	30	1,500,000	450,000
Total	1500		4,100,000	100		4,250,000

### *Hire Valuation*

We have obtained an efficient allocation of machines on the basis of the saving achieved per machine-day for each type of operation. We had to stop at type V operations on which the utilization of each machine-day furnished a saving amounting to 600 roubles. The latter figure is of great importance: it shows that each spare machine provides a daily saving of 600 roubles. Conversely, the absence of one machine for one day results in an increase in cost equivalent to this amount. Thus, under the given conditions, it would have been worth while spending 600 roubles in order to obtain an additional machine-day. The latter figure will be designated as the *hire valuation of a machine-day under the given conditions*. It should be taken as the valuation of the use of a machine for one day. Such a valuation is objectively determined and is entirely realistic since each additional machine-day produces, in fact, a saving of 600 roubles. Conversely, a machine-day may be saved by increasing expenditure by 600 roubles.

We use the term "hire valuation" as this is a valuation of the cost that would be justified if such a machine were hired (*rented*) for some time. It may also be considered as the amount of rent on

equipment which is calculated but not paid. Moreover, we believe that this ought to find its reflection in economic accounting.

The economic meaning of the hire valuation of equipment as a factor influencing the productivity of labour is characterized by the extent to which this potential economy in labour may be attained under given conditions by utilizing the factor in question (the use of a machine). For this reason, when calculating cost or savings of labour, expenditure on this factor must be accounted for on the basis of such a valuation.† (Not to use a machine for one day means losing 600 roubles or a corresponding amount of labour.)

TABLE 30. COST CALCULATION WITH HIRE VALUATION OF MACHINES

Type of operation	Full cost when using machines (including hire valuation) (in roubles/unit)	Cost when working by hand (in roubles/unit)
I	0·8	0·6
II	2·4	3·0
III	13·0	7·0
IV	<b>0·11</b>	0·15
V	1·5	1·5

We shall now include this hire valuation per machine-day in calculating the cost per unit of each type of operation when using machines. Thus, for instance, for each operation of type I (for which the daily machine-day productivity equals 1000 units) the cost of 0·2 rouble will have to be increased by the hire valuation of  $600:1000 = 0·6$  rouble, giving a total of 0·8 rouble. Carrying out similar calculations for the remaining types of operations, we obtain (Table 30) the full cost per unit of each type of operation on the basis of o.d. valuations.

These are the revised costs with the hire valuations included. For purposes of comparison, the costs of operations carried out by hand are given as well. It is apparent that now the cost is lower when using a machine than with manual labour only for operations II and IV, and identical for V, which exactly agrees with the optimal plan (the figures corresponding to the optimum are in bold print in Table 30).

† We shall return to the question of the economic meaning of hire valuation of equipment in connection with the analysis of the related problem of rent which is carried out in more detail (see pp. 92-107).

Hence, if in the calculation of cost a correctly determined hire valuation is included in the expenditure on a machine, it is possible to be guided in the choice of equipment by the principle of least cost (principle of profitability) and each operation will be performed by the method showing the lower cost.

Only by taking into consideration the hire valuation obtained in agreement with actual conditions (volumes and types of operations, existing fleet of machines) is it possible to solve problems of the use of equipment. This valuation shows that a machine standing idle for one day causes a loss of 600 roubles, that the use of the machine on type III operations produces a daily loss of  $50 \times (13.0 - 7.0) = 300$  roubles. Conversely, the performance of type II operations by hand results in a loss of 0.6 rouble per unit operation. If the hire valuations are ignored, both could prove fully justified: the use of machines on type III operations if such use results in very low operating cost; manual performance for type II operations if it proves impossible to ensure the performance of all the operations by machine.

Let us give a further example. Assume that by adding one more worker to operate a machine on type V operations, its efficiency may be increased by 10 per cent. The valuation of the working day is 30 roubles. Would this be advisable? Let us calculate the cost in the usual manner. The expenditure per day on type V operation amounted to  $500 \times 0.3 = 150$  roubles (see Table 28). Now it will amount to  $150 + 30 = 180$  roubles. The efficiency of the machine increased by 10 per cent will represent 550 units, consequently the cost per unit of type V operation will be  $180/550 = 0.33$  rouble compared with 0.30 rouble, which means it will increase. Also, the productivity of the workers operating the machines decreases. Thus, the measure appears to be inadvisable. We shall now carry out calculations allowing for the hire valuation of equipment. As a result of such calculation we find that the previous cost equals 1.5 roubles (Table 30). Expenditure per working day amounted to  $500 \times 1.5 = 750$  roubles and with the additional worker it now amounts to  $750 + 30 = 780$  roubles. Since output equals 550, the cost per unit of type V operation will be  $780/550 = 1.42$  roubles. A decrease in expenditure is obtained and consequently the above change is advisable in the given conditions. It should be mentioned, however, that in practice such a change will not always be accomplished. If at a given production unit a machine is available, the

question as to how it could be used in other production units does not enter into consideration, and without such consideration a change appears of no advantage and is not likely to be carried out.†

On the other hand, incorporating in the calculation the hire valuations enables one to allow for the general situation when carrying out the economic analysis of the use of such equipment at any one production unit. The calculation of the hire valuation is of particular importance in questions of the use of high-capacity and expensive equipment (blooming mills, walking excavators, generators, chemical machinery) operating at a small unit when the efficient use of such equipment frequently determines the results of the operation of a whole factory. It is clear that measures causing an increase (however insignificant) in the efficiency of such equipment would be justified (for instance, the introduction of a 4-6 hours working day for the staff operating such machinery or the introduction of special training for them if this were to lead to an increase in the productivity of the machinery even by only a small percentage). Let us say that for this reason greater importance is rightly attached in metallurgy to the indices of metal output than to costs. With the inclusion of the hire valuation in calculating expenditure similar measures would have a favourable effect on costs.

The calculation of the hire valuation in determining the economic efficiency of automation is particularly important. If automation ensures an increase in productivity of complex, expensive and fully used machinery, even if its completion and exploitation should entail very considerable expenditure, sometimes even higher than the costs involved in the operation of the machinery up to the introduction of automation, economic accounting allowing for the important hire valuation of such equipment would confirm the justification of automation. Current economic analysis may show otherwise.

EXAMPLE. A machine has to be sent for repair. The time taken for the repair is ten days and the cost 2000 roubles. The repair may be carried out by a quicker method in two days but the cost will then be 3000 roubles (payment of overtime, the use of more expensive material). At first sight it is not clear whether this is justified.

† Typical examples of this kind were frequently quoted in the press. Thus, the existing indices of labour productivity and the system of payment frequently prove to be an obstacle in the intensive use of complex machinery in the coal industry.



A correct solution may be obtained through the hire valuation. Eight days during which the machine cannot be utilized (as a result of the slower repair) produces a loss in the hire valuation of the order of  $8 \times 600 = 4800$  roubles, while the difference in cost of repair amounts to only 1000 roubles.

Thus, if there is a shortage of machines, speedy repair seems advisable in spite of the increased cost this may entail.

The hire valuation of equipment must be taken into account in solving problems relating to the possibilities of fulfilling the plan if the means of production are changed. Thus, in the present example there were 1500 manual workers and, let us say, 300 people attending to the machines, altogether 1800 workers. The total time of operation was 100 days. To what extent would this time be changed by adding another 300 workers? Of course, it would be incorrect to say that by increasing the number of workers by  $300 : 1800 = 16.7$  per cent the time would be reduced by  $16.7 : 116.7 = 14.3$  per cent, therefore, to complete the work would take 85.7 days since the number of machines has not increased. The correct calculation is as follows. The total daily productivity of existing means is valued at 83,500 roubles and, accounting for the hire valuation, at 143,500 roubles. Productivity of one worker for type V operation where supplementary labour may be used rationally would amount to  $20 \times 1.5 = 30$  roubles/day. Productivity of the additionally engaged labour is  $300 \times 30 = 9000$  roubles. The increase in productivity is  $9000 : 143,500 = 6.3$  per cent. In agreement with this, the working time may be reduced by  $6.3 : 106.3 = 5.9$  per cent, i.e. the work can be completed in 94.1 days.

This is often not taken into account. To quote a typical example of this kind: after the decree of the Supreme Soviet of 26 June 1940 many industrial managers thought that the change from 7 to 8 hr working day would automatically result in an increase in production of 14 to 15 per cent. In fact, at the majority of factories an increase of only 6 to 7 per cent was attained. An increase of 14 to 15 per cent was only achieved at those factories which had reserves of equipment that could be utilized. In those estimates the higher hourly productivity of the worker as a result of a shorter working day was not allowed for either.

At present, in the economic analysis of problems relating to the change-over to a working day of 7 hr and later on of 6 to 5 hr, this

aspect could be of special significance. In particular, this calculation is most essential in questions of the productivity of workers operating group and expensive equipment when the shortening of the working day makes it possible at the same time to increase the intensivity of use of the equipment. A correct account of this economic effect must include the hire valuation of equipment, and in many cases it may reveal the positive economic advantages of a reduced working day. Let us assume, for instance, that on type V operations on changing to a 7 hr working day, the hourly productivity of a machine is increased by 5 per cent (fuller utilization owing to increased attention and the possibility of working at higher speeds). We shall assume that as a result of a change to a 7 hr working day, cost of operating the machines (per hour) rises by 14 per cent. Accounting for the increase in productivity we calculate a new o.d. valuation per unit operation of this type. We find that daily expenditure amounts to  $150 \times 1.14 = 171$  roubles and together with the hire valuation to 771 roubles. But in so far as  $500 \times 1.05 = 525$  units of type V operation are produced, expenditure per unit will amount to  $771 : 525 = 1.47$  roubles instead of 1.5 roubles; thus there is a decrease, and the shortening of the working day has a favourable economic effect in these conditions.

It is clear that the change to a shorter working day is economically advisable in factories operating on continuous production processes, using complex and highly efficient equipment, where labour resources are available that make it possible to ensure the full operation of such equipment. The calculation of the hire valuation is also of importance in the analysis of other similar problems, such as the introduction of shift work.

Further, the calculation of the hire valuation makes possible a correct approach to the problem of evaluating the national economic cost (labour cost) of each type of operation. For operations I and III which are carried out by hand in the optimal plan (see Table 29), this valuation is in essence determined by the cost per unit operation, using the manual method, i.e. (see Table 28) it amounts to 0.6 rouble for operation I and to 7 roubles for operation III.

Passing to type V operation which is partly performed by hand and partly by machine we obtain two values in Table 28 for the cost (1.5 roubles and 0.3 rouble). If an operation has already been completed and is similar in quality, it is of no importance how it has

been performed: the valuation should be identical since the individual unit of a commodity "is to be considered as an average sample of its class".†

What figure should one adopt? We maintain that the figure should be 1.5. In fact, if it were necessary to complete an additional unit of type V operation, if all the machines were utilized, this operation would have to be completed by hand at a cost of 1.5 roubles. Equally, a reduction by one unit of the quantity of type V operation would enable one to reduce expenditure by 1.5 roubles.

Thus, the quantity of socially-necessary labour for the performance of one unit of work of type V determines the cost figure of 1.5 roubles.‡ Finally, for type II and IV operations which are carried out only by machines, a correct valuation must include the hire valuation of the machine, and it must be taken from Table 30 (the figures in heavy type). In order to perform an additional 500 units of type II operation, the use of a machine (which is more advisable) entails an expenditure of  $500 \times 1.2 = 600$  roubles. Moreover, in order to save a machine-day, 500 units of operation V have to be performed by hand which results in an increased expenditure of  $500 \times 1.2 = 600$  roubles; the total additional expenditure will amount to  $600 + 600 = 1200$  roubles, from which the unit cost is  $1200 \cdot 500 = 2.4$  roubles, i.e. the same cost as that shown in Table 30.

The example quoted above may also be interpreted in another way. In the metal-working industry the use of certain types of equipment (stamping machines—presses, automatic equipment) results in a substantial increase in productivity and in a reduction in cost in comparison with others (the usual apparatus for mechanical processing). However, such equipment is frequently overloaded to such an extent that the more productive method of operation cannot

† Karl Marx, *Capital*, vol I, p. 39. Foreign Languages Publishing House, Moscow, 1954.

‡ Usually, in such cases the average cost and not the cost of production is used, for instance, in the given case for type V operation ( $1,000,000 \times 1.5 + 1,500,000 \times 0.3$ ):  $2,500,000 = 0.78$  rouble. Such a figure may be found but it is useless; to use it in the allocation of labour and the control of the current plan of work is wrong as the expenditure necessary for the performance of a unit of operation V and the saving realized are not determined by this figure. Similarly, the mean velocity of a projectile for the whole course of its motion may be determined; however, in order to determine its penetration, this speed is of no importance but rather that which the projectile will have at the moment of impact.

be used in all cases where it could be applied and produce a favourable result. For which components would it be most advisable to use this method? Let us assume that the relative data for productivity and cost are given by the figures of Table 28, where column 5 gives the data of high-efficiency equipment (as mentioned, presses or automatic equipment).

In this case the method of solving the problem given above will enable one to determine exactly for which components it may be advisable to use the more efficient equipment. A correct allocation will be achieved by calculating the hire valuation of such equipment. If there are several of such efficient and scarce types of equipment, a hire valuation for each will have to be applied.† Such a hire valuation must also invariably be included when the o.d. valuation (the correct "cost") of production is to be determined, otherwise a meaningless relationship will be obtained. Thus, a more complex component produced with an automatic machine may have a lower cost than a simple component manufactured with ordinary equipment.

The hire valuation is also important in the allocation of equipment. A greater hire valuation for the same type of equipment at one factory compared with another factory indicates that the first is more in need of additional equipment than the second. The results obtained may be formulated in the following manner.

**CONCLUSION 19.** In problems of the use of equipment the amount of use in the manufacture of a given product or in any operation must be taken into account by making an allowance for the hire valuation of the equipment. Its magnitude, equal to the saving of labour obtained from an additional unit of equipment in an optimal plan, is determined by all the actual conditions: the volume and type of operations which are to be completed, the supply of such type of equipment. In the choice of means for the completion of the work under an optimal plan, the principle of least expenditure is observed (provided that costs include the degree of utilization of equipment in terms of the hire valuation). The magnitude of the hire valuation of equipment must be taken into consideration in order to arrive at an o.d. valuation of each type of production and operation.

The hire valuation has the same properties and the same application as other kinds of o.d. valuations.

† The methods of calculating hire valuations simultaneously for several types of equipment are given in Appendix II.

*Utilization of Equipment*

The question of the proper and full utilization of equipment is extremely important and is solved less satisfactorily. If idleness of labour is an isolated phenomenon, the case of some types of equipment standing idle or being under-utilized is met everywhere. At the same time the shortage of this equipment leads to heavy losses elsewhere. This problem was duly discussed at the XVIII Communist Party conference when it was pointed out that the non-use or irrational use of equipment was widespread. But even now this problem has not yet been solved satisfactorily. The need to utilize equipment rationally and to the fullest extent was given particular attention by N. S. Khrushchev in his paper at the Session of the Supreme Soviet (January 1957) on the problems of the reorganization of management in industry.

The target figures for the coming seven years specify as one of the basic tasks a considerable improvement in the use of productive capacity in existing factories.

At present, equipment is far from being fully utilized. Fairly frequently a crane or a conveyor works systematically only 5-10 per cent of the time. Yet heavy loading and unloading is being carried out by hand elsewhere. At one place excavators and scrapers are used occasionally to dig small pits and level small areas, at other places heavy excavations (for the building of irrigation canals or highways) are mainly carried out by hand or by machines of low efficiency. What is the cause of such a situation? A factor of primary importance—the degree to which complex and often scarce equipment is being utilized at a given undertaking—is not accounted for quantitatively and is not given a sufficiently high value. Hence, calculations make the use of such equipment appear advantageous—furnishing lower operational cost—even when it is only utilized to some limited extent and not to its full value. Consequently, the use of equipment appears advisable much more often than warranted by its actual supply. The difficulty of deciding where it should be sent is shown even by the simplest example discussed above. For this reason, equipment is often allocated arbitrarily. Furthermore, if equipment is in balance, a factory may keep it for even if it is not used to any measurable degree some increase in productivity and a reduction in cost are obtained; and in addition it helps in the organization of

production. If equipment is not used at all, the undertaking itself does not incur any expenses on it.† The fact that this equipment could be much more efficient at another factory is frequently not taken into consideration.

In the "levelling-out approach" as applied to the allocation of vehicles to motor transport columns, those with a sufficient number of vehicles were frequently receiving new vehicles while others, suffering from an acute shortage of vehicles, were going without. One of the motor transport columns experienced an acute shortage of vehicles and was clearly unable to carry the heavy goods traffic. At the same time, at the neighbouring motor transport column vehicles in perfect condition were standing idle owing to lack of work. Similar situations have occurred in the past in all parts of Russia.

By taking into account the hire valuation it is possible to ensure a better use of equipment. Even if the hire valuation were determined very approximately, its calculation would make it possible in the last resort to avoid the most serious misuse of equipment which still frequently occurs. The incorporation in the amount of expenditure of the hire valuation of equipment would at once force one to stop using this equipment where it is hardly or never utilized, just as no one would keep a labour force for a long time without using it since otherwise there would at once be an excess of expenditure over earnings. This would enable one to ensure the supply of equipment to those production units where it may be used at 80-100 per cent

† Connected with this point is the existence of large quantities of equipment which are neither allocated nor utilized. "On 1 August 1958 the following unallocated equipment existed in the national economy of the Russian Federation: 60,000 metal-cutting lathes, more than 15,000 units of forge-press equipment" (see the paper of A. P. Aristov at the XXI Congress of the Communist Party of the Soviet Union, *Report*, vol. I, p. 503). The facts referred to by A. P. Kirilenko also merit attention: "However, unfortunately a portion of manufactured machines and equipment lie for a long time at building sites without being put into operation. For instance, people at the Ural Machine Tool factory calculated that of the equipment manufactured by the factory in the years 1953-7 for factories and buildings of the country the following were not put into operation up to the present: bloomings, thick plate and rolling mills, powerful presses weighing more than 80,000 tons and worth more than 500 million roubles which equals almost three years' work. The fault lies with the local authorities, but it seems to us that the Gosplan of the U.S.S.R. should assume an organizing function and plan more accurately the manufacture of equipment in relation to the time necessary for the construction of workshops and factories." (*ibid.*, p. 199.)

capacity. The use of equipment at those points is advantageous and can lower costs despite the inclusion of a considerable hire valuation. With such a procedure in accounting, the motor transport column in the example mentioned above would not needlessly hold vehicles required by others as it would burden its finances heavily.

When the hire valuation of equipment is included in the amount of expenditure which is at the basis of the production valuation of an optimal plan in which the economic agreement between the requirement and the production of a given product has been established, the concept of scarcity of the product loses its meaning. In fact, scarcity as a rule results from a relatively low price owing to inadequate cost accounting that leads to a demand for this product which is unjustified economically. But even in cases of a real scarcity of the product—a radical change in the economically justified requirement and volume of its production—a low price would not lessen the scarcity as it does not promote a speedier expansion of production and the replacement of the scarce product by another.

In considering applications for new equipment the calculation of the hire valuation plays also an essential part. Inclusion of the hire valuation would show the claims of those factories unfounded in which the equipment is not sufficiently fully and efficiently utilized, although without such an accounting they may appear fully justified and liable to be accepted (as the applications for machines for operations of types I and III in the above example). At the same time, such a reduction in the requirements would make it possible to meet—in due course—those applications in full where the given equipment would produce the highest results. Many factories, instead of wanting additional equipment which in this case would sharply raise their costs (by including the hire valuations in expenditure), would aim at the fullest and most efficient use of the available equipment, at a reduction of the time necessary for repairs, etc., which in turn would produce a reduction in expenditure. On the other hand, the factory at which the given machine tool operates at full efficiency and where its absence reduces production, would as a rule receive it immediately.

As regards mobile means of production (such as construction machinery), factories and building operations would endeavour to keep them during the period when they could be fully occupied and

would arrange to organize work in such a manner as to reduce this period as far as possible. In this connection it may be appropriate to mention that building was too slow and expanded too little (in particular before World War II) partly because the hire valuation of equipment was not allowed for in calculated costs. The system, introduced recently, of increased depreciation rates for equipment supplied to building organizations (such as cranes) works essentially in the right direction and affects its use favourably.

Finally, the calculation of the hire valuation of equipment will substantially settle the problem of the valuation of production. At present, a greatly differing cost is often obtained if production is carried out with simple or with specialized equipment. Sometimes this inconsistency remains, sometimes an average cost is adopted. Both these solutions are not very satisfactory. If the cost per cubic metre of excavation is expressed by two different indices—for the work carried out by hand and by mechanical operation—then in solving a planning problem without knowing in advance what means will be supplied at the given building it is impossible to arrive at a more or less correct idea of the magnitude of the necessary expenditure (on comparing several solutions, etc.). Neither will the mean figure reflect the expenditure on a given operation or production (see footnote on p. 84) appropriately and may frequently lead to confusion. In fact, if at a factory with new specialized equipment the cost of an article is 8 roubles and at an old one 12 roubles, and if, say, the average selling price is fixed at 10 roubles, then the second factory will systematically be considered as unprofitable and superfluous in spite of substantially good work. When solving the problem of whether the use of such an article is advisable its price of 10 roubles will be taken into consideration and also the fact that abandoning its production (at the second factory) may reduce expenditure by 12 roubles. The calculation of the hire valuation of equipment would save introducing differing prices (or an unjustified average price) and provide a price which reflects the actual extent of expenditure based on the production of a given volume.

The absence of such accounting may cause confusion in the use of output. Let us imagine that two machines can be manufactured at a more modern factory at a cost of 600,000 roubles each and at an older one at a cost of 1,200,000 roubles. It is apparent that the price of these machines should be identical. However, if one of them is



produced, in fact, at the first factory only and the second machine at the second one and if the price is fixed on the basis of cost, their prices will sharply differ which will lead to confusion in economic solutions relating to their use.† The calculation of a hire valuation at a higher scale for the more efficient factory would remove such an unjustified difference.

It should be pointed out that beside the possibility of determining the hire valuation of given means of production used in this example—from the calculation of the losses incurred as a result of the change to manual work—there exist other possibilities. In particular, the efficiency of using given means may be calculated from the valuation of production obtained by the use of such means, if such a valuation is known from other data. For instance, the latter is determined by the known level of expenditure on production by some other method providing the same basic output, or by the conditions of use and of equivalent substitution in production (compare Section 4).

To illustrate such a possibility let us turn to the example in Chapter I, Section 1, where the allocation of the production programme was discussed. Let us assume that for a set of articles (two articles No. 1 and one article No. 2) the valuation, say 65 roubles, is known. Then, if expenditure on materials for the set equals  $2 \times 10 + 15 = 35$  roubles, the manufacture of the set should be valued at  $65 - 35 = 30$  roubles and to the extent that the ratio of o.d. valuations for the manufacture of articles No. 1 and No. 2 was 1:4 (see pp. 5-6), these valuations should equal 5 and 20 roubles respectively ( $2 \times 5 + 20 = 30$  roubles), so the full valuation of article No. 1 is:  $10 + 5 = 15$  roubles; for article No. 2:  $15 + 20 = 35$  roubles. Further, if expenditure on the operation of each factory, apart from material costs, is known, then it is possible to calculate the planned profitability from its output under a rational production plan; and if the only kind of expenditure not accounted for is the hire valuation of the factory, the latter will be found automatically. Thus, let us assume that for a factory of type A this monthly expenditure amounts to

† These conclusions are not of general validity. It may be apparent that a more efficient factory is already equipped for the manufacture of a given machine and cannot be utilized for the time being for other purposes and the output of this machine is still needed. Then the costs shown will correspond more closely to the actual economic expenditure and may prove an acceptable basis of analysis.

300,000 roubles; if the valuation of net production carried out at this factory (the manufacture of 100,000 articles No. 1) amounts to  $5 \times 100,000 = 500,000$  roubles, then the hire valuation for this factory will amount to  $500,000 - 300,000 = 200,000$  roubles per month. In this manner the hire valuation of the remaining types of factory may be found.

Introducing the valuation of the rent from equipment (hire valuation) for entire factories in the form of planned targets of profitability will make factories more interested in increasing the range of the plan and in obtaining a larger number of orders; and the calculation of this type of expenditure in production will level out the conditions of production at various factories and remove cases of planned unprofitability.

In the subsequent pages we shall use the terms hire valuation, rent valuation, rent from equipment synonymously without specifying the field of their application. The term rent may properly be used as all-embracing, as the hire valuation for mobile equipment (lorries, cranes) and the rent valuation for stationary equipment. These terms correspond also to three possible forms of using the rent from equipment in economic accounting: hiring out, renting and finally the planned target of profitability which has just been the subject of discussion.

It should be noted next that—as pointed out above—the hire valuation which is to be used as a guide is determined by all the circumstances. For instance, during World War II the situation changed radically for many types of equipment. The shortage in metals must have resulted in a very high hire valuation of equipment in metallurgy and metal-working industries. Conversely, equipment of the textile industry which was not used to full capacity must have had a comparatively low valuation.

The calculation of the hire valuation of equipment must be reflected in the valuation of production. Such materials (metals, coal, oil), for the production of which complex equipment is used at over-full capacity, should receive a higher valuation in the calculation of rent than at present.

There exists the view that the role of equipment in the valuation of production is fully taken into account by including depreciation in costs. This view is quite incorrect. The share of depreciation in costs is for the majority of types of production insignificant (3-7

per cent). At the same time, the difference in costs of production when the correct equipment is used reaches 50 to 100 per cent and more in comparison with production by simpler equipment which frequently operates side by side with the other. For this reason, to confine cost of equipment to depreciation and not to account for its actual enormous role as a factor of production determining the productivity of labour leads not only to its improper use but distorts the whole system of prices and costs: the latter does not reflect correctly the real relationship of economic cost.

The position is less satisfactory in accounting for the use of equipment than for labour. The difference in the intensity and skill of labour is taken into account in one way or another through the medium of wages. The difference in regard to the conditions of labour as expressed by the supply of equipment is in no way considered.

It should also be mentioned that—as will be seen in Chapter III—the calculation of the hire valuation of equipment, the necessity for which was shown above, is not the result of its temporary scarcity but should appear as a regular element in economic accounting. The temporary scarcity of a given type of equipment increases only the magnitude of its hire valuation.

## Section 6. Rational Utilization of Natural Resources. Calculation of Rent

### *A Rational Crop Plan*

In the preceding section we considered the problem of utilizing equipment and emphasized the importance of a correct solution in the construction of an optimal plan. In the construction of a production plan there arise also problems of the rational utilization of natural resources when these are available in limited quantities.

Let us explain this position on the following example.

EXAMPLE. Three plots of land are available—one fertile, one ordinary and one poor (the resources of the latter are unlimited). The yield of wheat, rye and oats for each plot of land is known, as is

the expenditure of labour (in days per hectare) required for the production of each crop.† All the data are given in Table 31.

In accordance with the planned target (or on the basis of a plan constructed earlier which is subject to improvement), 5000 quintals of wheat, 3500 quintals of rye and 5000 quintals of oats must be produced. A cropping plan is to be drawn up which will ensure the completion of the task with the least expenditure of labour.

TABLE 31. FERTILITY AND LABOUR REQUIREMENTS FOR VARIOUS CROPS BY PLOT

Land	Area (in hectares)	Crop	Yield (in quintals per hectare)	Labour requirements (in days per hectare)
More fertile	100	Wheat	30	10
		Rye	25	8
		Oats	28	7
Ordinary	200	Wheat	20	10
		Rye	20	8
		Oats	26	7
Poorer	300 (and more)	Wheat	15	10
		Rye	15	8
		Oats	25	7

In solving the problem of allocation of land under cultivation the basic criterion will be the saving of labour which may be attained by raising a certain crop on a given plot of land as compared with another crop (compare Conclusion 19). Let us devise an optimal cropping plan by the following reasoning.

† Instead of expenditure of labour we could take cost. In the given example all the expenditure is expressed in man-days in order to reveal more distinctly the meaning of o.d. valuations and their agreement with labour expenditure.

If in the cultivation of land other expenditure is incurred besides labour, for instance on fertilizer, we suggest that this expenditure expressed in labour should be added to expenditure on labour. All the numerical data (fertility, expenditure on labour) are arbitrary as in other examples.

Of course, similar calculations may be applied to other crops—food-crops, fodder and industrial crops (maize, sugar-beet, cotton) and also to problems of the rational utilization of larger areas of land (a group of districts, provinces).

production method may exercise on expenditure on other types of production. This influence is operative in the expenditure or engagement of factors of production which include scarce and more productive natural resources, in the present instance in the utilization of the more fertile and ordinary land; such expenditure remains unaccounted. Meanwhile, these factors should also receive definite valuations and their use should influence the allocation of expenditure on production.

The calculation in the latter situation (consisting essentially in the computation of the differential rent and its incorporation in expenditure) makes it possible to obtain o.d. valuations for the present problem.

From the foregoing it is clear that in the given example the more efficient natural resources, the more fertile and ordinary land, should also receive a definite o.d. valuation. The poorer land will have a zero valuation as this factor is available in excess (compare Appendix I, Theorem 3).

Moving upwards in Table 32, the o.d. valuation is found (expressed in man-days) per quintal of each crop and per hectare of the more fertile and ordinary land.

To produce 25 quintals of oats 7 working days are required (see Table 32); in the valuation of oats, expenditure in terms of more fertile land does not enter as oats are sown only on poorer land, and for this reason the valuation of 1 quintal of oats will equal  $7:25$  days. The valuation of 1 quintal of rye on poorer soil will amount to  $8:15 = 0.533$  of a day.

One hectare of ordinary land produces 20 quintals of rye which is consistently valued at  $20 \times 0.533 = 10.67$  days, while direct expenditure amounts to 8 days. Thus, the use of 1 ha of ordinary land results in a saving of labour of  $10.67 - 8 = 2.67$  days. This figure must be taken as a valuation of the use of 1 ha of ordinary land.

One hectare of ordinary land produces 20 quintals of wheat. To the apparent expenditure of 10 days must be added the valuation of 1 ha of ordinary land (2.67 days). The figure obtained—12.67 days/ha—represents the full cost of production of 20 quintals of wheat on ordinary land. Hence the o.d. valuation of 1 quintal of wheat equals  $12.67:20 = 0.633$  of a day.

Thirty quintals of wheat obtained from 1 hectare of the more fertile soil are valued at  $30 \times 0.633 = 19$  days. The visible expenditure

amounts to 10 days, therefore the valuation for the use of 1 ha of the more fertile land equals  $19 - 10 = 9$  days.†

We have obtained valuations for the present plan. We shall now ascertain whether the plan is optimal. For this purpose we shall compare total expenditure (including rent) in the production of each crop. The results of the simple calculations are given in Table 33.

TABLE 33. COST STRUCTURE WITH RENT INCLUDED

Crop	Land	Yield in (quintals/hectare)	Cost of labour per hectare (in days)			Cost of labour per quintal (in days)		
			Direct	Indirect (rent)	Total	Direct	Indirect (rent)	Total
Wheat	More fertile	30	10	9	19	0.333	0.3	0.633
	Ordinary	20	10	2.67	12.67	0.5	0.133	0.633
	Poorer	15	10	—	10	0.667	—	0.667
Rye	More fertile	25	8	9	17	0.32	0.36	0.680
	Ordinary	20	8	2.67	10.67	0.4	0.133	0.533
	Poorer	15	8	—	8	0.533	—	0.533
Oats	More fertile	28	7	9	16	0.25	0.321	0.571
	Ordinary	26	7	2.67	9.67	0.269	0.103	0.372
	Poorer	25	7	—	7	0.280	—	0.280

From this table it is apparent that in the plan those methods are used for which the total expenditure on the production of 1 quintal of each crop is minimized (the corresponding data in the table are in heavy type). For the methods used in the plan, total expenditure equals the valuation of production, but for those methods which are not used it is higher than the valuation shown.

Hence one can state with certainty that the plan constructed above is optimal and the valuations obtained from it are o.d. valuations (see Conclusions 11, 12, and 14).

† In these simplified arguments rent of land was calculated on the basis of the yield of the individual crops. In fact, the efficiency of using a given piece of land is determined not by a single product but by the totality of products under the given system of management.

Let us note that the proposed accounting methods may also be applied in the calculation of rent under more complex conditions (see Appendix I).

It is clear that if only direct expenditure were considered (see column five of Table 33) this would lead to confusion in the various economic accounts. Thus, in many cases wheat would receive a lower valuation than rye; judging by the valuations of visible expenditure (if production is evaluated on the basis of average costs), the cultivation of poorer soil is not profitable at all. Let us explain with some further examples the importance of obtaining a valuation of the use of more fertile land, and of limited natural resources in general, that is, of differential rent.

Let us consider the following problem. Assume that with the conditions of our example it is possible to increase the yield of wheat on the more fertile land by 2 quintals/ha as a result of more intensive working of the land with a 10 per cent increase in the expenditure on labour. Is it advisable to introduce the new method?

The calculation of the direct expenditure on labour leads to a negative answer. In fact, an increase in the expenditure on labour by 10 per cent gives an increase in output of only  $2/30 = 6.7$  per cent—labour productivity decreases.

The calculation should be carried out on the basis of the valuations obtained by allowing for rent (see Conclusion 15). The additional output of 2 quintals of wheat is valued (in man-days) as  $2 \times 0.633 = 1.267$  days. A comparison with the increase in labour expenditure (1 day) shows that the new method produces a saving of labour, and its application appears economically justified. Let us once more emphasize that a calculation without allowing for rent would turn against the introduction of the more intensive method of cultivation.

Let us consider a second problem. Under the conditions of our example, a certain quantity of grain (wheat or rye) has to be allotted for consumption by the farms themselves (feeding). What is more advisable to use if wheat is 10 per cent more effective than rye (if 10 quintals of wheat are equivalent to 11 quintals of rye)?

The calculation on the basis of valuations allowing for rent shows: a valuation of 10 quintals of wheat would amount to  $0.633 \times 10 = 6.33$  days; a valuation of 11 quintals of rye  $0.533 \times 11 = 5.87$  days. It is more advisable to use rye for the given purpose as this provides a saving of labour.

If calculations were carried out with valuations of direct and visible expenditure only, this again would lead us to an incorrect

result. Ten quintals of wheat would be valued at  $0.4 \times 10 = 4$  days; 11 quintals of rye at  $0.46 \times 11 = 5.06$  days. The cost of wheat would prove lower than that of an equivalent amount of rye and the choice of wheat would appear more advantageous.

From the foregoing it is clear how essential the role of rent is in economic analysis. The calculation of rent makes it possible to solve the problem of the allocation of the more efficient natural resources. Rent must also be taken in consideration when the full cost of production is determined.

All the above considerations enable us to arrive at the following conclusion.

**CONCLUSION 20.** In solving problems of the use of natural resources which are more efficient but in short supply, their use must be determined by allowing for differential rent. The magnitude of the latter is determined by the saving of labour obtained from the use of these resources in the optimal plan. If rent is included in expenditure, then the principle of least cost is observed in the optimal plan. The amount of the rent should be allowed for in determining o.d. valuations of production.

Let us note that rent possesses the same properties and applications as other types of o.d. valuations.

Natural resources which are in short supply but are efficient may include not only land as in the above example, but also forests, water reservoirs for irrigation and fishing and deposits of useful minerals.

A correct calculation and a systematic account of the rent will make the most expedient use of the natural resources possible and prevent their improper, incomplete and irrational use. Moreover, the calculation of the rent in the valuation of production equalizes, first of all, the conditions of production for various resources, ensuring profitability wherever production is rational; secondly, it furnishes a relatively higher valuation of the kinds of output, which use scarce and more favourable natural resources, and promotes the most efficient use of such outputs.

Consequently, the calculation of the rent should play an important role in questions of price formation.

Meanwhile, any likely increases in the price of some products are fully offset by receipts obtained by society in the form of rent. In this



manner, calculation of the rent will only lead to a reallocation of the means of production among the various constituents of the national income. At the same time the inclusion of rent in economic accounting and in the working out of economic indices will produce the most correct solution of economic questions about the use of such natural resources considered from the point of view of the whole community.

For instance, the collection of rent, in one form or another, of land and of irrigation waters in the southern districts, with an appropriate increase in the fixed prices of cotton and other industrial crops, of fruit (without a change in retail prices), and without reduction in the budget, would create favourable conditions for rational and intensive use of these lands, would stimulate the growth of production as well as the more rational use of the outputs mentioned; it would result in an increase in the income of the *kolkhozes* and would also create ultimately the conditions for a reduction in the price of these products. The calculation of the rent would also reveal more fully the economic superiority of crops whose production is dependent upon a more intensive use of land. cotton, rice, maize.

Calculation of rent would show the advantage in costs of grain grown on virgin soil better than ordinary economic calculations. For this reason, such an account shows a greater efficiency of investments for the development of virgin lands (compared with ordinary accounting), justifies decisions made by the Communist Party in 1953-4 regarding the development of virgin lands.

Now that this has taken place, the rent obtained from them in whatever form presents a substantial portion of accumulation by the state.

Thus, a rent from forests and from fishing reservoirs would prevent their uneconomic use. Of great importance is the calculation of rent in mining and other sectors of the extracting industries.

It should be emphasized that a socialist differential rent is of an entirely different nature from capitalistic rent.

A capitalistic rent is a form of income from exploitation—a portion of the surplus value appropriated by the owner, which can conflict with other costs.

In a socialist society, rent represents a portion of social expenditure on labour and a portion of the social product which belongs to the country as a whole and does not conflict with other components.

It stands out from the total as a specific form which—for reasons given above—should be accounted for separately so as to ensure the most efficient use of natural resources.

Great importance is attached at present to this latter task. “Within the coming seven years there will be a marked improvement in the use of land as the principal factor of production in agriculture.”†

In a capitalist society private ownership of natural resources and rent can be an obstacle to their right and efficient use. In a socialist society it is possible to utilize them in the best and most efficient manner; the inclusion of rent in economic accounting is the means of ensuring rational use of these resources. And, conversely, failure to account for rent may lead to waste of natural resources, and to their use in a haphazard, rather than optimum manner.

The magnitude of capitalist rent is determined spontaneously in the market. A socialist rent must be determined and allowed for in the process of economic planning. Its magnitude must be established in such a manner as to ensure the fullest use of natural resources.

The hire valuation considered in the preceding paragraph represents in itself essentially a specific form of differential rent—a rent from equipment. The difference consists in that equipment, as distinct from natural resources, can be reproduced. However, this difference becomes only apparent in long-term planning; in short-term planning reproduction of equipment within a short time is not possible and for this reason the difference is not obvious.

### *Analysis of Labour Expenditure*

The example above dealt with the calculation of expenditure of labour for various types of agricultural production.

In the process of analysis the important fact emerged that expenditure depends upon the conditions of using labour. For instance, expenditure on 1 quintal of rye grown on ordinary and poorer land is in a relationship of  $15:20 = 3:4$  (Table 32), that is, in terms of efficiency a working day on the poorer land in the growing of rye corresponds to  $\frac{3}{4}$  of a day on ordinary land.

In the above calculation of labour expenditure on various types of production, productivity of a unit of labour on poorer land was

† Target figures for the development of the national economy of the U.S.S.R. for the years 1959–65. *Report of the XXI Congress of the Communist Party of the U.S.S.R.*, vol. II, p. 493.

adopted as the unit—that is, when favourable natural resources could not be secured for labour. In general, it would be proper to adopt as a basis of calculation work performed under average conditions. As no sufficient data are available, we shall adopt as a unit a working day under average conditions for a given community. However, the general approach in converting labour to an average will be explained as we proceed.

The total valuation of production in the arbitrary units adopted (a working day on poorer soil) amounts to

$$5000 \times 0.633 + 3500 \times 0.533 + 5000 \times 0.280 = 6430,$$

and the general expenditure of labour amounts to 5000 days. Thus, expenditure of (average) labour per conventional unit amounts to  $5000 \cdot 6430 = 0.778$  of a day.

Consistent with this, expenditure of average labour per unit production constitutes:

For 1 quintal of wheat  $0.633 \times 0.778 = 0.492$  of an average labour day

For 1 quintal of rye  $0.533 \times 0.778 = 0.415$  of an average labour day

For 1 quintal of oats  $0.280 \times 0.778 = 0.218$  of an average labour day

The valuations of production of these three crops have now been expressed in average labour, but it is evident that they maintain their previous ratios. It is not difficult to show that they are not only expressed in terms of labour, but that they correspond in fact to that expenditure of average labour which is necessary to achieve production under the given conditions. In other words, these valuations of production are fully consistent with the labour theory of value. Of course, when considering the estimate of labour outlays on production in complex conditions, it is necessary to account for the interdependence of the production of various types of output and thus the interdependence of the necessary expenditure.

The correctness of the method adopted for the calculation of expenditure on production and of the valuations obtained was already evident as these valuations conform to a rational production plan and lead to the right solution of various problems of economy of labour. In order to explain this method more fully and to justify it, we shall analyse those particular features of the calculation of labour expenditure which have been adopted in the assessment of expenditure.

(1) *The calculation of labour expenditure (and of the valuation of production) is based on the methods of cultivation adopted in the optimal plan, that is, it takes into consideration the necessary (the minimum, realizable and rational) expenditure of labour (in terms of the national economy—the socially-necessary expenditure).*

(2) *Allowing for the fact that labour productivity depends upon the conditions of its use, it is very important, if labour expenditure deviates from the average favourable social conditions, to take this fact into consideration in the calculation and to convert expenditure to the average favourable conditions. In the given analysis such conversion to average labour was actually carried out in the conditions of the problem considered.*

Once expenditure on production has been calculated it is easy to obtain the value of the *coefficients*† which convert actual to average expenditure of labour:

when wheat is produced on more fertile land

$$k_1 = 0.492: \frac{1}{3} = 1.476$$

when wheat is produced on ordinary land

$$k_2 = 0.492: \frac{1}{2} = 0.984$$

when rye is produced on ordinary land

$$k_3 = 0.415: \frac{2}{3} = 1.038\ddagger$$

when rye (and oats) are produced on poorer land

$$k_4 = 0.778$$

Thus, it appears that the value produced at a given production unit is not determined by direct expenditure on labour at that unit, but that it may be higher or lower according to the extent to which labour conditions there differ from average conditions and the extent to which production factors are favourable to labour. The use of

† The coefficient which converts individual expenditure of labour to average labour cost is determined as a ratio of the calculated average outlays of labour and the magnitude of direct expenditure of labour under given actual conditions. It is clear that this coefficient is greater than 1 when labour is spent under more favourable conditions than the average, and less than 1 under less favourable conditions. In the above account, the magnitude of labour outlays under the conditions given is taken from Table 32. (This may easily be found from the first and second columns of that table.)

‡ The conditions of production of wheat and rye on ordinary land are not equally favourable everywhere: in the first example, one man-day is related to  $\frac{1}{16}$  ha of ordinary land, in the second one to  $\frac{1}{8}$  ha.

However, such reallocation of production usually takes place for a single product. Under socialist conditions of production (which is unified) it appears correct and in accordance with the scheme described to give reality to such accounting of the difference in the conditions of labour utilization, and to achieve a reallocation of value also for several interdependent products such as are characteristic of present-day industry.

(3) Considering that socialist social production is a single whole, one may attempt to determine directly *social expenditure on a given product as the expenditure of labour which is necessary to make one unit of a product under given conditions*. Such an approach, as may be shown, leads to the same values of expenditure as those obtained with the methods developed above.

Let us demonstrate this on the above example of a production unit. Let us, for instance, calculate expenditure of average labour necessary for the production of 1 quintal of rye.

Let us increase labour resources by one per cent, i.e. 50 days. In order to maintain the same labour conditions it is necessary to provide for an equivalent increase in the associated factors, an increase in resources: by 1 hectare for the more fertile land and by 2 hectares for the ordinary land. What increase in the production of rye could be achieved as a result? On 2 hectares of ordinary land with an input of 16 days we obtain  $2 \times 20 = 40$  quintals of rye. On 1 hectare of more fertile land 25 quintals of rye could be obtained, yet as was shown its use for the cultivation of rye was not rational. It is more rational, with an outlay of 10 days, to obtain 30 quintals of wheat. This would save 1.5 hectares of ordinary land and 15 working days from the cultivation of wheat. Spending 12 days on the production of rye on this ordinary land, we obtain  $1.5 \times 20 = 30$  quintals of rye. Finally, if the remaining supplementary sources of labour  $50 - 16 - 10 + 15 - 12 = 27$  days were utilized on poorer land, then by cultivating  $27 \div 8 = 3.37$  ha, we would obtain additionally  $15 \times 3.37 = 51$  quintals of rye, a total of  $40 + 30 + 51 = 121$  quintals. Consequently, expenditure on 1 quintal will amount to  $50 \div 121 = 0.415$  days of average labour. In other words, we shall obtain the same figure as above.

Thus, we can see, that the *values of necessary expenditure of labour obtained from calculating o.d. valuations is nothing else than the expenditure of the production unit as a whole and not individual or*

sectional expenditure. But this type of expenditure is decisive in the analysis of problems relating to the allocation of social labour under conditions of unified socialist production. At a *kolkhoz* total results are important rather than that one group should reach higher results than the others. In an economic district, the progress made by the district as a whole is decisive and the high indices of one factory can in no way be satisfying if they are attained at the expense of others.

(4) Finally, without going into further detail (since the relevant analysis has already been carried out above), let us note that the amount of this necessary expenditure may also be obtained by calculating expenditure at a given production unit, provided it is rational and not only visible and direct expenditure of labour is accounted for but also indirect expenditure, so as to reflect the use of factors which save labour. Such an account is given in Table 33; in this table, indirect expenditure is shown as the calculated efficiency of more fertile sections of land (rent). In this table a working day on poorer land was adopted as the conventional unit. A similar calculation could be carried out in average working days; for this purpose, the valuations of ordinary and of more fertile land would have to be converted to average working days.

We should obtain:

using 1 ha of ordinary land (rent)

$$2.67 \times 0.778 = 2.077 \text{ average working days}$$

using 1 ha of more fertile land (rent)

$$9 \times 0.778 = 7 \text{ average working days}$$

In such a calculation indirect expenditure on labour would have to be converted to average labour by using again the conversion coefficient  $k_4 = 0.778$ , and taking into consideration that the unsupported labour corresponds to labour on poorer land. Then, for instance, expenditure per quintal of rye on ordinary land would amount to  $(8 \times 0.778 + 2.077) \div 20 = 0.415$  of an average working day, i.e. to the same value as before.

Let us note that in this case the more fertile land (as compared with the poorer one) appeared as a factor that saves labour, the amount of such land being limited; in other cases, as was shown (Section 5), these factors may not be natural resources but rather

the use of equipment in limited supply, the valuation of which will amount to its hire valuation or rent from equipment.

The calculation of these factors is necessary in order to arrive at a correct determination of their use, at a full valuation of costs of production and correct allocation of the latter among the various types of production.

In conclusion, the treatment of the quantitative application of the theory of value developed here is not the only one that can be applied in a socialist society. The valuations of production as obtained here are the result of calculating the necessary divergence from costs or of some form of standardized costs. However, it is indisputable that only by fully calculating the cost of labour, by taking into consideration the conditions of using labour or the calculation of the factors that save labour, can one obtain valuations with which the problems of labour allocation can be solved correctly. There is no doubt that such an objective method of calculating could hardly disagree with the law of value when this is correctly applied in a socialist society.

The analysis in Sections 5 and 6 of the importance of the use of equipment and natural resources in production and their inclusion in expenditure and the valuation of production may on superficial acquaintance remind one of certain theories of vulgar bourgeois schools of political economy about the three equally important sources of value: labour, land and capital. The radical difference lies in the fact that the construction developed here is in full agreement with the labour theory of value, labour being considered the only source of value. Natural resources and equipment appear only as factors influencing productivity of labour and the saving of labour. For this reason, the calculation of expenditure of these factors and their valuations must be looked upon only as means for an optimal allocation of labour in order to attain its highest productivity, and also as a basis for the comparison of labour costs incurred under various conditions. Thus, these factors can in no way be independent sources of value. Only their indirect effect on the productivity of the labour force is included in the calculation.

**Section 7. Planning of Transport. Production Problems  
connected with Transport. The Appropriate  
Railway Tariff**

It is known that the transport of products plays an important role in economic problems. Varying conditions of production at different points, depending upon the location and quality of the sources of raw material, the supply of stocks of equipment as well as other circumstances make it necessary to move output produced at one point to another. Here the best use of transport is of very great importance. This is particularly so in Russia owing to the great diversity of natural conditions and great distances on the one hand, and the shortage of means of transport and their overloading on the other.

If it were not necessary to take into consideration the limited capacity of railways and rolling stock, and also the limited productive capacity of factories at any given moment, then the solution of economic problems concerned with transport would present no difficulty. Let us, in fact, assume that the cost of transport from point A to point B amounts to 150 roubles per wagon. Then, if at point A the cost of some product amounts to 500 roubles per wagon† and at point B 250 roubles, and if production capacity at B is not fully utilized, it is advisable to obtain the requisite quantity of this product from B instead of producing it at point A since the saving of expenditure on production ( $500 - 250 = 250$  roubles) exceeds expenditure on transport (150 roubles). But if the cost of this product at point B amounted to 350 roubles this would no longer be advisable.

If there is also a point C where the cost of a wagon amounts to 150 roubles and the cost of transport to A 200 roubles, it is clear that it is more advisable to supply A from C than from B as the cost of a wagon of products at point A in the first instance will amount to  $150 + 200 = 350$  roubles, in the second instance  $250 + 150 = 400$  roubles. However, if transport capacity and volumes of production are limited, the solutions obtained in this manner will prove impracticable and useless. The quantity of all loads, the transport of

† For the purpose of calculation a wagon has a rated capacity of 16.5 tons throughout.



which may be advisable on the basis of such calculation, can by far exceed the capacity of the railways. The requirements of a product at those points of consumption which from the calculations should be supplied from a given point of production may considerably exceed available production there.

Thus, under real conditions, the simpler methods described above are not applicable and any attempt to use them may only lead to confusion. The proper method of solving such problems will be demonstrated by us on two simple examples.

If railway transport is in great demand, it is impossible to solve problems of its utilization simply by starting from costs and yields calculated in the usual way. However, this should not force one to give up economic calculation; but it must be done differently. The aim of the calculation must be the choice of a solution which would ensure the smallest possible losses to the economy arising from overstrained transport. We shall consider the basic idea of such a method in the following example.

EXAMPLE 1. Let us envisage a railway line linking points A and B, where transportation is carried out only over the whole distance. The capacity of the line is 1200 wagons in 24 hours. The cost (and the tariff for simplicity's sake is taken as equivalent to it) is 200 roubles per wagon.

The volume to be transported is 2000 wagons per 24 hours (*see* Table 34) which considerably exceeds capacity. We shall divide these loads into three categories: (1) *Unconditional*—requiring the full supply of wagons for loading; (2) *Variable*—required to be moved without fail, but admitting (at a constant cost) a reduction in the number of wagons needed for transportation. Timber may be quoted as an example of such a load under the following conditions. If the timber cannot be moved from A to be sawn up at B, but may be transported after sawing, the number of wagons necessary could be reduced by 40 per cent, but this would entail a considerable increase in expenditure on the working of the timber, say, because at A there is only an overworked, badly equipped sawmill, while at B there is one underworked and fully mechanized. As a result, savings for each wagon and load moved will amount to 400 roubles; (3) finally, there are *conditional* loads which are not absolutely needed, for which transport need not be supplied, although here too considerable losses are incurred (for instance the substitution of a more

expensive material for some raw material). All the loads are shown in Table 34.

TABLE 34. LOADS REQUIRING TRANSPORTATION

Loads	24-hourly requirements (wagons)	Possible economy (wagons)	Losses per wagon saved (roubles)
Unconditional 1st type of load	500	—	—
Varying 2nd type of load	200	80	300
3rd type of load	300	150	250
Conditional 4th type of load	100	100	1500
5th type of load	500	500	500
6th type of load	400	400	250
Total	2000		

As may be seen from the table, the losses per wagon saved in all cases substantially exceed the cost of transportation (200 roubles) and therefore the lack of means of transportation for conditional loads entails great losses; and in the case of variable loads an economy in wagons is not advantageous. In spite of the fact that in all cases transport is profitable and the demand for wagons entirely legitimate it is clearly impossible to meet this demand fully. The magnitude of economic losses (unavoidable in the given conditions) owing to the lack of wagons will depend upon how the available rolling stock is allocated.

In the first of the plans quoted (Table 35) transportation is fully provided for loads which have to be transported; among these are the variable loads. The remainder made it possible to meet the other demands up to 200 per cent, which was done. The total losses of undertakings as a result of lack of wagons amounts to 400,000 roubles per 24 hr. In Table 36 the optimal allocation plan is shown. As may be seen, the losses amount here to 246,500 roubles: they have been reduced by 38 per cent.

The latter plan may be found by determining an o.d. valuation (under the given conditions) of conveyance of one wagon between

points A and B. This valuation is obtained by the method used in the preceding sections and equals 500 roubles. Taking this as a basis we arrive at the optimal plan. In those cases where lack of wagons entails expenditure exceeding 500 roubles for the saving of one wagon (4th load, *see* Table 34), wagons are fully supplied. Where expenditure per wagon saved is less than 500 roubles, all possible economies in wagons can be made (2nd, 3rd and 6th loads, Table 34). Finally, where the losses per wagon saved equal the valuation—500 roubles (5th load), those wagons yet to be utilized can be made available.

TABLE 35. ARBITRARY PLAN

Loads	Available wagons	Losses due to lack of wagons (roubles)
1	500	—
2	200	—
3	300	—
4	20	120,000
5	100	200,000
6	80	80,000
Total	1200	400,000

TABLE 36. OPTIMAL PLAN

Loads	Available wagons	Losses due to lack of wagons (roubles)
1	500	—
2	120	24,000
3	150	37,500
4	100	—
5	330	85,000
6	—	100,000
Total	1200	246,500

It should be stated that in the given conditions it is not possible to adopt a valuation other than that of 500 roubles. If a valuation exceeding 500 roubles were adopted, then proceeding as above, the wagons would not be fully utilized. If a valuation below 500 roubles were taken, then the demand for transport facilities would exceed available capacity—once again it would not be possible to make the best choice. Only the o.d. valuation makes it possible to arrive at the optimal plan.

This o.d. valuation—500 roubles—differs from the cost (200 roubles) in that the latter (if it has been correctly determined by including the o.d. valuations of materials and fuel) does not take into consideration the hire valuation (rent) of the railway system, equipment and rolling stock, or that it accounts for this only to an insignificant extent. Thus, the rent in the given example calculated per wagon transported should amount to  $500 - 200 = 300$  roubles. On a more thorough analysis—for which there is no need at present—this sum could be split into portions corresponding to: (1) the

permanent way and equipment of the railways, (2) locomotive stock and (3) rolling stock.

Starting from o.d. valuations of transport it is possible to obtain with comparative ease a valuation of the measures that would enable one to achieve an economy in the use of transport and, in addition, transport measures leading to increased capacity of railways in the light of their ultimate advisability for the economy as a whole. Such a valuation enables one to establish whether the given measures will result in a reduction in the losses of the national economy due to a shortage of transport. Here are a few such calculations.

(1) The weighing of a load before it is put into a wagon presents a labour consuming operation without mechanized means of weighing. Let us assume that its cost would amount to 15 roubles per wagon. Weighing makes it possible to send off fully loaded wagons and thereby to reduce the number of wagons used by 5 per cent. Is mechanization advisable?

One wagon in twenty will be saved; expenditure needed to save this wagon will amount to  $19 \times 15 = 285$  roubles which is lower than the o.d. valuation of transport (500 roubles). Thus, mechanization is advantageous from the point of view of the national economy although of disadvantage to the undertaking producing it which for an expenditure of 285 roubles will only realize a total saving of 200 roubles per wagon.

Of course, the introduction of such a measure may be recommended only in the case where it is possible to provide a sufficient number of workers for loading so that no wagons remain standing idle.†

(2) For an "unconditional" load, provision of special railings and flaps for loading will increase the capacity of the wagons by 50 per cent. The cost of such equipment would be 150 roubles per wagon: afterwards it becomes worthless. Is it advisable to use such additional equipment?

The load from the three wagons could be placed in two. Their equipment will cost  $2 \times 150 = 300$  roubles. Expenditure per wagon saved amounts to 300 roubles (less than 500); consequently, the measure is advisable.

Of greater advantage should be an increase in the carrying capacity

† A typical example of this kind was quoted in the press: incomplete loading of wagons with grain results in losses amounting to 130 million roubles (see the *Promyshlennno-ekonomicheskaya gazeta*, 18 September 1957).

of wagons which was achieved by a decrease in the weight of a wagon through the partial use of modern materials such as duraluminium and plastics instead of ferrous metals and wood in its construction. To all appearances, in spite of the high cost, their use in the given instance may prove justified economically if in the calculation allowance is made for the o.d. valuation of transport and the intensive use of such wagons on the fully used main lines is taken into account. The same applies to the use of wagons carrying larger loads.†

(3) A factory at town A uses ballast from the sand quarry situated near town B (requiring 20 wagons in 24 hr). The cost of this high quality sand is 100 roubles per wagon at B. Near A it is possible to obtain only low quality sand and the quarrying of this is much more complicated. The operations necessary to ensure the possibility of its use (washing, etc.) would result in a cost of about 300 roubles per wagon. In addition, capital investment amounting to 800,000 roubles is necessary (for the building of an access road, etc.). Is it advisable to forgo the use of the quarry at B?

Expenditure per wagon saved amounts to  $300 - 100 = 200$  roubles. If the valuation is 500 roubles, the economic saving will be equal to  $500 - 200 = 300$  roubles per wagon, and  $20 \times 30 = 600$  wagons per month constitute a saving of 180,000 roubles. Thus, capital investment will be repaid in a little over four months. It follows that the measure is advisable.

(4) Ore is an "unconditional" load. Mechanical dressing on the spot (for instance, sizing) reduces weight by 10 per cent. The cost of dressing amounts to 30 roubles per wagon. Thus, the cost of saving one wagon amounts to 300 roubles. A comparison with the o.d. valuation of transport (500 roubles) shows that this measure is advisable from the national economic point of view although its operation may cause some increase in the cost of ore.

In all the examples quoted it was possible to reduce the transport load and this was in the interest of the national economy. At the same time, since on many busy main lines the existing railway tariff is too low, putting these measures into operation would increase costs and would consequently not be to the advantage of the factories which are in a position to carry them out. In fact, a factory which succeeds in meeting its transport requirements does not take them if they are not to its advantage.

† See the *Promyshlennno-ekonomisheskaya gazeta*, 18 July 1958.

For instance, supplies of moulding sand are being transported although sand is available locally, while frequently the lack of wagons entails high losses elsewhere.

The calculation of an o.d. valuation of railway traffic in economic accounting, and in particular a railway tariff that approximates this valuation, will help a better use of other types of transport. Such a calculation would render it economically advisable for factories to utilize those other means of transport where they are advantageous (waterways—where available—and mixed transport, or motor transport over short distances); this would at the same time exclude the use of those means of transport which are clearly inadvisable (water transport for perishable freight, motor transport over long distances). Furthermore, this calculation would make it possible to solve the problem of choosing the right type of transport in a practical manner, taking into account both the national economic interest and the interests of the factory.

The existence of an improved system of tariffs is particularly important at present when many construction plans are moved to other countries. It is therefore very important that the requisite economic indicators be available to establish these relations correctly in agreement with the public interests.

(5) At a factory in town A, castings for the state being made with unsuitable equipment costs 160 roubles per ton. If the metal were directed to a factory in town B where the foundry is better equipped and suitable for making this type of casting, the cost would amount to 130 roubles per ton. Is it advisable to send the metal for casting at the factory in town B?

For a wagon the loading capacity of which is 16.5 tons, the saving would amount to  $16.5 \times (160 - 130) = 495$  roubles. The expenditure for the round trip of a wagon would be  $2 \times 500 = 1000$  roubles (on the basis of the o.d. valuation). The change is not advisable in these conditions.

This is a typical example. When correct calculations of transport costs are made, the co-operation of factories needing additional transport of heavy loads over long distances appears frequently inadvisable. As a result, the narrow specialization of factories (in particular of rolling mills) is often not advisable when it is calculated on the basis of such co-operation and involves transport over long distances. The incorrect calculation of such aspects of transport was

one of the principle causes of unnecessary and superfluous specialization, instances of which have been noted already at the XVIII Congress of the Communist Party of the Soviet Union.

In many other cases, however, specialization and co-operation are fully justified in spite of the fact that this entails an increase in the load of transport. To solve this problem it is always necessary to have a correct national economic valuation of transport costs.

(6) In town A there is a soap works operating on a large scale. The cost of manufacturing a cake of soap amounts to 50 kopecks.

There is a proposal to set up a handicraft soap works in town B (for the supply of the town and district) so as not to overload transport. In other words, it is proposed to produce soap on the spot instead of sending the raw material to the factory at town A. The cost of manufacturing a cake of soap by methods almost amounting to handicraft would amount to 1 rouble. Is it advisable to organize production in such a way?

The weight of a cake of soap is 100 g. Allowing for packing, it may be assumed that the capacity of a wagon will represent 100,000 cakes. The difference in their cost will be 50,000 roubles. The conveyance of a wagon in both directions (according to the valuation) is 1000 roubles. Conclusion: the organization of a handicraft soap works at B is inadvisable. Thus, if conveyance of the product and materials does not involve overloading of transport, production costs should be of decisive importance in the choice of location of production. In particular, use should be made of the advantage of reducing costs which mass production and specialization offer.

In spite of the undoubted advantages of specialization and concentration of production, such measures were not always taken for various reasons. Frequently this was justified by the load on transport and by the difficulty of transportation over long distances, especially when this involved consumption goods.

Failure to allocate or to use transport in the case of such valuable goods can in no way be justified. This can only be explained by the absence of any sense of proportion. The reasoning is approximately as follows. What is more important, coal or buttons? Of course, coal. This means that a wagon of buttons should be allocated second rank of importance. But meanwhile one forgets that in order to supply a large town with buttons one wagon per year would suffice,

for coal thousands of wagons are necessary and if it was a question of one wagon of coal this could easily be replaced by wood.

It is advisable, therefore, to refer to another calculation which is evidence of the justification for our conclusion.

Let us assume that the problem is to transport loads of a high cost per unit weight both of producer goods and of goods widely consumed. The supply is of great importance to the country: for the normal continuity of production ensures the vital requirements of the population, and furthers the mobilization of the population. At the same time, the number of wagons necessary for the transport of goods of this kind is insignificant.

Let us consider expensive products the price of which is not below 10 roubles/kg (including packing and, for light loads, allowing for the volume taken up). On the average, their price may be assumed to equal 15 roubles/kg or about 250,000 roubles per wagon. Then, even if the aggregate cost of this type of load available for transport over a year was assumed to amount to 30 billion roubles, the number of wagons for loading this every 24 hr (in the whole Union) would amount to 300 ( $30,000,000,000 \div [250,000 \times 360] = 330$ ). Therefore, even reducing this type of transportation by 50 per cent would result in heavy losses to the country, but would in fact not alleviate the operation of railway traffic to any noticeable extent (150 wagons represent approximately 0.2 per cent of one day's load).

In view of this, it would be inadvisable not to allocate transport facilities to the categories of costly loads such as, for instance, small instruments, dyes and varnishes, footwear, cosmetics, books in small editions.

(7) Carrying out certain costly changes on the railways (improving communications, increasing the staff of inspectors, speeding up traffic as a result of automation), involves an increase in the average cost of transport to 210 roubles per wagon, but increases the capacity of the railway to 1300 wagons per day. Should these changes be made?

The cost of transportation of an additional 100 wagons will amount to:  $1300 \times 210 - 1200 \times 200 = 33,000$  or 330 roubles per wagon while the valuation is 500 roubles per wagon. Consequently, it should be realized that it is advisable from a national economic point of view to introduce such changes.

This, of course, does not mean that there is no need to reduce costs of railway transport. It is necessary to find measures which



would make it possible to reduce costs but would not affect the capacity of the railway; such possibilities are very important.

In this connection it is proper to turn our attention to the economic advantages of large scale conversion to diesel-electric and electric locomotive traction contemplated in the Seven-Year Plan which become even more striking when the o.d. valuations of railway transport are included in the economic calculations, as the use of such traction results in a sharp increase in the capacity of the railways owing to higher speeds and faster turnover of the rolling stock (how this affects economic efficiency is clear from the calculations in the example quoted). Usually, through economic valuation of transport, it is possible to solve more correctly various problems connected with the introduction of technically advanced means of traction (choice of the type of traction, order of application, etc.).

(8) Let us also consider the following problem. In the vicinity of B coal of high calorific value is mined and its cost is 1500 roubles per wagon; this coal is being used at A. Would it be advisable to replace it by electricity generation low-calorie coal mined in the neighbourhood of A, considering that the cost of the calorific equivalent of the latter will amount to 1800 roubles? On the basis of the fixed railway tariff this would appear inadvisable, but if we allow for the o.d. valuation of transport, we see that the cost of imported coal which must be valued at  $1500 + 500 = 2000$  roubles exceeds expenditure on local coal (1800 roubles). It is more advantageous to utilize local coal.

Let us now assume that the problem is the railway itself. The section of this railway in the vicinity of A uses 20 wagons of imported coal daily. Is it advisable to turn to local coal for the running of locomotives? We must take into consideration that, as a result, owing to the reduced speed of the locomotives, the increase in time of getting fuel, etc., the capacity of the railway will be reduced by 5 per cent or 60 wagons. Let us calculate the effects of such a change. The saving resulting from the substitution of local coal for imported coal will amount to:

$$20 \times (2000 - 1800) = 4000 \text{ roubles.}$$

The loss of the hire valuation of 60 wagons owing to reduced capacity of the railway will equal:

$$60 \times 300 = 18,000 \text{ roubles.}$$

Such substitution is clearly not advantageous under these conditions. As far as one can see, it is rarely advantageous.

It is evident that in practice it is generally quite realistic that even an increase in consumption of fuel and with it an increase in the cost of transport may prove advisable if this increases the limited capacity of a line in heavy demand; and for such a line, for instance, electric traction may prove more efficient than diesel-electric traction even if this should entail a higher consumption of fuel.

Let us formulate the results obtained as follows.

**CONCLUSION 21.** If the given loads do not fill the capacity of the railway, the costs of transport (correctly calculated) truly reflect the national economic outlay on transportation. In the case where the capacity is exhausted, the o.d. valuation of transport should be used instead of the cost in order to include also the rent from equipment of the railway calculated per wagon in the account. Such valuation is obtained by taking into consideration all the conditions: the quantity and nature of the load to be moved, the capacity of the railway and the possibilities of increasing it. With the aid of such an o.d. valuation, the correct solution of the problem of advisability of any kind of transport is obtained by comparing the saving per wagon (the national economic efficacy) that can be obtained by it with the above-mentioned valuation.

The o.d. valuation of the load transported consists of the o.d. valuation of such a load at the point of despatch and the o.d. valuation of transport.

It should be pointed out that the calculation made in this scheme cannot be easily accomplished with the same precision in practice, although methods have been developed for the calculation in very complex cases. The reasons for this are either the lack or incompleteness of the necessary data.

In view of the fact that selling prices and costs do not reflect fully and precisely national economic expenditure on the output of a product at every point, calculations should be carried out by using o.d. valuations. Without them, however, we may be obliged to base ourselves on cost data. In such a case our conclusions will be much less precise and well founded.

However, even very rough o.d. valuations of transport and their

use would enable us to draw certain definite conclusions, which would doubtlessly be useful in practice.

Let us assume that the economic valuation of transport per wagon (for an average journey of the order of 1000 km and average demand on the railways) is very rough, say, as lying within the limits of 400–2000 roubles. Then, on the basis of such a valuation, two practical conclusions may be shown (in both cases, the average conditions mentioned above are to be borne in mind).

(1) It is advisable to put into operation measures to reduce the demand for wagons, or measures increasing the capacity of the railways, provided expenditure amounts to less than 400 roubles per wagon.

(2) It is always inadvisable to refrain from allocating wagons if expenditure of a factory amounts to more than 2000 roubles for each wagon not allocated.

\* \* \*

Let us consider the question of a rational connection of production points to consumption points where the volume of production is fixed.

EXAMPLE 2. A certain load is produced at points A and C, and consumed at points B and D. In the scheme below (Fig. 7) daily production (+) and consumption (–) in wagons are shown in brackets. Cost of transport per wagon between each of the two points is also shown: if the railway is not used to capacity, this represents cost; if capacity is exceeded it is the o.d. valuation of transport per wagon. An optimal transport plan has to be drawn up such that the total expenditure on transport will be minimized.

In one of the possible plans of transportation (Fig. 7), all 50 wagons are sent from A to D, 30 wagons are sent from C to D and 30 wagons to B. The total expenditure on transport amounts to  $50 \times 700 + 30 \times 200 + 30 \times 400 = 53,000$  roubles. How can one determine whether such a plan is optimal?

To solve the problem, we shall once again use the method of o.d. valuations. We shall try to find o.d. valuations per wagon of a given load at all points. Let us assume that at A this valuation equals  $\alpha$  roubles, then at D it will equal  $\alpha + 700$ , since the load is being transported from A to D (see Conclusion 21). When a load is also sent to D from C at a cost of 400 roubles, then the valuation of a

wagon at C should be 400 roubles less than at D, i.e.  $(a+700)-400 = a+300$ . Finally, as a load is sent from C to B, its valuation at B should be  $(a+300)+200 = a+500$ . At the same time, if a load were sent from A to B, this would be obtained at a lower valuation ( $a+300$ ).

The difference obtained at once shows that the plan of Fig. 7 is not optimal. Point B has to be supplied from A. The appropriate plan is shown in Fig. 8. Total expenditure on transport under this plan comes to:  $20 \times 700 + 30 \times 300 + 60 \times 400 = 47,000$  roubles, or 12 per cent less. This plan is optimal. If the o.d. valuation per wagon at point A equals, say, 1000 roubles, then at D it will equal 1700 roubles,

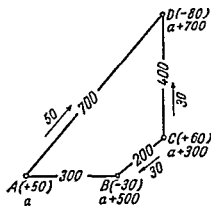


FIG. 7

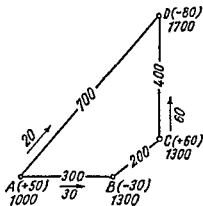


FIG. 8

at C 1300 roubles and at B 1300 roubles. On the basis of such valuations it is clear that it is not advantageous to supply point B from C; and this is not done in the new plan. Such a system of valuations accounts correctly for these aspects of transport, favours transportation of loads that are advisable and does not allow those which are not advisable.†

At present, a uniform price is adopted for many kinds of important materials—known as “free at the station of despatch”. Such a price

† A fuller account of this method of solving problems of the planning of transport is given in a special study by the author in co-operation with M. K. Gavurin: *Problems of Increasing Efficiency in the Operation of Transport*, Academy of Sciences of the U S S R., pp. 110–38. This gives the computational methods suitable for any number of points and several types of load, allowing for round trips (see also Appendices I and II).

system does not ensure an optimal plan of transport. At such a price it is more advantageous for point B to obtain supplies from C than from A, as point C is situated nearer to B and the material will cost less. But in this case a plan is obtained, as shown in Fig. 7, which was shown to be non-optimal. The same shortcomings arise with a uniform price—"free to the station of destination". Therefore, a system of zonal prices, correctly drawn up by calculating a rational plan of economic transport, should be looked upon as more advisable.

**CONCLUSION 22.** To the optimal plan of transport corresponds a definite system of o.d. valuations of loads at various points. This system is conceived in such a manner that if a load is transported from one point to another, the difference of the o.d. valuations at these points equals the expenditure incurred on transport between these points; if nothing is transported between these points the difference does not exceed expenditure on transport.

The existence of such a system of valuations shows that the transport plan is optimal—it does not involve wasteful movements. Conversely, when it is not possible to draw up such a system of valuations, this is evidence of the plan not being optimal.

Other systems of valuations of loads, in particular a uniform price—"free at the station of despatch", do not help to remove wasteful movements from the plan.

Similar methods may be applied in the analysis of the problem of transport and the determination of the volume of production if the latter is variable.†

### **Section 8. The Best Use of the Available Production Base. A General System of Objectively Determined Valuations and its Importance**

#### *General Propositions*

In the preceding sections we analysed the problem of the role and the advisable use of various factors of production: labour (which is the basic production factor), raw materials, other materials, natural resources, factors upon the use of which will depend the efficiency of labour (electric energy, equipment, transport). Here the only source

† An example of this type is analysed in Appendix II, pp. 336-41.

of value was seen to be labour: in the form of the labour force, in its material form or in the form of services. The effect of each of these factors was analysed separately. However, the conclusions obtained as to the optimal plan and the o.d. valuations hold also in the case when all these factors operate jointly. With the above results it is possible to note the principles of the method of solving the whole series of economic planning problems relating to the use of the available production base.

The results obtained apply now not only to a limited production aggregate but to the national economy as a whole.

All the problems of economic accounting and planning may be divided into two groups.

To the first group we assign the problems of the use of resources provided by the available means of production (equipment, machinery, transformed natural resources, the transport network and sources of electric energy, specialized labour).

In such conditions the task is that of the best possible allocation of existing means, and of the selection of methods of production which are feasible under the given conditions, that is, those which do not involve excessive time and considerable capital investment for completion. Therefore, the planning and allocation problems envisaged here relate to comparatively short periods of time (a year, a quarter, a month)—*problems of short-term planning* and of operational economic solutions.

To the second group we assign problems in which the time factor and the need for considerable new capital investment are essential: the choice of the type of new factories, plans for road expansion, plans for training specialized labour, etc. These are *problems of long-term planning*.

The problems of the first group also determine the task of rational utilization of the available production base. They are the subject of investigation in the whole chapter and they are considered in this section in their broader outline. The second group of problems is discussed in Chapter III.

The task of constructing a plan that ensures the best utilization of the available production base proceeds as follows. In the existing situation and a given period, the composition of the final product required is determined—its distribution by type of products. In the final product are included the means of personal consumption of the

population, means of consumption for state requirements (army, cultural and welfare organizations, housing, government, communications), means intended for the expansion of the production base—for capital investment in industry, agriculture, road and municipal building. Consumption or accumulation of reserves of raw materials are also decided upon.

Starting from these requirements and the available means (the labour force, equipment, methods of production in use or known) an optimal plan has to be constructed, a plan which will ensure the highest possible final output of the requisite composition. Of course, the fact that a plan is optimal does not mean that in the process of completion it could not be over-fulfilled; this may be attained by finding new resources, new production methods and new ways of organizing production, by improving technical indices and working methods. It has to be borne in mind that the available resources, the known production methods and the targets reached must be calculated and correctly used in the preparation of a plan, otherwise it could not be considered optimal.

As in the simpler special cases analysed above, an optimal plan must also exist for the fuller and more complex task. The problem of finding such a plan in practice will be further considered below.

A definite system of objectively determined valuations is associated with the optimal plan. This system of o.d. valuations incorporates: (a) a valuation of labour covering various specializations and qualifications, (b) valuations of production (final and intermediate), (c) valuations of various types of materials and raw materials, (d) a valuation of electric energy, (e) o.d. valuations of equipment (hire valuations of movable types of equipment and the planned profitability of whole factories), (f) rent of natural resources, (g) valuations of various types of services (transport, communications). At the same time, all these valuations may vary in different economic districts. Along with these general valuations, there may be valuations that are calculated and applied chiefly within a factory: valuations of individual components or of work performed on them, valuations of semi-manufactures, valuations of fixed and specialized equipment to be used. This system of o.d. valuations is related to the optimal plan in a manner analogous to the special cases, as was pointed out above on several occasions.

(1) At each factory, or in any production process, if a method of production is used in the optimal plan the sum of expenditure must equal the general value of production (and both are calculated on the basis of o.d. valuations)—in other words, production must be justified by being profitable.†

For unused methods of production the aggregate valuation of expenditure is higher than or equal to the aggregate valuation of production.

(2) If some product appears in one process as a raw material and in another (at the same point) as a finished article, then its o.d. valuation in both cases must be the same.

(3) The difference in the valuations of one and the same factor at two different places, where transfer from one place to the other is possible, should not exceed the valuation of losses that such a transfer entails. In particular: (a) the difference in the valuation of some product at two points should not exceed the valuation of expenditure on transport; (b) the difference in the hire valuation of equipment must not exceed the losses which may entail its possible transfer (transport, assembly after arrival, non-use during the time of transfer); (c) the difference in the valuations of labour when this is mobile should not exceed expenditure on transfer (transport, the time not worked during moving, decrease in the productivity of labour during the first period in the new location).

All the above refers only to those possible methods of production and to transfers which were considered in the preparation of the optimal plan

The system of an o.d. valuation is *concrete*, it relates to all the

† The reader may be surprised to find that in applying the system of o.d. valuations consideration is given only to the question of the justification of production by its profitability which is equal to zero. This is firstly due to the conclusions being given for a schematized statement of the task and secondly because the *concept of profitability is somewhat unusual*, namely.

(a) Expenditure includes the hire valuation—rent—of an enterprise; in fact, it represents in itself a net income at the disposal of the state (in practice, a portion of this must probably remain at the disposal of the factory and is included in its income). The same applies to other deductions which appear as expenditure. rent from land (Section 6), payment for the use of scarce types of labour (Section 3, p. 68).

(b) Only planned profitability is envisaged as zero. In fact, owing to the over-fulfilment of the planned targets of production and the lowering of standard costs, the actual profit should be positive even after allowing for expenditure mentioned under (a).



conditions which determine the planning task (composition of the final product, the relations between available resources, technical progress, the whole set of production methods used). Substantial changes in these conditions cause certain changes also in the system of o.d. valuations, basically for those factors which are particularly affected by these changes. At the same time, valuations are to a certain extent *stable*: single individual changes in the conditions cannot cause significant changes in the system of o.d. valuations.

Further, o.d. valuations are realistic, their ratios correspond to real relationships in economic costs and can in fact be realized. If these valuations are converted to ordinary labour, then if the valuation of a component equals 100 (man-days) it shows that the plan can be changed so that an additional output of these components is obtained by spending an additional 100 man-days of ordinary labour for each component. Conversely, by removing from the plan a certain number of these components, a corresponding number of man-days of ordinary labour could be released.

The proportion of 2:5 in the o.d. valuations of two types of product means that by reducing the plan by 2000 of the first component it is possible to increase it by (approximately) 5000 of the second. The position is similar with valuations of materials, services and equipment. Against this, the existing system of prices does not produce the realization of such relationships.

This is not so noticeable in an industry where monetary calculation in the plan, as a rule, is registered by the corresponding movement of material funds (if in the plan of a factory expenditure of 1,000,000 roubles is envisaged on raw materials, the supply of these raw materials to the factory is ensured). However, in the construction of capital goods, where the balance of material resources was not planned with such precision, this had an adverse effect. Although the sums envisaged in the plan were allocated, a large part could not be used; of those that could be used the actual cost of operations by far exceeded that planned. The expected prices could not be realized; it was impossible to obtain the necessary materials at those prices (or even to obtain the full quantity at all).

This explains to a large extent why the first and second five-year plans of construction were not completed in time.

*Application of O.D. Valuations in the Analysis of the Economic Efficiency of Production Methods*

If the o.d. valuations were found, they could be applied to the solution of numerous problems of the use of the available production base that arise in the process of fulfilling and improving the plan. In all such cases, a simple calculation on the basis of o.d. valuations would enable one to select objectively the right solution while taking into consideration the whole situation and also the interests and requirements of other factories. To explain how this could be done and what changes such accounting entails in comparison with the usual one, let us quote a few examples. Practical details are used only for greater clarity, and for this reason we shall not enter into the technical details and take the data (in particular, the values of o.d. valuations) quite arbitrarily.

EXAMPLE 1. At one of the units of a machine-building factory a whole series of components is manufactured on a universal machine tool. The transfer of a specialized machine tool to this unit would result in a considerable increase in labour productivity for some 20 per cent of the total number of components manufactured, resulting in a halving of their cost. Is it advisable to transfer the specialized machine tool to the given unit?

Let us assume that 500 components are manufactured per shift and the cost of the manufacture of components on the universal machine tool is 30 kopecks. The hire valuations of the universal and specialized machine tools at the given factory equal 35 and 60 roubles per shift.

The total manufacture on the universal machine tool is valued at:

$$500 \times 0.3 + 35 = 185 \text{ roubles.}$$

The second variant will give an expenditure of:

on the universal machine tool	$400 \times 0.3 + 35 = 155$	roubles
on the specialized machine tool	$100 \times 0.15 + 60 = 75$	roubles
Total		230 roubles

The second variant is less economical and must be rejected. On the other hand, with the usual approach starting from the calculation of

cost, it would appear quite advisable as it would seem to result in a reduction in cost without any additional expenditure.

EXAMPLE 2. At another unit of this factory obsolete machine tools are employed in the manufacture of components. Four hundred components are manufactured per shift and the cost of manufacture of a component is 0.45 rouble. The hire valuation of the machine tool is 20 roubles per shift. Is it advisable to replace the obsolete machine tools by specialized tools of the same type as in Example 1, if the latter should result in an increase in productivity by one and a half (or a reduction in cost of manufactured components by a third)?

Let us evaluate expenditure on the manufacture of components:

Obsolete machine tool	$400 \times 0.45 + 20 = 200$ roubles	(400 components)
Specialized machine tool	$600 \times 0.30 + 60 = 240$ roubles	(600 components)
Calculated cost for 400 components	160 roubles.	

The net saving amounts to 40 roubles per machine tool for one shift. Thus, the change is advisable and should be carried out. In so far as the valuation of the specialized machine tools is realistic, they should be released from another unit and supplied to replace the obsolete machine tools.

Evidently, if both questions in Examples 1 and 2 were considered together, then even on the basis of a simple comparison it would be recognized that in the second instance the specialized machine tool is more necessary. But in fact this does not happen: in solving one problem (with the aid of the normal indices) it is in practice not possible to calculate requirements elsewhere. However, the results of calculations carried out on the basis of o.d. valuations would show to the management of a factory in the first instance that at some other unit the specialized machine tool is more urgently needed since the calculation showed the inadvisability of the transfer. In the second case they would show that in spite of the scarcity (and the high hire valuation) of specialized machine tools their use is advisable and that the transfer should be carried out.

At present, in so far as the transfer appears to be advisable in both cases under calculation of cost, both requirements will be justified and it is highly likely that the first unit will succeed in obtaining the

*specialized machine tool and not the second. In other words, the available specialized machine tools will be far from being used at their maximum efficiency. Without calculation it is not possible to find the correct answers to such questions on the basis of qualitative considerations alone as with some data one solution is correct and with other data another.*

Of course, similar problems may arise in the allocation of equipment on a much wider scale. In solving the problem of where to direct new equipment a *sovmarkhoz*, let us say, must take into consideration its hire valuation in different activities.

EXAMPLE 3. It is planned to manufacture two articles by the stamping method. Instead of 15 min working time, as previously required for each article, the stamping method takes 5 min for the first article and 2 min for the second one. Let us assume that as a result the full cost of manufacture of the first article will be reduced by one-half and the second by three-quarters. However, the presses are working to full capacity. In such a case, both suggestions will probably be considered *advisable, but impracticable at the time.*

Comparison of the previous method and the stamping method on the basis of the *o.d. valuations may show that allowing for the o.d. valuation of the time previously taken by the metal cutting machine tools and the high valuation (in the light of requirements) of the time for the press operations, the total expenditure on manufacture will increase for the first article and decrease for the second one. This will show that it is not at all advisable to transfer the first article to stamping under the given conditions and that the second should be so transferred by relieving the presses from some other operation (satisfying the *o.d. valuation of the press*). Here, the losses will be smaller than the gain.*

EXAMPLE 4. By using less efficient machine tools, and increasing the number of those in operation by 20 per cent it will be possible to increase production by 15 per cent; expenditure on materials per unit output remains unchanged.

Is this advisable? On the basis of the cost of production calculated in the usual manner, such a change would be disadvantageous. Would it be advisable, however, if the question was considered from the point of view of the general plan? This will depend upon the *o.d. valuations and other data.*

Let us assume, for instance, that the *o.d. valuation of a unit of*

output amounted previously to 10 roubles and that it was composed as follows:

Materials and other related expenditure (excluding labour)	3 roubles
Fixed costs	1 rouble
Labour expenditure	3 roubles
Rent from equipment	3 roubles
(all on the basis of o.d. valuations)	

Output is 100,000 articles per month, expenditure amounts to 1,000,000 roubles. By engaging additional labour, expenditure on the output of 115,000 articles will be composed as follows:

Materials	$115,000 \times 3 = 345,000$ roubles
Fixed cost	100,000 roubles
Expenditure on labour (300,000 + 20 per cent)	= 360,000 roubles
Rent from equipment	300,000 roubles
(equipment is the same)	
Total	1,105,000 roubles

Hence the cost per unit production will amount to:

$$\frac{1,105,000}{115,000} = 9.61 \text{ roubles}$$

which is below the present valuation. The change is advisable.

At the same time, on comparing the usual indices an increase in cost and a reduction in labour productivity would be obtained, and the change would appear inadvisable.

It should be pointed out that, in fact, measures similar to those discussed are often taken even when this results in an increase in cost. The criterion here is the importance of production and the desirability of increasing output. However, in such cases it is thought that the measure is taken regardless of economics. In fact, if the measure is advisable, an economic calculation that is correctly carried out by allowing for existing conditions through the o.d. valuations should show its profitability. The usual estimate of cost is one-sided—it incorporates only visible expenditure on labour and does not

account in an appropriate manner for such an important aspect as the better use of equipment.

**EXAMPLE 5.** In mass production of articles on an automatic machine tool waste strip is obtained which remains unused. It is proposed, because of the need to save metal, to put this remainder on to the usual machine tools and thus obtain one more item. *Is this advisable?*

The solution will depend upon circumstances. Let us assume that 100 g of sheet metal is used per item and that the o.d. valuation of its manufacture on automatic equipment amounts to 12 kopecks per item. In their manufacture, 0.5 of an hour of machine tool and working time is necessary per item which is valued at 2 roubles.

Thus, to save 100 g of sheet metal, the additional expenditure will amount to  $2 - 0.12 = 1.88$  roubles. Even if the o.d. valuation of a ton of sheet metal is 2000 roubles, 100 g would still only amount to 0.20 rouble and the measure would be clearly inadvisable. It would only be advisable with a quite extraordinary o.d. valuation of the order of 20,000 roubles per ton of metal.

Of course, in the present case it would be more correct to use these waste bands in repair shops or at other factories if they were not to be re-smelted.

*It should be pointed out that sometimes any measure resulting in the economy of a scarce material is recommended without any calculation and to an unlimited extent, even though only minute amounts are saved at the cost of considerable outlay in labour and a reduction in output. The practical solution of such questions is largely determined not by the general situation but by the vagaries of supply at a given factory. If the supply of sheet metal were stopped today, this factory would resort to such measures of which an example was quoted. If the metal were supplied tomorrow the factory would not incur an expenditure on labour to save even 2-3 kg of metal (instead of 100 g), which would already be advisable.*

Even if in individual cases such measures are advisable, their systematic use may only cause harm, the more so since frequently some materials are considered scarce which can be produced with some extra expenditure of labour if necessary. (This may be considerably smaller than the expenditure of labour required to economize on such materials.) Of course, these materials too must be economized, but not at any price.

On the other hand, many measures to save materials which by the usual calculation show an increase in cost must not be considered merely in response to accidental difficulties, but must be applied systematically.

The solution of these problems may be clarified by applying o.d. valuations.

In connection with Examples 3-5 it may be relevant to consider some of the present inadequacies, despite certain improvements, in questions of the valuation and implementation of various rationalization proposals and proposals for the organization of production.

First of all, as a result of the shortcomings in the valuation of the effects of rationalization proposals (undervaluation of a whole series of factors in the usual calculation of cost) many proposals furnish considerable savings, but their acceptance is inadvisable under given conditions (stamping of the first article in Example 3). As a result, there are among them other proposals which are not adopted and frequently discarded, proposals which could furnish a real economic saving (and which do not always coincide with savings as calculated). Even if a proposal yields a considerable economy as calculated it does not necessarily follow that it would be adopted. A whole series of proposals is sacrificed, and although their benefit is undoubted some time elapses before they are introduced.

Further, many important proposals leading to a saving in scarce materials and an increase in the volume of production without decreasing cost do not produce a saving on the basis of existing calculations. In spite of this many of these may lead to a large economic saving (Example 4). Frequently such proposals are looked upon as temporary measures only caused by circumstances. That valuation and also the system of incentives that existed stimulated neither the introduction of such proposals nor their continued implementation.

As a result of a one-sided valuation of contradictory elements it appears that perhaps no proposal can be justifiably accepted or rejected. The same applies to a certain extent also to inventions.

EXAMPLE 6. Additional mechanical enrichment of ores makes it possible: (a) to reduce the smelting time by 10 per cent, (b) to reduce consumption of coal per ton of metal by 10 per cent. It requires the installation of additional equipment and increased consumption of electric power. Is this measure advisable?

This will depend upon the o.d. valuations. Let us assume, for instance, that the o.d. valuation of coal consumed in the production of one ton of metal is 120 roubles; 2 tons of ore (not enriched) are used per ton of metal. The hire valuation of equipment for enriching ores amounts to 6 roubles/ton, the valuation of electric energy consumed in the enrichment of one ton of ore amounts to 2 roubles 50 kopecks. The hire valuation for a blast-furnace and other equipment is calculated as 200 roubles per ton of metal (all on the basis of o.d. valuations).

Let us carry out the calculation of the change in cost per ton of metal. *Additional expenditure for the enrichment of two tons of ore:*

Equipment	$6 \times 2 = 12$ roubles
Electric energy	$2.50 \times 2 = 5$ roubles
Total	17 roubles

The smelting time is shortened and thereby the hire valuation of the blast-furnace is reduced by 10 per cent (by 20 roubles); coal consumption is reduced by 10 per cent = 12 roubles. The saving will amount altogether to 32 roubles.

As a result, the valuation of expenditure per ton of metal is reduced to  $32 - 17 = 15$  roubles, and hence the introduction of enriched ores is advisable. Of course, such a result is obtained only with a given system of o.d. valuations. Under different circumstances, with no scarcity of metal, the metallurgical equipment would not be used to full capacity and correspondingly the hire valuation of blast-furnace equipment would be lower and the result could be different.

The problems of intensification of production processes appear similar to those considered—for instance, the question of oxygen blowing in metallurgy. At present, owing to the high cost of oxygen, its application often leads to an increase in the cost of metal which hampers the introduction of new techniques. It is known at present that the use of oxygen leads to an increase of productivity by 20 to 30 per cent in Martin ovens and by 8 to 10 per cent in blast-furnaces.

Without full data it is difficult to arrive at a definite conclusion on this problem, but no doubt if the hire valuation of metallurgical equipment were included in the economic accounts, the calculation



(as in the example considered) would have shown a much higher economic efficiency of the new technique in comparison with the results obtained by the usual calculations.†

It should be pointed out that in complex problems of this nature it is proper to choose a solution merely with the help of qualitative valuations, and for this it is necessary to have uncommonly good intuition. It may be objected that the managements of factories, of the Councils of the National Economy (*sovmarkhozy*) and of ministries always apply such measures and do without this type of accounting. However, this by no means proves that the adopted solutions are the best (or approximately so) and that no better system of economic solutions is possible such as might ensure a higher output and its speedier growth. Numerous losses and unused potentialities and on the other hand examples of the operation of leading factories show that the advantages of socialist production are far from being fully reaped.

*The Importance of O.D. Valuations in Questions of Economic Accounting, of Operational Indices of Factories and of Price Formation*

No matter how well a plan may be drawn up, its full use is only possible in two cases:

- (1) If in the process of fulfilling the plan changes are introduced consistently with the changes that take place in world conditions.
- (2) If the executives are given the necessary incentives to follow the plan.

The solution of these tasks for the optimal plan is made easier by the fact that the plan is accompanied by a system of o.d. valuations of production and of its factors. We have already seen above how it is possible by using these valuations to introduce changes in the plan by changing the target—changes in the composition of output as a result of the emergence of new methods of production which so far have not been allowed for. Meanwhile, the realistic nature of o.d. valuations makes it possible to adopt solutions that take account of the actual situation—for instance, of the scarcity or surplus of a

† The exceptional importance of using oxygen in metallurgy and the role of the correct economic analysis of this problem were brought out in the speeches of A. I. Gaev and L. I. Brezhnev at the XXI Congress of the Communist Party (*Report*, vol. I, pp. 349 and 426).

given type of equipment. The dynamic character of o.d. valuations enables one to follow the situation with flexibility and to supply factories with the necessary data for their guidance, which ensures the maintenance of the plan at its optimal level when changes occur.

However, providing a plan and o.d. valuations that enable factories to make an economic choice of solution consistent with the situation and the public interest is very important, but not yet enough. It is necessary also to introduce such changes into the system of economic accounting and financing and into the statistical and economic indices which characterize the operation of the factories as would stimulate the awareness of factories and other economic bodies as to the optimal plan and in correct economic solutions.

The system of o.d. valuations furnishes the necessary data. Thus, whatever the form of the o.d. valuation of using equipment may be (lease or hire valuation, planned level of profitability), its inclusion in accounting between factories (in the case of a temporary transfer of equipment, method of work in the case of co-operation) will help the most intensive and best use of equipment. The collection of an additional payment for the use of scarce categories of labour will also provide incentives for its best and fullest use. The same applies to rent for natural resources. Finally, the construction of basic indices characterizing the operation of a factory on the basis of valuations of net production and planned profitability (including the hire valuation of equipment in costs) will arouse the interest of factories and economic bodies in the best and most correct selection of the composition of output in the plan of production and the volume of output (see Example 4) and also in raising the plan and in obtaining new orders; this will lead to overcoming prolonged unprofitability of the factories.

The completion of the optimal plan which ensures the fullest and the most advisable use of resources at all levels of economic management simultaneously in the whole country so as to achieve the necessary output would lead to the drawing up of a general economic system of o.d. valuations of all types of output. According to the principles of their construction it is clear that these valuations should then correspond fully to the economic expenditure on the manufacture of a unit output (in the current rational production plan) at a given moment and under given conditions; in other words, they

must correspond to the socially necessary expenditure of labour. They also correspond to the economic efficiency in the use of a unit output under given conditions.

In fact, in so far as the system of o.d. valuations must agree with the real relationships of economic expenditure on various types of production, these indicators should be included in the price formation.† In principle, prices should approximate to o.d. valuations.‡ Of course, the foregoing refers only to wholesale prices operating within the state-controlled sector; retail and other prices may substantially differ from them. As regards wholesale prices, they, too, need not strictly agree with o.d. valuations since frequent changes in these prices are for various reasons not desirable. However, even an approximate agreement of prices with the o.d. valuations would mean that both prices and valuations should reflect hire cost, rent, etc. Here it is essential to show that such changes in the price formation are connected with two facts. In the usual price formation on the basis of cost some essential types of expenditure are not considered at all, and, as previously explained, these must be included. This is a systematic structural distinction in the price formation. The second difference consists in that the o.d. valuations reflect those deviations which are due to temporary deficiencies or to the existence of reserves of one type of equipment or another or to an abrupt increase in the demand for the given type of output, etc.

No doubt, this would result in a change in price relationships in comparison with existing prices—in particular, a certain relative increase in prices for those types of output (and of services) in the production of which large, specialized and also scarce equipment is being used, namely, prices of metal, petrol, coal, cement and railway transport. The question arises as to whether such increase in prices of those types of output will cause difficulties in so far as they are used by state factories. It is clear that this will not be so. The increase in prices will be determined by incorporating in the valuation of production a corresponding share of the hire valuation of equipment

† The importance of price controls and of a correct solution of the problems of price formation on a uniform basis was stressed by A. N. Kosygin in his speech at the XXI Congress of the Communist Party (*Report*, vol. I, pp. 171–3).

‡ Prices for which the approximation to o.d. valuations is of the highest importance are those used in planning and in economic calculations. It is possible that here the direct use of the o.d. valuations will be the most rational procedure.

used. But in so far as the whole hire value constitutes state revenue, this will signify only a reallocation of resources among the various items of the budget.†

Such relative increase in prices of these types of output, consistent with their real national economic value (and cost), in a socialist economy without crises in no way hinders the full use of the entire possible volume of production. At the same time, in so far as such a system of prices should promote a more appropriate use of the given kinds of output, economy and their rational replacement by other kinds and the growth of their output, the operation of such a system would lead in the final result to a reduction in prices although with different relative values.

The above paragraph refers to that part of production of department I which is used internally. However, this production is partially used in the production of means of consumption. This circumstance and also the appearance of new types of expenditure (hire valuations) must result in some increase in the cost of consumer goods. However, this should not cause an increase in their prices. The appearance of new items of revenue (hire valuation, rent) which go to satisfy social requirements and accumulation will allow the redistribution of the national income in the form of the turnover tax to be reduced considerably. Because of this reduction the prices of consumer goods will not be increased at all, in spite of increased costs. Meanwhile, a certain reduction of the gap in the price levels of the two departments will follow.

Finally, bringing prices nearer to o.d. valuations will produce a much more exact agreement between the material and monetary balances—owing to the real and practical nature of these valuations—which will lead to raising the function of the rouble in economic analysis and economic solutions. The importance of economic criteria in the valuation of the activity of factories and sectors will

† Even the highest-paid citizens of the Soviet Union do not acquire for their own various purposes excavators or rolling mills. For this reason it is impossible to believe that a change in prices of the means of production could impoverish or enrich anybody. All it amounts to is a reallocation of means between the categories of state expenditure and income. Nevertheless, it is important to know at what level prices of the means of production will be fixed: the choice of economic solutions may depend substantially upon it. But from the point of view of the choice of solutions which lead to the determination of an optimal plan we also have to consider the question of determining the prices of means of production.

also be increased. The profitability of factories will become decisive in such a valuation which must replace numerous and frequently contradictory partial indices used for the valuation of the operation of the factory, leaving to them only an auxiliary role. At the same time, if the valuations of production of the factors of production were actually obtained, profitability calculated on the basis of o.d. valuations would be consistent and harmonize substantially with the higher profitability—the interests of the national economy as a whole.

*The Influence of O.D. Valuations on the Changes in Production Tasks and the Composition of Final Output*

In the definition of the task of finding an optimal production plan we assumed that the target plan was already given. However, after the production plan and the o.d. valuations showing the distribution of economic expenditure on various types of production are found they may furnish some indication as to the advisability of certain changes in the production task itself.

Thus, if it were found that the production of some article at a given factory is relatively expensive—having a high o.d. valuation—it is essential to consider removing it from the plan and of transferring it to other factories where expenditure is lower. It may prove that this article may be taken out of production altogether as there is a possibility of replacing its use by others involving less expenditure. Conversely, it may also be found that some article is given a lower o.d. valuation than proposed (for instance, it is obtained as a by-product from waste materials). In this case it is essential to increase its output and use. But now there must be a demand for it as any expenditure must in the first place be useful.

Of course, in order to be able to assess correctly all these calculations it is important to know the valuation of production which reflects exactly economic costs of production, or their o.d. valuations.

The same applies to the plan task for final production as a whole by which the national economic plan is determined.

The basic structure of this plan—the relationship of its parts to various requirements—is objectively the outcome of the general economic and political target, and the actual composition of production of the social and individual requirements and, among these, of the demand of the population for the different types of goods.

However, economic facts may necessitate certain adjustments to that composition. For instance, the proportion of meat and fish in consumption will be dependent upon the possibilities of developing meat production and fisheries, and the cost involved in the supply of these products. The same applies to the proportion of multi-storey and low-storey building, or to the ratio between sanatoria and rest homes.

When determining the plan for the production of goods for individual consumption, which will depend upon the demand of the population, cost of production is also of importance for the demand itself will depend upon the price structure.

It is necessary, however, to stress that although in the choice of methods of production economic factors will be of decisive importance and in particular the calculation of expenditure by means of o.d. valuations, their function in the question of composition of the final output, while still of importance, will be merely auxiliary. It may be said briefly that *relative costs*—the valuation of costs—are used basically *not for deciding what to produce but how to produce*.

However, the latter refers to final output; production and the use of intermediary products are determined both by the composition of the final output and the choice of production methods for its realization. For this reason, the requisite volume of output of these products and the ratio of individual kinds (various categories of fuel, different building materials) will depend very substantially upon expenditure involved in their manufacture, i.e. upon their o.d. valuations.

The composition of the share of final output intended for capital investment will also depend to a large extent upon the practical circumstances reflected in the o.d. valuations. Thus, scarcity (and the resulting high valuation) of metal and cement may be reflected not only in the types of installations but even in the choice of items, and through it also in the final allocation of the end products within this sector.

Briefly, the ultimate solution of the problem of allocation of final output must be constructed in such a manner that "the useful effects of various articles of consumption compared with one another and with the quantities of labour required for their production will in the end determine the plan".† The approach we developed above aims

† F. Engels: *Anti-Dühring*, Foreign Languages Publishing House, Moscow, 1959, p. 427 (English edition).

at achieving this object. Here the use of o.d. valuations should help in a proper valuation of labour costs.

### *Practical Ways of Obtaining O.D. Valuations*

We have already said that as regards a production plan for the economy as a whole, all the conclusions quoted above apply and that there exists an optimal plan and a system of o.d. valuations. However, it is hardly possible to find a plan and valuations by such methods as applied above to simplified and schematized problems. This would require considering the valuations of tens of thousands of products, and analysing simultaneously thousands of factories and numerous possible methods of production. Of course, this cannot be accomplished owing to the difficulty of compiling and using the necessary data.

Without aiming at presenting a method for obtaining o.d. valuations and an optimal plan on a national scale (since their preparation presents a task calling for much research and practical work) we should like to show possible ways of arriving at o.d. valuations even if they are only approximate.

### *Partial Improvements in the Plan and Approximate Valuations*

First of all, let us note the great importance of an analysis aiming at a more appropriate use of individual types of resources or of resources of a given concern. Of this nature are the problems considered above: the allocation of the plan, the electrical energy balance and the balance of individual materials, the allocation of land for crops, the planning of transport, etc. Besides the direct value of such an analysis for the purpose of improving the plan (increasing production or reducing expenditure), the data used in it are essential in drawing up a general plan and its indicators.

It must be pointed out that in practice schemes of analysis of this type can only rarely be applied directly. This is due to the necessity of accounting in practical tasks simultaneously for a very great number of factors, and also to the fact that the conditions which we postulated in analysing individual tasks (a clear division of expenditure into two types in the allocation of the plan, Chapter I, Section 1; homogeneity of loads in planning transport, Chapter II, Section 7, etc.) are not always even approximately satisfied. Nevertheless, the

methods of drawing up an optimal plan may find sufficiently wide application provided an attempt is not made to apply these schemes literally from their description here.

First of all, it is important that there should exist a sufficient diversity of ways of fulfilling the plan. Next, it is essential that it should be possible to allocate some main factors for which these variants would differ so that the choice of a variant would show neither the *influence on other indicators nor where these indicators themselves would be inessential.*

These conditions ensure some autonomy of a given system which makes it possible to neglect the influence of other parts of the plan, and to analyse the system by itself. At the same time, this autonomy is usually only relative and incomplete. For instance, in the analysis some inessential expenditures are not included directly but by costs. Then, the corresponding items are found from the valuations used (*prices, cost, or o.d. valuations*) of other factors. Requirements in terms of resources, and the composition of the plan, are conditional and may be changed by taking other units into account. Finally, the basic initial data of analysis are usually very approximate. In view of this, both the solutions obtained from the analysis of the system and the indicators of the plan (*o.d. valuations*) are only approximate and would change by a more accurate calculation of the influence of other factors and units.

These valuations are frequently also incomplete (a valuation is obtained for the work on the manufacture of a component but it is not a full valuation of the component); this type of valuation is only *of local and partial application.*

Greater autonomy of the schemes and with it a broader basis for the conclusions derived from their analysis may be attained by extending and unifying them. For instance, it is advisable to consider the problems of planning in transport not in isolation but together with the problem of the volume of output and the allocation of the programme.

To give this analysis a more practical appearance the initial indicators must be more accurate and detailed; for instance, the allocation of costs of a given component in a complex product.

In spite of greater accuracy and other improvements, the results of the analysis of individual sets of problems will be unavoidably approximate and relative.



For this reason, in the application of such results there is no need to follow literally the most advantageous allocation obtained: the more so as the system of analysis—the use of o.d. valuations—enables one to perceive various solutions which approach it in their effect. Taking this into consideration, it is possible in the choice of a plan to bear in mind a whole series of supplementary circumstances (such as the desirability of retaining certain production methods used and economic relations established).

At the same time, certain conclusions from this analysis—for instance, of the economic advantages of one method in comparison with another—are frequently found to be sufficiently definite to serve as a basis for practical solutions. The ratios of o.d. valuations derived from such an analysis may already be reliable within certain limits if it is possible to estimate the possible changes caused in them by unexpected and external data.

We suggest that on the basis of analysis of individual problems it is possible to obtain if not the national economic valuations themselves, at least some important data for their construction: the relative valuations of operations in the manufacture of some types of goods, rough estimates of hire valuations or extra transport charges for individual types of production, etc.

### *Calculation of O.D. Valuations on the Basis of Models*

In addition to the relative valuations obtained from the analysis of individual schemes of planning, it is necessary to have o.d. valuations, even if only approximate ones, for basic kinds of output and its factors on a national economic scale. The first path which appears possible for this purpose consists in constructing a highly simplified model of the economy. By taking the total of indicators and commodity groups (grain crops, conventional fuel, ferrous metals) we consider for each group of production some typical production methods. For each of these it is necessary to evaluate the total natural outlays, the degree to which the method is actually applied, and the likely reserves for its expansion. Bearing also in mind labour resources and natural factors, the required model will then be obtained. The calculation of an optimal plan for this model will furnish rough o.d. valuations for these aggregated types of production referred to and for the basic factors of production. Data

*approaching real conditions more closely would be obtained if the analysis was carried out for large economic regions, including transport and communications between them, and by taking into account the capacity of the existing main railway lines.*

It must be mentioned that in order to follow this path it is necessary to overcome considerable difficulties entailed both in the choice of the model and in the finding and treatment of actual data necessary for ascertaining the initial parameters of the model.†

#### *Approximate Determination of Valuations on the Basis of the Analysis of an Actual Plan*

A second, perhaps more realistic way of arriving at approximate guiding values of the o.d. valuations consists in the analysis of actual economic relationships resulting from the working of our national economy.

When considering the schematized tasks, we saw that if in an optimal plan the o.d. valuations are not given they can usually be determined on the basis of the analysis of such a plan from the solutions actually adopted by determining the consistency or disparity in these valuations (Chapter I, Section 1, pp. 16–18).

The thesis that there are o.d. valuations for an optimal production plan should also hold in principle for real production plans, not excluding the general plan for the national economy.

We may assume that the existing economic plan allows for actual conditions and that it is correctly drawn up in its basic outline. In other words, as a first approximation, it may be looked upon as a rational plan. For this reason, the economic solutions which were firmly established in the working plan are typical and justified by long economic practice. They may be utilized for extracting from it a rough system of o.d. valuations.

Let us give an illustration as to how o.d. valuations could be constructed on a few simple examples. We shall be interested here not in the absolute but only in relative values of these o.d. valuations. The purpose of these examples is to give some illustration of the method used in such calculations. For this reason all the numerical data in the examples are taken quite arbitrarily.

† The data relating to the co-operation between various branches of industry may find only extremely limited application for this purpose. For further details see Appendix I, pp. 278–81.

(1) Let us assume that in the northern provinces of the Soviet Union lands producing grain crops of 8 quintals per hectare are systematically cultivated. Let us say that labour expenditure per hectare amounts to 10 days at 20 roubles per day, while the remaining expenditure adds another 80 per cent to labour costs. The valuation of a quintal of grain may be taken as equal to 45 roubles  $[(18 \times 20) \div 8]$ . In the southern provinces a crop of 20 quintals per hectare is obtained at an expense of 12 days per hectare and, including other expenditure, of 20 days. Then, assuming the valuation of a quintal of grain to be identical (transport costs may at first be disregarded) and adopting an identical valuation for the working day, it is possible to determine the o.d. valuation (rent) of 1 hectare of land in the southern districts. It will amount to:  $20 \times 45 - 20 \times 20 = 500$  roubles.

(2) In the southern regions cotton is grown. If the yield of this crop is 15 quintals per hectare, if for the cultivation 30 days are required at 30 roubles per day, and if the remaining expenditure adds 100 per cent to the expenditure of labour, then the valuation of a quintal of cotton may be taken as equal to  $(60 \times 30 + 500) \div 15 = 153$  roubles.

The use of irrigated and non-irrigated lands would have to be considered separately in more detail.

(3) Coal is mined in the coal basin of district A in mechanized pits at a cost of 50 roubles per ton and in less efficient pits at a cost of 80 roubles per ton; the latter pits are not being fully utilized and extraction there could be increased. Here, the o.d. valuation of coal in district A may be assumed to equal 80 roubles per ton.

(4) At district B, situated 1000 km from A, low-calorie brown coal is mined at a cost of 75 roubles per ton, which allowing for its calorific value would be equivalent to the cost of 150 roubles of coal from district A. However, because of the load on the railways, brown coal is systematically used side by side with the coal imported from A. This enables us to determine the o.d. valuations for the transport of a ton of freight from A and B at  $150 - 80 = 70$  roubles per ton (see the previous example). This figure may prove to be considerably higher than the cost of transportation and the current tariff.

Confirmation of this valuation may be found, say, in motor vehicle transport operating systematically parallel with the railways—because of their overload—at an even higher cost, of the order of 100 roubles per ton.

(5) In the same manner, o.d. valuations of metal may be obtained, on the one hand by starting from production conditions, for instance, from the fact that obsolete factories are being utilized, resulting in an expenditure considerably above average, say, at 750 roubles per ton, or exceeding its price by 200 roubles (if the latter were to equal 550 roubles). On the other hand, because of the scarcity of metal, the saving that could be achieved by more widespread application of metal constructions is systematically ignored. In building, for instance, wooden and reinforced concrete frames are widely used although the use of metal would in many cases result in reduced costs of building, of 300 roubles for each ton of metal consumed. Consequently, the use of metal here would be justified even if its price were 300 roubles higher than the current price. Comparing both, we may assume that the o.d. valuation of a ton of steel equals, say, 800 roubles.

(6) Starting from the calculated o.d. valuations of production valuations of individual types of equipment may be derived. The hire valuation in such a case may be determined not only for an individual kind of equipment but for the whole factory. Let us assume that we have a group of metallurgical factories with a productivity of 300,000 tons of steel per month. Let us assume that the o.d. valuations have already been determined, for instance, at 800 roubles per ton of steel. Thus, the valuation of production for the group of factories will amount to:  $800 \times 300,000 = 240,000,000$  roubles per month. Further, let us calculate its expenditure on the basis of o.d. valuations. Let us assume that it equals:

coal 400,000 tons at 120 roubles per ton	48,000,000
(90 roubles local valuation plus 30 roubles for transport)	
ore 600,000 tons at 50 roubles per ton	30,000,000
(mined locally)	
limestone 200,000 tons at 40 roubles per ton	8,000,000
(10 roubles local valuation plus 30 roubles for transport)	
workers' earnings from production, and other earnings	38,000,000
(the labour valuation is taken as roughly equal to earnings)	
other expenditure	17,000,000
	Total 141,000,000

The difference in the amount of valuation of production and of expenditure is  $240,000,000 - 141,000,000 = 99,000,000$  roubles per

month and also gives the approximate magnitude of the hire valuation of the whole equipment of the group of factories.

If this hire valuation were included as a constant item (for some length of time) in the total expenditure of the group of factories, the actual profitability could serve as a sufficiently accurate index of its working progress at a given period. By actual profitability is understood the net profit—the difference between the valuation of production of a factory for the given period and the sum of expenditure (including in the latter the constant hire valuation, as above). Then, should the work of the factory remain unchanged, profitability will equal zero. In the actual realization of the plan, it may be expected that if a factory operates well as a result of intensification of processes, a positive profit will be obtained. In particular, the following measures will help:

(1) By increasing output, even where the expenditure on an additional ton of production would exceed the average, but would not be higher than the o.d. valuation which equals 800 roubles.

The average valuation of expenditure per ton of steel is  $141,000,000 \div 300,000 = 470$  roubles, but allowing for the hire valuation it is 800 roubles. For this reason, even if expenditure for an additional ton amounted to 700 roubles, and provided the hire valuation which has already been fully calculated is not added in again the production of such an additional ton would increase profit.

(2) By carrying out measures which ensure a reduction in expenditure per ton, especially on raw materials and particularly by using raw materials—for instance, limestone—from places situated near by.

It should be mentioned that these measures may lead to increased profit only if their operation does not entail a reduction in output.

(3) By increasing the use of scrap metal in the composition of the charge. In this case such an increase will prove profitable even if the price of scrap considerably exceeds its present fixed price.

In determining o.d. valuations we started from the assumption that the existing plan was optimal.

However, the plan operating in practice is not fully optimal. In individual problems irrational solutions occur. For this reason, already in the process of determining o.d. valuations tentatively on the basis of the existing plan can contradictory results be found.

This will reveal some particularly significant errors and disparities in the plan, while at the same time it will become clear by what changes and transfers they could be removed. It will also show the possibility of increasing the volume of production or of making improvements in the plan.

The valuations obtained in this way will be determined within a likely error of 30 to 40 per cent. Such valuations may only serve for tentative calculations although even their use may be of great importance and may help to remove particularly bad shortcomings in the plan.

#### *Development of Methods for Drawing up an Optimal Plan and for Obtaining O.D. Valuations*

The practical realization of the task of the simultaneous construction of optimal plans for all individual factories, economic districts, sectors and of the national plan, and of a system of o.d. valuations, presents a problem of the highest complexity and requires the development of special methods. Such methods should be prepared by the joint efforts of scientists in various fields of specialization and of practical workers, and should include the construction of a system of the necessary technical and statistical-economic indices, a method of processing, co-ordinating and fitting of the data obtained, model schemes as well as the necessary computational methods, planned organization and sequence in the carrying out of this work. The development of such methods is a task of the future. Here we should only like to note some considerations regarding its possible properties and ways of approaching its construction.

Let us try to calculate the more important features of these methods. It may be expected that such methods:

(1) will consist in the simultaneous drawing up of outlines of a plan and of economic indicators (o.d. valuations);

(2) will be graduated, or calculated for the simultaneous and consistent carrying out of the planned work at various levels, territories and sectors. at factories, in individual management of the *sovnarkhozy*, at the level of an economic district as a whole, at the level of individual sectors and on a national scale;

(3) will proceed by consecutive stages, by a gradual improvement,

greater precision and consistency of the plans and indicators, and also by co-ordination of short-term with long-term planning;

(4) will extensively use data from the results of productive activity of the preceding period;

(5) will furnish planning solutions of a flexible rather than a final character. They should be adjusted in the process of plan fulfilment, and be supported by economic accounting and a system of incentives.

This is not very far from the existing order of planning. The basic difference which should be emphasized is the systematic aiming at attaining the optimum of the plan and the simultaneous determination of a system of o.d. valuations and with it the systematic use of computational methods for the construction of an optimal plan.

The task of planning by such methods may be described approximately as follows.

As a starting point in the construction of the plan a fixed target is set in terms of the composition of the final product for a given period. General circumstances, social and individual requirements, including the needs of expanded production, are borne in mind.

Further, it is necessary to find tentative o.d. valuations for the basic types of production and its factors by making use of the methods of approach described above, or by adjusting the valuations from the preceding period.

Individual firms compile data of the probable growth of production and of new types of output, including the necessary expenditure. Clearly uneconomical variants are rejected on the basis of the preliminary figures of the o.d. valuations obtained. This also determines basically all the possible methods of production.

Next, the total of the preliminary plans is carried out by local associations of factories of each sector, setting an initial allocation of the plan between factories on the basis of the data of their capacities. Furthermore, local balances of labour, electric power, fuel, raw material are compiled, while at the same time the o.d. valuations are determined.

If it appears that the requirements for some type of service or raw material exceed the volume of their production the use of such material should be avoided at the point where it may cause the lowest losses, and the increase in its output considered where this can be achieved without an increase or with the smallest possible increase

in cost per unit of product. The o.d. valuation of such type of material or service increases correspondingly. Allowing for the changed valuations, alterations in the plans and in their allocation between the factories are made. This leads to consistent local balances; in individual cases the clear need and economic advisability of increasing the supply of one or another type of raw material, other materials and sometimes also of labour resources from other districts may become evident.

Simultaneously with the draft plan individual systems of o.d. valuations are established for various types of output and factors within the boundaries of each economic district.

Next, established economic relations between the districts and demand for transport are taken into account, and o.d. valuations are determined for transport services. Here it is already advisable to carry out the analysis without reference to individual types of production but with more aggregated data.

Then comes the analysis of the allocation of the programme by means of the local o.d. valuations and those of transport, which may prompt a certain redistribution of the plan among the districts. The analysis of o.d. valuations may also show whether changes in the allocation of raw materials, other materials, fuel, electric energy and the establishment of different economic relations between the districts are advisable.

As a result a general optimal plan may be constructed as a first approximation and its o.d. valuations established.

Then, the plan will have to be made more accurate, starting by adjusting the composition of the final output. The calculation of the established valuations and of the possible growth of output will suggest some change as well as changes in its composition, and it will also make greater accuracy of the projected growth of the volume of production possible, and in particular of the volume of capital investment achieved in the plan. Defining all the plans with greater precision at the second stage may proceed in approximately the previous order. Meanwhile, not only is greater accuracy of planning achieved on the basis of more accurate o.d. valuations and of changes in balances and relationships, but the initial technical data for the types of production and its volume which were already considered in practical terms are also improved. At this second stage the plans are again substantially adjusted so as to make them more accurate.



Further improvements in accuracy and adjustment in the plans must take place in the course of their completion.

Of course, this represents only a bare preliminary scheme. The drawing up of such a plan must present an enormous and extremely complex task. However, this work is fully feasible provided that its methods, the order of its application, and the necessary technical and statistical indices have been fundamentally thought out and developed.

In particular, consideration must be given to the favourable circumstance that for the preliminary stages in the drawing up of such a plan quite considerable time may be available, and that this work will be conducted by a whole series of government bodies. If carried out systematically it will be possible to utilize many technical and statistical data found from the preceding period, and in any case a method for finding them will exist.

The use of electronic computers for the processing of extensive information and the performance of calculations necessary for the construction of optimal plans and for the calculation of o.d. valuations at individual stages will not only greatly reduce the time required but will ensure the very feasibility of such work.

The methods of planning described here will be of value in so far as they will make it possible to co-ordinate general planning with the planning and economic accounting of individual factories more easily and accurately. The analysis carried out in the drawing up of the national economic plan will, as a result of establishing o.d. valuations, furnish individual factories with a summary of the whole situation in an extremely convenient form which should be used as a guide. For instance, a metal works in solving the problem as to whether it is worth substituting three tons of aluminium for one ton of lead need not analyse production and consumption of lead and aluminium on a nation-wide scale, but be guided simply by the o.d. valuations given and calculate whether such a measure results in a reduction of expenditure. Should requirements not correspond to the balance it will in turn be possible, on the basis of plans of individual factories, to carry out not a mechanical reduction of requirements but that reduction which will be least painful by means of o.d. valuations. Moreover, the very necessity of such substitutions will show that the o.d. valuation of a given factor was not determined quite correctly.

Control of the plan may be achieved by way of a corresponding increase in the o.d. valuations and hence a review of the use of a given factor.†

Consequently, the method of o.d. valuations will make it possible to introduce changes in the plan with greater flexibility and effectiveness in accordance with the requirements of the moment and of circumstances, while keeping the plan practically optimal throughout (relative to the new requirements).

We submit that the system of objectively determined valuations should present a system of indicators that is consistent, simple in meaning, sufficiently universal and convenient to use, a system which would furnish a synthetic economic characterization of the national economy at any given moment. The use of these indicators, once they have been compiled, will prove simpler and will facilitate the search for an optimal solution as against the application of numerous, often mutually contradictory, systems of indices which are being used at present. The application of the system of o.d. valuations will enable us to utilize continuously all the available productive capacity in the most efficient manner.

Thus, the importance of the principles of drawing up an optimal plan by having recourse to o.d. valuations consists in the organic combination of the balance and the value approach. In planning these two usually become separated.

The process of gradually increasing the accuracy of o.d. valuations, by taking into account the balance of a product as described above, is outwardly reminiscent of the process of competition in the capitalist world. Of course, in actual fact the one differs radically from the other. Here the problem, instead of the actual competition on the market, is one of competition among plans and methods in the process of planning calculations. Thus, the process is being accomplished without any material losses and may be brought to a balanced optimal plan which will be realizable. The process of capitalist competition is linked with continuous oscillations leading to constant disproportions and large losses (over-production, incomplete use of equipment, unemployment), and to the periodic appearance of crises.

Consequently there can be no question of consistent balances and

† See the description of the method of adjusting valuations (multipliers) in Appendix II, p. 322, *et seq.*

a planned utilization of resources in the interests of the national economy in a capitalist system. A socialist system, on the contrary, makes it possible in the process of preparing a plan to find the best agreement between the needs for a given product and its production, which ensures the highest development of productive forces and the maximum satisfaction of the material and cultural needs of the members of a socialist society.

For this reason, further improvement of planning, the transition to a system of optimal planning with valuations of production corresponding to the full national economic costs, should lead to a fuller realization of the advantages of a socialist system and to further increases in the rate of growth of its productive forces.

The unfounded propositions of individual economists from some People's Democracies regarding "improving" the planning system by allowing elements of spontaneity and competition between factories are due to their underestimating the enormous progress already made in planning and the development of socialist countries and also to their underestimating the great potentialities of further improvements in planning and of the economy that are inherent in socialist methods of production—the most highly developed in the history of mankind.

## CHAPTER III

# PROBLEMS CONNECTED WITH THE EXPANSION OF THE PRODUCTION BASE.

## EFFICIENCY OF CAPITAL INVESTMENT

THE problem of the efficient use of means for capital investment is of primary importance in the development of the national economy. *The speed of development of productive forces, the progress of Soviet industry and agriculture and the full realization of the advantages of a socialist economic system will depend upon a correct solution of this problem.* A profound and practical analysis of the importance of a proper choice of objects for capital investment for the rate of development of the economy was made by N. S. Khrushchev in his speech at the inauguration of the Volga hydroelectric power station named after V. I. Lenin.

The diversity of possible technical solutions and modes of development in present-day industry, interdependence between various sectors within the national economy and the close connection of the problem of capital investment with other fundamental economic problems together with questions of technological policy, make this an extremely complex task. It is difficult to think that it could be satisfactorily solved in theory or in practice by old-fashioned methods. A thorough and comprehensive scientific analysis is required.

Under capitalism the efficiency of capital investment is determined on the basis of the maximum profit obtained; this is calculated from the system of prices formed spontaneously in the market and from normal profits.

In a planned socialist economy problems of the efficiency of capital investment are solved in the process of drawing up the plan of development of the national economy and of adopting particular

economic measures as part of this plan. The central problem in the allocation of means of capital investment and the choice of individual investments is to ensure the best possible development of the national economy in conformity with the tasks and requirements of society. Although the national economic plan and the economic indices utilized in drawing it up are based on the decisions of planning and economic organizations as a whole, neither the measures nor the indices are arbitrary. They are objectively determined from the state and the tasks of the national economy and the totality of economic laws governing a socialist society. The task of Soviet economic science is to discover these laws and the mechanism of their operation and to utilize them in solving the economic planning tasks confronting society.

The correct and most efficient choice of objects of capital investment is of quite exceptional importance considering the enormous sums, running into astronomical figures, spent by our country on capital investment: "In the years 1946-1958 alone, the volume of capital investment by the state at current prices amounted to more than 1,600,000 million roubles. During this period, approximately twelve thousand large state industrial factories and a large number of medium and small factories were put into operation."†

"The volume of state capital investments will increase in the years 1959-1965 to 1,940,000-1,970,000 million roubles, or by 1.8 times as compared with the preceding seven years. This almost equals the volume of capital investment in the national economy since the Soviet government came into power."‡

The determination of the efficiency of capital investment under socialism differs radically from the solution of similar problems under capitalism. It would, therefore, be inadmissible to transplant mechanically the methods of calculating efficiency as utilized in a capitalist economy; an accurate and direct analysis of this problem in a socialist society is necessary.

An attempt is made in this chapter to develop a method of calculating the efficiency of capital investment under socialism from the analysis of an optimal investment plan.

† Official target figures of the development of the national economy of the U.S.S.R. for the years 1959-65. *Report of the XXI Congress of the Communist Party of the U.S.S.R.*, vol. II, p. 466.

‡ *Ibid.*, p. 501.

We first consider problems relating to short-term investment for production and give a few numerical examples of their analysis (Section 1). We then consider carefully the problem of the extent to which the conclusions thus obtained may be applied to long-term investments (Section 2). We further consider the problem of the ways and means of using the methods arrived at in practical planning (Section 3). Finally, we compare the proposed method with other proposals for the calculation of efficiency (Section 4).

### Section 1. Short-term Investments. Normal Efficiency

In order to ensure the growth of the volume of production, it is essential, besides improving the use of the available production base, to expand this base by capital investments. For this reason, a certain portion of final output is set apart each year for capital investments, to meet depreciation and for further expansion of the production base. "The Communist party attaches paramount importance to the most effective movement of capital investments which would make it possible to increase productive capacity and industrial output with *reduced expenditure on means, in the shortest possible time, by rapidly raising labour productivity and lowering costs of production.*"†

What is the criterion to be adopted in considering whether a given *capital investment, such as the use of a given machine or device, is advisable?* The first necessary condition is that the use of the machine during its entire period of operation should produce a saving in labour amounting at least to the cost of labour entailed in its manufacture.

However, the number of such machines and, generally, of objects which may become the aim of capital investments, is extremely large, while the resources available for this purpose are limited. Consequently, out of all possible objects those must be chosen in which the use of available resources would produce the maximum effect. How should this problem be approached? First of all, the valuation of the saving achieved in the national economy by equipment obtained as a result of capital investment is of great importance.

† Target figures of the development of the national economy of the U.S.S.R. for the years 1959-65. Stenographic report of the XXI Congress of the Communist Party, vol. II, p. 501.

However, this question was discussed in Chapter II, Section 5, where it was shown that the magnitude of such saving was given by the hire valuation of such equipment.

The principle of solving this problem can now be easily stated. Let us assume that we have two machines with an objectively determined valuation of their manufacture of 100,000 roubles each. The hire valuation of the first is 5000 roubles per month, that of the second 12,000 roubles per month. It is, therefore, clear that means must first be made available for the second machine since by putting it into operation a monthly saving of 12,000 roubles is achieved, while the operation of the first would only produce a saving of 5000 roubles. Consequently, in solving the problem, the competing positions of the investments must be arranged in the order of magnitude of the efficiency of investments, which equals the ratio of the valuation of hire (the saving to be achieved) to the necessary expenditure on resources (in the example, this is 5 per cent for the first machine per month and 12 per cent for the second). It therefore follows that means should be secured for those investments for which the efficiency is highest. This will give the greatest possible rise in net production.† Broadly speaking, this conclusion also furnishes the correct solution to the present problem. Without any fundamental reservations, it may be applied to solve problems of investments which can be completed or recovered within a short period. For longer periods, the analysis of the problem becomes complicated for a number of reasons. The chief one is the fact that investments assume a definite form and may only be transformed into some other form of investment at a great loss or not at all, if the form in which they were made ceased to be efficient. For instance, in the course of time changes occur in the system of o.d. valuations (in particular, the valuation of hire of a given type of equipment may change substantially) as a result of which the degree of efficiency of the investment also changes.

Further, it should be taken into account that for investments

† The following doubt may arise: could this increase in output fail to correspond to the production required? Such a doubt is not justified. If, with the aid of the machine put into operation, a certain type of product can be increased by 12,000 roubles, an increase which we do not require, the output of this product may be maintained at its previous level as it affords the fulfilment of the o.d. valuation, and by releasing manpower, materials, etc., produce a corresponding amount of some other product that is required.

involving a longer period of service, the recovery of costs and the achievement of saving frequently covers so long a period as to render the investment inadvisable on some occasion. Finally, the investment itself requires a certain time to be carried through. When solving the problem of the advisability of investments, the future position must be taken into consideration since the investment starts yielding a saving only after the appropriate machine or equipment has been completed.

We shall show by examples how the influence of some of these factors may be accounted for; and from them the problem will be further analysed. The first of these examples relates to the case of short-term investments in production planning.

EXAMPLE. In a metal-working factory a large quantity of instruments, tools and punches are necessary in order to ensure that current output is maintained and increased in the future. All the tools required are divided into groups according to their kind (Table 37). The saving effected by each tool or the corresponding losses caused

TABLE 37 SAVING IN CURRENT EXPENDITURE, COST OF TOOLS AND NET SAVING

Instruments and tools		Saving in current expenditure per month (in roubles)	Cost of tools (in roubles)	Net saving	
Type	Number			in roubles	as percentage of cost
(calculated per tool)					
I	100	800	500	300	60
II	100	1200	1000	200	20
III	500	300	100	200	200
IV	1000	56	40	16	40
V	200	250	200	50	25
VI	30	2000	1000	1000	100

by its absence are also shown in the table. We suppose that in both cases this figure is obtained in accordance with the o.d. valuations of the basic production. A month is taken as the period of service of all the instruments and tools and the saving achieved is also calculated for this period. These tools and instruments are produced in a proper instruments workshop of the factory. The table gives the cost of each tool on the basis of the o.d. valuations.



Further, on subtracting the cost of the tool from the saving effected in production, the net saving resulting from its use is obtained. In all cases this saving is positive. Therefore, the use of all the tools seems advisable. The total cost of these tools and instruments is 310,000 roubles, the saving that may be effected by their use is 516,000 roubles, and the net saving 206,000 roubles. However, the sum that may be invested in tools is limited to 150,000 roubles, so that this figure determines the productive capacity of the instruments workshop† for the given month. Thus, not all the instruments and tools ordered can be manufactured. In order to select those which should be produced, the efficiency of each investment is calculated. For this, a ratio is established between the net saving resulting from its use in production which agrees with the hire valuation (Conclusion 19) and the magnitude of its cost of production (o.d. valuation of expenditure for its manufacture). The value of this efficiency, expressed as a percentage, is given in the last column of Table 37.

TABLE 38. WORKING PLAN OF THE INSTRUMENTS WORKSHOP

Tools and instruments	First month				Second month			
	Number of tools	Cost of production	Saving in current expenditure	Net saving	Number of tools	Cost of production	Saving in current expenditure	Net saving
I	100	50,000	80,000	30,000	100	50,000	80,000	30,000
II	—	—	—	—	—	—	—	—
III	500	50,000	150,000	100,000	500	50,000	150,000	100,000
IV	500	20,000	28,000	8,000	1000	40,000	56,000	16,000
V	—	—	—	—	150	30,000	37,500	7,500
VI	30	30,000	60,000	30,000	30	30,000	60,000	30,000
Total	1130	150,000	318,000	168,000	1780	200,000	383,500	183,500

Thus it can be seen that an expenditure of 100 roubles (and consequently, the allocation of a corresponding portion of the productive capacity of the instruments workshop) for the manufacture of tool I gives a net saving in basic production of 60 roubles per month, while for tool II this figure amounts to only 20 roubles. It is clear,

† We assume that in this case the volume of output of each item expressed in money reflects with sufficient accuracy the portion of productive capacity utilized in the instruments workshop on its manufacture.

therefore, that first the tools should be manufactured in the following order: III, VI, I, and IV. It is found that they (not all of IV) already exhaust the productive capacity of the instruments workshop. Its plan for the first month is drawn up on this basis (Table 38).

Although we are only able to meet approximately 50 per cent (by volume) of the demand for instruments and tools, by an accurate (best) selection, we have achieved about 80 per cent of the possible total net saving (168,000 roubles out of 206,000 roubles).

The plan for the second month is drawn up in a similar manner, on the assumption that the productive capacity of the instruments workshop is planned to be increased to 200,000 roubles. All the tools IV as well as a portion of tools V are completed for this month. The appropriate plan is given in Table 38.

### *Normal Efficiency*

While selecting investments consecutively according to their efficiency, we stopped (for the first month) at tool IV, the efficiency of which is 40 per cent. This magnitude constitutes for us the measure of the advisability of an investment which leads to an optimal plan. By making investments the efficiency of which exceeds 40 per cent (I, III, and VI), and refraining from those with less than 40 per cent, we arrive at the best plan. We shall call this magnitude the normal (or objectively determined) efficiency of investments. It is one of the forms of the o.d. valuations. The fact that normal efficiency equals, say, 40 per cent per month shows that an additional sum for capital investment can be used with the efficiency referred to, i.e. the allocation of an additional 100 roubles for investments affords a net saving of 40 roubles in the course of a month; in other words, an increase in production or a saving in expenditure of 140 roubles will be achieved in the course of a month. Briefly: 100 roubles assigned to investment today will yield 140 roubles within a month. On the other hand, a reduction in the means of investment by 100 roubles today will result in a reduction of output of 140 roubles for the next month. Thus, normal efficiency characterizes the extent to which, under given conditions, the presence or absence of means of investment may influence the progress of labour productivity in the future. More precisely, it shows the saving that could be achieved in the future by an expenditure of labour now.

Normal efficiency like any o.d. valuation is concrete. This is evident from the example already considered. An increase in means of investments lowers normal efficiency in the second month to 25 per cent (efficiency of investments in type V tools). It displays equally the other properties, such as stability and realism. The above may be formulated as follows.

**CONCLUSION 23.** With limited means for short-term investments, there exists a definite normal efficiency of investments. If this were adopted as a guide, if an investment was made when its efficiency (the relationship between the net saving per month achieved as a result of the investment and the amount of the latter) exceeded the normal, and not made when its efficiency was below normal, then an optimal investment plan will be obtained. The latter means that such a plan produces the largest total saving in expenditure that can be achieved with the given means of investment. Normal efficiency of investments is practical. It is determined by all the conditions: the volume of means of investment, the possibilities of using investments and their efficiency. This efficiency diminishes as the means available for investment increase, and conversely rises as the latter diminish. Normal efficiency shows to what extent, under given conditions, the presence or absence of means of investment may influence the progress of productivity of labour in the future.

It must be mentioned that our conclusion that a single measure of efficiency should be used in ascertaining the advisability of an investment was substantially based on the fact that both saving in production and the volume of investment are determined according to the o.d. valuations. We emphasize this reservation, as the application of this proposition is unfounded when using current prices or prime cost in the calculation. An attempt to apply it in such a form leads sometimes to incorrect conclusions which may arouse doubts as to whether the proposition is valid at all.

For instance, it may be argued that a particular instrument does not produce a considerable saving, yet it must be manufactured, as without it it is impossible to manufacture certain components, and as a result the output of some type of production may have to be cut down. However, if we calculated correctly the losses due to the lack of a given instrument, allowing for the fact that if the component in question is in short supply its valuation rises sharply and with it the

hire valuation of the corresponding instrument, it will be found that the efficiency of this investment is unusually high and will certainly be included in the plan.†

Another possibility, although a particular tool may produce a considerable saving and operate at an efficiency above normal, it cannot be manufactured as its manufacture would involve the use of scarce materials or because it would entail the use of machines in the instruments workshop which are already working to capacity. In this case, taking into consideration the high o.d. valuation of scarce materials together with the high hire valuation of the machine in operation in the instruments workshop, the calculation of the cost of the given tool will show a considerably higher figure (and a considerably lower one of its efficiency) than that obtained by the usual calculation. Should the efficiency be below normal its use under the given conditions would indeed be inadvisable. Should it prove to be higher, the use of the tool would be advisable and it should be manufactured irrespective of the difficulties mentioned, and both the materials and the machine should be made available at the expense of some other work.

We do not know whether such a calculation has actually been applied in such and similar problems; it is indeed difficult to apply it without o.d. valuations. The use of cost of production in their place would furnish inadmissible results chiefly because it does not account fully for the load factor of the available equipment. However, such a calculation is very important for solving this kind of problem.

If normal efficiency is not taken into account, demands for instruments and tools the use of which appears profitable usually by far exceed the capacity of the instruments workshop. The lack of method which would provide an objective valuation and comparison of the effects of each of these instruments has the consequence that these demands of factories are reconciled with actual capacities largely by a mechanical or accidental reduction; or else if they are maintained, the instruments workshop leaves part of the assignment deliberately unfulfilled. In the first instance considerable losses are incurred in comparison with what extent to have been expected had the problem been suitably solved by means of ample, with a uniform, activity in the future.

† Nevertheless, absolute demands could be excluded from the analysis from the very beginning, as in the example of unconditional loads in Section 7 of Chapter II.

mechanical reduction in the demands, the net saving from tools would constitute 101,000 roubles instead of 168,000 roubles of the plan in Table 38, and instead of 206,000 roubles which could be achieved if production by the instrument workshop had not been limited). In the second case, the discrepancy between the assignments and the capacity of the instruments workshop may often lead to delays in the delivery of instruments and tools by the latter, causing stoppages and interrupting the work of the main workshops.

Increased demands on the instruments workshops frequently restrict basic production. The fullest possible utilization of the capacity of the instruments workshops and the choice of a plan for this purpose following the method given above could reduce the losses due to insufficient productive capacity.

In considering the example quoted the idea naturally comes to mind that whilst it is not possible to take all the measures that would ensure a considerable saving it is advisable to expand the instrument workshop even if this has to be done at the expense of current production: if the given machine should yield less than 100 roubles'-worth of basic production, it could—if used in the instruments workshop—produce 100 roubles'-worth of instruments which would in turn provide 140 roubles'-worth of production (at the same expenditure) instead of the previous 100 roubles. However, such reasoning will not always hold. The fact is that 100 roubles'-worth of production can be obtained immediately, while a production worth 140 roubles could only be achieved in the course of one or one and a half months. It is quite possible that if production is required for immediate use or its delivery is necessary to ensure the operation of other factories, it may be more important to have 100 roubles'-worth of production immediately than even  $1\frac{1}{2}$  times more in one or two months. For this reason, the advisability of expanding the instruments workshop at the expense of the remainder is not absolute and will depend upon practical conditions. However, it is indisputably important to use its available productive capacity in the best possible manner.

The reasoning given should also be applied to other problems. For instance, although the construction of a new railway or sidings for a factory may enable production to rise and thus to recover the expenditure incurred with interest, this additional production will only be achieved after some months or a year, while the metal for the

rails must be used immediately and is not available for other activities where it might be required. This is not always possible or advisable, and hence not all the investments which may afford an increase in production can be realized. Only a definite proportion of final products is allotted to capital investment as determined by the general situation. These means cannot be increased indefinitely and must be utilized in the best possible manner.

The analysis of the problem carried out on the example is general, and therefore the conclusions obtained here on the basis of the analysis of the investments plan are of general significance.

In other words, the solution of this problem in a general case is the same and is based on the same considerations as in the example quoted. The normal efficiency of investments is determined by a given concrete situation (available means of investment, possible objects) at a given period which should be taken as a guide in the solution of individual problems. In particular, with two possible investments, that investment of which the efficiency is higher should be secured first of all. In fact, if an investment of 1000 roubles was contemplated in the plan for the manufacture of product I with an efficiency of 20 per cent, but meanwhile the possibility was discovered of investing 1000 roubles in the manufacture of type II at an efficiency of 100 per cent then generally speaking, it would be feasible and advisable to change over from one to the other. In view of the practical nature of the o.d. valuations, material resources of the value of 1000 roubles for the first investment can be replaced by other material resources of the same value that are suitable for the second investment. In turn, products of type II obtained in the following month due to this investment and valued at 2000 roubles can be replaced by products of type I to the same amount. As a result, without any change in expenditure for the given month, a larger quantity of the required production will be available after a month.

This reasoning shows that the proposition about normal efficiency is applicable in so far as substitutions by equivalents are admissible, determined as a rule by o.d. valuations. This does not apply to very large investments which change fundamentally the conditions and the system of valuations.

Thus, there exists at a given moment an objectively determined normal efficiency of investments. Its magnitude is so determined that all the objects of investment with an efficiency exceeding the

normal one can be realized with the means allocated for the purpose which, in turn, are exhausted by these investments. Normal efficiency may differ in various sectors of the economy, but only insignificantly. It should also be adopted as a guide when solving particular problems relating to the use of the means of investment.

These propositions may be used, under appropriate conditions, by the various units that have to determine an optimum investment plan: a group of factories, a sector, an economic district.

### *Accounting Technique Using Normal Efficiency*

The possible magnitude of such a measure of efficiency will be considered below. We shall assume now that the normal efficiency has been determined for the economy as a whole (or for an economic district, sector, factory) for the present and for subsequent time periods, and we shall show how various calculations relating to investments can be carried out with its aid.

Basically, the calculation is as follows: if normal efficiency at a given moment is 20 per cent per month (quarter, year), then there exist unused possibilities where an investment involving an expenditure of 100 roubles now will lead to an increase in production to 120 roubles in the following month. In other words, expenditure on labour made rationally in the given period will afford a considerably greater saving of labour in the subsequent period (by increasing its productivity). This also provides a starting point for converting the expenditure of the next period to the given period. Thus, the sum of 100 roubles today equals the sum of 120 roubles after a month. If normal efficiency remains unchanged in the next month, the sum of 100 roubles will correspond to the sum of 144 roubles (120 roubles + 20 per cent) or roughly 140 roubles (100 roubles +  $2 \times 20$  per cent) after two months. Conversely, the sum of 100 roubles after a month corresponds to  $100 \div 1.20 = 83$  roubles now. This should, therefore, be the starting point. Let us consider a few examples.†

EXAMPLE 1. In the example considered above a tool with a two months' period of service is proposed. Its cost is 2000 roubles, and the monthly saving 1400 roubles. Of course, its cost is fully

† An efficiency of investments of 20–40 per cent per month which appears in the examples is fairly seldom met in practice. However, such a level of efficiency makes the example more obvious, while the method of calculation does not depend upon it.

recovered, but is its manufacture advisable when normal efficiency is taken into account, when other possible objects of investment are considered?

We carry out the calculation by converting everything to the last month of the period under consideration (the third month) and allowing for normal efficiency (40 per cent for the first month and 25 per cent for the second—see pp. 158–9). We obtain:

for the sum of investments:

$$2000 \times (1 + 0.40) \times (1 + 0.25) = 3500 \text{ roubles,}$$

for the sum of saving:

$$1400 \times (1 + 0.25) + 1400 = 3150 \text{ roubles.}$$

This shows that this investment is not justified.

Thus, both the extent to which an investment made is recovered and the time within which this is achieved (the period of service) are of paramount importance.

Thus, if normal efficiency each month is 30 per cent, an investment of 1000 roubles that gives a monthly saving of 300 roubles and will therefore be recovered almost twice over within six months will prove inadvisable. In practice, the calculation can be carried out with a sufficient degree of accuracy by using simple instead of compound interest.

The average period for the recovery of expenditure is three and a half months. Therefore, at the given normal efficiency (30 per cent), we should arrive at  $1000 \times 3.5 \times 0.30 = 1050$  roubles of net saving, but we only have  $6 \times 300 - 1000 = 800$  roubles. The investment is inadvisable since there will be more efficient investment possibilities.

**EXAMPLE 2** Normal efficiency is 20 per cent per month. A tool costs 1000 roubles, the period of service is four months, the monthly saving 400 roubles, the time necessary for its manufacture two months, while the initial expenditure is incurred in the first month.

After two months, at the moment when it enters into operation (in the period of the third month), its cost should be  $1000 \times (1 + 2 \times 0.20) = 1400$  roubles. Subsequently, the (mean) period for the recovery of expenditure is one and a half months. It should produce, therefore, a saving of  $1400 \times 1.5 \times 0.20 = 420$  roubles. In fact, the saving amounts to  $4 \times 400 - 1400 = 200$  roubles. Thus, if a saving is not achieved immediately after the expenditure has been incurred,



the efficiency of an investment must be considerably higher before it can be recommended.†

Let us note that calculations similar to those given above may be given a somewhat different form. We could enter for each month an appropriate valuation (coefficient) in order to convert the sums (of production or expenditure) of the given month to the first month on the basis of normal efficiency. It is then sufficient to calculate the effect of the investment and the expenditure involved in it in accordance with these valuations in order to determine whether it is justified or not or whether it is efficient enough under the given conditions. Thus, in the example under consideration and with a normal efficiency of 20 per cent, these conversion coefficients have values as shown in Table 39.

TABLE 39. COEFFICIENTS FOR CONVERTING EXPENDITURE AND ECONOMIC EFFECT TO THE INITIAL PERIOD (THE FIRST MONTH)

Months	1	2	3	4	5	6	7
Conversion coefficients	1.00	0.83	0.69	0.58	0.48	0.40	0.33

In fact, 100 roubles in the second month correspond to  $100 \div (1 + 0.20) = 83$  roubles in the first month, and in the third month to  $100 \div (1 + 0.20)^2 = 69$  roubles, etc.

By calculating the saving obtained and the expenditure in the given example and by converting them to the first month as in Table 39, we obtain  $400 \times 0.69 + 400 \times 0.58 + 400 \times 0.48 + 400 \times 0.40 - 1000 \times 1.00 = -140$  roubles, i.e. the investment is not justified and should be rejected as not efficient enough (this is somewhat different from the preceding calculation in that it was carried out with simple percentages and by taking average values).

† The calculations given may remind one of the usual calculations of standard profit (as a percentage of capital). The main difference consists in that we do not look upon investments as capable of yielding a new product by themselves, as do some bourgeois economists, but only as a means which increase productivity of labour, reduce the costs of production and thereby cause a certain rise in the efficiency of labour. Surplus value could perform such a function both under capitalism and socialism. The difference between capitalism and socialism does not lie in the fact that surplus value performs this function and is accounted for in the cost of production, but in the fact that under capitalism this surplus value is turned into capital which is the private property of capitalists and is used as an instrument of exploitation.

**EXAMPLE 3.** *Normal efficiency is 20 per cent. Twenty-five instruments of a certain type are required each month, and these requirements are unconditional. If a batch of 25 instruments is produced, the cost of each will be 40 roubles, for a batch of 50 it will be 35 roubles and for a batch of 100, 32 roubles each.*

Which batch size would be the best to choose?

To produce an additional 25 instruments in the given month,  $50 \times 35 - 25 \times 40 = 750$  roubles will have to be spent. If they were to be produced after a month, the expenditure involved would be  $25 \times 40 = 1000$  roubles, i.e. 33 per cent more. Since normal efficiency is 20 per cent, a batch of 50 instruments would be better than one of 25. Again, passing from a batch of 50 instruments to a batch of 100, i.e. producing an additional 50 pieces, we shall spend  $100 \times 32 - 50 \times 35 = 1450$  roubles instead of  $50 \times 35 = 1750$  roubles after two months. The efficiency is  $300:1450 = 21$  per cent for two months, which is insufficient when the normal efficiency per months is 20 per cent.

Thus, under the given conditions, the batch of 50 is the best choice.

It should be emphasized once more that in calculating the efficiency of investments, the outlay on investment and the saving achieved must be determined by taking into account the o.d. valuations, and the normal efficiency must be applied according to given conditions and the existing situation. Otherwise, if an arbitrary value or one derived under entirely different conditions were adopted as normal efficiency, the whole calculation would turn into a meaningless play with figures.

We have now shown the application of the method of calculating the efficiency and advisability of investment in more complex cases than envisaged in Conclusion 23. We shall describe this method in the following conclusion.

**CONCLUSION 24.** *If the normal efficiency and the system of o.d. valuations are known for all intervals of time into which the period under consideration has been divided, then in order to ascertain the advisability of a certain investment it is necessary: (1) to calculate the expenditure involved in the completion of an investment and also the efficiency achieved as a result of its use in each interval of time on the basis of o.d. valuations; (2) to convert all these sums to one common interval of time by means of normal efficiency; (3) to compare the sums of expenditure and the general effect, or the total saving (theoretically, compound interest should be used in the conversion to*

a common time interval, but in practice it may be possible to confine oneself to simple interest).

NOTE 1. Let us note that in the particular instance when an investment is completed in the following period, that is, under the conditions of Conclusion 23, Conclusion 24 furnishes an identical criterion. Indeed, if  $n$  designates the value of normal efficiency,  $C$  the amount of investment and  $D$  the net saving attained (the gross saving =  $C + D$ ), then using the conversion coefficient  $1/(1+n)$ , the comparison of the sum of expenditure with the efficiency achieved gives the formula

$$C \leq (C+D) \frac{1}{1+n}, \quad \text{or} \quad \frac{D}{C} \geq n.$$

NOTE 2. Another important simple case occurs when normal efficiency and the saving are constant and the object of investment does not depreciate (has a very long period of service), or where the economy achieved coincides practically with the net saving. In this case, the criterion of choice is that the efficiency of an investment must not be below normal. In fact, with this notation, if  $D$  represents the annual net saving, the comparison of expenditure and the total saving converted to the first year gives the condition:

$$C \leq D \frac{1}{1+n} + D \left( \frac{1}{1+n} \right)^2 + \dots = D \frac{1}{1+n} \cdot \frac{1}{1 - (1/(1+n))} = \frac{D}{n},$$

or

$$\frac{D}{C} \geq n.$$

The ratio  $D/C$  again represents the efficiency of the investment.

NOTE 3. When the final period of service of an investment equals  $k$ , and the cost of completing the object of investment at the end of the period of service is designated by  $C^*$ , the condition for the justification of an investment may be expressed in the following form:

$$C \leq \bar{D} \frac{1}{1+n} + \dots + \bar{D} \frac{1}{(1+n)^k} + C^* \frac{1}{(1+n)^k}$$

where  $C$  is the volume of an investment and  $\bar{D}$  the yearly saving. This may be rewritten as:

$$\frac{\bar{D}}{C} \geq n + \frac{n}{(1+n)^k - 1} \left( 1 - \frac{C^*}{C} \right).$$

Particularly, in the case where there is no depreciation of the investment ( $C^* = C$ ), the second term disappears, and we have the same expression as in Note 2 (here  $\bar{D} = D$ ). If, on the other hand, the cost of completion can be disregarded,  $C^* = 0$ , the condition becomes:

$$\frac{\bar{D}}{C} \geq n + \frac{n}{(1+n)^k - 1}.$$

Usually, the term for the efficiency of investment in the final period of service is used (starting from the normal efficiency or from the standard period of recoupment) in the form given under Note 1, wherein  $D$  denotes the net saving, i.e. the annual saving less deductions for recoupment (for renewal). In other words, we take  $D = \bar{D} - C/k$ . Consequently, the condition showing whether an investment is justified is given in the form:

$$\frac{D}{C} = \frac{\bar{D} - C/k}{C} \geq n \quad \text{or} \quad \frac{\bar{D}}{C} \geq n + \frac{1}{k}.$$

This condition differs from that obtained above (which is more accurate) and provides an approximation to the latter only if  $kn$  is small, or the period of service is short in comparison with the period of recoupment, or if  $kn$  is very large. Thus, for more accurate calculations the last condition cannot be used. The calculation of expenditure on capital repairs made by equal annual deductions suffers to a lesser extent from the same shortcomings. It would be more accurate to compute all the costs by the period of its realization and to convert them to a single point of time.

NOTE 4. The concept of efficiency of a given investment may also be introduced in the case where the period of service is not unlimited or when annual savings are not constant. We shall understand by efficiency of a given investment the highest attainable value of normal efficiency at which the given investment is justified. In the cases considered in Notes 1 and 2, this concept agrees with the previous one since the highest value of normal efficiency is:

$$n = \frac{D}{C}.$$

As an illustration, we shall calculate the efficiency of the investment described in Example 1. Its value is obtained from the equation (all converted to the first month):

$$2000 = 1400 \frac{1}{1+n} + 1400 \left( \frac{1}{1+n} \right)^2.$$

By solving this quadratic equation, we find  $n = 0.257$ . The efficiency of investment is 25.7 per cent.

NOTE 5. When normal efficiency varies from year to year, the condition for the advisability of an investment assumes a more complex form, namely:

$$\begin{aligned} C &\leq \frac{D_1}{1+n_1} + \frac{D_2}{(1+n_1)(1+n_2)} + \dots + \frac{D_k + C^*}{(1+n_1)\dots(1+n_k)} \\ &= r_1 D_1 + r_2 D_2 + \dots + r_k (D_k + C^*) \end{aligned}$$

where  $D_i$  is the saving in the  $i$ th year,  $n_i$  is the normal efficiency from the  $(i-1)$ th year to the  $i$ th,  $r_i$  is the conversion coefficient of the  $i$ th year to the initial year (year zero).

NOTE 6. For the same purpose of calculating the efficiency of capital investment, the method of determining the period of recoupment of additional capital investments is widely used. If with one method of producing a given output, current (annual) expenditure amounts to  $C_1$  and capital investment to  $K_1$ , and with another method  $C_2$  and  $K_2$  respectively, the period of recoupment of the additional investment is expressed by the formula:

$$\frac{K_2 - K_1}{C_1 - C_2} = t.$$

The comparison of the efficiency of various additional investments is made on the basis of this period; by taking it as the normal one the admissible level of efficiency is determined.

If the period of service of capital investment is very long and if current expenditure and capital investments are evaluated accurately (in accordance with o.d. valuations), then in so far as capital investment  $(K_2 - K_1)$  affords a yearly saving  $(C_1 - C_2)$ , the efficiency of a given investment as stated above (Note 2) would equal:

$$n = \frac{C_1 - C_2}{K_2 - K_1} = \frac{1}{t};$$

it would represent the inverse of the period of recoupment. For this reason, the comparison of recoupment periods and their normalization under such conditions are equivalent to a comparison of efficiency and the setting of a normal efficiency. For instance, a five-year period of recoupment would correspond to a normal efficiency of 20 per cent. The fundamental distinction (and shortcoming, in our view) of this method in comparison with the method described above (Conclusions 23 and 24) consists in the following: (a) its application becomes difficult for short periods of service, by the changes in the value of efficiency over several years, when capital expenditure, etc., effected at different times has to be taken into account; (b) it relates to supplementary and not to basic investments; (c) usually the normal period of recoupment is fixed conventionally and not objectively, depending upon the conditions of the situation, in a manner similar to normal efficiency; (d) the values of  $C_1$ ,  $C_2$ ,  $K_1$ ,  $K_2$  are calculated on the basis of cost or of current prices and for this reason they do not always reflect accurately the actual national economic expenditure so that a change in that respect could completely alter the value of  $t$ .

#### *Calculation of Change in O.D. Valuations*

When the relative o.d. valuations change in the course of time these changes must be allowed for in the calculation of the efficiency of capital investments.

To give an illustration as to how an analysis should be carried out in such a case, let us turn back to the example of the manufacture of instruments and tools which we shall now consider under more complex conditions. We shall assume that the manufacture of the necessary instruments involves the use of some scarce material the possible expenditure on which is limited to 68,000 roubles for the first month and to 88,500 roubles for the second month. The data for the expenditure on this material and the optimal plan that allows for this condition, are given in Table 40.

Owing to the necessity of observing the limit for the expenditure on the scarce material some changes had to be made to the plan as compared with Table 38 by substituting to some extent method IV for method I which involves a smaller expenditure on the material in the first month, and by substituting method II for method V for the second month. The extent of the saving is consequently somewhat reduced.

In order to verify that the plan shown is optimal, it is sufficient to determine the existing relative valuations. We shall show that the following valuations can be adopted for the first month: 0.76 for the conversion coefficient of the saving achieved (realized in the second month) to the first month, and 1.27 for the scarcity coefficient of the

TABLE 40. PRODUCTION PLAN OF THE INSTRUMENTS WORKSHOP  
(ALLOWING FOR SCARCE MATERIALS)

Tools and instruments	Cost of instruments I, including scarce materials (roubles)	Net saving on one instrument (roubles)	Number of instruments		Cost of instruments, including the material (roubles)		Net saving attained (roubles)	
			Month					
			1st	2nd	1st	2nd	1st	2nd
I	500 (400)	300	80	100	40,000 (32,000)	50,000 (40,000)	24,000	30,000
II	1000 (200)	200	—	20	—	20,000 (4,000)	—	4,000
III	100 (45)	200	500	500	50,000 (22,500)	50,000 (22,500)	100,000	100,000
IV	40 (10)	16	750	1000	30,000 (7,500)	40,000 (10,000)	12,000	16,000
V	200 (120)	50	—	50	—	10,000 (6,000)	—	2,500
VI	1000 (200)	1000	30	30	30,000 (6,000)	30,000 (6,000)	30,000	30,000
Total					150,000 (68,000)	200,000 (88,500)	166,000	182,500

material in short supply. In other words, production at the instruments workshop amounting to 100 roubles should ensure a saving in the following month of  $100 : 0.76 = 132$  roubles; an increase in expenditure of 100 roubles on the scarce material should correspond in efficiency to 127 roubles of other expenditure.

We shall not deal with the determination of these valuations;† to establish that the plan is optimal for the first month we shall verify

† The methods given in Chapter I (pp. 16-20) should be used. The valuations may be obtained by balancing production and expenditure for methods that are not fully utilized in the optimal plan. Designating these valuations by the letters  $r$  and  $d$ , we shall find them from the equations:  $100 + 400d = 800r$  (I);  $30 + 10d = 56r$  (IV); hence  $r = 0.76$ ;  $d = 1.27$ .

that in accordance with these valuations the production of tools included in the plan for the first month is justified: the given valuation of expenditure does not exceed the saving. For instance, for tool I:

$$(100 \times 1) + (400 \times 1.27) = (500 + 300) \times 0.76;$$

for tool III:

$$(55 \times 1) + (45 \times 1.27) < (100 + 200) \times 0.76.$$

Similar valuations for the second month give: a conversion coefficient of the saving obtained (achieved in the subsequent third month) of 0.84 against the expenditure in the given (second) month; the scarcity coefficient of the material equals 1.1.

All these valuations can be converted to a single unit—expenditure in the first month. Thus, the saving achieved in the third month will be converted to the expenditure of the first month with a coefficient of  $0.76 \times 0.84 = 0.64$ . As regards the material for the second month, the expenditure for which as related to all other expenditure in the second month has a conversion coefficient of 1.1, and from it we obtain as conversion coefficient  $1.1 \times 0.76 = 0.84$ , in terms of expenditure in the first month.

In the final account we arrive at the system of valuations given in Table 41 (the figures in brackets are explained below).

TABLE 41. RELATIVE VALUATIONS CONVERTED TO THE FIRST MONTH

	Months		
	1st	2nd	3rd
Valuation of expenditure and saving	1.00 (1.00)	0.76 (1.00)	0.64 (1.00)
Valuation of material	1.27 (1.27)	0.84 (1.1)	--

With these valuations it is easy to solve the problem of the advisability of one or another method of production on which expenditure is incurred and which becomes effective at different times.

Let us assume that a tool costing 2000 roubles (including the cost of the scarce material of 800 roubles) can be manufactured in the course of two months (50 per cent of costs in each month), and will save 3000 roubles in the course of the third month. Is such manu-



facture advisable? Converting the expenditure and saving to the first month we have:

expenditure:

$$600 \times 1.00 + 400 \times 1.27 + 600 \times 0.76 + 400 \times 0.84 = 1900 \text{ roubles}$$

saving:

$$3000 \times 0.64 = 1920 \text{ roubles.}$$

The use of the tool is advisable.

This calculation can be carried out in another form, namely, by starting from the valuations for each period and calculating the expenditure and efficiency for each period on that basis, and thereafter convert the net effect to a common period by the same conversion coefficients. Thus, adopting a rouble of expenditure and saving as a unit for each interval of time (besides the scarce material), we obtain the figures of the relative valuations for each month as given in Table 41 in brackets. The coefficients for conversion to the first month will be: 0.76 for the second month, and 0.64 for the third (0.84 when the values of the third month are converted to the second). These coefficients correspond to a normal efficiency of 32 per cent ( $1:0.76 = 1.32$ ) for the first month and 19 per cent ( $1:0.84 = 1.19$ ) for the second.

The calculation of the expenditure and saving will then appear as follows:

expenditure:

$$(600 \times 1.00 + 400 \times 1.27) + (600 \times 1.00 + 400 \times 1.1) \times 0.76 \\ = 1900 \text{ roubles}$$

saving:

$$3000 \times 0.84 \times 0.76 = 3000 \times 0.64 = 1920 \text{ roubles.}$$

It should be noted that where in the present case relative valuations change, the coefficients of conversion of expenditure to one period or the corresponding level of efficiency of investments will substantially depend upon the choice of unit of valuations for each period. Thus, in the given example expenditure (of 1 rouble) was chosen as a unit of the valuations excluding the scarce material; with the choice of another unit, the value of the coefficients of conversion and efficiency would have been different.

The method of calculation given here may be applied in the analysis of short-term investments when changes in the relative valuations have to be taken into account.

The problem of the efficiency of short-term investments is directly linked with the preparation of an optimal production plan for several periods of time which takes into consideration the available resources of productive factors and the required composition of output. The difference here as against the tasks of production planning for a single period is that methods of production have to be considered in which a portion of the expenditure is made within one interval of time (capital investment) and the output is obtained at other intervals. Both the conditions of the availability of the resources and the demands for the output obtained are also connected with these periods. Of course, it does matter in which period a given kind of output will be produced. For this reason each type of productive factor and of output in each period must be considered by itself. Then, the task of drawing up a production plan for a whole series of periods will be found to be similar to the usual task of drawing up an optimal production plan in which the number of types of output and of productive factors is multiplied by the number of planning periods. Therefore, it is natural that here too the optimal plan should be characterized by a system of valuations determined for each type of production and of factors (which differ for each period), and that when these are taken into account the methods of production used in the optimal plan must be justified.

These valuations will be interrelated and will include the conversion of various types of production and expenditure to a common equivalent together with the conversion of expenditure and efficiency to a common time period. It is possible to proceed in a different manner, choose an identical measure for all the periods, adopting for instance the valuation of some single product, factor or of some fixed set of products as the unit for each period, and change proportionally the valuations of all the products and factors accordingly. Then the method must no longer be judged directly on the basis of the valuations obtained, but coefficients for converting all periods to one period must be worked out, or, what comes to the same thing, the values of normal efficiency of investments when passing from one period to another must be determined. These conversion coefficients represent nothing but the valuations of the standard set in the original system (and will consequently depend upon the choice of standard). An example of two such systems of valuations was given above (Table 41).

We summarize the above analysis in the following conclusions.

**CONCLUSION 25.** An optimal production plan drawn up for a series of periods (capital investment plan) is characterized by a dynamic system of valuations, i.e. by a system of valuations of all types of output and factors of production for each period. These valuations, generally speaking, change both absolutely and relatively on transition from one period to another. They can be given in two forms: either converted to a common period, or in the form of relative valuations for each period, showing the common coefficients for conversion to one single period (which is equivalent to determining normal efficiency on transition from each period to the next).

In accordance with these valuations, all the production methods (calculated for a series of periods) used in the optimal plan are justified (profitable), and those which are not used are no more than justified.

**CONCLUSION 26.** If a dynamic system of valuations characterizing a given plan has been determined, then in order to judge the advisability of applying some method of production, as calculated for a series of periods (usually relating to capital investments), it is sufficient to compare expected production for the whole period as well as planned expenditure, and to convert them to one period in accordance with the dynamic system of valuations. The calculation of output and expenditure can also be made by starting from the valuations of each period, converting thereafter the data obtained to a common period in agreement with the conversion coefficients or the norms of efficiency. In the case where the relative valuations may be considered constant, the calculation becomes simplified and can be completed when only the norms of efficiency and the valuations for one period are known (compare Conclusions 23 and 24).†

#### *Further Examples of the Calculation of the Efficiency of Investments*

We shall give some further examples of the analysis of short-term investments in which it is assumed that the value of the normal efficiency or, where necessary, the dynamic system of valuations are known in one way or another.

† A mathematical analysis of these problems is given in Appendix I (pp. 284–287) and an example of the calculation in Appendix II (pp. 336–41).

**EXAMPLE 4** A boiler operates on oil supplied from a long distance, consuming 20 tons of oil in twenty-four hours. It is contemplated to replace it by a boiler working on gas, consuming  $25,000 \text{ m}^3$  of gas in twenty-four hours. The expenditure on the new boiler and its installation amount to 250,000 roubles (on the basis of o.d. valuations); the period for the completion of the work is four months. The o.d. valuation of oil including delivery is 200 roubles per ton, that of gas 20 roubles/ $1000 \text{ m}^3$ . Normal efficiency is 10 per cent per month. The advisability of such a replacement is to be determined

Monthly saving according to the o.d. valuation is  $30 \times (20 \times 200 - 25 \times 20) = 105,000$  roubles. In view of the great length of service of the boiler the gross saving will here in practice correspond to the net saving. The valuation of the expenditure on the boiler at the moment of putting it into operation is  $250,000 \times (1 + 4 \times 0.10) = 350,000$  roubles. The efficiency of the investment at this moment is  $105,000 / 350,000 = 30$  per cent. The replacement is clearly advisable.

If the valuation of gas was 120 roubles per  $1000 \text{ m}^3$  for instance, and if the gas was used entirely by the chemical industry, the measure would prove inefficient.

**EXAMPLE 5.** The structure of expenditure for some articles at a given machine-building factory is as follows: materials 25 per cent, expenditure on labour and operating expenditure of the factory 25 per cent, hire valuation 50 per cent.

The building up of two weeks' supply of articles and the consequent introduction of continuous working make it possible to increase output by 10 per cent without additional labour and without any change in the expenditure of the factory on work and operating costs, but with a proportional increase in the expenditure on material. At what value of the normal efficiency is this change advisable?

Let us assume that monthly production of the factory amounts to 1,000,000 roubles. After the change has become operative it will be 1,100,000 roubles. There will only be an increase in the expenditure on materials by 25,000 roubles. Thus, we shall obtain a net saving of 75,000 roubles/month. The cost of the two weeks' supply of articles which is somewhat less than the cost of two weeks' finished production (since articles enter stocks at the initial stage of processing) constitutes, say, 350,000 roubles. Then, the efficiency of the change

will be  $75,000:350,000 = 21$  per cent per month. Consequently, it is advisable to put the change into operation if normal efficiency does not exceed 20 per cent per month.

This example was taken with arbitrary, but sufficiently realistic, figures. Of course, at many factories normal stocks of articles are lacking and the creation of these could substantially increase output.† Indeed, as shown by the calculations, the investment of resources in such conditions is many times more efficient than many other investments which were made and are being made. For this reason, the possibility of these resources being immobilized is unjustified.

Frequently direct instructions were even given as to the necessity of building up normal stocks and in many cases such stocks were actually created at the factories. However, very often they were consumed by the end of the month to ensure the fulfilment and over-fulfilment of the plan. As a result, at the end of each month and at the beginning of the following one the work did not proceed normally. The cause of this was not only the interruption of supply, but also the methods of calculating the fulfilment of the plan. Doing this only by commodity production favours such consumption of stocks.

Of course, it is an advance in comparison with the calculation by gross production whereby in individual factories an unlimited accumulation of semi-finished and incomplete products may occur. However, more precise than both these methods would be an account in which the movement of semi-finished production were linked with commodity production, but only within the limits of planned stocks. Such an account of semi-finished products within the framework of the plan could also be made when passing from the

† For instance, in 1956, at the factory Baku Worker, owing to discontinuity of operation, output was reduced by 16 per cent in some months and the cost of production of certain articles increased by 2.6 per cent to 106.1 per cent (see A. M. Alibekov: "The influence of continuous operation on the cost of production", in the collection *Organisation and planning of steady work at machine producing factories*, Mashgiz, 1958, p. 130). Greater possibilities of expanding production by improved planning are also recorded in the statement of V. I. Gorbunov at the XXI Congress of the Communist Party: "It may be asserted with complete certainty and without any exaggeration that the timely delivery of stocks of components, assembly parts, equipment, metal and other materials will make it possible, in accordance with the plan and charts, to increase the production of ships by 20-30 per cent, without any additional capital investment at the same shipbuilding yards and with the same number of workers" (*Report*, vol. I, p. 372).

index of commodity production to net output, the advantage of which was noted in Chapter I, Section I.

A similar problem is at present that of taking whole units out of operation for repairs and for preparatory work even when these measures entail some reduction in the output of the current period. In such a case, when the given short-term investment also covers loss of production, even though its value is taken into account by adopting a high o.d. valuation, we usually still obtain an investment of very high efficiency. A failure to do so at a time when a whole series of investments is completed at a much lower efficiency can in no way be justified and may lead to great losses in the near future. At the same time an incorrect solution of these problems is sometimes still encountered in the activities of individual factories. This is also partly due to some shortcomings in the operational factory indices: the preparatory work carried out is not reflected in commodity production or in gross output.

Finally, it is often essential to stock a certain supply of finished products for some time in order to ensure the operation of a given factory or factories. The gain thus achieved of meeting requirements in a more satisfactory manner may frequently exceed the damage caused by the immobilization of these means. A typical example of this is when the type of a book is kept for printing additional copies if this should become necessary after the book has been published. The resultant elimination of possible losses (overstocking with too big an edition, or an insufficient supply of the book to readers who may need it in the case of an insufficient edition) will exceed many times the losses entailed in the immobilization of metal for a few months. Even with a very high valuation of type metal the calculated efficiency of retained type will constitute at least a few hundred per cent per month.

EXAMPLE 6. The construction of a thermal power station may be carried out in one or in two years. Rapid construction entails some additional expenditure (building in winter, providing material for a greater number of builders, etc.), but then electricity generation will begin one year earlier. The data of expenditure and production for the two variants are given in Table 42.

It is sufficient to make a comparison of the variants over a period of three years as after that period the station is in operation at full capacity (from the second year after completion of the construction)

TABLE 42. EXPENDITURE DATA ON THE CONSTRUCTION OF AN ELECTRIC POWER STATION AND THE GENERATION OF ELECTRIC ENERGY

Construction variants	Expenditure on building and equipment (thousand roubles)		Expenditure on generation of electric energy				Generation of electric power (thousand kWh)	
			Coal (thousand tons)	Other operating costs (thousand roubles)				
	Year							
	1st	2nd	2nd	3rd	2nd	3rd	2nd	3rd
One year	110,000 (30,000)†	—	300	500	10,000	15,000	600,000	1,000,000
Two years	50,000 (20,000)	50,000 (20,000)	—	300	—	10,000	—	600,000

† The figures in brackets show costs of hire valuation for construction machinery.

under both variants, and further results of its activity are practically identical.

To complete such an analysis it is necessary to have dynamic valuations of the types of expenditure and production listed here. We shall assume that these data are known for the whole economy (or for an economic district) and are given by the figures of Table 43.

TABLE 43. DYNAMIC VALUATIONS

Types of production and expenditure	Valuations converted to the 1st year and conversion coefficients		
	Year		
	1st	2nd	3rd
Coal per ton	—	80 (100)†	64 (100)
Electric energy per 1000kWh	—	84 (105)	61 (95)
Construction and operating costs (roubles) (conversion coefficients)	1.00 (1.00)	0.8 (1.00)	0.64 (1.00)

† The figures in brackets show the valuation and coefficients for the given period.

Table 43 shows valuations converted to the first year, consistent with a normal efficiency of 25 per cent per year (the coefficient of conversion to each preceding year is 0.8), and the valuations for each year in brackets. The data of the table for electric energy show a reduction of its valuation (a lesser scarcity) from 105 roubles to 95 roubles. Conversion coefficients to the first year are given for expenditure, expressed in money terms.

Starting from these figures, as was done above, we shall calculate the expenditure on construction and the planned profitability of the operating station for both variants by converting everything to the first year.

#### *Construction costs*

Variant I: 110,000 thousand roubles = 110 million roubles

Variant II:  $50,000 + 50,000 \times 0.8 = 90,000$  thousand roubles  
= 90 million roubles

#### *Planned profit for three years*

Variant I: 2nd year.

$$600,000 \times 84 - 300,000 \times 80 - 10,000,000 \times 0.80 = 18,400,000 \text{ roubles}$$

3rd year:

$$1,000,000 \times 61 - 500,000 \times 64 - 15,000,000 \times 0.64 = 19,400,000 \text{ roubles}$$

Total 37,800,000 roubles

Variant II: 3rd year:

$$600,000 \times 61 - 300,000 \times 64 - 10,000,000 \times 0.64 = 11,000,000 \text{ roubles}$$

Total 11,000,000 roubles

It may be seen from the calculation that by increasing the construction costs in variant I by  $110 - 90 = 20$  million roubles, as compared with variant II, its planned profit is  $37.8 - 11.0 = 26.8$  million roubles higher (all the sums have been converted to the first year).

Thus, in these conditions, accelerated construction is preferable.

In this calculation, on the basis of dynamic valuations which include normal efficiency, the basic considerations which are essential in the choice of such a solution are reflected: the factor of immobilization of means of production, the importance of speedier generation of electric energy (taking into account that it is in particularly short



supply in the earliest period). Such accounts make it possible (if the necessary data are available) to calculate in an objective and quantitative manner all those details which are usually allowed for only qualitatively. It is possible that the solution of a faster construction might have been adopted without such a calculation, but then it would have been thought that it was accepted on the basis of the above considerations even though it was unprofitable. A correct calculation (which allows for the efficiency of capital investments and other factors) shows that in reality speedy construction is more profitable.

It should be noted that if the analysis is to be accurate it is not enough to take into account normal efficiency of investments alone; the use of the o.d. valuations is also important. Otherwise, the fact that electric power is in short supply and has a high o.d. valuation would not be included; and without that the results of the calculations would be quite different. Even the cost of the investment would be different. Thus, in the calculation for the construction of variant I it was assumed that, apart from the larger expenditure on other items, expenditure on the hire valuation of construction machinery would be less. This is natural since in the speedier construction the machinery is used the whole year round and more intensively. In the usual calculation, which does not take the hire valuation into account, the speedier construction would prove less profitable. The calculation of the norms of efficiency of investments (which is covered by the given calculation) is, of course, also of fundamental importance in the solution of similar problems since it reflects the immobilization of means of production and the possibilities of utilizing them elsewhere.

The example quoted is typical although worked with arbitrary data. As was shown in the resolutions of the party and government on problems of construction, the importance and consequences of accelerated construction for the national economy were very frequently underestimated, and the great damage caused by the immobilization of means as a result of their dissipation on many objects under protracted construction was not taken into consideration. The application of accounting methods based on the calculation of normal efficiency of investments and also the dynamic valuations would further a more accurate determination of the periods of construction and the sequence of objects, together with

the advisable allocation of means so as to attain the highest economic effectiveness.

In this connection it is appropriate to say that the popular view that in the solution of a problem economics must often be ignored is greatly exaggerated. As an example, it is argued that very necessary investments must be completed in the shortest possible time regardless of economics. In reality, in all such cases accurate economic accounting leads to the same conclusions. For instance, it is known that the first electronic computers produced an annual contribution of the order of hundreds of million roubles, not to mention those effects that cannot be evaluated economically.† For this reason it is clear that even an increase in the cost of manufacture of 50 to 100 per cent, or of some millions of roubles, was fully justified if it made it possible to put them into operation half a year sooner. Only with a very superficial analysis (based on the cost of production of electronic machines) could it be found to be uneconomical. An accurate analysis (similar to that provided above) would prove that in this case an investment leading to shortening of the period of production shows an efficiency of some thousand per cent per year!

## Section 2. Long-term Investments

### *Features of Long-term Investments*

Until now we have considered short-term investments, or such investments for which the means invested are fully recovered within a short interval of time. For this reason it is possible to confine the analysis to this short interval in assessing the result of their use. Investments producing results also after the period under consideration and which may be justified even by the analysis of their results over a short period of time (Examples 4 and 6) can be considered on the same bases as short-term investments. An interval may be considered short if there is no radical change in the situation and, in particular, in the nature of the planned task—the allocation of final output and of methods of production.

† For 1958, the saving resulting from electronic machines in operation has already amounted to about 1000 million roubles. See the speech of F. R. Kozlov at the XXI Congress of the Communist Party (*Report*, vol. II, p. 135).

From a theoretical point of view, the analysis of problems of long-term investments does not differ substantially from the analysis of short-term investments. In particular, Conclusion 24 in the general form in which it is stated holds again fully in the evaluation of the advisability of long-term investments, as we shall see below. However, in practice its application presents considerable difficulties.

Particular solutions involving an analysis of the efficiency of capital investments cannot be applied separately but must be related to the overall national economic plan. For this reason, in analysing a given capital investment we must be guided by such indicators of the national economic plan as the o.d. valuations and the normal efficiency determined by the plan for the national economy as a whole.

With short-term investments where no radical changes took place it was possible to utilize the o.d. valuations and normal efficiency valid at the moment, or to anticipate approximately the changes which they may undergo within the short period of time considered. As regards long-term investments, it is immeasurably more complicated to do this with any satisfactory degree of accuracy and reliability, and for this reason the calculation of the efficiency of investments is much more difficult.

The fact is that the values and the dynamics of the o.d. valuations are fundamentally tied up with all the conditions, in particular with the nature and the allocation of the planned task by types of the final output. The latter will depend upon many circumstances, including the political decisions adopted and the general situation.

The socialist path of industrializing the country calls for the development of large-scale industry and in the first place of heavy industry—of the means of production—by the country's own means, and determines the large economic requirements for metal and its high valuation. For this reason, the investment of resources in the ferrous metal industry during the period of industrialization was not only unavoidable, but on an accurate economic analysis would have been found both very effective and profitable, even though the calculation based on current prices which did not correspond to the situation may have shown them to be unprofitable at the time.

Only by bearing in mind the decision on the collectivization of agriculture was it possible to foresee a sharp rise in the demand for tractors and other agricultural machinery in the years 1931-3. As a result they would have carried a high economic valuation in the

calculations, and therefore investments made in the years 1928-30 in factories making tractors and agricultural equipment would have proved to be very efficient, given the appropriate calculations.

From the foregoing it is clear that the efficiency and the o.d. valuations of production are, as other cost indicators, not a regulator determining the movement of capital investments, but on the contrary, are themselves determined by economic tasks and objectively obtained on the basis of the general situation and by the basic economic measures necessary for the fulfilment of such tasks. Or, as aptly expressed by A. F. Zasiadiko, the criterion of efficiency should not be looked upon as a regulator but as an operational, well-based and accurate mechanism in the planning of the national economy.†

For this reason, the data of economic accounting obtained on their basis do not only not contradict the decisions referred to but, on the contrary, furnish the best means of turning them into reality.

It may be briefly stated that if economic accounting is of secondary importance in questions of what to produce (final production), in a question of how to produce, of the choice of the most economic methods of achieving the required output, these indicators are essential.

Thus, the basic character and movements of long-term investments can only be determined within the framework of general political and economic decisions.

At the same time, in the process of fulfilling the plan as set by the general party line, the calculation of efficiency must play a very important role—in particular, when more specific, although still fundamental, problems of the kind such as the choice of the kinds of raw material being utilized and the technological processes, the type of factory, the degree of concentration and specialization are considered—and these problems too must be solved with the general plan in mind.

If the calculation of efficiency cannot be carried out with great accuracy, then even a rough calculation of o.d. valuations, and in particular of normal efficiency, would have helped in avoiding a whole series of mistakes that occur frequently in problems of this nature, viz. that the plan included a whole series of investments of low efficiency, recovered only in the course of many years, whereas

† *Report of the XXI Congress of the Communist Party of the U.S.S.R.*, vol. II, p. 77.

meanwhile many possibilities of investing with an extremely high efficiency were not being made at all or only very slowly because of the shortage of resources. The fact that some old factories were preserved even after considerable means had been made available and that some newly built factories were found not to be working to capacity is evidence of the presence of such unsuccessful investments.

The proposition that in solving the problem of the efficiency of producing a given object the magnitude of the necessary capital investments is of importance as well as the cost of production was hardly seriously disputed by anybody.

But contradictory opinions and different practices were encountered in the question of the extent to which the volume of capital investments was essential and how it should be taken into account.

Opinions were also expressed that capital investments are included in the cost of production by calculating depreciation and that this synthetic index reflects fully this aspect of the problem. However, to the majority of economists and technicians it was clear that including depreciation of an object does not reflect fully the time factor and the load factor of the means employed in capital investments. For this reason, in addition to the cost index, an index of individual capital investments was usually worked out, but it was frequently given secondary importance or else was allowed for only qualitatively. This procedure made the determination of a correct and objective solution extremely difficult, for in solving any particular problem it was cumbersome to consider the whole economic situation and all the other possibilities of investments simultaneously, and therefore the attractiveness and efficacy of one aspect or another of a project could overshadow the economics of the problem.

In a quantitative analysis, the problem of the principles of using indices for each separate capital investment became particularly acute if the comparison of cost and of individual capital investments led to contradictory conclusions. Of course, at the same time efforts were made towards establishing an objective approach so as to unify these indices in one form or another into one synthetic whole. Such attempts were made on many occasions.

At one time, for instance, a fixed percentage of the investment involved was added to the cost of electric energy when the efficiency of a hydroelectric power station was determined. However, its

magnitude was taken quite arbitrarily (2.5 or 6 per cent) rather than on the basis of a general analysis of the problem of efficiency of investments in the country. For this reason, such a calculation brought, unfortunately, more loss than benefit as it gave the impression that the full effect of committing means for investments had already been allowed for when in fact this was not so.

This type of accounting was applied in the planning of railways.† Many planning organizations used the index of the recoupment period for the additional capital investments in their analysis of variants (*on this index, see above, p. 169*).‡ This method found much wider recognition in recent years when detailed recommendations for its use were introduced.

From the foregoing it is clear that the basic initial data for the drawing up of a capital investment plan are the volume of resources available for capital investments as well as the total and the composition of the final output, and with it the general direction of capital investments.

The problem of the efficiency of capital investments cannot be analysed in isolation from other problems of economic planning.

First of all, the problems of efficiency of capital investment and of the comparison of some investments with others can be accurately analysed only in conjunction with long-term planning as a whole. Of course, on the one hand a general long-term plan is drawn up in the final account from particular decisions on capital investments. On the other hand, in an isolated analysis of any one capital investment it is not possible to ascertain the need for such an investment and its practicability compared with other possible solutions; nor is it possible to take into consideration the balance of available means for capital investments and the need for them.

In the calculation of the efficiency of capital investments current planning data cannot be disregarded since only with their aid is it possible to ascertain to what extent the available means of production are utilized fully and correctly, what needs there are for their expansion in a given direction and what results they will produce.

Finally, in order to assess the economic effect of a capital investment and the output achieved by it, as well as the expenditure

† See M. M. Protod'yakonov: *Surveying and Planning of Railways*, 1934.

‡ We return to this discussion and the comparison with our propositions on p. 236.

entailed in its realization, it is important to arrive at an accurate national economic valuation of individual types of production and services. Consequently, the problems of capital investment are more closely related to problems of calculating national economic expenditure, the valuation of production and the problems of price formation.

However, the problems of capital investment are, above all, connected with the structure of the long-term plan of development of the national economy.

In a socialist society, a long-term national economic plan aims at the development of productive forces, ensuring the highest possible growth of output and of productive capacity in accordance with the task of meeting the requirements of the society in the best possible manner by the fullest and most appropriate use of resources—it should in principle represent the optimal plan.

The nature of a socialist system of society makes it possible to ensure the fullest and most rational use of resources. For this reason, such an optimal plan emerges as an attainable reality and the consistency of an optimal plan represents real economic conformity with the laws of the national economy.

Schematically, the problem of drawing up a national economic plan may be represented in broad outline as follows.

The resources at the beginning of the planning period are known. On the basis of the general political and economic situation, the tasks confronting the national economy lie in studying social requirements and hence in determining the allocation of the final production for consumption by individuals and for the needs of society as a whole, as well as for accumulation. On the basis of known technical and production data plus data of the possibilities of developing production—in particular, on the basis of forecasts of further developments of technology and exploitation of available resources (prospecting for useful minerals, etc.)—it is possible to describe quantitatively the existing methods of production. On the basis of all these data a plan should be drawn up which would provide for a faster growth of production and productive capacity in the required direction in future, while ensuring the necessary current consumption as determined in accordance with the needs of the society.

It is clear that in such a situation the problem of long-term planning is in the main similar in character to the problems of production planning of short-term investments, analysed in the

preceding section, which leads to the idea that the analysis given above could be used here. The far-reaching character of this problem, the enormous quantity of initial data, the immense number of conceivable methods of production, the practical difficulty of obtaining information about all these matters, the need to foresee further developments in technology and the organization of production, the relationship between needs and the plan, determined by a whole series of non-economic aspects (especially where final production is concerned), all these do not permit us to rely here on a direct and literal application of the scheme of accounting and of the construction of a plan and of the related valuations, as described in Section I (we shall return to this problem).

Nevertheless, it may be expected that the characteristic features of the solution of the problem of optimal planning over time—the existence of numerical valuations which characterize an optimal plan, as explained above in the analysis of the practical problems of short-term planning—must be preserved in the much more complex situation under consideration as here too the solution represents an optimal plan over time.

*In view of the foregoing, the statement may be considered sufficiently substantiated: that associated with the optimal long-term plan there exists a definite dynamic of objectively determined valuations as well as a unique value of normal efficiency of capital investments, which may change in the course of time.*

They may be introduced as above in two forms: the dynamic system of valuations converted either to one common period or to one unit in each period together with the introduction of normal efficiency (which depends upon the choice of such a unit).

Furthermore, if the valuations change relatively little, it may be sufficient for an approximate characterization of the optimal plan to obtain only one system of valuations for the production together with the value of normal efficiency on transition from each period to the next. The characterization of the optimal plan becomes still simpler when normal efficiency can be considered constant.

Assuming that a long-term national economic plan is under consideration and that its basic economic indicators (in particular, the dynamic valuations and the value of normal efficiency) have been determined, we shall show how it is possible to carry out the analysis of the efficiency of individual capital investments and to arrive at a



conclusion as to their advisability by starting from such indicators (compare Conclusions 23-6).

This analysis becomes particularly simplified if the dynamic of relative valuations can be disregarded and (approximate) calculations made by taking into consideration the value of normal efficiency alone.

In the majority of examples quoted below this is precisely the assumption used, and normal efficiency is also assumed to be constant.

### *Examples of the Calculation of the Efficiency of Investments*

We shall show on several examples of long-term investments how the problem of the advisability of an investment can be approximately solved.

**EXAMPLE 7.** Two types of machines of a given capacity are available. For machine A the initial expenditure is 500,000 roubles and the length of service five years. Operating costs amount to 500 roubles per day. For machine B the initial expenditure is 1,000,000 roubles and the period of service ten years. As regards operating costs, machine B is 20 per cent more economical than machine A. It is necessary to determine under what conditions it is advisable to use one or the other machine, and also the conditions in which it would be advisable to replace machine A if already in operation by machine B.

At first sight, it appears that machine B is always preferable, for capital expenditure averaged over the length of service is identical in both cases and the operating costs in the second instance are considerably lower.

To solve the problem in a more detailed manner, we shall carry out the calculation for several values of normal efficiency.

We shall take normal efficiency as equal to 20 per cent per year. First of all we calculate the annual saving. It will amount to  $360 \times 500 \times 0.20 = 36,000$  roubles. We then calculate the difference in the investments in the course of ten years by converting them to the initial point of time. Selecting machine B, the investments constitute 1,000,000 roubles. Taking machine A, they amount to 500,000 roubles now and 500,000 roubles after five years, which, converted to the present, comes to 250,000 roubles (since 100,000 roubles

today at a normal efficiency of 20 per cent correspond to 200,000 roubles after five years), † altogether 750,000 roubles.

Thus, 250,000 roubles are saved in investment. An investment of this size with a normal efficiency of 20 per cent produces a yearly saving of 50,000 roubles. For this reason, with a normal efficiency of this magnitude, the expected saving of 36,000 roubles is not enough, and consequently it is more advisable to use machine A.

If machine A is already in operation and the question is to replace it by the better machine B, the expenditure for the construction of the first machine A need not be included, and the difference in investments will be 750,000 roubles. This shows that, at a normal efficiency of 20 per cent, such a change is highly inadvisable. Carrying out a similar calculation for other values of the normal efficiency (or an algebraic analysis) we arrive at the conclusion that the solution of the problem set will depend upon the value of normal efficiency. Namely:

(1) If the normal efficiency exceeds 16 per cent it is more advisable to use the machine of type A.

(2) If the normal efficiency is below 16 per cent but more than 6 per cent, then it would be more advisable to use the machine of type B in new installations. However, the replacement of the existing machines A in good working order by machines B is inadvisable. Thus, although machine A is obsolete it is advisable to continue using it where available.

(3) If normal efficiency is less than 6 per cent, it is advisable to use machine B not only in new installations but to substitute it for machine A.

Let us note in connection with this example that owing to inadequate understanding of the function of normal efficiency the use of very complex and expensive equipment was frequently advocated, even though it produced a relatively small absolute saving in labour or materials. Even if the efficiency was calculated, a magnified value was frequently obtained because the cost of equipment was not fully allowed for in the calculation and the shortage of materials, etc., was not borne in mind. Frequently the use of such equipment by some leading factories abroad was taken as a basis which can in no way

† It would be more accurate to use compound interest. However, allowing for the probability of some reduction in normal efficiency in the course of time, we would arrive at approximately the same result.

be considered as adequate. It should be mentioned that the actual efficiency was found to be even lower when such equipment was not used to full capacity.

Further, on the grounds that such equipment was being used for new factories some were inclined to support the needs of quick replacement and the dismantling of old types of equipment as depreciated. As the example quoted shows, one conclusion can in no way be taken as a basis for another.

However, it should be mentioned that these endeavours of individual managers met the proper resistance from central bodies, and in fact dismantling was carried out only to a comparatively minor extent. The inadvisability of dismantling too soon was confirmed in particular at the time of World War II, when much of this obsolete and even dismantled equipment was successfully put into operation.

Thus, at present, in spite of the change to diesel and electric locomotives, the use of available steam engines is looked upon as well founded for many years to come (mainly on lines with less intensive traffic).

Such problems can be solved correctly only by allowing for the value of normal efficiency.

EXAMPLE 8. There are two variants of a projected bridge: a wooden one and a stone one. Both meet the technical requirements.

The cost of the wooden bridge is 1,000,000 roubles, the period of service is ten years, and the cost of repairs 20,000 roubles per year on the average. The cost of the stone bridge of the same capacity is 2,500,000 roubles, the period of service fifty years and the cost of repairs 5000 roubles per year.

Under what conditions is one or the other project more advisable?

Let us calculate the cost involved in one year's service for both variants:

the wooden bridge:

$$(1,000,000 + 10 \times 20,000) \div 10 = 120,000 \text{ roubles/year,}$$

the stone bridge:

$$(2,500,000 + 50 \times 5,000) \div 50 = 55,000 \text{ roubles/year.}$$

The advantage of the stone bridge seems undoubted.

In reality the problem is not so simple and its solution should depend upon the value of normal efficiency.

Let us take, for instance, an annual normal efficiency of 10 per cent. Let us compare costs: (1) of a stone bridge for a period of ten years; (2) of construction of a wooden one at a given moment, its immediate use, and the construction of a stone bridge in the course of ten years.

In both cases we shall obtain approximately the same result in meeting needs.

When comparing costs, they must be converted to the given point of time. We obtain:

*In the first case*

the cost of the stone bridge	2,500,000 roubles
cost of repairs 50,000 roubles, converted to the initial instant†	35,000 roubles
Total	2,535,000 roubles

*In the second case*

the cost of the wooden bridge	1,000,000 roubles
cost of its repairs 20,000 roubles, converted to the initial instant	140,000 roubles
the cost of the stone bridge in the course of ten years 2,500,000 roubles, converted to the initial instant	1,250,000 roubles
Total	2,390,000 roubles

If the value of the normal efficiency were greater, the advantage of the wooden bridge would be even larger. Thus, if the normal efficiency is 10 per cent or more, the proper solution at that moment will be to construct the wooden bridge. If the 1,500,000 roubles saved today were used, for instance, to make a new mine which could not be started because of the lack of means, and if the efficiency of investment in the mine is 40 per cent, the additional production resulting from this in ten years will exceed several times the costs involved in the construction of the stone bridge in the course of ten years. We are not prepared to conclude from this that wooden bridges rather than stone bridges should always be constructed. Of course, the ratio of costs may also be different, in which case the

† These are average costs incurred in the course of five years. And the sum of 35,000 roubles today with a normal efficiency of 10 per cent is equal to 50,000 roubles after five years. A more accurate calculation may be made with the aid of compound interest.

construction of the wooden bridge may not prove the most economical on the basis of the present calculation; also, a most important fact is that the wooden bridge will often not meet the same technical requirements as the stone one, which may force one to forgo the wooden bridge even if it were more economical. However, in the latter case, when selecting the stone bridge variant (or one of metal or of reinforced concrete), it must be taken into account that it is less economical and not more (as was shown in the initial calculation), and these economic losses should be compared with the technical advantages. It may be generally noted here that the enthusiasm for gigantic solutions without any particular need from which many of our planners suffer is harmful as it deprives the other more efficient investment outlets of the necessary means. One of the causes of such inaccurate solutions was incorrect economic accounting which failed to include normal efficiency.

The lack of due attention to the economic side of the operation and an unsatisfactory analysis of it were one of the causes of excesses in the field of architecture.

We shall elucidate the nature of efficiency accounting on the basis of dynamic valuations on the following example.

EXAMPLE 9. Let us assume it is necessary to determine the efficiency of a capital investment aiming at an expansion of the productive capacity of article A by 100,000 units per year, this being achieved by equipping a new workshop for a certain production process. The cost per unit of article A with this process entails ten units of material B and ten days' work. The efficiency of the given investment has to be determined, given the general long-term optimal plan and the dynamics of the relative valuations. They are given in Table 44.

It should be pointed out that the valuations in this table have been converted to a common period (the first year). Thus, the reduction in the valuation of labour by no means signifies a reduction in its remuneration. In reality, since the valuations of production decrease considerably faster than the valuations of labour, the data in the table show that a significant rise in the productivity of labour is expected, which of course makes it possible to foresee a certain increase in real earnings.

Such a reduction in the valuation of labour is, in actual fact, consistent with the statement that a rational expenditure of labour

today enables one to obtain a much higher saving of labour in the future; and hence one must assign a higher valuation to a unit of labour for any given year than for the following one.

Let us note that the value given for the valuation of the equipment in the workshop after five years does not show the cost of its production at that moment but the valuation of the equipment already used up, taking into consideration its physical and economic depreciation.

TABLE 44. VALUATIONS CONVERTED TO THE FIRST YEAR  
(VALUATIONS FOR EACH YEAR IN BRACKETS)

Product or factor	Years				
	1st	2nd	3rd	4th	5th
Article A (per unit)	500	420 (503)	350 (507)	280 (465)	200 (380)
Material B (per unit)	10	8 (9.6)	6 (8.7)	4 (6.6)	3 (5.8)
Labour (per unit)	20	18 (21.3)	16 (23.1)	14 (23.2)	12 (23.1)
Equipment of the workshop (roubles)	70,000,000	—	—	—	15,000,000 (29,000,000)
Normal efficiency (%)	20	20	20	15	15
Coefficient for conversion to the first year	1.0	0.83	0.69	0.60	0.52

We must also mention that we assume, for simplicity, that costs for a given technological process are constant.

In order to calculate efficiency as indicated above we calculate the total saving (profitability) of a given workshop for the period under consideration (five years) converted to the given instant—the first year. Subtracting from the valuation of production the cost for each year, adding these and multiplying by the number of units we have:

$$100,000 \times [(500 - 10 \times 10 - 10 \times 20) + (420 - 10 \times 8 - 10 \times 18) + (350 - 10 \times 6 - 10 \times 16) + (280 - 10 \times 4 - 10 \times 14) + (200 - 10 \times 3 - 10 \times 12)] = 100,000 \times (200 + 160 + 130 + 100 + 50) = 64,000,000 \text{ roubles.}$$

At the same time the cost of investment amounts to 70,000,000 - 15,000,000 = 55,000,000 roubles.

Thus, the calculation shows that this investment yields a positive efficiency.

A similar computation could be made using the valuations for each

period and converting these subsequently to a common period by means of conversion coefficients or normal-efficiency values.

Let us note that here too the accuracy of efficiency calculations does substantially depend upon the use of o.d. valuations. Thus, if, for instance, the cost of production or the price derived from it were taken in the usual manner instead of the o.d. valuation of the article (which included the hire valuation of the equipment used as well as the relation between its demand and the volume of production) the result of the calculation of efficiency would be different. The investment might turn out to be quite inefficient or barely efficient.

The problem of the efficiency of capital investments can only rarely be solved on the basis of an isolated analysis of the given problem. As a rule, it is related to the general economic situation and the economic plan for the country as a whole. In the preceding examples this was taken into consideration by using the economic indices (of normal efficiency and of the dynamics of valuations) of the overall national economic plan. In other cases, consistency with the general plan may be achieved by planning the targets of a given sector (output, supply, amount of resources for capital investment). Stating the problem in this manner may be practical if no long-term plan for the economy as a whole nor a system of indices (normal efficiency and o.d. valuations) existed as yet, or if it were a question of improving a plan drawn up earlier for the given sector. In such a case, it is necessary to set up an optimal plan for the given sector directly, while allowing for initial conditions.

When drawing up such a plan, the system of valuations or the value of normal efficiency are determined intrinsically within the limits of the given sector (group of factories). These valuations can be used to adjust the plan within a sector. Moreover, the comparison of these indicators with similar ones for the national economy as a whole or for other sectors (groups of factories) may even give rise to a revision of the planning problems (a redistribution of targets and resources).

We shall consider an example of this nature.

**EXAMPLE 10.** In a branch of the mining industries of a given economic district processing is carried out by simple means. The number of workers employed is 2000. Annual extraction per worker is 1000 tons of ore, the annual earnings of a worker 10,000 roubles. The local price of one ton of ore is 20 roubles, i.e. the value of the annual production of one worker is 20,000 roubles. To increase

extraction it has been decided to mechanize it. The initial investment funds allocated amount to 80,000,000 roubles; further investment in machines must be made by the sector from its own funds. The number of workers employed in the sector remains unchanged. Two types of machines A and B are available for the mechanization of extraction with the relevant data given in Table 45.

TABLE 45. TYPE OF MACHINES AND EFFICIENCY OF THEIR USE

Machines	Number of workers required	Annual production (roubles)	Operating costs other than wages (roubles)	Length of life (years)	Cost (roubles)	Extraction per worker (roubles)	Operating costs and wages per 1000 roubles of production (roubles)	Recoupment per 1000 roubles'-worth of production (roubles)	The cost of 1000 roubles'-worth of production (roubles)
A	10	800,000	60,000	5	800,000	80,000	200	200	400
B	20	3,200,000	60,000	20	8,000,000	160,000	81	125	206

Which type of machine should be chosen in order to carry out the mechanization? It may be seen from this table that, as regards productivity of labour and the cost of production, machine B is superior to machine A. Apparently it should be given preference. However, such a solution of the problem would be superficial as it ignores the more important matter—the volume of investment on machine B. In this case, normal efficiency is not given but, since total means of investment are fixed, it can be determined. We calculate the hire valuation and the efficiency of each machine. For machine A we have: 10 workers servicing the machine when operating with simpler means would produce  $10 \times 20,000 = 200,000$  roubles'-worth of production. Thus, the cost of the additional production obtained as a result of mechanization will be:

$$800,000 - 200,000 = 600,000 \text{ roubles.}$$

Annual costs:

$$160,000 + 60,000 = 220,000 \text{ roubles}$$

(recoupment and operating costs, excluding wages).



The net saving (and hire valuation) resulting from the use of machine A is:  $600,000 - 220,000 = 380,000$  roubles per year.

The efficiency of investment is  $380,000 : 800,000 = 47$  per cent.

For machine B the additional extraction is:

$$3,200,000 - 400,000 = 2,800,000 \text{ roubles.}$$

The net saving (hire valuation) is:

$$2,800,000 - 400,000 - 60,000 = 2,340,000 \text{ roubles.}$$

The efficiency of investment is therefore

$$2,340,000 : 8,000,000 = 29 \text{ per cent.}$$

To simplify the calculation, we did not convert recoupment to a common period of service as otherwise we would have had to calculate it in a somewhat different form. However, this would not have made any substantial difference. The result obtained shows that the proper solution in this case would be to start mechanization by putting machine A into operation, 100 units of which may be thus acquired and thus represent a mechanization for half the number of workers. One thousand workers will continue to work with simple equipment. Since the possibilities of more efficient investment in machine A are not exhausted, normal efficiency will be 47 per cent. Annual accumulation (without allowing for deduction on recoupment) will constitute  $1000 \times 20,000 + 100 \times 800,000 - (2000 \times 10,000 + 100 \times 60,000) = 74,000,000$  roubles. This will enable the sector to acquire another 90 machines A and to equip with these 95 per cent of the sector. In the following year, accumulation will enable it to acquire machine B. From the moment when the possibilities of investment in machine A will be exhausted normal efficiency will become 29 per cent. Thereafter, the sector will begin to be re-equipped with the superior machine B.

It would not be right to begin the conversion by introducing machine B at once. Only 10 of these machines could be acquired, and 1800 workers would have to continue extraction with simpler equipment. Annual accumulation would be:  $1800 \times 20,000 + 10 \times 3,200,000 - (2000 \times 10,000 + 10 \times 60,000) = 47,400,000$  roubles; the process of re-equipment of the sector would proceed much more slowly than in the first instance. It may be easily found that, starting by introducing machines A, the complete re-equipment of the sector

with machines B in the first instance will ultimately proceed faster than if machines B were introduced at once.†

An examination of this example shows that where the sources for investment are scarce one should not be influenced too much by particularly large and efficient units which are complex and expensive. This important gain in productivity of labour which will be achieved where such equipment is used cannot provide as much production as could be obtained from the equipment elsewhere with an inferior and less efficient but cheaper machine, if the most primitive means continue to be used in numerous other sectors which cannot be supplied with the better machine. As may be clear from the last calculation, such consecutive carrying out of mechanization, starting from less capital-intensive means, ensures both a more rapid rise in output in the early years and its faster growth in future. Of course, this qualitative conclusion is not universal and should be checked in every case against the results of economic accounting that takes into consideration the value of normal efficiency.

In the present example capital investment per unit output was higher for machine B than for machine A. It should be mentioned that even if a large unit or factory should not be found in a worse position in this respect than a medium one it should not always be given preference. This is particularly due to the fact that a large factory involves a large number of indirect investments which must also be allowed for in the calculation. The following considerations must, therefore, be borne in mind in this context.

(1) The period of equipping and of putting into operation a large factory is much longer than that of a small or average one.

How this helps to lower the efficiency of investments was seen above in the analysis of Example 2.

Moreover, the longer construction period of a large factory does not enable it to meet the requirements for a given production in the

† The calculations for this example could be made more accurate by taking into consideration the change in the value of o.d. valuations and normal efficiency in the course of time, in accordance with Conclusion 24. In the present case such improvement in the accuracy of the calculation would not influence the results of the analysis very much. A more accurate calculation of recoupment is also essential. Here it was taken as proportional to the cost of investment. In a more accurate calculation, if the necessary data are available, depreciation must be included in the account of planned costs of capital repairs and renewal, taking into consideration the periods of their realization and converting the costs to a common point of time (compare Note 3, p. 167).

early years when the demand for it may be most acute and its o.d. valuation particularly high.

(2) While a small and medium factory may start operating after a short time and base its operations on available auxiliary resources (supply of electric power, means of transport, housing fund, municipal network), a large factory needs large auxiliary investments simultaneously.

(3) In a large factory or unit it is difficult to ensure a constant and full load (for instance, at certain periods for meat packing plants, individual hydroelectric power stations), and it is more difficulty to adapt its planned capacity; and with plants not working at full capacity efficiency drops sharply.

(4) More transport needed for large factories increases production costs greatly (if the o.d. valuation of transport is high), and thereby reduces their efficiency.

(5) Since the period of achieving the possible economic effects is usually much longer for a large factory it is more difficult to ascertain the changes in the situation and technology which may take place during the period of realization (possible reduction in the demand for a given type of output, the introduction of new techniques for the given type of output or for its substitutes, etc.).

The resolutions of the XVIII Party Congress condemning the mania for gigantic factories were adopted at the time chiefly because of these considerations.

Economic accounting which allows for normal efficiency would have confirmed the validity of these conclusions at that time and would have furnished the means of calculating the most appropriate size of factories.

The calculation of the efficiency of a given capital investment and its implications may, in accordance with Conclusions 25-6, be obtained on the basis of the dynamics of economic valuations, if the changes connected with the inclusion of this investment are so small that they do not substantially influence the system of o.d. valuations and its dynamics. However, it is often found that the incorporation in the plan of a given investment at its appropriate size does influence perhaps not the national economy as a whole and the entire system of valuations, but the position of some sector, the production of a product everywhere or in a given economic district, and hence some of the o.d. valuations. Thus, the new production on a large scale of

some article in a given district may significantly change the cost and the balance ratio for this article as well as its o.d. valuation. The above applies in particular when a given investment, by its nature, cannot be broken up: a large hydroelectric power station, a canal, an irrigation installation, a new railway, etc.

For this reason, with large capital investments one must not begin the calculations immediately from the existing system of valuations. It is necessary to allow in one way or another for the changes which will result from the inclusion of such investments in the plan. Here it is necessary to compare the optimal plan in its initial form with the changed optimal plan drawn up so as to incorporate this investment.

However, in such a case there is usually no need to reconstruct the whole plan; changes can be confined to those aspects which the given investment strongly affects, and also to the valuations of factors and products concerned, while for the rest the previous values can be used.

The following example may give some idea of this kind of calculation.

**EXAMPLE 11.** The problem of the construction of a railway linking points A and B is under consideration. For a single-track railway the carrying capacity of the line will be 600 wagons per day, the cost of the permanent way and rolling stock being 350 million roubles; for a double-track railway the carrying capacity will be 1000 wagons per day and the full cost 480 million roubles. In both cases the direct cost of transportation amounts to 200 roubles per wagon. The period for the construction of the railway is one year, and normal efficiency is 30 per cent per year.

To solve the problem, it is necessary to assess the goods traffic the line will have to carry in the first years. This goods traffic is given in Table 46. The saving achieved by the line is also calculated. The third column shows the saving made in other sectors of the national economy as a result of transporting one wagon on the railway, the fourth—the net saving (after the deduction of cost).

Thus, type I of goods is in fact moved at present by motor transport at a cost of 2000 roubles for the transportation of one wagon. The transfer of this goods traffic to the projected railway will give a net saving of  $2000 - 200 = 1800$  roubles per wagon per day. If a single-track line should be constructed, the carrying capacity will make it possible to transport altogether the first three types of goods. The

annual saving will be  $64.8 + 64.8 + 43.2 = 172.8$  million roubles. The cost of the railway is 350 million roubles, but taking into consideration that the construction will take a year, its cost at the moment of entering into operation must be considered 30 per cent higher (normal efficiency equals 30 per cent), i.e. 455 million roubles. Consistent with this value of the normal efficiency, the annual saving for such an investment will be  $455 \times 0.30 = 137$  million roubles. The calculation proves that the investment is entirely advisable.

TABLE 46. GOODS TRAFFIC AND SAVING

Types of goods	Volume of transport per day (wagons)	Saving per wagon (roubles)	Net saving		
			Per wagon per day (roubles)	On the total goods traffic	
				Per day (roubles)	Per year (roubles)
I	100	2000	1800	180,000	64,800,000
II	200	1100	900	180,000	64,800,000
III	300	600	400	120,000	43,200,000
IV	200	400	200	40,000	14,400,000
V	200	300	100	20,000	7,200,000

A double-track railway would make it possible to include transportation of goods of types IV and V which would give an additional saving of  $14.4 + 7.2 = 21.6$  million roubles per year. The necessary additional investments will amount to 130 million roubles, which at the moment of opening the railway should be calculated as 169 million roubles. The saving for such an investment, with a normal efficiency of the order of 30 per cent, should constitute approximately 50 million roubles, but an economy of 21 million roubles only is obtained. This leads to the conclusion that the immediate construction of the second track is not advisable; but it may become appropriate in future if the demand for the movement of goods should increase or if normal efficiency should decrease, i.e. if the possibilities of more efficient investments become exhausted.

However, it would be correct to carry out a fuller calculation, bearing in mind, besides the possibilities envisaged of using the railway for the factories already in operation, such new factories the construction of which would become advisable and efficient after such a

railway starts operating, and to calculate the additional economic effect which could help to justify them in relation to normal efficiency. Such a calculation must be carried out tentatively both for the given year and the subsequent years.

In particular, allowing for the additional goods traffic, the construction of the second track could be found advisable even now or after a shorter period of time.

*Let us note that generally in the evaluation of efficiency of large objects of capital investment which cannot be broken up, we consider it admissible to include in the calculation also those related projects which can be carried through because of the main object. In many cases this may rightly increase the value of the efficiency of the latter, since its economic effect may be raised by the economic effects of the related investments as far as it exceeds the effect corresponding to normal efficiency.*

*We stated above that although normal efficiency is of an entirely different nature than the profit norm in a capitalist society, they may be compared in so far as both are of importance in problems of the allocation of investment.*

It is necessary to underline the radical difference in principle between normal efficiency and profit norm as regards their origin, social meaning, manifestations and function in the economy.

The first is determined by an objective planned calculation; it represents an objective numerical indicator of one of the laws of socialist economics, consciously applied in economic calculations and planning, and enables us to show the most efficient economic use of capital investments in order to achieve a fast rise in the *productivity of labour*.

The second is determined in the capital market in the pursuit of the highest profit; it represents a numerical index of the law of average profit norms, formed spontaneously in the process of capitalist competition, and acts as a regulator in the distribution of the surplus value among various financial groups. It manifests itself in the movement of capital from one sector to another; and not only does it not afford efficient use of surplus value but on the contrary, it is the direct cause of over-production, continuous disequilibrium, and crises leading to the dislocation of productive forces.

It must also be emphasized that there is a radical difference between them not only in principle, qualitatively, but also a quantitative

difference, affecting the numerical results of the calculation and the conclusions derived from them.

This becomes apparent in the analysis of the last example. Let us assume that an industrialist or a joint stock company in a capitalist state is considering the problem of efficiency of investing in the construction of a railway under the same conditions. Supposing that the profit norm of 30 per cent is the same as the normal efficiency above. The fundamental difference in the calculation consists in that the industrialist would not calculate the saving for the national economy resulting from the investment, which is of little interest to him, but the profit he could make by carrying out a given investment.

The ratio of this profit will depend upon what tariff is fixed. The highest tariff at which a single-track railway will run at full load is 600 roubles.

The profit on each wagon moved will be  $600 - 200 = 400$  roubles, altogether  $600 \times 400 \times 360 = 86.4$  million roubles per year. For an investment of 455 million roubles, the profit should amount to 137 million roubles at a standard profit of 30 per cent, i.e. the investment will be found unprofitable by private capital.

It is of interest that the profit obtained is somewhat higher with the second tariff—1100 roubles. At such a tariff, the railway will carry only goods of types I and II (300 wagons per day), i.e. 50 per cent of the load, but the profit will be found to be higher, amounting to 900 roubles per wagon or  $300 \times 900 \times 360 = 97.2$  million roubles, which is still insufficient.† Let us note in passing that this is one of the fairly frequent cases of economic anomaly in a capitalist society where it is quite possible that only half the carrying capacity of the railway is used, while the transport of some goods, however desirable, does not take place as it is not worth while at the current high tariff, whereas the low tariff produces a smaller profit for the owner of the railway.

The example quoted is typical. A large number of investments which could be made in a capitalist society and which are efficient from the point of view of the whole economy (even if the current profit norm is taken into account), are not being realized; for within the framework of private property it is not possible in practice to secure the whole national economic benefit in the form of private profit—the only result in which the capitalist is interested. This

† We do not allow for the possibility of differential tariffs for various loads.

applies above all to various social and cultural measures and to the construction of large installations such as roads, canals, large hydro-electric power stations, housing construction, irrigation work, afforestation, etc.

Furthermore, with the national economic need to increase the production of many commodities and notwithstanding the actual possibilities for such an expansion, the latter often cannot be realized under capitalism since no market can be found for the available goods as a result of the low purchasing power of the population. Under socialism such a situation cannot arise.†

Thus, if normal efficiency and profit norm are the same, many possibilities of investment which are unprofitable (which do not ensure normal profit) in a capitalist system, may be found advisable, i.e. they show a high enough efficiency (above normal), in a socialist system.

It should be mentioned that this important proposition was expressed by many in a much more general form, viz. that in a capitalist society investments which can be realized are limited to those which ensure normal profit while in a socialist society any investment which gives a positive saving of labour is advisable.

Of course, in this form the proposition is incorrect as it does not take into consideration that investment funds are limited and that it is necessary to find the most efficient way of using them. Also, it is in practice harmful as it may lead to the immobilization of funds in objects of low efficiency.

Turning back to the example considered, it is important to note one more property of large investments of such a nature.

The completion of such an investment often makes it possible to meet both the more necessary and acute needs for a given type of

† Thus, a group of British engineers describing the Soviet metallurgical industry write in their report published in the journal *Steel Review*:

"This industry owes its success in many respects to one factor alone which may be obviously considered unique. The metallurgical industry was evidently planned in Russia. The Soviet steel smelting industry enjoyed over the whole period a ready outlet for its production, and there was always a stimulus both inside and outside compelling it to strive for the maximum development of production and productivity. The capital necessary for new installations is forthcoming without difficulty and the growth of foundries is accompanied by the supply of all possible means of transport and also of power and raw materials. Under such conditions each unit is ensured a full load, and with such a combination of circumstances these brilliant achievements may be considered a natural result." (See *Kommunist*, 1958, No. 6, p. 83.)



output or services and also the less important ones, the satisfaction of which would in itself not justify the investment. Hence, in meeting these needs as well (the latter is advisable), the hire valuation for the completion of the installation, which is derived by taking into account the low efficiency of such investments, may be found to be very small, and sometimes if the possibilities of its use are practically unlimited (such as a canal, a bridge) it may be near zero. Nevertheless, the appropriate investment may have, as we saw, the necessary efficiency if its effect on the national economy as a whole is considered.

However, here it would be inadvisable to attempt to realize directly the effect of the investment (corresponding to normal efficiency) by increasing the hire valuation, by incorporating the hire valuation in the value of production, etc., as this would make it impossible to obtain all the possible efficiency.†

Thus, in such cases a commercial approach to the realized efficiency of capital investment is inadvisable.

In a socialist society capital investments must be justified directly both for the sector in which they are made and in the national economy as a whole (or in an economic district). Let us note in passing that the number of projects of subsequent investments in a given sector need not necessarily be made in accordance with the accumulation realized in this sector.

### *Level of Normal Efficiency*

If the degree of economic development and of total means of investments are the same, the number of possibilities for investment at a high degree of efficiency are much greater in a socialist society

† A typical example of this is the principle of fixing prices of books in the U.S.S.R. In spite of the great difference in the cost of production according to the number of copies printed books of a certain type are sold in our country at approximately the same price (calculated per folio) irrespective of the number of copies printed. This may be explained by the fact that in our country the problem of the edition of a book is not solved by such considerations as profitability. For instance, the national economic efficiency of the issue of an educational, technical scientific book is not essentially determined by the sum received from the sale. If the question of publication has already been decided and consequently the editorial and publishing costs can be excluded from the calculation as well as the payment for type-setting the cost of a copy of a book will only slightly depend upon the number printed.

than in a capitalist one where the system of private property holds back the development of productive forces. In view of this the level of normal efficiency in a socialist society must be much higher than the profit norm in a capitalist society at the same level of economic development. Hence, although some investments, some types of machines used in capitalist countries ensure there the current profit norm, it may be inadvisable to use them in socialist countries where many other possibilities may exist offering still higher efficiency which could not be achieved under capitalism.

It is necessary to emphasize that when speaking of the efficiency of capital investments, we have in mind their efficiency calculated on the basis of o.d. valuations and not of current prices or of cost of production. The change to o.d. valuations alters radically the usual ideas of profitability and efficiency, and furnishes a new valuation of production and the volume of capital investment. The relative efficiency changes too. More efficient investments as determined by the usual calculation may in fact be found less efficient (as determined by the use of o.d. valuations); the opposite may also occur.

It is essential that an accurate computation (on the basis of o.d. valuations) should not show reasonable investments in the heavy industry to be less efficient than investments in the light industries.† Thus, the proposition that the calculation of a high level of efficiency may hinder the realization of these investments is not valid. All the more so because, as already mentioned on several occasions, the movement of capital investments and the allocation of means to sectors of industry are basically predetermined by the planning of

† Thus, when computing the annual hire valuation (rent on the hire of equipment) of a metallurgical group of enterprises (p. 144) we obtained the value  $12 \times 99,000,000 = 1188$  million roubles. If the cost of construction (o.d. valuation) of the group of enterprises amounts to 3 billion roubles, or 4 billion roubles if the period of construction is taken into consideration, its efficiency will be  $1,188,000,000 : 4,000,000,000 = 29.7$  per cent. Thus an electric power station with an annual output of electric energy of 5 billion kWh and requiring 1 billion roubles of capital investments (including investments in allied branches) at a cost of 9 kopecks/kWh, then with a price of 10 kopecks/kWh, its efficiency will be  $0.01 \times 5,000,000,000 : 1,000,000,000 = 5$  per cent. If the o.d. valuation of electric energy is determined by accounting for its productive efficiency and the availability of a considerable volume of generated energy at less efficient power stations and is equal, let us say, to 18 kopecks/kWh, then this efficiency of investment will already represent 45 per cent; if on the basis of o.d. valuations the cost of the electric power station should be found to be even higher, say 1.5 billion roubles, the efficiency would still be 30 per cent.

final production, and the calculation of efficiency serves only to choose the best system of solutions for the completion of this task.

This shows that in a socialist society there exist possibilities of much faster growth than in a capitalist one even in a period of prosperity of the latter. Evidence of this is the unprecedented growth of industrial production in most sectors which was attained in our country both in the years of the five-year plans before World War II and recently, in particular, the growth contemplated by the target figures for the next seven years.

The potentially realizable growth may in practice be attained only with accurate planning and plan fulfilment. This is confirmed by the fact that in certain sectors where economic management was weak the growth over many years was insignificant.

Even of those sectors in which production was growing fast enough, it is in no way possible to say that all the possibilities offered by the most progressive method of production, the socialist method, were utilized. The use of improved methods of planning could ensure a faster growth. This is confirmed, in particular, by the results obtained from the reorganization of management in industry following the formation of economic districts.

It should be mentioned that although the importance of debiting means for capital investments was to some extent allowed for qualitatively, a certain apparent similarity of the calculations of efficiency and of interest on capital was a barrier to a systematic quantitative account of this basic factor, and more systematic discussions of this problem have begun only in recent years.

Meanwhile, there are no real grounds for such fears. In practice the introduction of such computations will promote the growth of productive forces and can in no way change the nature of a socialist society. The presence and great importance of normal efficiency in our country is only a feature of the enormous possibilities of developing the productive forces and for this reason it should be stressed to the utmost.

Doubtless, owing to the great need for capital investments, the great possibilities of their very effective use on the one hand, and the shortage of means for this purpose on the other hand (in so far as investment had to be confined to internal accumulation), normal efficiency in the U.S.S.R. during all the years of reconstruction, and industrialization of the five-year plans preceding World War II would

have exceeded several times the value of the profit norm in capitalist countries and in pre-revolutionary Russia. During World War II it should have been even higher since with the extreme scarcity of means of investment and the need for supplying a very high part of output to the front, more important resources would have been required for the reorganization of the industry and the reconstruction of the districts destroyed.

Normal efficiency remained at a high level also during post-war reconstruction. Further, the satisfaction of the most urgent needs and the carrying out of some of the most efficient investments as well as the growth of accumulation as a result of the development of production which enabled the country to use considerable resources for capital investment (since in our country there is no unproductive consumption by the "idle" classes), should have led to a certain reduction in the value of normal efficiency and justified objects of investment of longer recoupment. Meanwhile, the fast progress of current technology and the reorganization of many sectors by automation which open up the possibility of particularly efficient capital investments at comparatively small costs produce a rapid increase in productivity and a rise in the value of efficiency. This leads one to think that in our country the value of normal efficiency continues to remain fairly high.

We shall refrain from giving even an approximate indication of the level of normal efficiency at present as this requires the selection and analysis of the corresponding data but we shall return to this problem later in the book.

Setting a definite level of normal efficiency means that as a rule all the investments of a higher efficiency should be carried and those of a lower efficiency should not be undertaken.

However, the principle of a single normal efficiency was not applied widely enough in planning practice.

In fact, some investments of very high efficiency were often not made for a long time even though this efficiency was known to exceed the normal one.

Simultaneously, a whole series of large investments are realized for which the calculation showed a very low degree of efficiency. As to these investments, it should be mentioned that for many of them an accurate computation taking into account o.d. valuations of production (taking into consideration its significance, the demand for

it) would show a much higher real efficiency of these both at the moment of construction of the given objects and in subsequent years.

These investments (for instance, the Magnitogorsk group of factories) have justified themselves brilliantly. Other investments for which an accurate calculation would have furnished a low percentage of efficiency were not justified (the giants of the fishing industry, certain hydroelectric power stations which in the course of many years were little used, etc.).

It should be mentioned that the high value of normal efficiency has not always been taken into consideration in our country by the planning bodies. The immobilization of means in constructions taking many years and showing low efficiency is to a large extent bound up with this fact.

A correct economic computation which takes account of a realistically high normal efficiency would be of assistance here, since it could show the harm of such planning solutions.

Another important problem must be considered, in which failure to use a single normal efficiency in the accounts leads to considerable losses. This is the problem of allocation of means for capital investment between long-term and short-term investments.

While for long-term investments resources were frequently allocated even if their efficiency was not great, of the order of 5–10 per cent per year, the circulating capital of factories was greatly reduced which hampered the creation of normal stocks, a normal supply of materials which would ensure uninterrupted work, the realization of numerous temporary improvements in production involving even comparatively small costs. As a result, many investments that could have been made with an efficiency of 10–20 per cent per month were not carried out because the necessary means, monetary or material, were lacking. We mentioned one of these matters above when discussing Example 5, namely, the supply of normal stocks.

A systematic computation of the normal efficiency would help to remove in time bottlenecks and disproportions. In the first place, it would allow investments to be directed to the most retarded sections which hold up the development of the remaining ones, to correct the disproportions between branches and to increase the production of those branches in which a particularly great shortage is felt, since precisely for these cases the efficiency obtained by including o.d.

valuations should have been particularly high.† Similar considerations were decisive in the allocation of means for capital investment. However, qualitative considerations supported by a scientifically based analysis and computation would promote a more accurate use of means.

Let us summarize our conclusions.

**CONCLUSION 27.** When solving problems of the advisability of long-term investments, the principle of the comparison of their efficiency with the normal one remains valid. However, in distinction from short-term investments, in the calculation of long-term investments it is strictly necessary to allow for changes in the ratio of o.d. valuations and normal efficiency during the period of the operation of the investment. Such a calculation can and should only be carried out on the basis of the general political and economic solutions contemplated for the period under consideration.

**CONCLUSION 28.** Normal efficiency differs fundamentally from the profit norm in a capitalist society, although both relate to the problem of capital investments. The results of the calculation based on the profit norm differ also substantially from the calculation of normal efficiency.

If the degree of economic development of a capitalist and a socialist state is the same, the profit norm in a capitalist system is lower than the normal efficiency in a socialist system because of the greater possibilities of development of productive forces in a socialist society than in a capitalist one.

### *Normal Efficiency and Valuations of Production*

In Chapters I and II, when considering the problem of short-term optimal planning, we separated it from the problem of long-term

† Thus, for instance, supposing that for a railway working at full capacity the hire valuation for transport is 850 roubles per wagon. The carrying capacity of the railway is restricted to one junction. The rebuilding of the junction at an investment of 10 million roubles would increase the carrying capacity of the whole railway by 100 wagons per day. Then the saving that could be attained by the investment will be  $360 \times 850 \times 100 = 30,600,000$  roubles and its efficiency would be of the order of 400 per cent per year. Without taking into consideration the o.d. valuation of transport, this exceptional efficiency characterizing the urgent need and timeliness of a given measure could disappear among the other important and necessary ones—and with resources being scarce it could be found that this measure could not be carried out.

planning. Such an approach, on the assumptions adopted there (resources, and in particular productive capacity, and more or less accurate targets of the composition of final production and the movement of supplies being given), is legitimate. This is confirmed by the fact that the analysis carried out was shown to be soundly based. It was possible to arrive at useful economic indicators of the optimal plan—the o.d. valuations characterizing the relative costs and efficiency of the productive use of various types of output at a given moment.

Such an analysis is not complete. A plan for a given period, when not limited by such strict conditions, is linked to other periods. The available productive capacity will depend upon the results of the activity of preceding periods and the output produced in a given period determines the resources of the subsequent one. The plan for the current period aims to a large extent at the production of means of production intended for use in future. Thus, if the setting of the planned task were included in the current planning of a given period, it would be impossible to draw up this plan without taking the long-term plan into account. The same applies to the movement of supplies.

In turn, when objects of capital investment in long-term planning are chosen, the general direction of economic development is important as well as the results of the analysis of the state of the economy and of the data for the current plan so as to show the requirements for particular kinds of output or productive capacity, and the valuation of the costs involved in their manufacture by various methods of production.

The indicators of the current and long-term optimal plan are also interdependent. An analysis of this relationship helps one to understand more profoundly the character and meaning of the o.d. valuations. Thus, while the data of valuations at a given moment may be determined in the analysis of the current plan, their dynamics and trends can only be revealed by relating them to the long-term plan.

In Chapter II we showed that the o.d. valuations of production must substantially differ from the cost of production which basically reflects only the direct outlay on labour and ignores the indirect costs. This difference is linked above all by the inclusion in the o.d. valuations of rent (when using limited natural resources) and of the

hire valuation (when using equipment in short supply). While the use of limited natural resources is not essential for all kinds of production the use of equipment is always essential. It is therefore necessary that the hire valuation should be included in the o.d. valuations of production quite systematically. At the same time, when we speak of such an adjustment in the price formation, there is no question of relating the level of prices for means of production more closely to prices of consumer goods; the level of prices for consumer goods, and in particular of retail prices, we do not touch upon at all. Basically, the problem is the desirability of changes in the price structure of various means of production in such a manner as to ensure their closest agreement with national economic costs.

The level of the hire valuation of equipment is determined by the actual situation as a whole and this fact becomes evident in the drawing up of the *optimal current plan*. However, by bringing into the analysis of this problem the long-term plan and the analysis of capital investment it is possible to arrive at some important general conclusions

If the hire valuation of a certain machine is so large that the investment in this machine is advisable even when normal efficiency is taken into account the manufacture of such machines will be increased in the optimal plan. Thus, the o.d. valuation of its output and its hire valuation will decrease. As a result, further investments in this machine will not yield an efficiency exceeding the normal one which will, as a rule, prove the inadvisability of further increasing its production under the plan, and then the level of the hire valuation for the given machine may be considered normal. This normal value is equal to that cost of manufacture of the given machine which corresponds to the magnitude of normal efficiency.

In this manner we obtain the first fundamental conclusion, that the necessity of including the hire valuation of equipment in costs does not represent in itself an isolated phenomenon caused by a temporary special shortage of some type of equipment to be overcome soon. On the contrary, it is constantly determined by the operative real economic factor of limited means of capital investment in comparison with the objects for which they could be used, the quantitative characteristics of which is normal efficiency. Furthermore, we have reached the second fundamental conclusion, that the level of hire valuation is to some extent determined by the value of



this normal efficiency. More precisely, its average (normal) value is determined by normal efficiency to which the hire valuation should tend in principle. This normal value equals  $nK$ , where  $n$  is normal efficiency and  $K$  the volume of capital investment in the given machine.

The hire valuation (per unit output) enters into the valuation of production and for this reason the average value of the latter will approximate some value which may be called the *normal valuation*. The normal valuation of production includes, besides direct costs, the proportion of capital investment per unit of the given production (corresponding to the value of normal efficiency). In other words, the normal valuation of a unit of production is determined by the formula  $p = C + nK$ , where  $C$  is the cost of production and  $K$  the particular capital investment. It is to be borne in mind that the valuation of the materials used, raw materials, etc., were constructed on the same principle; the volume of capital investment should also be calculated from such valuations. The structure of this normal valuation is reminiscent of the cost of production but differs fundamentally from it in that it includes instead of the profit norm, normal efficiency which is of an entirely different character.

With the best possible allocation of means of investment, a constant value of the normal efficiency and a constant composition of the final production the o.d. valuation should approach its normal magnitude as shown. This normal valuation itself changes with the change in the value of normal efficiency and with the introduction of new technical processes. In this respect it is also to a certain extent similar to the cost of production.

However, in an actual optimal plan the o.d. valuations will systematically deviate from these normal values.

Although the conditions stated above, for instance the stability of the value of normal efficiency,† are, as a rule, not observed, there are many causes, both permanent and temporary, which in reality operate systematically and which produce considerable deviations of o.d. valuations from the normal values.

† The latter circumstances and also the complex interdependence of normal valuations renders the very concept of a system of normal valuations and of a method of their computation far from simple. We shall refrain from defining this problem more precisely, and we shall not deal with the change in the concept of normal valuation when normal efficiency is not constant.

The permanent causes of the deviation of o.d. valuations from their normal values shown are as follows:

(1) The influence of the factor of the limitation of particularly useful natural resources, of the varying efficiency of resources actually used and of the necessity of allowing for this effect by including the rent in the cost, which makes the valuation deviate from the normal one in those kinds of production which involve the use of such resources (coal, oil, ore, timber, grain, cotton).

(2) The basic significance of the influence of the dynamics of relative valuations on the valuations of certain types of production, in particular the calculation of a certain relative increase in the valuation of labour and with it the relative increase of the valuations of types of production, in the production of which human labour is extensively used.

(3) Particular features of the valuations of production and their determination where single large investments are involved which by their nature are indivisible (irrigation and shipping canals, bridges, railways and highways, hydroelectric power stations). In this case, in spite of the large-scale investment, the o.d. valuations of services or of production may even be equal to zero (bridges). To some extent, the above is of significance in the valuation of production of large factories which have already been constructed and in the production of components involving large costs of setting up.

(4) The supply of products, the processes of which are interdependent and complex (chemistry, ores containing more than one metal, timber and its by-products, steel, cast iron, scrap metal, electric and thermal energy). The o.d. valuations of these products are also interdependent and the ratio of these valuations will depend in particular upon the ratio of requirements for such products.

(5) The need for reserves of productive capacity for certain kinds of output which in normal times exceed requirements. This results in a systematic reduction of the hire valuation of this capacity as compared with the normal one together with the reduction of the o.d. valuation of production which is used in its manufacture.

In addition to these permanent causes of deviations, there are many temporary ones such as the situation arising in the course of economic development and the structure of the national economy as well as many others, in particular:

(1) Freely available productive capacity of a given kind or its shortage. For instance, the availability of considerable capacity for the production of aluminium after the war must have reduced its hire valuation and also the o.d. valuation of aluminium which was consistent with the economic advisability of its wider utilization.

(2) The relationship between the productive possibilities and the objectively justified economic needs for some product at a given moment.

(3) Shortage of a certain kind of labour needed for a given product or the availability of reserves which cannot otherwise be used to their full value.

(4) The appearance of a substitute for a given product of a much lower normal valuation.

(5) An increase or reduction in the need for the given kind of product.

(6) The effect of local conditions (in addition to natural ones).

Usually, such divergences should gradually disappear in the course of a few years. However, their existence is an objective fact which cannot be ignored. It is all the more essential to take them into account since their emergence must be considered an unavoidable phenomenon which occurs systematically and particularly so in the present situation.

These divergences are consequences of a whole series of causes, such as changes in the composition of final product brought about by changes in existing conditions; the appearance of new requirements; the appearance of new products replacing old ones; the appearance of new and improved techniques which, while not put into general application immediately (Section 2, Example 7), reduce the hire valuation of previously manufactured equipment (moral depreciation).

Further, we obtained the very concept of normal valuations by assuming that the plan has already been optimal for a long time. In actual fact, however, the use of resources for capital investment is still far from perfect. But even in applying a system of optimal planning a plan actually in operation over a number of years will inevitably not be optimal since changes in the situation and new data require continual changes of plan, and therefore the planned solutions already operating are frequently not optimal.

Since economic decisions must be taken in a given practical situation and not for some idealized conditions we cannot ignore all these factors which cause deviations of the o.d. valuations from the normal values. For this reason, when drawing up an economic account for the current plan we consider it inadmissible to replace the o.d. valuations by their normal values, although the construction of the latter may be useful. The direct use of the system of o.d. valuations is also necessary in the analysis of the efficiency of capital investment.

It is important to know these valuations when selecting objects of investment. The excess load in the balance of electric energy in a given district, characterized in particular by its o.d. valuation, will enable one to solve the problem as to whether a new electric power station should be built in a given year or the following one, by the usual or by a speedier method. By computing this valuation in the calculation of efficiency the account will reflect this overload in the balance which will assist in the choice of a correct solution.

A knowledge of the o.d. valuations of building materials, of different categories of labour, of the hire valuations of construction equipment will help to choose the correct variants of the solution and the right methods for its realization.

It is also necessary to calculate the changes in the valuations which are a result of the expected increase in productivity and the reduction in the cost of operations as a result of their mechanization. When calculating the efficiency of the organization of some industry it is essential to allow for the possibilities of the future appearance of cheaper types of substitute product.

The inclusion of rent where natural resources are involved is of paramount importance in the calculation of the efficiency of capital investments and is in no way reflected in the normal valuation. Would it be possible, without taking rent into consideration, to evaluate accurately, for instance, the efficiency of capital investment for the irrigation of land?

It is also essential to change relative valuations systematically, and in particular to increase the relative valuations of labour and labour-consuming products.

At the same time, in view of the impossibility of forecasting with some accuracy the dynamics of the valuations, the calculation of the effect of an investment over a prolonged period is particularly com-

plex and unreliable. For this reason, we consider it here permissible to use normal valuations instead of the dynamics of o.d. valuations.

Since in the case of normal valuations it is easier to forecast the dynamics of a single indicator such as normal efficiency, it would appear that more soundly based values could be obtained.

The question could arise in what respect the relations determining the o.d. valuation differ from prices in a capitalist society. Without raising the general problem which requires independent and complete investigation we shall restrict ourselves to a single comment.

Although the valuations have a different meaning and origin, namely, that they must be computed on the basis of the analysis of an optimal plan, as against prices, formed spontaneously on the market, both are objectively determined by the conditions of production and by production costs.

Despite the radical difference in the relations of production under capitalism and socialism the quantitative relationships which determine both the o.d. valuations and the market prices have one feature in common. Both are determined by the necessary expenditure of labour though they do not coincide directly with it but show deviations. Under capitalism prices oscillate round prices of production, while with an optimal plan in a socialist society the o.d. valuations approximate on the average to the normal valuations.

It was noted above that it is not enough to replace the o.d. valuations by their normal values, but that it is strictly necessary to find the o.d. valuations by a direct analysis of the optimal plan. At the same time, in analysing the economics of a capitalist society where market prices constantly diverge from prices of production, Marxist political economy does not deal in detail with these deviations. It seems that this distinction is justified and is due to two causes.

Firstly, by the subject matter of the analysis. In a spontaneous capitalist market price oscillations are of so random a character, and are caused by so many factors that their accurate scientific investigation is hardly possible. "Under capitalist production, the general law acts as the prevailing tendency only in a very complicated and approximate manner, as a never ascertainable average of ceaseless fluctuations."<sup>†</sup> On the other hand, in a planned socialist economy an accurate analysis is possible in so far as the deviations of o.d. valuations from the normal ones can be calculated.

<sup>†</sup> K. Marx: *Capital*, vol. III, p. 159, English edition, Moscow, 1959.

Secondly, by the aim of the analysis. Marxist analysis of the capitalist economy aimed at a more general, fundamental investigation of capitalist production and the study of its basic laws, and for this reason could, of course, abstract from all the temporary transient factors and influences.

Economic accounting (and analysis) in a socialist economy serves as a basis for practical solutions and for this reason it must be more accurate and detailed. It must take into consideration the concrete situation including temporary and accidental circumstances.

### Section 3. Ways of Realizing Optimal Long-term Planning

#### *The Problem of Constructing an Optimal Long-term Plan and its Indicators*

The values of normal efficiency and the dynamic system of valuations necessary in the calculation of the efficiency of capital investment are represented by the economic indicators which characterize an optimal plan for the national economy; and for this reason their determination is directly connected with the setting up of this plan.

The construction of an optimal national economic plan is a problem of exceptional complexity, and the working out of methods for its solution involves extensive and intensive investigations and practical work.

We shall endeavour here to describe the possible approaches to this problem, its general character and its stages, while abstracting from those immense difficulties and complications which their fulfilment entails. Simultaneously, we refer only to some special considerations of the possible ways of a practical realization of certain stages of this process and of a rough, preliminary determination of the indicators of the plan.

As previously mentioned, the problem of long-term planning may be represented schematically and in the abstract as follows: (1) data are available of all types of resources which are at the disposal of the country and of the prospects of the growth of labour and natural resources; (2) the various possible technological processes of production and methods of organization of production are known, for the

existing basis of production as well as for its expansion and for capital investments: for these too accurate data are available as to the necessary costs and the volume of output attained; (3) similar data are available for technological processes which will be used later in the course of the planning period; (4) from the study of the requirements of the society in the light of the actual situation (at the present moment and in the future) the following have been determined: the composition of final output for consumption by individuals and by society, the necessary volume of productive capacity for certain kinds of output and the requirements for their location (in particular cases), the movement of stocks and also the share of output allocated to capital investment at every interval of time during the planning period.

It is required to draw up an optimal long-term plan which would ensure the most rapid development of productive forces and the maximum volume of production.

If all the data mentioned were actually available the construction of an optimal plan would represent a problem similar to the one considered above for which definite methods of solution are given. For this reason, if we were not limited by computational means an optimal plan corresponding to the numerical data and the tasks could be drawn up with a determinate system of dynamic valuations (for all kinds of output and factors of production, for each interval during the planning period, for each place), and the value of the normal efficiency of capital investments at each interval could also be determined.

Of course, the practical realization of such a plan directly in the form described is inconceivable: it is not possible to obtain accurate data for all types of resources, for the many millions of conceivable technological processes possible in the production of hundred thousands of products, including those processes which will emerge during the planning period; it is not possible to indicate and assess accurately the demand for all types of output over a longer period, all the more so as the data of costs of production and of the realized volumes of output, etc., must be allowed to influence allocation by type of output. If it were possible to obtain all these data, their compilation and computational treatment would hardly be technically feasible even if modern computational techniques were used.

And finally, if such a plan were constructed for the whole period, it may encounter in its realization quite considerable difficulties since the emergence of new and unknown circumstances would each time require a fresh computation and a change in the whole plan.

However, despite the practical impossibility of carrying through such a scheme to the letter, its presentation and, in particular, the specific knowledge of the data which it may be desirable to have, the properties and characteristics of such a plan, may substantially help and give direction to the practical determination of a plan and its indicators as well as in the understanding of particular economic problems.

An optimal long-term plan can hardly be constructed straight away in its final form. It is evident that its construction must represent a process of successively drawing up and improving the plan, consisting of a whole series of stages at which the plan itself and its indicators together with the initial data and tasks are simultaneously refined.

The ultimate improvement of the plan must take place in the process of its realization.

### *Some Features of Long-term Planning*

Without attempting to give a method for the actual construction of an optimal plan and its indicators let us only note certain aspects which show possible ways of removing the obstacles to such a construction.

(1) In order to determine roughly the outline of the plan and its main indicators such as the level of normal efficiency and the dynamic system of valuations, a simplified model of the national economy may be used. Such a model constructed for a combined group of products and services (conventional fuel, metals, machine tools, electric energy, individual types of productive capacity, transport services), on the basis of data obtained by averaging and sampling methods, may already cover some tens or hundreds (but not hundreds of thousands) of products and factors of production as well as cost data for various technological processes (including processes requiring capital investment).

Such a model may be analysed by the methods described on page 175 (Conclusions 25, 26). Using electronic computational techniques this analysis can be carried out in a relatively short time.



The results obtained in this manner will furnish only very tentative data for the optimal plan. The indicators found in the course of the analysis will be of great importance for the plan (normal efficiency, dynamic valuations). Although they will be of a very approximate and generalized character and will change substantially when the plan is actually worked out the knowledge of even such rough values should be of great significance as a guide in the process of planning.

(2) The analysis of the current plan should be of very great value in the drawing up of a long-term optimal plan. The data of the current plan may furnish initial values in the analysis of social requirements and of the structure of the final product, and the plan for the immediately following period may be considered as a variant of it.

It is also possible to derive from the analysis of the current plan factual data of the inputs of natural resources in the production of various products and services, and also of the capital investments that are necessary in order to create productive capacities. In the determination of costs it is essential to take into account not only material and labour costs, but also costs such as the use of natural resources, the hiring of equipment and of production space (rent, hire valuations) which are frequently omitted in the analysis. For instance, when comparing the relative costs of synthetic and natural fibres we know that we must obtain a distorted picture of costs by not accounting in the first instance for the hire valuation of equipment in one case and for the ground rent in the other.

Thus, when considering the problem of the possibility of using waterways instead of railways for the delivery of ores or any other product of the mining industries which entails setting up six months' stocks of such products, we can obtain an accurate solution only by a numerical computation of the load of railway transport (by means of the hire valuation) and the immobilization of resources in the form of stocks (by using normal efficiency).

It is evident that the data of the initial resources must be derived from the analysis of the structure of the national economy and the indicators of the current plan. Finally, on the basis of the data of the current plan and the productive experience a preliminary selection of those technical processes may be carried out which should be considered in the drawing up of an optimal plan both for the forthcoming period and for the future.

In this manner, the construction of an optimal plan may be accomplished by a process of successively improving and unifying plans earlier drawn up for the current period and preliminary plans for future periods.

Finally, the analysis of the current plan and the technical and long-term solutions adopted in it may be used (together with the analysis of the model) to obtain tentative values of the indicators of the optimal plan, as described earlier in the discussion of the indicators of an optimal plan in the problem of current planning (Chapter II, Section 8, p. 143). However, here it is necessary to bear in mind the limited possibilities of such an analysis.

The existence of consistent valuations is characteristic of an optimal plan. In practice, a current plan is not optimal and the direct construction of valuations from it may lead to contradictory relationships. For this reason, approximate valuations can be obtained by retaining in the analysis only methods which are justified, systematically used and advisable, and which are consistent with optimal use, and by rejecting arbitrary methods which are unjustified and do not correspond to the present level of development of the productive forces and the concrete situation.

The contradictions revealed during this analysis are of a real, vital but not of an antagonistic character and are solved in the process of improving the plan and of removing its shortcomings.

(3) One of the problems which causes great difficulties in the realization of this scheme of constructing an optimal plan is the need for data for numerous feasible technological processes (including the processes relating to the expansion of the productive base). To obtain such data entails much work, such as the listing of technical projects, accounts, experiments and the compilation of experimental data. Meanwhile, the number of processes which may be found advisable in certain conditions and for which such data would be necessary is very great. In practice only a small part of these processes and solutions will be used in the optimal plan, i.e. *data relating to a large number of processes will be found unnecessary*. When constructing a plan by the method of successive improvements it is possible to prevent such unnecessary work. Already on the basis of the tentative valuations (compare (1), (2) from the preliminary data and draft projects certain technical processes and planning solutions may be rejected as uneconomical

in the given conditions, and the later detailed plan and the construction of complete data will only be needed for those processes and technical projects for which the possibilities of application are sufficiently realistic.

(4) In drawing up a long-term plan it is necessary to bear in mind and to allow somehow not only for known technical processes but also for the technical progress which should take place during the period under consideration. On the whole, the calculation of technical improvement entails certain forecasts of technical development. In some cases, the calculation of technical progress may be carried out by starting from the proposed introduction of some concrete new technique; in others it will be necessary, on the basis of available experience in the given sector, to plan tentatively for some improvement of the applied technical processes (increase in productivity, reduction of specific costs). Of course, these data will be extremely inaccurate and may be partly wrong.

However, it should be mentioned that this is not so important because, while it may influence the valuations of future years, the changes in these valuations will not have any marked influence on the solutions of the first years.

(5) It appears that the value of normal efficiency may be forecast more easily and with greater accuracy than the whole dynamic system of valuations.

At the same time, if its level is known it is possible to obtain approximate normal valuations which may replace the use of the dynamic system of valuations in preliminary computations. Of course, the interdependence of normal valuations of various products must be borne in mind since one kind of product enters into the cost of production of another kind and this entails finding these valuations jointly. However, although this difficulty involves substantial complications, on the whole it would seem simpler to obtain an approximate system of normal valuations than to compute a complete dynamic system of valuations.

It is necessary to emphasize at the same time that the normal system of valuations is of limited application. The construction of this system of valuations is carried out in the abstract, without allowing for the required composition of production, or without considering the given initial resources of productive capacity.

For this reason, it is natural that since the normal system of valuations does not reflect the practical requirements of the plan, it cannot be used directly in the solution of problems of the following years. All the same, the construction of this system is valuable as a guide. Normal valuations can be used as rough approximations of the dynamic system. It is evident that they may be used in the calculation of the efficiency of investments for future years when a more or less accurate construction of a dynamic system of valuations is hardly feasible, as previously mentioned.

As regards costs and output of the next period, it would probably be more accurate for the purposes of an approximate comparison, to use the o.d. valuations of the given moment (the current plan).

It is also important to note that the use of more efficient and scarce natural resources is not at all reflected in the normal valuations. For this reason, these valuations can only be used, even approximately, for those industrial products for which this aspect plays no important role.

(6) When formulating the problem of drawing up an optimal plan it was assumed that the required composition of the final output for the whole period is initially given.

To some extent this requirement is indispensable in so far as the composition of the final output and also the part intended for accumulation depend upon the needs of society (for individual consumption and social needs) and may only be determined by allowing for a whole series of non-economic factors, starting from the general political and economic situation and the tasks confronting the national economy. However, these requirements for final production may not bear the detailed character of well-defined tasks or ratios for all outputs over many years in advance.

First of all, in so far as they relate only to the final output (used directly for consumption), the ratio of intermediate products, productive capacity,† which are to some extent predetermined by the final output, become defined nevertheless and take on a concrete appearance in the process of constructing the optimal plan. In this respect, requirements as to the composition of the final output in the long-term plan are less rigid than for current planning where the tasks of the production of means of production are given a definite concrete form.

† Productive capacity may occur directly in the tasks in certain cases only.

Secondly, many tasks in terms of final outputs may be defined by generalized indicators (textiles, foodstuffs, housing). A more detailed division of production by types may be carried out in drawing up the plan by including the valuations of necessary social expenditure for any product which are brought out by this process.

The composition of the final output intended for individual consumption should also partly be adjusted to the demand by the population. However, we shall not touch upon this problem here.

(7) The co-ordination of the long-term and current plan must represent an important aspect of the planning process. The long-term plan determines to a large extent the problem of the composition of output for the current plan of a given period, namely, the part of output for means of production and also the movement of stocks. However, the elements and indicators of the current plan, in particular the o.d. valuations determined in this plan (and the direct data for the cost structure) which are more accurate and realistic, can be used in the estimation of valuations, and in the choice of solutions for the long-term plan. Thus, their use is particularly important when determining the sequence of introducing productive capacity, the computation of costs for various planning solutions and the choice of economically optimum variants among them.

(8) A long-term plan must, of course, be less accurate and detailed for more distant periods, and more accurate and complete for the immediate future. The same applies to its economic indicators, such as the o.d. valuations.

The elements of the plan must take on a more concrete form and become more accurate the nearer the period to which it refers. The plan is perfected in its final form while it is in operation. Here, the use of the system of o.d. valuations will enable one to proceed with the process of improvement in a more flexible and accurate manner.

(9) Making the overall plan for the national economy consistent with the plans for sectors, economic districts and individual factories, and state and collective farms (*sovkhosy* and *kolkhozy*) presents a very important aspect of short-term and long-term planning.

From the overall plan for the national economy which takes into account the basic tasks as well as balanced inter-relations of sectors and districts a more detailed long-term plan for individual sectors, economic districts and particular factories is drawn up. Here the use of the methods of optimal planning and of constructing its

indicators should substantially improve the internal consistency of these plans. On the one hand, even an approximate knowledge of the general dynamics of valuations would make it possible, while drawing up a local plan, to consider accurately the national economic situation as regards both the composition of production and the use of raw materials, and other materials produced by other sectors and districts. On the other hand, the construction of a system of local o.d. valuations and their dynamics in the drawing up of a plan for a given district or sector will furnish in a convenient form data which ensure that these plans are consistent with the targets of the overall plan (the production plan, the resources for capital investment). Thus, the difference in the ratio of o.d. valuations of certain outputs (or the difference in normal efficiency) may suggest the advisability of a reallocation of certain types of resources, targets, etc., which would improve the overall plan for the national economy.

(10) Finally, it is essential to stimulate the fulfilment of the optimal plan by a system of economic accounting and of incentives for economic leaders. For this purpose, it is very important to construct a system of economic accounts and of indicators for the assessment of economic activity in such a manner that it favours the observance of the targets of the optimal plan and its overfulfilment by an appropriate use of resources, and by the discovery of new resources not anticipated in the plan. It may be expected that the construction of the basic characteristics of the short-term and long-term plan (o.d. valuations, normal efficiency) will also furnish the necessary data for the preparation of such indicators. We have discussed this in the context of short-term planning.

Of course, there is also the question of the usefulness of calculating the value of normal efficiency in the allocation of circulating capital and bank credits for short-term capital investments, in economic agreements, deliveries ahead of time or delayed deliveries, or in encouraging accelerated construction.

#### *Practical Use of the Method of Computing the Efficiency of Capital Investments*

The methods of computing efficiency of capital investments in the preceding exposition is directly linked with the system of optimal long-term planning of the national economy and with the indicators

of this plan (dynamics of valuations, normal efficiency). It is clear that the determination of such a system is a complex and long task involving considerable preparatory research work and an accumulation of experience (certain ideas about the approach to this problem have been given above). The question arises naturally whether this analysis and the method of computing efficiency could not in some measure be used in the realization of the general system of optimal planning.

Of greatest value appears in the first place the understanding of the quantitative interrelation of the various factors determining the efficiency of capital investments, and also of the principles of an advantageous choice of investments, which may be attained by the study of the basic assumptions of these methods. Particularly relevant are the following aspects, the importance of which is revealed by the analysis: the link between the problem of the efficiency of a particular investment and the overall optimal plan; the valuation of the effect of investment from the point of view of the national economy as a whole and not from the point of view of narrow, local interests or of individual indicators; consideration of the actual situation at the time; revealing the incompleteness of the computation of efficiency from costs, the necessity of bringing to light the basic factors in an objective and quantitative manner, determining the efficiency of the use of labour; inclusion of indirect costs (the hire valuation, ground rent); conversion of costs and of output at different points of time to a common period; the function of the dynamics of valuations; the function of factors determining the value of normal efficiency; features of the analysis of large, indivisible investments.

An understanding of these matters and their inclusion in the analysis, even without data for the direct use of the method developed here may serve as a guide to those aspects of the problem that ought to be taken into consideration and will frequently enable one to calculate them in a quantitative manner even if only approximately, and sometimes not directly but in an indirect way.

As previously mentioned, it is possible to obtain sufficiently accurate values of the indicators of the optimal national economic plan (normal efficiency, dynamic valuations) only by using an appropriate method in the construction of this plan. For this reason it is not possible to count on their use in the very near future.

However, rough values of these indicators can be obtained by more accessible means and may in many cases be sufficient for the purpose.

The first way is to use a simplified model of the national economy. As was shown, the results obtained in this way can give only very general data for the optimal plan. The indicators of the plan (normal efficiency, dynamic valuations) so obtained will be of the highest value and of great importance as a guide in the analysis of the efficiency of capital investments, in particular when the interdependence of the analysed object with the national economy as a whole or with other sectors is taken into account. Another way of arriving at general indicators of the optimal plan is the direct study of the solutions applied systematically in the current plan.

Finding an accurate value of normal efficiency and its dynamics is of particular importance. This value may be determined tentatively, it would appear, both from the analysis of the model and on the basis of the study of the current plan. Here, on the one hand, summary data of the national economy may be used, such as the growth of the productivity of labour, the growth of national income, the volume of capital investments and their dynamics; however, there is no ground for adopting any of these indicators directly (even if only approximately) as a value of normal efficiency. On the other hand, an analysis of typical and comprehensive examples of realized investments is important, and also of investments which remain unfulfilled as a result of limited means.

Starting from the value of normal efficiency on the basis of data of individual capital investments and their structure, it is possible to construct approximately normal valuations of those outputs the production of which does not substantially involve the use of more productive natural resources. The computation of these should be perfected by taking interdependence of these valuations into account. When these normal valuations are only approximate, data from the analysis of the interdependence of sectors† may also find use. The standard valuations obtained can be used in the calculations as tentative o.d. valuations, in particular for more distant periods.

The direct use of the method of optimal planning seems more practicable in the construction or even in the improvement of a long-term plan of development of any sector, say, of metallurgy, or electric power, in a large economic district. In such a case it is possible to

† On this problem see Appendix I (p. 278).



start from the tasks given by the national economic plan for the development of the given sector, resources to be made available year by year, expressed both as a sum of money and in terms of labour, as well as the more important materials and productive capacity in their natural form. Then, starting from the planned objects in their various alternatives (by technical processes, the volume, the time of the beginning and ending of operations), it is possible to draw up an optimal plan and to construct its basic indicators. It may be expected that this optimal plan will furnish an appreciable increase in output and an intensification of its rate of growth with the same costs. Moreover, the valuations of outputs, of materials used, the hire valuations of productive capacity and finally the implicit value of normal efficiency found in the course of its construction may show the necessity of introducing changes in the dependence of a given sector on other sectors, for instance the allocation of additional capital for one type of material by reducing capital for other types, or by increasing capital investments in the given sector as a whole against some other sector.

An analogous situation may arise in the setting up or improvement of a long-term plan for the development of some economic district. In such a case, its interdependence with the national economy or the connections between districts may also initially be taken to be in accordance with the accepted plan data and the question of their adjustment raised only on the basis of the construction and analysis of the optimal plan.

Finally, an interesting problem arises from the possibility of using the methods developed for the evaluation of the advisability of some capital investment when we have not at our disposal the indicators of the optimal plan for the national economy.

Although for the time being the experience of the practical use of such an approach is still lacking it seems to be applicable in principle. Of course, its application under such conditions can only be realized approximately but it may lead to more accurate conclusions than the methods usually used.

We shall give some reasons for the possibility of applying such a method.

The more important aspect of this approach consists in that when envisaging a particular capital investment it must not be considered by itself but in conjunction with the current national economic plan.

For this reason, *in analysing the problem whether a given capital investment is justified, the comparison of the effect of its realization for the national economy under given conditions (it is particularly important to consider local conditions) and the costs involved in its realization are of decisive importance.*

The numerical expression of efficiency of a given investment which was introduced in Section 2 would appear to be a sufficiently objective characteristic. This value is a sufficient indication of a given investment being advisable in itself and also for a comparison of two competing investments.

However, the actual determination of the value of efficiency is made difficult by the lack of necessary data, above all of the values of o.d. valuations and their dynamics. We believe that an approximate computation of this value may often be achieved more or less satisfactorily in using data for the usual calculation of costs with such adjustments and additions as may be necessary. In particular, the following appear to be especially necessary (in determining them we shall assume that the project refers to a factory intended for the manufacture of a given product):

(1) On the one hand, the periods of putting new capacity into operation and of starting production must be determined more accurately than is usually done in planning; and on the other hand, the time over which basic costs are spread during the construction and assembly of the factory. Here, the first should be early and the second as remote as possible (without changing the date of starting production).

(2) It is necessary to forecast and calculate the dynamics of a decrease in costs of the more important types of raw material and other materials used in the production, and also of building materials and labour. Sometimes such a lowering of costs may be tentatively estimated from data of past years, in others—for particularly essential items—the valuation may be made on the basis of actual local data together with projects for the organization of production of these materials or for the carrying out of operations. For instance, for a large volume of excavations, the actual cost of the project must be used and not their normative cost.

(3) An accurate determination, as far as possible, of the economic valuation of the basic production of the factory is particularly

important. Here, two fundamental ways of calculation seem realistic: (a) on the basis of the valuation of the effect of using the products in other branches; (b) on the basis of the costs necessary to obtain a given production (or its equivalent) by other methods. In the first case, the valuation may be carried out by calculating the saving which will be obtained from the use of a given output or on the basis of cost of production which it replaces. This seems the only possible way if the given production can be carried out only at the planned factory. In the second case, the costs of production of a given output at another place or of its substitutes must be calculated not from average data but by discovering how the additional production which could replace the production of the planned factory would actually be obtained.

In some cases these costs will be found to be higher than average (transport for longer distances, the need to use inferior raw materials or deeper mines, bigger mines for coal and ore). In other cases, these costs will be below average (available unused productive capacity, the existence of more advanced methods than now used). For instance, when comparing artificial fibre with natural wool or cotton, it is necessary to take into account that an expansion of the production of the latter would entail the use of additional (poorer) land or the transfer to such lands of other crops involving a corresponding increase in their costs (calculation of rent), or finally, capital investment for the irrigation of new areas. These reasons may also play a part in the construction of valuations for raw materials and building materials.

(4) In the valuation of raw materials or other materials costs and prices can be adjusted by scarcity coefficients which characterize the complexity and load of the equipment used in the production of a given material. In some cases these coefficients may be constructed by taking averages and using the deviation from the average of individual capital investments for a given type of production (taking into account the tentative value of efficiency). In other cases they may be constructed by calculating the load in a more concrete manner, or the under-utilization of the relevant equipment or the actual costs involved in the manufacture of the products necessary for the completion of a given project.

Even more efficient would be the centralized processing of the preliminary system of valuations of the more important types of

production by such bodies as the *Gosplan*, the Central Statistical Administration and the Scientific-Technical Committee (for the purpose of improving price formation generally), corresponding to full social costs (or correction factors for current prices of various groups of products). In the initial stage, such a system would be intended for use in technical-economic and project estimates alone (but not for the national economic account—*khozrashchet*). Such a system could be tested and adjusted in the process of application.

(5) Analogous principles may be used in the valuation of costs of the use of building machinery in construction. Their hire valuation may be determined either on the basis of their cost of manufacture, or more accurately (particularly for machinery intended for special use), by taking into account their supply and the possible effect of their use in other places. For instance, it is possible to take into account building machinery which becomes available with the completion of a building.

(6) In practice, costs of transport must also be included, particularly in the calculation of the load on railways, waterways, and other means of transport.

(7) Scarcity coefficients can also be introduced for labour costs when those categories (qualifications, physical condition), which are in particularly short supply in the country at a given period or in a given district are to be allowed for. Conversely, in other cases the impossibility of using fully trained and organized cadres on other production units may require the use of a scarcity coefficient below one.

The approach described is, of course, of an indeterminate and imprecise nature, and is in no way a substitute for the improvement in analysis which the use of the system of optimal planning may provide. It may only be considered as a temporary palliative aimed at some adjustment of the computations by taking into account those aspects of the problem which the system clarifies. Nevertheless, we believe that the proposed approach may be of advantage in the economic analysis of capital investment. It is more particularly justified in a relative comparison of two factories manufacturing the same products, since in such a case the difficulties and inaccuracies involved in the evaluation of the basic production are removed, and other conditions used in the computation are sometimes more justified if they apply to both variants.

In any case, it seems that the introduction of these adjustments is an advance on the usual comparisons of the periods of recoupment. However, even the latter approach must be considered more perfect in comparison with a purely qualitative valuation of efficiency.

Finally, it seems to us that even for the commonly adopted forms of substantiating a solution economically it may be of advantage to ascertain (numerically or even qualitatively) the more important considerations brought out by the analysis of the optimal plan: calculation of the period of construction and concentration of investments (provided that efficiency is used instead of the period of recoupment), calculation of recoupment over the actual periods, the need for a special method of calculating large indivisible objects of investment (calculation of ancillary investments). It is also essential to understand the relative and somewhat arbitrary character of the results obtained from the calculations carried out in this manner and to understand what has been omitted from this computation so as to make a more accurate use of the resultant conclusions possible.

#### Section 4. Comparison with other Proposals for the Calculation of the Efficiency of Capital Investments.

##### Conclusion

The constantly arising need for a factual analysis of the efficiency of capital investment to support the contemplated solutions in planning from an economic point of view entails a complete specification of the chosen approach and the method of calculation on which the analysis is based. Many economic and technical indices are of great importance in the choice of a solution: the cost of production, productivity of labour, individual capital investments, labour input, individual outlays on raw materials. A direct comparison of the variants, simultaneously for a whole series of indices, cannot furnish a solution of the problem although such an approach is still being put forward by certain economists.† However, in real life it may

† G. D. Bakulev: *Problems of economic science in the field of production automation in industry*, 1956: Report to the plenary session on scientific problems of automation of the Academy of Sciences of the U.S.S.R.; *Basic criterion and indices for the determination of economic efficiency of capital investment in industry*, Moscow, published by the Academy of Sciences of the U.S.S.R., 1958. P. Orlov and I. Romanov: "Problems of methodology in comparison of variants of construction projects" (*Voprosy Ekonomiki*, 1956, No. 1).

only find a very limited application. With this approach the choice is indisputable (and usually correct) only when one variant is found to be better than the other with respect to all indices. This is the case when any efficient technical improvement is introduced into the production process without involving considerable costs. But in such a case there is essentially no need for an economic analysis. Usually a comparison of different indices leads to contradictory results, and then their comparison and the choice of a solution is qualitative and to some extent subjective. In practice, however, an objective, quantitative comparison of various indices is strictly necessary and above all a principle of synthesizing two basic cost indices: the cost per unit product and the cost of individual investments. This has found its expression in planning practice and in the work of many economists. Various combined economic indices were proposed: the period of recoupment, the efficiency of investment, present value (allowing for interest on fixed capital). The standardization (or comparison) of these indices is proposed as a means for an accurate choice of a solution. These proposals stem from the needs of life itself and aim at improving the use of capital investments.

At present, this principle has won fairly wide recognition.† For this reason, we shall not deal at all with the work of the economists (Levin and others) who deny that such computations of efficiency are admissible since their views have been completely discarded although in their own time they had done much harm to the efforts towards an efficient use of means for capital investment.

As previously noted, our approach to the problem of the efficiency of capital investment in its simpler form, based on the use of normal efficiency, is to a certain extent similar to the proposals listed and for this reason their comparison seems useful. It is not our intention to give here a survey of the whole literature or a somewhat detailed critical analysis, especially since the proposals discussed are not as a rule published in sufficient detail. We should like to point out only certain fundamental differences of these proposals from the construc-

† See "Provisional standard method for the determination of the efficiency of technological improvements", *Gostekhnika of the U.S.S.R.*, 1956. In connection with this see the paper by I. G. Kurakov: "Some problems of technical development under socialism" (*Voprosy Filosofii*, 1956, No. 1, p. 14), also the discussion on it in the *Promyshlennno-ekonomicheskaja gazeta*, 1956.

tions and conclusions described above and to note the following features of the approach adopted by us.

### *First Feature*

The existence of normal efficiency and of criteria for the advisability of capital investments, based on the comparison of expected efficiency with the normal one, is scientific and rests on the analysis of the long-term optimal plan.

This criterion is established directly by analysing the conditions of production in a planned socialist economy. The analysis shows that only by taking normal efficiency into account is a maximum of the socially necessary production attained together with a faster growth of productive forces. This criterion is also linked indivisibly and organically with the system of indicators determined by the optimal plan which compare costs of various kinds and at different points of time (o.d. valuations and their dynamics).

It must also be emphasized that, in the interpretation given, the principle of normal efficiency is not self-sufficient but only derived, and plays a subordinate role. The long-term plan is not determined by the investments chosen on the basis of this criterion, but on the contrary, the basic indicators of this criterion (normal efficiency, the dynamics of o.d. valuations) are determined by the economic situation and by the problems which are solved by the optimal plan. The given criteria are only a means for arriving at their best solution.

The justification of the criterion of efficiency seems less convincing in the other proposals. It is true that sometimes a reasoning similar to our own is used (pp. 155, 162). However, in our case the basis is that the o.d. valuations are realistic. When prices or costs are used this reasoning does not apply.

Sometimes people appeal to the need for combining the indices of costs and capital investments into one index; however, such consideration does not by any means justify the aggregation of these indices in the form of the period of recoupment, or of efficiency, since various other forms of aggregation are conceivable.

Finally, in the construction of an index of present value the inclusion of interest stands on a somewhat different footing as compared with the calculation of costs under capitalism. Reliance on

Karl Marx's theory of prices of production is also unconvincing (I. S. Malyshev, L. A. Vaag)† in this context.

It is not clear why the law of value in the modified form of prices of production, as formulated under capitalism, can be transplanted mechanically to a socialist society. The designation of the product used for society by the same letter  $m$  as the surplus value does in no way render them identical and does not provide any basis for the assumption that the surplus product under socialism must also be proportional to the invested capital.

It must be emphasized that our objections are directed not so much against the conclusions of the writers mentioned as against their reasons. But a proper substantiation is not unimportant. Without it, it is generally impossible to understand fully and to formulate accurately a given proposition or to make proper use of it. This will be evident also from further comparisons.

### *Second Feature*

The normalization of efficiency is applied in a special form by converting costs and output to a common time period which makes it possible to unify the most varied cases and conditions.

The approach described here (Conclusions 24, 25, Notes 1-6, pp. 167-70) permits a consideration of the most diverse circumstances in the valuation of efficiency: the period of construction and the allocation of costs during this period, change in the volume of production and in current costs during the period of adaptation, actual periods of recoupment of costs (renewal and repairs of capital goods), the dynamics of the normal efficiency. Although this advantage is rather of a technical nature than a matter of principle, yet to us it seems fundamental.

As previously mentioned, the indicators generally applied are: the period of recoupment and the efficiency of investment

$$t = \frac{K_2 - K_1}{C_1 - C_2}, \quad v = \frac{1}{t} = \frac{C_1 - C_2}{K_2 - K_1}.$$

Their normalization in simpler cases (when the appropriate data are identical) leads to the same conclusions as our computations (if such

† I. S. Malyshev: *Voprosy Ekonomiki*, 1957, No. 3. L. A. Vaag: *General problems of the valuation of the economic efficiency of capital investments*, Academy of Sciences of the U.S.S.R., Moscow, 1958.



radical differences as the principle of establishing a norm and valuations are not considered; these will be dealt with below). but in complex cases their application is inadmissible. Thus, even in the case when the annual saving is constant but the period of realization is neither very short nor very long, these formulae produce already different and moreover obviously incorrect results (compare Note 3 on p. 167).

The method of comparing variants in terms of the so-called present cost of output†

$$p_1 = C_1 + nK_1. \quad p_2 = C_2 + nK_2$$

(here  $C_1$  and  $C_2$  represent the costs of production of a unit of output, and  $K_1$  and  $K_2$  are particular capital investments) seems somewhat more flexible; it enables one to compare simultaneously several variants and to allow for the period of construction. However, even this method does not cover all the cases that are of practical importance.

An endeavour to improve the calculation of efficiency in individual cases is made in the works of Z. F. Chukhanov.‡ Basically, the author introduces a continuous interest rate (calculation of normal efficiency for indefinitely small intervals of time), and converts the costs to a common time interval with the aid of differential equations. Although Z. F. Chukhanov's computations are correct and the author of this book is in no way against the use of higher mathematics in economics, such use seems unjustified in this case.

The greater accuracy attained by such continuous calculation is in practice quite immaterial since the adjustment is many times smaller than the unavoidable errors in the indices themselves.§ At the same time, the use of the appropriate formulae involves a whole

† Such comparison was proposed in the works of V. V. Novozhilov (quoted below), and also Z. F. Chukhanov and L. A. Vaag.

‡ Z. F. Chukhanov: *The process of gasification of coke and the problem of underground gasification of fuel* (Chapter V), Academy of Sciences of the U.S.S.R., Moscow, 1957.

§ For instance, 100 roubles at a normal efficiency of 10 per cent becomes with continuous calculation 110 roubles 54 kopecks in the following year and not 110 roubles. Nevertheless, even if the need had arisen for such an improvement, the computation could have been carried out by the usual method half-yearly or quarterly. The use of a differential equation to establish a criterion is of no avail since percentages were introduced in its setting up.

series of simplifying hypotheses (constant normal efficiency, constant amount of saving, a uniform schedule of outlays on construction) which restrict their field of application, for they entail the averaging of data leading to immeasurably greater deviations from real conditions.

However, the above remark refers only to this particular problem and is no evidence of a negative assessment of Z. F. Chukhanov's work as a whole. It deals very thoroughly and clearly with the importance of the calculation of efficiency of capital investments for the proper use of resources, and a whole series of important technical-economic problems are boldly stated. It is true that in the course of this the author makes an unnecessary fetish of the results of his calculations without considering that the data and the indices used are imperfect, and states in a categorical form in addition to the correct conclusions a whole series of unfounded and sometimes implausible propositions.

### *Third Feature*

When efficiency is computed (in calculating the cost of investment and also the cost of the completed production), the valuations of production, objectively determined by the situation, and the optimal plan are taken as a basis rather than current prices and costs. The criterion of efficiency based on the comparison of the normal efficiency with the expected one is inseparably and organically linked with the system of indicators determined by the optimal plan for the measurement of various types of costs at various points of time (o.d. valuations and their dynamics). Without such a system of indicators, reflecting correctly the full national economic cost of production, the measurement of cost and effect of a concrete investment and the comparison of its efficiency with the normal one is not correct. For this reason, although the criterion of efficiency described in this chapter for a basically simpler case (pp. 168-9) has the same form as the indices which figure in other proposals (normalization of the period of recoupment or of the level of efficiency), it was essentially given an entirely different content.

In fact, in their proposals the other authors envisage that the period of recoupment (or the efficiency) be calculated from the cost of production or current prices, which are as a rule based on the cost

of production. However, as noted on several occasions, o.d. valuations may differ very noticeably from these prices and costs of production. In this case, if they were proportional to the o.d. valuations, it would be of no importance since the period of recoupment would be given by a relative index; but the problem is precisely that they are far from proportional. Thus, for one product, the ratio of the o.d. valuation to costs may equal two, and for another product four.

The construction of these criteria directly on the basis of cost is devoid of inner logic and is eclectic.

Actually, if the criterion of efficiency is in the form of values converted to a common point of time (as was shown, this is equivalent to the normalization of efficiency or the period of recoupment), then as a criterion of comparison of national economic costs this converted value  $p = C + nK$  is used. At the same time, current costs and capital investments are calculated on the basis of costs of production or prices, and not of values converted to a common base. Consequently, in such an economic analysis two entirely different scales are used simultaneously. For certain important factors of production (metals, oil, electric energy), this difference in valuations, the conversion coefficient, equals 2, 3 or 4. However, the wages in both cases are the same, their coefficient being 1. It is clear that for accuracy such an economic calculation may be compared with a technical calculation, in which one portion of a length would be measured in units of versts, another in inches, a third in centimetres and the data were utilized without converting all measurements to a common unit.

At any rate, it must not be thought that for these problems such inaccuracies are immaterial. On the contrary, in the calculations of efficiency a certain degree of accuracy is absolutely indispensable; without it they lose all meaning. In practice it is possible that such inaccuracy would be of no importance if it were a question of comparing variants among which one has a great and undisputed economic and technical advantage over the other. But then, such a variant, as a rule, shows better figures of costs and of capital investments, and then the need to compare these indices and compute the period of recoupment generally disappears. The very structure of the formula for the period of recoupment is evidence of some subtleness of the criterion of efficiency which represents the ratio of difference in the pairs of similar numbers. And in such cases, as

known from experience in physics and mathematics, particular accuracy is necessary in the measurement of the given magnitudes and in their manipulation.

Meanwhile, it is clear from the above that we are concerned here with very inaccurate measurements. For this reason, in certain cases such measurements cannot be trusted when comparing indices either with one another or by themselves. For instance, a situation is possible when a certain variant gives a lower computed cost and smaller investment in comparison with another, yet must be rejected since in the computation of the structure of costs it proves worse as regards the actual national economic effect (for instance, a metal-using variant when metal is short).

Since the o.d. valuations represent a measure different from costs (and prices), the computation of the index of efficiency on the basis of o.d. valuations (which furnishes an accurate quantitative picture of the social effect), may lead to entirely different results and conclusions than the computation by cost, although in simpler cases both calculations are carried out by one and the same formula. We shall explain the foregoing by a numerical example.

Let us assume that on the basis of o.d. valuations we have for two variants  $C_1 = 800$  roubles;  $C_2 = 1000$  roubles, and  $K_2 - K_1 = 1000$  roubles. It is clear that in this case no reduction in the current national economic costs is attained as a result of additional capital investments and the second variant must be rejected.

Let us assume, however, that the computations are carried out on the basis of cost of production without taking into consideration, in particular, indirect costs. Let us assume that indirect costs amount to 30 per cent in  $C_1$ , 70 per cent in  $C_2$  and 20 per cent in  $K_1$  and  $K_2$ . According to this, in the computation on the basis of cost of production (or current prices), the following data, for instance, would be obtained

$$C_1^0 = 560 \text{ roubles, } C_2^0 = 300 \text{ roubles, } K_2^0 - K_1^0 = 800 \text{ roubles;}$$

from this the efficiency will be found equal  $(560 - 300) : 800 = 32$  per cent (the period of recoupment is three years). Thus, on the basis of the usual calculation the investment will appear to be very effective, although in actual fact it is inadvisable. The converse may also happen.

From the foregoing it follows that it is impossible to carry out any

reliable and accurate analysis of efficiency if instead of o.d. valuations costs or current prices are used; and it is also apparent to what extent the indices of efficiency obtained may be inaccurate.

Can one conclude that in general it is not possible to apply the criterion of recoupment on the basis of cost data and that consequently it is useless as long as no changes have been introduced in the price structure?

We make no such categorical statement. If the conventional and approximate character of the criterion of efficiency were clearly understood, if the disparity between apparent costs and the full national economic costs were taken into consideration and if the necessary adjustments were made in the data used, as far as possible, then the criterion could often be useful and helpful in the analysis of efficiency.

Indeed, the costs (and current prices) represent also objective indices, characterizing (even if not fully) national economic costs and for this reason they may furnish to some extent the means for an evaluation of the national economic effect.

Above all, when all the costs  $C_1$ ,  $C_2$ ,  $K_1$ ,  $K_2$  show a similar structure in factors and in their ratios (the ratio of labour costs, costs of metal, fuel, electric power, etc., being identical), then costs will differ from o.d. valuations by approximately the same coefficient of proportionality:

$$C_1^0 = \lambda C_1; \quad C_2^0 = \lambda C_2; \quad K_1^0 = \lambda K_1; \quad K_2^0 = \lambda K_2.$$

For this reason, it can be easily seen here that the period of recoupment or the value of efficiency calculated on the basis of costs will produce the same result as calculations with o.d. valuations (or come close to it), and will, therefore, accurately characterize the national economic effect of additional investments. However, such cases are evidently fairly rare. More real is the case when  $C_1$  and  $C_2$  have the same (or similar) structure, and  $K_1$  and  $K_2$  another structure. We shall have here one multiplier, let us say,  $\lambda_1$  for the first two and  $\lambda_2$  for the second two. Then, the value found for the period of recoupment will differ from the actual one by a certain multiple ( $\lambda_2/\lambda_1$ ), but will still give some indication of it. It seems to us that such cases arise not infrequently where it is a question of comparing two variants of one and the same machine differing in their structural properties or parameters.

As a rule, when endeavouring to construct indices of efficiency starting from costs, we are forced to make adjustments to these computations in one way or another such as to bring the results of the computations closer to the valuation of the actual national economic effect of an investment. Metaphorically speaking, when analysing the structure of costs, it is necessary to clarify which part is measured in units of versts, which in inches, which in centimetres and to allow for it.

One such adjustment is the calculation of supplementary capital investment in related sectors; a whole series of other ways has been discussed above (pp. 226-33). A centralized determination of special valuations or adjusting multipliers (compare p. 231) would be a very effective means for plan computations.

However, it should be mentioned that when the problem is to compare two variants of production which have an entirely different cost structure (natural and synthetic rubber, thermal and hydro-electric energy), the adjustments are somewhat large and the indices constructed on the basis of costs are generally not reliable. Here it becomes necessary to evaluate somehow or other the national economic effect on the basis of complete o.d. valuations of costs.

As a means of overcoming shortcomings connected with the use of costs and current prices, many economists (L. A. Vaag, I. S. Malyshev and others) put forward the proposal of using prices constructed on the principle of prices of production (with the surplus product taken proportionally to the fixed capital on the basis of a "percentage coefficient").

This proposal raises the following objections.

It has no adequate basis in theory since in essence it amounts to a mechanical transfer of the law of price formation in a capitalist society to a socialist society.

The principle of the formation of this type of prices is not fully explained. In the construction of prices it is not shown satisfactorily what discounting coefficients must be established and how. Were they to be constructed by the method recommended by L. A. Vaag, by evaluating fixed capital on the basis of current prices, a certain confusion of principles would still obtain. If the authors have in mind a successive revaluation of capital (although they are not very explicit about it), the actual manner of constructing such prices is not clarified.

However, even if it were possible to carry out such a calculation it would lead to values close and similar to those which we called normal valuations (with the difference that the percentage coefficient may differ from normal efficiency). Hence, in their application in the computation of efficiency, all the objections which we put forward (pp. 220-6) to the use of normal valuations instead of o.d. valuations remain valid in any case: the influences of more favourable natural resources, of the required volume of output, of available productive capital have not been taken into account.

Moreover, since the given proposal involves the construction of a new system of sales (or planned) prices, its realization is no less complex than the construction of a system of o.d. valuations which make optimal planning possible by taking all the factors more fully into account.

Nevertheless, the use of prices constructed according to this principle (with the surplus product proportional to the fixed capital), even if they are calculated in a simple manner, may be of some advantage in leading to the adjustment of economic computations in the necessary direction. In particular, these prices lead to relatively higher valuations of those products and services (metals, electric energy, oil, etc.) which involve larger fixed capital. This will be reflected more accurately in the actual national economic costs (see p. 135). Similar prices may also be used in adjusting efficiency computations of costs when cost structures differ.

It is clear from the foregoing that a radical improvement in the computation of efficiency may only be achieved by using prices constructed on the basis of the full national economic cost determined by the optimal plan, rather than on the basis of immediate costs.

It must be mentioned that the problem of changes in the price structure has been under discussion in our press but it seems to us that this was not done in a sufficiently realistic manner. Namely, the problem was a change in relative prices for Departments I and II.† However, as is well known and has been frequently noted here, prices of consumption goods (in particular, retail prices) should be formed according to different principles (which were not treated by us here) than those of means of production. For this reason, the proposals of bringing the respective price-levels closer were not sufficiently substantiated.

† *Voprosy Ekonomiki*, 1958, No. 8, pp. 106-7.

Another problem appears to us of great importance: the determination of more accurate relations between prices of various means of production, and the closest possible approximation of these to the real relations among national economic cost (o.d. valuations). Thus, the problem is not of a wholesale increase in these prices, but of some relative changes within a given department. As we already mentioned here, at first it may be possible for us to confine ourselves to the construction of a system of accounting prices to be applied only in economic planning estimates.

#### *Fourth Feature*

It has become possible to establish the (absolute) efficiency of the entire investment as a whole, and not of the supplementary capital investments alone

In the calculation of the efficiency of capital investments two types of problems occur in practice. First, when the problem is that of a relative valuation of efficiency, i.e. of the comparison of two variants of investments, of two possible processes of production for the manufacture of identical products, and in particular, the comparison of one variant with another which is better for current production but requires larger means for its realization. In such cases, the task may be reduced to the problem of efficiency of supplementary capital investments. Secondly, in practice, it is necessary throughout to analyse the efficiency of some investment when no similar projects are available for a direct comparison as, for instance, when the problem is to construct a factory for the manufacture of a new product. In this case, it is necessary to analyse the efficiency of the investment as a whole. In other cases too it is important to know, in addition to comparative effects, the national economic effect of the selected object in itself, how far its realization is justified in general.

As a rule, the use of the method of calculating efficiency or the period of recoupment was proposed only in relation to the first of the two problems mentioned, of supplementary capital investments. This is not surprising since in its usual order of application, on the basis of cost, it is unsuitable for the solution of the second problem, i.e. the valuation of the efficiency of the investment as a whole.

To substantiate this statement it is sufficient to attempt an evaluation of the efficiency of actual investments (or of investments in new



factories which are being planned at the same technical level, as often happens). When the price is constructed on a previous level of cost, we have no saving from the investment and its efficiency,  $O/K=0$ , equals zero. In other words, in such a calculation current investments are rated at zero efficiency and have an infinite period of recoupment! If some increase in value or gain is expected from the production, then this arbitrarily adopted percentage will determine the efficiency of the current investment which again does in no way reflect its underlying objective national economic effect. If such a calculation was carried out for a better process of production, the value of efficiency obtained, although positive, would be distorted and reduced.

These examples once again confirm how arbitrary the results obtained from the efficiency estimates based on costs are and how great the need for caution in referring to the results of such estimates.

The method described in this chapter may be used directly in the determination of the efficiency of investment as a whole. This was actually done in some hypothetical examples given above. For instance, efficiency was found to be: for a metallurgical combine 30 per cent, for an electric power station 45 per cent (p. 206, note).

It is essential here that those aspects which are usually considered from a qualitative point of view only should find an objective quantitative reflection in the calculations: tightness in the balance of outputs, shortage of construction materials, the load on transport.

In particular, the possibility of calculating the absolute efficiency of some capital investment as a whole makes it possible to account more accurately for its effect achieved on other sectors of the economy, and to compare this effect with the costs incurred. It is important to emphasize that accurate economic consequences are found precisely as a result of such a comparison and are not determined by the reduction of costs alone.† For this reason, such

† For instance, a certain increase in printing costs in order to achieve a higher quality of a book is economically justified since these costs are many times recovered by extending the circle of readers, by saving of time for the reader and by the greater benefit obtained from the book.

A typical example of this kind is given in the speech of Z. N. Nuriev at the XXI Congress of the Russian Communist Party: "On the basis of the data of the State Scientific-Technical Committee of the Council of Ministers of the U.S.S.R. an increase of the octane rating of motor fuel to 76-80 would lead to a saving of 6-8 milliard roubles in current and capital costs on motor vehicles in the country." (*Report*, vol. II, pp. 40-1.)

computation is helping to assess the relevant measures accurately from an economic point of view and must lead to an increase in those productions which are most urgent for the national economy at a given moment while ensuring at the same time high quality. It should also promote measures for modernization, and for the improvement and increase of the operational data of manufactured output.†

### *Fifth Feature*

There exists quite a different principle for the calculation of normal efficiency. The latter is determined by the saving (by an increase in the productivity of labour) that may be achieved per unit of capital (per unit time) invested under the optimal plan.

The importance of normal efficiency which is shown in the course of setting up the long-term optimal plan is determined by the whole situation—the present level of technical development, successive problems of economic progress, etc.

The problem of the actual computation of normal efficiency is complex and requires still further study. Nevertheless, as the principle of its computation was quite clearly shown above, some possible approaches to its actual approximate calculations were given.

In other studies of the problems of the efficiency of capital investments, different and, as a rule, insufficiently valid assumptions for the choice of a norm of efficiency have been introduced.

For instance, Z. F. Chukhanov proposes to adopt as a level of efficiency the ratio of the volume of manufactured output to the volume of fixed capital. It can be demonstrated that this is not an optimal criterion.

L. A. Vaag in the work quoted proposes as a value of efficiency ("a percentage deduction") the ratio of net income of the society to the sum of fixed and circulating capital (under net income of the society the author understands the aggregate surplus product). Without entering into the analysis of this statement which the author even believes to be the only scientific determination possible, let us

† In his brilliant article, the aircraft designer O. Antonov demonstrates quite correctly the shortcomings of the economic indices and economic computations used for this purpose at present

See *Znamia*, 1957, No 2; *Izvestia*, 15 February 1959.

note that ordinary common sense would suggest that such an approach is incorrect. For instance, with a decrease in personal consumption and a corresponding increase of net income and thus also of accumulation, the magnitude determined according to L. A. Vaag increases. Meanwhile it is clear that, on the contrary, an increase in the volume of accumulation makes it possible to realize also less efficient investments, and for this reason efficiency (per unit of investment) should in this case somewhat decrease.

S. G. Strumilin's approach is also interesting, distinguished by a peculiar dualism. On the one hand, transplanting to a socialist economy the theory of Karl Marx's commodity economy, he adopts the share of surplus product (the product for the society in the value-composition) as constant. In essence, this would mean that the values are found to be proportional to the costs of production and it would appear that no computation of the efficiency of the investments could be made on this basis.† However, S. G. Strumilin introduces efficiency artificially and in a roundabout way by calculating obsolescence of fixed capital as a result of a rise in the productivity of labour. This, in essence, is equivalent to the introduction of a normal efficiency equal to the growth of the productivity of labour. In other words, he introduces the principle of the commensurability of costs incurred at different times on the assumption that the products of the unit of average labour at different periods are equalized. Such a principle of commensurability—a single one for all cases without allowing for actual conditions—does not seem convincing and also does not conform to the real meaning and purpose of the index of normal efficiency.‡ Here, the same argument as that used in dealing with L. A. Vaag's proposal may be repeated.

† It is true that in S. G. Strumilin a certain difference is found in the period of recoupment when passing from the computation on the basis of cost to computation by value. However, the reason for this is an evident omission. When computing the manufactured production, the latter is reckoned in value and not on the basis of costs; when calculating fixed capital and recoupment the corresponding adjustment (for the product for society) is not introduced. (See the following footnote.)

‡ S. G. Strumilin: "The time factor in planning of capital investment", *Transactions of the Academy of Sciences of the U.S.S.R.*, 1946, No. 3; *Economic problems in the field of automation of productive processes*, Report to the plenary session of the Academy of Sciences, 1956; *On the economic efficiency of new techniques*, Academy of Sciences of the U.S.S.R., Moscow, 1958; "The efficiency of new techniques", in the journal *V pomoshch politicheskomu samoobrazovaniyu*, pp. 108-20.

Somewhat nearer to our approach is T. S. Khachaturov's† proposed construction of an index of national economic efficiency as a ratio of increase in annual output realized by capital investments to the volume of annual capital investments. However, as regards the computations of this index, the possible statistical methods of approach for this purpose are not clearly described by the author and are controversial. T. S. Khachaturov himself considers this index only as some characteristic of the national economy as a whole and not as a normal efficiency; he considers its use in the valuation of individual investments inappropriate and advises the setting up of special sectoral norms for this purpose.

In addition to the formula itself which determines the level of normal efficiency all the enumerated proposals, including the proposals of T. S. Khachaturov, differ from the proposal developed in the present work in that in the calculation of efficiency they envisage the construction of the ratios starting from the actual and not the optimal plan, and also by the use of costs and of current prices as a basis instead of o.d. valuations of production.

A. I. Notkin‡ and some others endeavour to determine the period of recoupment on the basis of the average period of service or the normative period of moral depreciation (obsolescence) of the machine. But this means the inadmissible substitution of a technical index of an entirely different nature for an economic index. It is also inadmissible to confuse the period of recoupment with the period of repayment of costs. An object with a short period of service may ensure a short period for the repayment of the investment even if its efficiency is very low. On the contrary, a much more efficient investment with a longer period of service will have a longer period of repayment. For this reason, short-term bank loans to factories do not promote the most efficient investments as investments with a short term of service. In particular, the conclusion of Z. V. Atlas§ that a three-years' loan by the State Bank and a six-

† T. S. Khachaturov: "Problems of economic efficiency of capital investments in a socialist economy", *Voprosy Ekonomiki*, 1957, No. 2, p. 106; *Economic efficiency of capital investments in the national economy of the U.S.S.R.*, Academy of Sciences of the U.S.S.R., Moscow, 1958.

‡ A. I. Notkin: *Problems of the determination of economic efficiency of capital investments in the industry of the U.S.S.R.*, Academy of Sciences of the U.S.S.R., 1953.

§ Z. V. Atlas: "Profitability of socialist enterprise", *Voprosy Ekonomiki*, 1958, No. 3, pp. 115-26.

years' loan by the Industrial Bank mean the practical acceptance of the period of recoupment as equal respectively to 3 and 6 years (or a normal efficiency of 35 and 18 per cent) is entirely unfounded.

It should be mentioned that in practice it is found that the period of recoupment is fairly frequently confused with the period of service and the period of repayment of the investment.

In many proposals the problem of determining normal efficiency or the period of recoupment was left open, and not only were these values, or the possible method of their calculation, not given, but even the very principle of their selection was not shown. Even when the authors specify a definite period of recoupment they do not usually give any reason for it.

However, the choice of a value for the normal efficiency is very important. The value and the possibility of applying the principle of normalization of efficiency depends essentially upon the correctness of the adopted numerical value of the normal efficiency (or the admissible period of recoupment when this period is normalized). For this reason, an unsound choice of these values is inadmissible.

Among the writers who put forward the thesis of the existence of a single norm of efficiency should in particular be mentioned V. V. Novozhilov who developed this thesis systematically in the years 1938-9. Of particularly great merit is his proposal to determine the value of normal efficiency by selecting the most efficient investments which can be realized with available means of capital investments, and also the justification of such an approach by considerations based on the analysis of the national economic effect.†

Thus, V. V. Novozhilov's proposals in their initial formulation are the nearest to those stated here. At the same time, the principle of comparing the efficiency of a given investment with the normal one in the form given by V. V. Novozhilov is in our opinion not sufficiently justified since he too proposes using costs computed in the usual manner for the calculation of the saving achieved and the cost of investment. Even certain adjustments in the computation of savings and costs to allow for scarcity which were introduced later by V. V. Novozhilov do not produce the effect required. Not individual

† V. V. Novozhilov: "Methods of comparability of the national economic efficiency of plan- and project-variants", *Transactions of the Leningrad Industrial Institute*, 1939, No. 4. A similar approach was developed even earlier but in a less detailed form, see L. P. Iushkov: "Fundamental problem of planning methodology", *Vestnik finansov*, 1928, No. 10.

adjustments but only systematically constructed valuations as determined by the concrete situation and the optimal plan, i.e. objectively determined valuations, can lead to this goal.

V. V. Novozhilov himself admitted the limited application of the principle of normal efficiency in the form originally proposed and advised its use only in the comparison of variants, apart from other reservations.†

Let us deal with yet another problem which was discussed in the literature, in particular in the works of V. V. Novozhilov and T. S. Khachaturov: whether normal efficiency should be applied as a limiting standard (by avoiding, as a rule, investments with lower efficiency) or as an average.

With our complex approach, when the optimal plan, the measurement of production and efficiency are all consistent, this problem does not generally arise. In the basic aggregate all the realized investments have approximately the same efficiency and this is compatible with the normal efficiency as determined. Only in the process of improving the plan can individual investments of greater efficiency emerge.

### *Sixth Feature*

The use of a single value of normal efficiency in all sectors of the national economy (provided that efficiency is accurately computed) is well based. In any case, a significant deviation from it is not advisable.

The validity of this thesis, which follows from the unity of a socialist economy, of its plan and its economic indicators, is sub-

† The following works by V. V. Novozhilov include already to some extent the developments by the author of this book and at times follow in parallel our investigations of this problem, but as a rule from a different aspect. See V. V. Novozhilov: "Practical methods of comparing costs and investments", *Transactions of the Leningrad Polytechnic Institute*, 1941; "Methods of arriving at minimum costs in a socialist economy", *ibid.* 1946; "Methods of arriving at maximum efficiency in a socialist economy", *Transactions of the Leningrad Financial-Economic Institute*, 1947; *Laws and methods of measuring costs and results in a socialist economy as a basis for the determination of the economic efficiency of new techniques*, Academy of Sciences of the U.S.S.R., Moscow, 1958. A similar approach is also developed in the works of A. L. Lure. See, for instance, "Methods of comparing operating costs and capital investments in an economic assessment of technical measures" in the collection *Voprosy ekonomiki zheleznodorozhnovo transporta*, 1948.

stantiated in sufficient detail in the text of this chapter. However, this conclusion must be explained since at first sight it cuts across the opinion of many economists on the necessity of establishing differential efficiency norms and periods of recoument by sectors.

The cause of this divergence of opinion is again due to the fact that efficiency computed on the basis of o.d. valuations from full costs represents an entirely different magnitude from efficiency computed on the basis of cost of production, by allowing only for direct visible costs. For this reason the objections usually raised against the principle of a single norm of efficiency do not arise in such an application.

We shall not deal with objections arising simply from misunderstandings which lead some to think that a uniform normalization of efficiency implies paying equal attention to various sectors and making equal capital investments in them. We have often stated that the importance of the sectors in the plan and the amount of their capital investments are basically determined by the planning tasks as to the composition of the final output and not by efficiency. However, the economic indicators and the plan are constructed with reference to these tasks. In a similar manner, a uniform efficiency norm certainly does not imply identical technical equipment of various sectors since sectors of the heavy industry, by their nature, require a different capital structure; with accurate valuations of outputs large scale capital investments in these sectors are very efficient. Only through the habit which it is difficult to give up (and which originates in particular in the under-pricing of production in these sectors) have we become used to the idea that investments in the heavy industry are less profitable than those in the light industry.

At the same time, to ignore the principle of a single normal efficiency and to set markedly different levels of efficiency for individual sectors may cause damage both to these and to other sectors. For instance, if mechanization (or automation) of a certain sector of the light industry or of agriculture makes it possible to release a considerable number of workers both in the light and the heavy industries for a relatively small volume of investment, it may produce a greater result in the latter than the cost of these resources with a low efficiency of investment in the same industry. However, it is understandable that such cases may occur in a small portion of the

aggregate volume of capital investments since the importance of heavy industry in the total volume of production and also the organic structure of capital in it determine the allocation of the major portion of capital investments to Department I. This will correspond to the realization of the most efficient capital investment (on the basis of the correct valuation).

Even when some productive capacity must be expanded to a larger extent than necessary for the manufacture of the required quantity of products, the capital investments necessary for the purpose will still be found efficient for then, as we have mentioned, the necessary productive capacity will be found from the final product and as such must be taken into account in the planned task for the composition of the final plan. Certain considerations of a non-economic nature may also make it advisable to use a somewhat higher level of efficiency in some sectors as compared with others.† However, a large difference would not be justified in any case.

Many other objections against a single norm of efficiency linked with the method of computing efficiency must also disappear when efficiency is properly calculated on the basis of o.d. valuations. This is related in particular to some other objections which were put forward by T. S. Khachaturov who has analysed this problem in detail. For instance, he shows that in sectors of the heavy industry less efficient investments may be made to overcome the need of using materials in short supply or with excessive costs of labour. In fact, if o.d. valuations were used in the computation, the scarcity of materials and the correct valuation of the labour released would be reflected in them; then the given investment (if it is indeed advisable) would be found from the computation to have an efficiency not below the norm.

So far our exposition has been related to the efficiency computed on the basis of o.d. valuations. What can be said if efficiency is computed on the basis of the usual calculations?

The use in such a case of a single efficiency norm seems unjustified in view of the defects in the calculation of efficiency, and therefore the values obtained deviate considerably from the actual national economic effect (determined on the basis of o.d. valuations). We

† See the work of T. S. Khachaturov (*Voprosy Ekonomiki*, 1957, No. 2, p. 118) quoted above.



think, moreover, that in this case the observance of a single efficiency norm within each sector is also not justified.

A very convincing argument against differentiation of the period of recoupment by sectors was given by M. A. Styrikovich. Since the partisans of sector norms admit the computation of investments in related sectors, they thereby level out the possibilities of an investment in various sectors, and in essence equalize also the norms of efficiency for various sectors in the estimates of particular capital investments.

In the arguments of the partisans of a differential norm of efficiency (T. S. Khachaturov, Z. F. Chukhanov and others), we do not find convincing grounds for the use of a single efficiency norm within a sector, and their objections against a single general norm remain valid to a large extent also for a single norm for each sector.

We have mentioned above that if the structure of cost in all the variants as well as the structure of capital investments is approximately the same, the efficiency obtained by the usual calculation will differ from the national economic one only by some constant distorting factor. Such a consideration would justify the use of differential norms by sectors if the same structure of costs were observed within each sector. However, it is clear that if it were a question of the comparison of essentially different variants such an assumption would be hardly plausible so that this consideration seems a very weak argument in support of differential norms.

Doubtless, any correct valuation of efficiency would be possible either with an accurate national economic valuation of costs or, in the usual calculation, by introducing into the computation adjustments which bring accounting costs closer to real costs. In the latter case, the objections against a single norm do not arise. The normalization of efficiency, even when differentiated by sectors, would, in our opinion, be of no advantage without adjustments in its computation.

We have said above that a socialist economy ensures a very high level of efficiency. This statement refers to an efficiency correctly evaluated. Conversely, the establishment of a high level of efficiency (even if differentiated by sectors) and its mechanical use in the computation of efficiency on the basis of costs may cause considerable damage. As a result of distortions produced by such calculations, certain investments which in fact are very important and

efficient for the national economy may appear irrational, and conversely, certain investments which in fact have a low efficiency may appear quite justified.

### *Seventh Feature*

To take account of the actual situation, in particular the temporary deviations in the computation of efficiency of a given investment.

The essential feature of the computation of efficiency, on the basis of the optimal plan with the use of o.d. valuations and their dynamics, is its concrete character. Owing to the use of o.d. valuations, constructed in accordance with the existing situation, those features of the state of the national economy are taken into consideration in the computation which relate to its actual development, reflect the individual discrepancies that are historically justified, and also the diversification caused by changes in demand and the emergence of new technological means, etc.

This leads to an increase in the o.d. valuations of those products that are in particularly short supply at a given moment, and to their reduction for products with an excess of productive capacity. Such assessments are important both in the valuation of production and of current costs—in particular, during the first years of the operation of the investment—and in the valuation of costs for the construction or manufacture of an object when the variant of its realization is selected and in particular when the materials to be used are chosen.

For instance, considerable productive capacity of the aeronautical industry which remained after World War II and exceeded the normal requirements of the country made it advisable economically to increase the use of aviation in the national economy. This must have been reflected in the reduced hire valuation of this capacity compared to its normal value, and thus in reduced o.d. valuations of aircraft and air traffic. Such deviations and sometimes also partial disproportions are objectively unavoidable, particularly in present-day conditions. To allow for them in the analysis through the o.d. valuations and their dynamics, even if they have been determined in a very approximate manner, is of fundamental advantage in the method of approach developed here for the calculation of the efficiency of capital investments. It is by this concrete nature that it differs not only from the usual computations based on cost or current prices, but also when additional items are added to these prices which take into

account the cost of particular capital investments per unit output (prices fixed on the basis of typical costs of production). even if the computation of the latter were carried out quite correctly.

The comparison of such valuations with the normal valuations mentioned above clarifies the problem fully. However, normal valuations have the advantage over costs of production in that they are based on normal efficiency and not on an arbitrarily chosen (or insufficiently substantiated) percentage of investment.

In spite of this, as previously mentioned (pp. 220-4), o.d. valuations deviate substantially from normal ones, and for this reason the latter may be used in the analysis of investments only as approximate and auxiliary valuations.

### *Eighth Feature*

Clear elucidation of the relative character of the criterion of normal efficiency, methods of improvement and the need for such improvement.

From the analysis made at the time it is evident that the criterion of normal efficiency (in its simpler form) was considered by us only as an initial tentative criterion, and as a precise one only under a whole series of simplifying assumptions: the absence of relative changes in valuations, its stability, etc. Simultaneously, from this analysis it was clear what changes should be introduced into the computation in order to achieve greater accuracy: calculation of the dynamic valuations, comparison of costs and outputs over the whole period of the operation of the investment. As was noted, another modified computation should also be used in relation to particularly large (indivisible) investments, and in addition adjustments should be made for non-economic considerations. Against this, the authors of other proposals for the computation of efficiency confine themselves as a rule to the formulation of the proposition of the use of normal efficiency and do not demonstrate its relative, organic character and the need for improvement, all of which leads to the inaccurate use of this criterion.

In concluding this survey it is necessary to emphasize that the computations on the basis of periods of recoupment (by efficiency) and of prices that are of the same kind as prices of production seem—in spite of their shortcomings—to constitute an advance on the methods of approach which in the economic analysis of capital

investments completely ignore the freezing (allocation) of means and the time factor, or which merely allows for them qualitatively.

In reality, in the first group of methods, the conclusions from the analysis of efficiency of capital investment derived from the systematic study of the optimal plan are nevertheless realized to some extent. For this reason, with an accurate, undogmatic and realistic use of these computations, such methods may be of some advantage when adjustments are made to the computations that aim at the full evaluation of the national economic effect and of the costs involved in the realization of any investment.

But an entirely satisfactory solution of the problems of efficiency would seem possible only by their analysis within the complex of problems of long-term planning and price formation from the point of view of an optimal national economic plan.

### *Conclusion*

Let us note some general, and at the same time realistic, practical conclusions derived from the analysis of the problem of efficiency of capital investment.

(1) The problem of the valuation of efficiency of capital investment is extremely acute *in the building of a communist society*.

(2) The existence of a real permanent possibility of using means of capital investments in a planned socialist economy without crises at a very high efficiency shows the particularly urgent need for their accurate use which must be checked by the calculation of the efficiency of capital investments. In particular, if such a computation is carried out systematically, even in a very approximate form, it would permit us to evaluate the damage resulting from investments of low efficiency, squandering of resources, extended periods of construction and of delays in starting operation; and it would also help to prevent these kinds of losses. At the same time such a computation would assist in showing the most efficient measures and their faster realization, including certain short-term investments and a speedier introduction of the most efficient new techniques.

(3) A systematic and accurate computation of the efficiency of capital investments is a basic factor in the solution of all the problems of long-term planning, valuation of the efficiency of a new technique, technological policy, allocation of means for capital investment, determination of size, location and type of factories.

(4) The computation of efficiency is essential for a correct solution of many problems relating to short-term investments in the field of production planning which can be quickly realized—in particular, the following: the best proportion of components, the volume and composition of stocks, assessment of the advisability of using special tools and instruments, comparison of technological processes with a varying length of the production cycle, allocation of the means between current production and preparatory work.

(5) The use of the analysis of efficiency would promote the best possible way of solving the following important problems: the determination of the desirable level of mechanization of particular sectors and processes (it would bring to light the inadvisability of an extreme difference in the levels of mechanization, it would reveal the advantages of complex mechanization), the evaluation of the economic effect of automation and the determination of the sequence of its realization, the determination of the length of periods of construction (it would often reveal the economic advantages of accelerated construction), allocation of the transport load between waterways and railways (it would frequently establish the economic advantages of waterways transport and the advisability of its increased use), the expansion of the network of roads (economic effect, priority, kind of roads).

(6) An accurate computation of efficiency—in particular, the computation of the absolute value of efficiency—is of fundamental importance in a full computation of the national economic effect of using production in other sectors of the national economy, in the comparison of the operational effect and costs of manufacture, especially the economic effect of improving the quality of output. The use of such a computation would provide an economic incentive for the immediate manufacture of the most urgently needed output, for production of high quality, and for the enforcement of measures aimed at perfecting and modernizing production.

(7) The analysis of the efficiency of capital investments must be carried out within the framework of the general long-term plan. This ensures the organic combination of the method of balances and of the method of costs of production.

(8) The improvement of quantitative computational methods in the analysis of capital investment and the effort to arrive at an optimal system of solutions require a comprehensive use of modern

mathematical methods of analysis of extremal problems (linear and dynamic programming), with the use of fast computers.

(9) The construction of a long-term optimal plan with the proper use of mathematical methods of computation should lead to the simultaneous determination of valuations of outputs of factors of production for each period of time.

These valuations measure the costs and outputs of all kinds in various time periods.

(10) In problems of the planning of capital investment it is of fundamental importance to know directly the normal efficiency (bound up with the dynamic valuations) for the national economy as a whole in each given time period. The level of normal efficiency used in the valuation of capital investments in individual sectors must, as a rule, approximate to this common normal efficiency.

(11) The realization of the methods of computing efficiency and optimal long-term planning calls for the improvement and a substantial extension of the system of basic statistical and economic indicators.

In order to characterize the volume of production, it is necessary to use an index of net production (constructed on the basis of o.d. valuations). It is also necessary to construct systematically indicators that characterize production reserves and potentialities, the extent to which equipment is being used and the economic saving achieved by it. These indicators, apart from computations relating to the construction of the optimal plan, must be reflected in the *khodzashchet*.

(12) The final characteristic of a given investment is the comparison of the effect achieved by it—its contribution to the output of the national economy in the course of many years—and the costs entailed in its realization.

(13) A simpler index of efficiency of a particular investment is the ratio of annual saving achieved to the cost of its realization, and the comparison of the value obtained with the normal efficiency, under given conditions and at a given moment. More accurate computations of efficiency must take into consideration the possible effects of a completed investment during the whole period of its operation, the prospects of developing productive forces as a whole, as reflected in the dynamic valuations of production.

(14) The comparison of variants and the assessment of the

advisability of additional investments on the basis of the period of recoupment, the computation of efficiency of additional investments and the comparison of costs converted to a common point of time (including a part of each particular investment), give similar results in simpler cases, but the last two methods should be given preference. The method based on converting costs and the effect of investment over the whole period of its operation to a common instant of time is the most accurate, convenient and universal one. For simpler cases it is equivalent to the other method mentioned, but in addition it makes it possible to allow for: the change in normal efficiency over the years; the period of construction and the allocation of costs in the course of a given period; the change in the volume of output and in individual costs; the national economic effect of the value of production realized and of its dynamics; costs of repair and renewal of capital goods to the extent and to the time that these take place; moral depreciation. It enables one to compute both the relative efficiency of additional investments and the absolute national economic efficiency of investment as a whole.

(15) In the computation of the efficiency of an investment it is of fundamental importance that in the valuation of its economic effect, the valuation of production, operating costs and costs of its realization should all be calculated accurately. For this it is necessary to use valuations of outputs consistent with the full social expenditure entailed in their realization (o.d. valuations), besides available reliable technical data; in particular, it is essential to account for the indirect costs (rent of land, hire of equipment) in addition to the visible costs. For this reason the construction of a system of such valuations is a very important problem, particularly for the purpose of economic planning calculations.

(16) Without the use of o.d. valuation, the principle of a common norm of efficiency can only be applied with extreme caution.

The substitution of costs of production (or current prices) for valuations of outputs is somewhat more justified in the computation of efficiency of supplementary investments if all costs have an identical structure. Otherwise it is necessary to make some adjustments in the computations so as to approximate the results of the computation (mainly by analysing the structure of costs) to the valuation of the actual national economic effect of a given investment.

(17) The determination of the valuations of production and of normal efficiency on the basis of the optimal long-term plan makes it possible to separate to some extent the problem of general planning from the actual economic solutions and to ensure at the same time a consistent solution for them. A general analysis of the national economic plan from the indicators thus computed makes it possible to acquaint individual factories and planning organizations in a convenient form with those facts of the general situation which should be taken as a guide in addition to the planned task. Moreover, this makes a certain decentralization of economic decisions possible, while at the same time the interests of the country are kept in view.

The use of these indicators will also enable one to make changes in the plan with greater flexibility and effectiveness in accordance with changes in the situation and in conditions, and thus to keep the plan optimal at all times (in relation to new requirements).

(18) The analysis of the efficiency of capital investments and in particular the proposition of the existence of a single normal efficiency under given conditions provide important conclusions about problems of price formation. Such an analysis shows in particular that the deviation of o.d. valuations from costs entering the calculation of indirect expenses for products which involve the use of complex and expensive equipment—coal, oil, gas, ferrous and non-ferrous metals, cement, electric power, transport services—is not a matter of chance, caused by their temporary scarcity, but is of a systematic character. For this reason, it is at all times necessary to allow for this situation in the structure of prices so as to reflect adequately national economic expenditure on more important products, to arrive at correct solutions for their manufacture and allocation, to encourage useful measures for raising their output, their economic use and replacement.

(19) The removal of systematic errors in the valuation of production if the conditions in the use of labour have not been taken into account (indirect costs), together with the construction of valuations of outputs conforming to actual conditions and the existing situation will ensure more realistic relative valuations for the various kinds of goods and services. This should lead to a better agreement between material and monetary balances, and to an increase in the role of the rouble in planning and in the *khozrashchet*.



(20) Improved methods of long-term planning and of the economic calculus of the efficiency of capital investments should lead to a more rapid development of the productive forces, the fullest opening up and use of the possibilities and advantages which are inherent in a socialist economic system.

## APPENDIX I

# MATHEMATICAL FORMULATION OF THE PROBLEM OF OPTIMAL PLANNING

THIS appendix provides a general mathematical formulation and analysis of the problems involved in constructing an optimal plan which were described in the main text of the book and were illustrated by means of numerical examples.

Such a generalization and more formal exposition, which requires a basic knowledge of mathematics, leads to a deeper understanding of the quantitative relations concerned and gives a clear conception of the field in which the methods are applied. The mathematics is necessary also for obtaining technical solutions of these problems in complex cases when there are a large number of different factors (Appendix II is devoted to the methods for finding solutions).

However, this appendix requires only elementary mathematical preparation to bring it within the grasp of those who are not specialists in mathematics. This and also the limitations of space have compelled us to abandon complete mathematical generality in this appendix, and some special cases have had to be left out.

### *The Problem of Programme Allocation†*

The general formulation of this problem, described in Chapter I, Section 1, is as follows.

$n$  different products (kinds of operations) in a given assortment are to be produced by means of  $m$  production units (factories, machines, machine-tools). The assortment consists of  $k_1, k_2, \dots, k_n$  units of products of types (1), (2),  $\dots$ , ( $n$ ) respectively. The productivity of each unit in each product is known: if the  $i$ th unit ( $i = 1, 2, \dots, m$ ) is

† This problem was studied by the author in reference [1] (see References to Appendices I and II).

assigned to the production of the  $j$ th product ( $j = 1, 2, \dots, n$ ), then  $a_{ij}$  units of this product are produced per unit of time. It is required to distribute the work among the production units in such a way that in a unit of time the maximum number of complete sets of products is produced.

If we denote by  $h_{ij}$  ( $i = 1, \dots, m; j = 1, \dots, n$ ) the fraction of the working time of the  $i$ th production unit spent on the production of the  $j$ th product, then the search for the optimal plan reduces to the following purely mathematical problem.

PROBLEM A. Given the non-negative numbers

$$\{a_{ij}\} \quad (i = 1, \dots, m; j = 1, \dots, n), \quad k_j > 0 \quad (j = 1, \dots, n),$$

with 
$$\max_{1 \leq i \leq m} a_{ij} > 0 \quad (j = 1, \dots, n)$$

(each product may be produced by at least one of the production units), find the set of numbers (the plan)  $\pi = \{h_{ij}\}$  ( $i = 1, \dots, m; j = 1, \dots, n$ ) satisfying the following conditions:

(1)  $h_{ij} \geq 0$  ( $i = 1, \dots, m; j = 1, \dots, n$ ) (the fraction of working time expended by a production unit for the processing of a given product is a non-negative number);

$$(2) \sum_{j=1}^n h_{ij} \leq 1 \quad (i = 1, \dots, m)$$

(the total working time of each production unit is limited by the planned calendar time);

(3) The quantity

$$\mu(\pi) = \min_{1 \leq j \leq n} \frac{x_j^\pi}{k_j}, \quad (1)$$

where 
$$x_j^\pi = \sum_{i=1}^m a_{ij} h_{ij} \quad (j = 1, \dots, n), \quad (2)$$

takes its maximum possible value (the numbers  $x_j^\pi$  express total output of the various products if the work is performed according to plan  $\pi$ , and the quantity  $\mu(\pi)$  shows the scale of production under this plan, the number of complete assorted sets produced per unit of time).

A plan  $\pi$ , which satisfies conditions (1-3), is said to be *optimal*, and a plan satisfying conditions (1) and (2) *feasible*.

First of all we note that in Problem A an optimal plan always exists (see Conclusion 1, p. 7).

In fact, let  $\pi_v = \{h_{ij}^v\}$  ( $v = 1, 2, \dots$ ) be a sequence of feasible plans such that  $\mu(\pi_v) \rightarrow \mu = \sup \mu(\pi)$ , where the exact upper limit ( $\sup$ ) is taken for all feasible plans  $\pi$ . Without loss of generality, we may obviously assume that the sequence converges to the limit

$$\lim_{v \rightarrow \infty} h_{ij}^v = h_{ij} \quad (i = 1, \dots, m; \quad j = 1, \dots, n)$$

(such convergence exists always for some subsequence). Then the plan  $\pi = \{h_{ij}\}$  is optimal.

Now we may formulate a proposition in general form about a characteristic property of an optimal plan—the existence of o.d. valuations for all kinds of products (see Conclusion 2, p. 8).

**THEOREM 1.** To determine whether a feasible plan is optimal it is necessary and sufficient that multipliers  $c_1, c_2, \dots, c_n$  (valuations for all kinds of products) exist such that

$$(a) \quad c_j \geq 0 \quad (j = 1, \dots, n), \quad \max_{1 \leq j \leq n} c_j > 0$$

(these valuations are non-negative, with at least one of the products having a positive valuation);

$$(b) \quad c_j a_{ij} = \max_{1 \leq t \leq n} c_t a_{it} = d_i, \quad \text{only if } h_{ij} \neq 0$$

(each production unit is used for the preparation of only those products for which its productivity valuation is maximum; the numbers  $d_i$  may be taken as valuation of the productive power of the production units);

$$(c) \quad c_j = 0, \quad \text{if } x_j^* > k_j \mu(\pi)$$

(for those products which are produced in excess, the valuations are equal to 0);

$$(d) \quad \sum_{j=1}^n h_{ij} = 1, \quad \text{if } d_i \neq 0$$

(production units with positive valuations of productive power are fully used).

Indeed, if for a given feasible plan  $\pi = \{h_{ij}\}$  such multipliers exist, then for any other feasible plan  $\pi' = \{h'_{ij}\}$ , we have, from (1), (2) and conditions (1), (2) (a-d):

$$\begin{aligned} \left(\sum_j c_j k_j\right)\mu(\pi') &\leq \sum_j c_j x_j^{\pi'} = \sum_j c_j \sum_i a_{ij} h_{ij} = \sum_i \sum_j (c_j a_{ij}) h'_{ij} \\ &\leq \sum_i d_i \sum_j h'_{ij} \leq \sum_i d_i = \sum_i d_i \sum_j h_{ij} = \sum_j \sum_i (c_j a_{ij}) h_{ij} \\ &= \sum_j c_j \sum_i a_{ij} h_{ij} = \sum_j c_j x_j^{\pi} = \sum_i c_j [k_j \mu(\pi)] = \left(\sum_j c_j k_j\right)\mu(\pi), \end{aligned}$$

which leads to the inequality  $\mu(\pi') \leq \mu(\pi)$ . In view of the arbitrary choice of a feasible plan  $\pi'$ , this inequality shows that the given plan  $\pi$  is optimal, and the first part of the theorem has been proved. The second proposition, that for every optimal plan there is a set of multipliers which satisfy conditions (a-d), will be proved below where a more general problem will be considered.

NOTE 1. If all the numbers  $a_{ij} > 0$  (all kinds of product may be produced by every production unit), then for every optimal plan  $\pi = \{h_{ij}\}$ , and the multipliers corresponding to it according to Theorem 1, the following conditions are satisfied:

$$(2') \quad \sum_{j=1}^n h_{ij} = 1 \quad (i = 1, \dots, m)$$

(all production units are fully utilized);

$$(3') \quad \frac{x_1^{\pi}}{k_1} = \frac{x_2^{\pi}}{k_2} = \dots = \frac{x_n^{\pi}}{k_n} = \mu(\pi)$$

(the given assortment of products is maintained);

$$(a') \quad c_j > 0 \quad (j = 1, \dots, n)$$

(all products have positive valuations).

Indeed in this case all the numbers  $d_i > 0$ ; and then from (d) we have (2'). Further, for all  $j$  we have  $h_{ij} \neq 0$ , for some  $i$  (each product is produced on some one production unit); therefore, (b) implies (a'). From (a') and (b) we obtain (3').

Consequently, in the case considered (when all  $a_{ij} > 0$ ) it is necessary and sufficient for the optimality of a feasible plan  $\pi$  that it

satisfies conditions (2'), (3'), and that there exists a system of positive multipliers which satisfy conditions (b) (see Conclusions 4, 5, pp. 13, 15).

NOTE 2. In the general case (when some  $a_{ij} = 0$ ) it is also possible to limit the consideration to so-called *assortment* plans, which satisfy conditions (1), (2') and (3'), since each feasible plan  $\pi$  is an assortment plan  $\pi'$  with the same scale of production,  $\mu(\pi') = \mu(\pi)$  (for obtaining such a plan it is sufficient to reduce certain  $h_{ij}$ , which correspond to those products which are produced in excess,  $x_j^* > k_j \mu(\pi)$ , and to increase other  $h_{ij}$ , corresponding to  $a_{ij} = 0$ ).

NOTE 3. Let all costs of production be sums of expenses, which are proportional to the volumes of outputs of each product, and of the outlays on work performed by the production units which are independent of the type of the products produced by them; then, for an optimal assortment plan, the costs of some one assortment set of products are minimal (see Conclusion 1, p. 7). For any feasible plan  $\pi'$  these costs comprise

$$\frac{1}{\mu(\pi')} \left( \sum_j p_j x_j^* + \sum_i r_i \right) \geq \sum_j p_j k_j + \frac{1}{\mu(\pi')} \sum_i r_i \geq \sum_j p_j k_j + \frac{1}{\mu(\pi)} \sum_i r_i \quad (3)$$

where  $p_j$  are the costs for producing a unit output of product ( $j$ ) ( $j = 1, \dots, n$ );  $r_i$  ( $i = 1, \dots, m$ ) are the operating expenses of the production unit;  $\mu(\pi)$  is the scale of production under an optimal plan; and the inequalities (3) become equations if, and only if,  $\pi'$  is an optimal assortment plan.

NOTE 4. Problem A may always be reduced to the case where  $k_1 = k_2 = \dots = k_n = 1$  (all products are required in equal quantities). In fact, if, for every product ( $j$ ), we take a new unit of measurement equal to  $k_j$  old units, the problem becomes one in which

$$a'_{ij} = \frac{a_{ij}}{k_j} \quad (i = 1, \dots, m; j = 1, \dots, n), \quad k'_j = 1 \quad (j = 1, \dots, n).$$

The case of output of multiple products. Suppose now that for every production unit ( $i$ ) ( $i = 1, \dots, m$ ) there are  $r_i$  methods of working; and when working according to method  $s$ ,  $a'_{i1}, a'_{i2}, \dots, a'_{in}$  units of the respective products are made by the production unit ( $i$ ). In this case the following more general problem arises, which was also considered in reference [1].

PROBLEM B. Given the non-negative numbers

$a_{ij}^s (i = 1, \dots, m; s = 1, \dots, r_i; j = 1, \dots, n)$ ,  $k_j > 0 (j = 1, \dots, n)$ ,

for which 
$$\max_{i,s} a_{ij}^s > 0,$$

it is required to determine the set of numbers (the plan)

$$\pi = \{h_{is}\} (i = 1, \dots, m; s = 1, \dots, r_i)$$

from the conditions

$$(1) \quad h_{is} \geq 0 \quad (i = 1, \dots, m; s = 1, \dots, r_i);$$

$$(2) \quad \sum_{s=1}^{r_i} h_{is} \leq 1 \quad (i = 1, \dots, m);$$

$$(3) \quad \text{the quantity} \quad \mu(\pi) = \min_{1 \leq j \leq n} \frac{x_j^\pi}{k_j},$$

so that 
$$x_j^\pi = \sum_{i=1}^m \sum_{s=1}^{r_i} a_{ij}^s h_{is} \quad (j = 1, \dots, n),$$

takes the largest possible value.

As above, a plan  $\pi$  which satisfies conditions (1–3) is said to be *optimal*, and one satisfying conditions (1) and (2) *feasible*.

It is not difficult to see that for this problem too an optimal plan always exists. The following theorem characterizes the optimal plan.

**THEOREM 2.** For a feasible plan  $\pi$  to be optimal it is necessary that multipliers  $c_1, c_2, \dots, c_n$  (valuations for all kinds of products) exist such that

$$(a) \quad c_j \geq 0 \quad (j = 1, \dots, n), \quad \max_{1 \leq j \leq n} c_j > 0;$$

$$(b) \quad \sum_{j=1}^n c_j a_{ij}^s = \max_{1 \leq t \leq r_i} \sum_{j=1}^n c_j a_{ij}^t = d_i, \quad \text{if } h_{is} \neq 0;$$

$$(c) \quad c_j = 0, \quad \text{if } x_j^\pi > k_j \mu(\pi);$$

$$(d) \quad \sum_{s=1}^{r_i} h_{is} = 1, \quad \text{if } d_i \neq 0.$$

We do not supply a proof of this theorem here since it will be obtained below as a consequence of the more general Theorem 3.

We note that Problem B may be interpreted in a way different from that above.

There are  $m$  kinds of composite raw materials, which are available in given proportions,  $p_1 : p_2 : \dots : p_m$ .  $r_i$  technical methods exists for the treatment of raw material of type  $(i)$ ,  $(i = 1, \dots, m)$ ; when working according to method  $(s)$   $(s = 1, \dots, r_i)$  the composite set  $a_{i1}^s, a_{i2}^s, \dots, a_{in}^s$  of units of products are obtained from the  $p_i$  units of this raw material. The required assorted set of products consists of  $k_1, k_2, \dots, k_n$  units of products of type  $(1), (2), \dots, (n)$ . We are looking for a plan  $\pi = \{h_{is}\}$  (the numbers  $h_{is}$  in this case show what part of the raw material of type  $(i)$  is treated by method  $(s)$ ) for which a maximum number of assortment sets of products is obtained from one composite set of raw material, which consists of  $p_1, p_2, \dots, p_m$  units of raw material of types  $(1), (2), \dots, (m)$ , or, what amounts to the same thing, the minimum quantity of composite sets of raw materials that must be used for one assorted set of products.

This kind of problem occurs regularly in various sectors of industry (treatment of metals, wood-processing, chemical, oil refining, non-ferrous metallurgy, etc.). As a characteristic example of such problems we may take the rational cutting out of industrial materials (sheet metal, profile rollings, pipes, wood and so on; see [1, 5, 6]).

In the special case when

$$r_i = n \quad (i = 1, \dots, m), \quad a_{ij}^s = \begin{cases} a_{ij} & \text{for } s = j \\ 0 & \text{for } s \neq j \end{cases}$$

(for every technical method only one product is obtained from a raw material), Problem B evidently coincides with Problem A. In another special case, when  $m = 1$  (there is only one type of raw material), we have the following problem.

**PROBLEM C.** Given the non-negative numbers

$$\{a_j^s\} \quad (s = 1, \dots, r; j = 1, \dots, n) \quad k_j > 0 \quad (j = 1, \dots, n)$$

for which  $\max_{1 \leq s \leq r} a_j^s > 0 \quad (j = 1, \dots, n)$ ,



it is required to determine the vector (plan)  $\pi = (h_1, \dots, h_r)$  from the conditions

$$(1) \quad h_s \geq 0 \quad (s = 1, \dots, r);$$

$$(2) \quad \sum_{s=1}^r h_s = 1;$$

$$(3) \text{ the quantity } \mu(\pi) = \min_{1 \leq j \leq n} \frac{x_j^\pi}{k_j},$$

$$\text{where } x_j^\pi = \sum_{s=1}^r a_s^j h_s \quad (j = 1, \dots, n),$$

takes the largest possible value.

In analysing this problem we shall pause to consider some details. We shall take, for example, the auxiliary problem  $C'$ , in which the surpluses (as compared with the required assortment),  $h_{r+1}, \dots, h_{r+n}$ , of each product appear as unknowns, as well as the levels of application of the various methods,  $h_1, \dots, h_r$ : in an optimal plan it may turn out to be necessary to provide for the possibility of such surpluses.

PROBLEM  $C'$ . With the data of Problem C, to find the vector  $\bar{\pi} = (h_1, \dots, h_r, h_{r+1}, \dots, h_{r+n})$  from the conditions

$$(1) \quad h_s \geq 0 \quad (s = 1, \dots, r+n);$$

$$(2) \quad \sum_{s=1}^r h_s = 1;$$

$$(3) \text{ the equations } \frac{x_1^{\bar{\pi}}}{k_1} = \frac{x_2^{\bar{\pi}}}{k_2} = \dots = \frac{x_n^{\bar{\pi}}}{k_n}, \quad (4)$$

$$\text{hold, where } x_j^{\bar{\pi}} = \sum_{s=1}^r a_s^j h_s - h_{r+j} \quad (j = 1, \dots, n);$$

(4) the quantity  $\mu(\bar{\pi})$ , equal to the common value of the ratios (4), is a maximum.

The required vector is said to be *optimal*, a vector  $\bar{\pi}$  which satisfies conditions (1) and (2) is called *feasible*, and one which satisfies conditions (1-3) is called an *assortment* vector.

It is not difficult to see that Problems C and  $C'$  are equivalent. Indeed, a plan  $\pi = (h_1, h_2, \dots, h_r)$  is optimal in Problem C clearly if,

and only if, the vector  $\bar{\pi} = (h_1, \dots, h_r, h_{r+1}, \dots, h_{r+n})$ , whose first  $r$  components coincide with the corresponding components of the vector  $\pi$ , and the remaining components are defined by the equations

$$h_{r+j} = x_j^* - k_j \mu(\pi) \quad (j = 1, \dots, n),$$

is optimal in Problem C'; then  $\mu(\bar{\pi}) = \mu(\pi)$ .

To make clear the geometrical meaning of Problem C', we shall consider an  $n$ -dimensional space  $R_n$ , whose elements  $x = (x_1, x_2, \dots, x_n)$  we shall call points or vectors, without elaborating these concepts. To each permissible vector we relate the point

$$x(\bar{\pi}) = (x_1^*, x_2^*, \dots, x_n^*) = \sum_{s=1}^r h_s a^s + \sum_{j=1}^n h_{r+j} e^j \in R_n, \quad (5)$$

where  $a^s = (a_1^s, a_2^s, \dots, a_n^s)$  are points characterizing the available technical methods, and

$$e^j = (0, \dots, 0, \overbrace{-1}^{j-1}, 0, \dots, 0)$$

are unit vectors along the respective co-ordinate axes.

It may easily be seen that the points (5), corresponding to all possible feasible vectors  $\bar{\pi}$ , fill the convex closed polyhedron  $M$ , which is spanned by the points  $a^s$  ( $s = 1, \dots, r$ ) and the negative orthant. Assortment vectors  $\bar{\pi}$  (and only these), given by (4), correspond to points  $x(\bar{\pi})$  situated on the axis†

$$Y = \{y \mid y = \lambda z, -\infty < \lambda < +\infty\},$$

where  $z = (k_1, k_2, \dots, k_n)$  is a vector characterizing the necessary assortment of products. With this notation the quantity  $\mu(\bar{\pi})$  (the scale of production) coincides with the corresponding value of  $\lambda$ .

Hence it is clear that optimal feasible vectors are those, and only those, which correspond to the extreme point of intersection of the  $y$ -axis with the polyhedron  $M$ , i.e. the point  $y^* = \lambda^* z$ , where

$$\lambda^* = \max_{\lambda z \in M} \lambda.$$

† This notation indicates that  $Y$  consists of points  $y$ , represented by the form  $y = \lambda z$ , where  $\lambda$  is any real number.

Figure 9 shows the polyhedron  $M$ , the  $y$ -axis and the point  $y^*$ , which correspond to the following numerical data:

$n = 2$ ,  $r = 5$ ,  $a^1 = (1; 6)$ ,  $a^2 = (4; 5)$ ,  $a^3 = (5; 4)$ ,  $a^4 = (8; 3)$ ,  $a^5 = (11; 0)$ ,  $z = (3; 2)$ .

The point  $y^*$  is evidently on the boundary of the polyhedron  $M$ . Therefore (according to a well-known theorem in  $n$ -dimensional

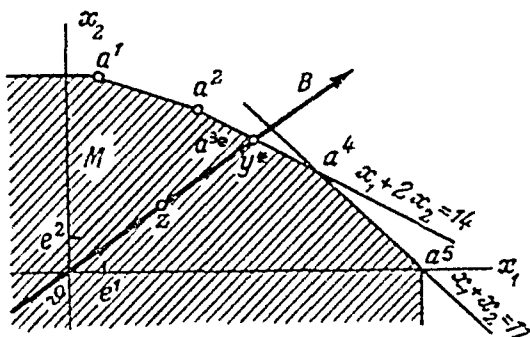


FIG. 9

geometry) there exists a supporting hyperplane,  $H$ , of the polyhedron  $M$ , which passes through the point  $y^*$  (see, for example, A. B. Aleksandrov, *Convex Polyhedra*, 1950). Let the equation of this hyperplane be

$$(c, x) = c_1 x_1 + c_2 x_2 + \dots + c_n x_n = d, \quad (6)$$

where†  $\max_{x \in M} (c, x) = (c, y^*) = d$ .

It is not difficult to check that the coefficients of the variable coordinates in equation (6) satisfy condition (a) (see p. 264).

Let  $\bar{\pi} = (h_1, h_2, \dots, h_{r+n})$  be an optimum vector; then

$$\begin{aligned} d = (c, y^*) &= \left( c, \sum_{s=1}^r h_s a^s + \sum_{j=1}^n h_{r+j} e^j \right) = \sum_{s=1}^r h_s (c, a^s) \\ &+ \sum_{j=1}^n h_{r+j} (c, e^j) \leq \max_{1 \leq s \leq r} (c, a^s) \sum_{s=1}^r h_s - \sum_{j=1}^n h_{r+j} c_j \leq d, \end{aligned}$$

† If this is not so, then it may be brought into this form by changing the sign of all coefficients in equation (6).

whence we have

$$(b') \sum c_j a_j^s = (c, a^s) = \max_{1 \leq t \leq r} (c, a^t) = \max_{1 \leq t \leq r} \sum c_j a_j^t = d, \text{ if } h_s > 0,$$

$$(c') c_j = 0, \text{ if } h_{r+j} > 0.$$

Conversely, if conditions (b') and (c') are satisfied for a given assortment vector  $\bar{\pi}$  and for certain numbers  $c_1, c_2, \dots, c_n$ , then equation (6) defines a supporting hyperplane of the polyhedron  $M$ , which passes through the point  $x(\bar{\pi})$ . Therefore in this case  $x(\bar{\pi}) = y^*$ , and consequently the vector  $\bar{\pi}$  is optimal.

The following theorem is proved in the same way.

**THEOREM 2'.** For an assortment vector  $\bar{\pi} = (h_1, \dots, h_r, h_{r+1}, \dots, h_{r+n})$  to be optimal, it is necessary and sufficient that multipliers  $c_1, c_2, \dots, c_n$ , exist which satisfy conditions (a), (b') and (c').

Taking into account the connection between Problems C and C', Theorem 2 may be easily obtained for the particular case when  $m = 1$ .

**NOTE 5.** From the given geometrical interpretation it is clear that in Problem C the optimal vector (plan) and the multipliers (o.d. valuations) corresponding to it always exist but, generally speaking, are not uniquely determined. In fact, if the point  $y^*$  permits various representations of the form (5), there exists more than one optimal plan; if, however, the point  $y^*$  lies on the face of a polyhedron  $M$  of dimensions less than  $n - 1$ , then the vectors as well are not uniquely determined (it is possible to draw various supporting hyperplanes of  $M$  through the point  $y^*$ ). However, multipliers which correspond to one of the optimal plans correspond to all others.

### *The Basic Problem of Industrial Planning (see [8])*

We direct our attention now to the more general problem studied in Chapter II

We consider production where there are  $N$  ingredients (different types of industrial factors, raw materials, intermediate and final products) There are  $r$  permissible technical methods (methods of organizing production). Each of these methods is characterized by a vector

$$a^s = (a_1^s, a_2^s, \dots, a_N^s) \quad (s = 1, \dots, r),$$

whose components indicate the volume of output of the respective ingredients per unit level of the particular method (negative com-

ponents denote use of inputs). A plan for the organization of production is determined by the choice of a vector  $\pi = (h_1, h_2, \dots, h_r)$  with non-negative components which indicate the level of application of the respective methods. For a plan  $\pi = (h_1, h_2, \dots, h_r)$ , the various elements are produced in quantities

$$x_i^\pi = \sum_{s=1}^r a_i^s h_s \quad (i = 1, \dots, N) \quad (7)$$

(elements, for which  $x_i^\pi < 0$ , are used in quantities  $|x_i^\pi|$ ).

In addition to technical methods, it is necessary in compiling an industrial plan to take into account also the resources available and the assortment of products required. These additional conditions may be stated in various ways. Here we consider one of the possible ways of stating them; however, the basic result that in an optimal plan there exists a system of o.d. valuations for all elements holds for any natural statement of these conditions (see Note 7, p. 275).

For some elements (certain final products) it may be necessary to achieve maximum output with the required assortment, while for others, there are limits of the kind,  $x_i^\pi \geq b_i$ , where  $b_i$  are given real numbers. (The positive  $b_i$ 's correspond to final products required in definite quantities; for the intermediate products, which do not have to be completely exhausted in the plan, the respective  $b_i$ 's are equal to zero; negative  $b_i$ 's correspond to factors of production and various types of raw materials of which not more may be used than the resources available, i.e.  $|b_i|$ .) In this case we pass on to the following problem.

PROBLEM D. Given the real numbers

$$a_i^s (i = 1, \dots, N; s = 1, \dots, r), \quad b_i (i = 1, \dots, m), \\ k_j > 0 \quad (j = 1, \dots, n; n = N - m)$$

it is required to determine the vector (plan)  $\pi = (h_1, h_2, \dots, h_r)$  from the conditions

$$(1) \quad h_s \geq 0 \quad (s = 1, \dots, r);$$

$$(2) \quad x_i^\pi \geq b_i \quad (i = 1, \dots, m);$$

$$(3) \quad \text{the quantity } \mu(\pi) = \min_{1 \leq j \leq n} \frac{x_m^\pi + j}{k_j}$$

to attain its maximum value (here and in the above conditions, the quantities  $x_i^\pi$  are defined by (7)).

A plan  $\pi$ , which satisfies conditions (1) and (2) is called *feasible*, and one which satisfies conditions (1-3) *optimal*.

This problem may be looked at as a mathematical model of current (short-term) planning of which the previous problems are clearly special cases. For example, in Problem B there are  $m$  factors of production,  $n$  final products and

$$r = \sum_{i=1}^m r_i$$

technical methods, by each of which one unit of the respective factor (use of a plant, factory or machine) is used up and a certain set of products is produced; the resources of each factor are here equal to 1, and therefore  $b_i = -1$  ( $i = 1, \dots, m$ ). The problems considered below also lead to Problem D. Hence the latter is called the basic problem of production planning.

The following theorems give the properties of an optimal plan and conditions for its existence.

**THEOREM 3** (see Conclusion 12, p. 49). For a feasible plan  $\pi = (h_1, h_2, \dots, h_r)$  to be optimal it is necessary that multipliers  $c_1, c_2, \dots, c_N$  (o.d. valuations for all elements) exist such that

$$(a) \quad c_i \geq 0 \quad (i = 1, \dots, N), \quad \max_{1 \leq j \leq n} c_{m+j} > 0$$

(these valuations are non-negative, at least one of the products included in the assortment set having a positive valuation);

$$(b) \quad \sum_{i=1}^N c_i a_i^s \leq 0, \quad (s = 1, \dots, r)$$

(for every technical method the valuation of the product cannot exceed the total valuation of the ingredients used up);

$$(c) \quad \sum_{i=1}^N c_i a_i^s = 0, \quad \text{if } h_s > 0$$

(for the methods used, the valuation of the products is equal to the valuation of the elements used up, the principle of profitability is observed);

$$(d) \quad c_i = 0, \quad \text{if } x_i^* > b_i \quad (1 \leq i \leq m)$$

$$\text{or} \quad x_i^* > k_{i-m} \mu(\pi) \quad (m+1 \leq i \leq N)$$

(for factors of production which do not limit production and for

products produced in excess, the respective valuations are equal to zero).

THEOREM 4. For the existence of an optimal plan it is necessary and sufficient that the following conditions be fulfilled.

( $\alpha$ ) a feasible plan  $\pi$  exists;

( $\beta$ ) no plan  $\pi$  (satisfying condition (1)) exists for which

$$x_i^{\pi} \geq 0 \quad (i = 1, \dots, m); \quad x_{m+j}^{\pi} > 0 \quad (j = 1, \dots, n).$$

NOTE 6. Condition ( $\beta$ ), indicating that no plan exists in which certain elements are produced (in positive quantities) without any kind of outlay, is clearly always satisfied in practical problems. Condition ( $\alpha$ ), generally speaking, may not be satisfied. Violation of this condition indicates that, with the resources available at a given production base, even the first  $m$  elements cannot be produced in the necessary quantities. However, if all the numbers  $b_i \leq 0$ , condition ( $\alpha$ ) is necessarily satisfied.

NOTE 7. Theorem 3 refers to Problem D, in which the available resources and required assortment of products were taken into account in a definite fashion. However, for any reasonable definition of optimality, plan  $\pi$  is clearly not optimal, if there exists a plan  $\pi'$  according to which all elements are produced in greater quantities (or where the available resources are used in smaller quantities), i.e.

$$x_i^{\pi'} < x_i^{\pi} \quad (i = 1, \dots, N).$$

This property is indeed sufficient to ensure that an optimal plan is associated with a certain system of non-negative valuations, which satisfy conditions (b) and (c).

NOTE 8. In the general case, valuations of all the ingredients are necessary in order to characterize an optimal plan in Problem D. However, if each technical method involves only one of the first  $m$  elements (or one of the last  $n$  elements), then an optimal plan may be specified in terms of the valuations of only the last  $n$  elements (first  $m$  elements). We have met such a situation in Problems A, B and C.

NOTE 9. Practical interest is provided by the case when the application of certain technical methods in Problem D is limited, when the only permissible plans are plans,  $\pi = (h_1, h_2, \dots, h_r)$ , in which

$$h_s \leq q_s \quad (s = 1, \dots, r; \quad r_1 \leq r),$$

where the  $q_s$  are given positive numbers. This case may be reduced formally to the basic one quite easily (it is sufficient to introduce the limitations on technical methods as additional resources). However, the problem may be treated also without such a reduction. In doing this though, it is true that in Theorem 3, which specifies an optimal plan, the inequality

$$\sum_{i=1}^N c_i a_i^s \leq 0$$

must be assumed for those of the limited methods which are being fully used ( $h_s = q_s$ ).

NOTE 10. In foreign literature, the special case of Problem D, in which  $n = 1$  (see [9, 10]), is usually considered as the fundamental problem of linear planning. The following problem will serve to illustrate this case.

PROBLEM E. Let there be  $m$  kinds of raw material in amounts of  $b_1, b_2, \dots, b_m$  units. From this raw material,  $r$  different products can be produced. The price of one unit of product ( $s$ ), ( $s = 1, \dots, r$ ), is  $a_s$ , and  $a_1^s, a_2^s, \dots, a_m^s$  units of the respective kind of raw material are spent on it. It is required to select a quantity of products of various kinds in such a way that, as a set, they may be manufactured from the available raw material and so that their total value is a maximum. In other words, a vector (or plan)  $\pi = (h_1, h_2, \dots, h_r)$  is to be found from the conditions:

$$(1) \quad h_s \geq 0 \quad (s = 1, \dots, r);$$

$$(2) \quad \sum_{s=1}^r a_i^s h_s \leq b_i \quad (i = 1, \dots, m),$$

$$(3) \quad \text{the quantity } \mu(\pi) = \sum_{s=1}^r a_s h_s \text{ to be maximized.}$$

In order to make clear the geometrical meaning of Problem D, and to prove the theorems given above, we shall consider the points in  $N$ -dimensional space  $R_N$

$$\begin{aligned} a^s &= (a_1^s, a_2^s, \dots, a_N^s) \quad (s = 1, \dots, r); \\ e &= (\underbrace{0, \dots, 0}_{i-1}, -1, 0, \dots, 0) \quad (i = 1, \dots, N); \\ y^0 &= (b_1, \dots, b_m, 0, \dots, 0); \\ z &= (0, \dots, 0, k_1, \dots, k_n). \end{aligned}$$



Let

$$K = \left( x \mid x = \sum_{s=1}^r h_s a^s + \sum_{i=1}^N h_{r+i} e^i; \quad h_s \geq 0, \quad s = 1, \dots, r+N \right)^\dagger$$

be a convex polyhedral cone, with vertex at the origin of co-ordinates, which is spanned by the points  $a_s$  ( $s = 1, \dots, r$ ) and  $e_i$  ( $i = 1, \dots, N$ ), and let

$$Y = \{ y \mid y = y^0 + \lambda z; \quad -\infty < \lambda < +\infty \}$$

be a line through the points  $y^0$ ,  $y^0 + z$ , directed towards the side of increasing  $\lambda$ .

For every feasible plan  $\pi = (h_1, h_2, \dots, h_r)$  the point

$$y^0 + \mu(\pi)z = \sum_{s=1}^r h_s a^s + \sum_{i=1}^N h_{r+i} e^i, \quad (8)$$

where

$$\begin{aligned} h_{r+i} &= x_i^\pi - b_i \quad (i = 1, \dots, m), \\ h_{r+m+j} &= x_{m+j}^\pi - k_j \mu(\pi) \quad (j = 1, \dots, n), \end{aligned} \quad (9)$$

clearly belongs to the cone  $K$ . Conversely, if the point  $y^0 + \lambda z \in K$ , there exists a feasible plan  $\pi$  for which the quantity  $\mu(\pi) \geq \lambda$ .†

Hence it is clear that Problem D amounts essentially to the study of the intersection of the axis  $y$  with the cone  $K$ . If this intersection is empty, or contains the point  $y^0 + \lambda z$ , no matter how large  $\lambda$  is, then in this problem no optimal plan exists.

If, however,  $Y \cap K \neq \Lambda$  ( $\Lambda$  denotes the empty set) and if

$$\lambda^* = \sup_{y^0 + \lambda z \in K} \lambda < +\infty,$$

then optimal plans exist; these optimal plans comprise only those feasible plans  $\pi$ , for which the point (8) coincides with the extreme point of intersection of the line  $y$  and the cone  $K$ , i.e. with the point  $y^* = y^0 + \lambda^* z$ .

† This expression indicates that  $K$  is a set of points  $x$ , belonging to the space  $R_N$ , which can be represented in the given form, where  $h_s \geq 0$  are arbitrary non-negative numbers.

Through the point  $y^*$  it is possible to draw a supporting hyperplane  $H$  to the cone  $K$  which does not contain points of the line  $y$  other than  $y^*$ . Let the equation of this hyperplane be

$$(c, x) = c_1 x_1 + c_2 x_2 + \dots + c_N x_N = 0, \quad (10)$$

where  $\max_{x \in K} (c, x) = (c, y^*) = 0$ .

Then, as may easily be shown, the numbers  $c_1, c_2, \dots, c_N$  (coefficients of the variables in equation (10)), satisfy conditions (a) and (b) of Theorem 3, and conditions (c) and (d) of this theorem are also satisfied for every optimal plan (i.e. for any plan such that  $y^0 + \mu(\pi)z = y^*$ )

On the other hand, if for a given feasible plan  $\pi$  there are multipliers  $c_1, c_2, \dots, c_N$ , which satisfy conditions (a-d), then the hyperplane  $H$ , determined by (10), is a supporting hyperplane of the cone  $K$  and intersects the line  $y$  at the point  $y^0 + \mu(\pi)z$ . Hence it follows that  $y^0 + \mu(\pi)z = y^*$ , and, consequently, that the plan  $\pi$  is optimal.

Thus Theorem 3 has been proved, and from it follow, in particular, Theorems 1 and 2, which were given above without complete proof.

As we have seen, for the existence of an optimal plan, it is necessary and sufficient that the conditions

$$Y \cap K \neq \Lambda,$$

$$\lambda^* = \sup_{y^0 + \lambda z \in K} \lambda < +\infty.$$

are fulfilled. The latter, as may be easily checked, is equivalent to the conditions of Theorem 4.

By means of the above geometrical interpretation it is also easy to prove Note 7 which is of fundamental importance.

#### *Relation to Leontief's Input-Output Matrices*

We shall pause to consider a particular problem in production planning.

Let there be  $n$  products and one factor of production (labour). The technical methods are such that only one product is produced by each of them, while other products and the factor of production are fully used. The supply of the factor of production is limited. It is assumed that there exists a plan by which all products are produced

in positive quantities (only the factor of production is consumed). In this case:

(1) The optimal plan problem (plan D) is soluble for any set of products (i.e. for any numbers  $k_j \dots 0, j = 1, \dots, n$ ).

(2) The collection of technical methods which are used in the optimal plan, and the values of the multipliers (o.d. valuations) are independent of the resources of the factor of production available and of the required set of products.

In the case here considered the technical methods are characterized by the vectors

$$a^{js} = (a_0^{js}, a_1^{js}, \dots, a_n^{js}) \quad (j = 1, \dots, n; s = 1, \dots, r_j),$$

where  $a_0^{js} < 0, a_j^{js} > 0, a_l^{js} \leq 0, \text{ if } l \neq j.$

The plan is determined in terms of a matrix

$$\pi = \{ \| h_{js} \| \} \quad (j = 1, \dots, n; s = 1, \dots, r_j),$$

whose elements indicate the level of application of the various methods. An optimal plan is sought from the conditions

$$(1) h_{js} \geq 0 \quad (j = 1, \dots, n; s = 1, \dots, r_j);$$

$$(2) x_0^\pi = \sum_{j,s} a_0^{js} h_{js} \geq b_0$$

( $-b_0$  denotes the available supply of the factor of production):

$$(3) \text{ the quantity } \mu(\pi) = \min_{1 \leq l \leq n} \frac{x_l^\pi}{k_l}, \text{ where } x_l^\pi = \sum_{j,s} a_l^{js} h_{js}$$

( $l = 1, \dots, n$ ), is a maximum (the numbers  $k_l$  characterize the required assortment of products).

For every product ( $v = 1, \dots, n$ ) we consider a plan  $\pi^v = \{ \| h_{js}^v \| \}$  (satisfying condition (1)), for which the input of the factor of production does not exceed one unit ( $x_0^{\pi^v} \geq -1$ ), and for which the products are produced in non-negative amounts ( $x_l^{\pi^v} \geq 0, l = 1, \dots, n$ ), the

product ( $v$ ) being produced in maximum quantity ( $x_0^{*v} = \max$ ). We assert, that for plan  $\pi^v$

$$x_0^{*v} = -1, \quad x_l^{*v} = 0, \quad \text{if } l \neq v.$$

In fact, if  $x_0^{*v} > -1$ , the volume of production of product ( $v$ ) may be increased by including partially a plan for which all products are produced in positive quantities; if, however, for a certain  $l_0 \neq v$ ,  $x_{l_0}^{*v} > 0$ , then it is also possible to include the above plan at the cost of reducing one of the numbers  $h_{l_0 s}^v$ .

It is not difficult to verify that, whatever the numbers  $k_j > 0$  ( $j = 1, \dots, n$ ), the quantities

$$c_0, \quad c_1 = \frac{c_0}{x_1^{\pi^1}}, \quad c_2 = \frac{c_0}{x_2^{\pi^2}}, \dots, \quad c_n = \frac{c_0}{x_n^{\pi^n}},$$

where  $c_0$  is an arbitrary positive number, represent a system of o.d. valuations, and the plan  $\pi = \|h_{js}\|$ , in which

$$h_{js} = \frac{-b_0 \sum_{v=1}^n c_v k_v h_{js}^v}{\sum_{v=1}^n c_v k_v} \quad (j = 1, \dots, n; \quad s = 1, \dots, r_j),$$

is optimal. From this proposition, (1) and (2) follow.

We note the important special case when, for every product ( $j$ ) ( $j = 1, \dots, n$ ), there is only one process of production, which is characterized by the vector

$$a^j = (a_0^j, a_1^j, \dots, a_n^j); \quad a_0^j < 0, \quad a_l^j > 0, \quad a_l^j \leq 0, \quad \text{if } l \neq j.$$

Here an optimal plan clearly uses all methods. Therefore the appropriate system of valuations may be found from the simultaneous equations†

$$\sum_{i=0}^n a_i^j c_i = 0 \quad (j = 1, \dots, n);$$

more exactly, from these simultaneous equations the valuations for all products may be expressed in terms of  $c_0$  (the valuation of one unit of labour).

This last case corresponds to Leontief's open production model,‡ which is frequently used in economic analysis. It has to be pointed

† Clearly these equations can be solved immediately.

‡ For Leontief's model, see, for example, reference [9], and also the last article in the collection [10].

out, however, that this model is only a very rough approximation to the real conditions of production which determine the current production plan. In fact:

(1) In real problems it is necessary to take into account also many factors other than labour which are available in limited quantities, in particular useful natural sources and especially certain production capacities. Moreover, since there are usually several categories of labour, this too must not be treated as a single factor.

(2) Output of multiple products not included in the above scheme constantly occurs.

(3) In the actual conditions of modern production there are a great many methods for manufacturing a particular product, and the application of the various methods used brings out in particular the limitations mentioned in (1).

Leontief's model appears useful for compiling matrices of inter-sector relations in which the coefficients  $a_{ij}$  are the costs of a product of the given type in terms of the inputs of products of other sectors. Although constructing such matrices of relations and obtaining total expenditures from them is of special interest this approach may not be considered sufficiently satisfactory for calculating valuations of products. Indeed, instead of actual methods of production, broad averages are used here and the results obtained depend essentially on the methods chosen for aggregation. Therefore there is no justification for thinking that the valuations of products obtained in this way will give realizable equivalent relations, and thus these valuations may not be directly used in the analysis of economic planning. Their basic shortcoming is that they do not take into account the limitations mentioned in (1) and the limitations on the indirect expenditures associated with them.

### *The Transport Problem*

In the simplest case this problem consists of the following.

PROBLEM F.† Let there be  $m$  points connected by a railway

† Special methods of solving Problem F have been considered in reference [2]; a mathematical analysis and general methods for solving this problem and also certain more general problems connected with transport planning (in particular Problem G which will be considered below) have been given in references [3, 4] and later in [9].

system consisting of  $r$  sections. Along the section ( $s$ ) ( $s = 1, \dots, r$ ) of the network goods may be conveyed from point  $i_s$  to point  $j_s$ ; the costs for conveying one unit of the goods (for example, one wagon) are  $a_s$  (in particular, the quantity  $a_s$  may be taken as equal to the distance between the points  $i_s$  and  $j_s$ ). At each point ( $i$ ) ( $i = 1, \dots, m$ ) there is a given demand  $b_i$  for a single kind of product (for points of demand  $b_i > 0$ , for points of production  $b_i < 0$ , for other points  $b_i = 0$ ), and

$$\sum_{i=1}^m b_i = 0$$

(the total amounts of production and demand are equal). The transport plan is determined by the choice of a vector  $\pi = (h_1, h_2, \dots, h_r)$ , whose components indicate the volume of transport in each section of the system. The problem is to find an optimal plan, which satisfies the conditions

- (1)  $h_s \geq 0 \quad (s = 1, \dots, r);$
- (2)  $\sum_{j_s=i} h_s - \sum_{i_s=i} h_s = b_i \quad (i = 1, \dots, m)$

(each point obtains the required net quantity of the product):

- (3) the quantity

$$z = \sum_{s=1}^r a_s h_s$$

is to be a minimum (the total cost of transport is to be minimized; for example, wagon-kilometres are to be a minimum).

It is not difficult to see that the given problem is a particular case of the basic problem of production planning. In fact, one may suppose that in this case there are  $(m+1)$  elements; the first  $m$  of these are the goods considered, situated at the various points, and the last one corresponds to the transport costs. The feasible technical methods are characterized by the vectors

$$a^s = (a_1^s, a_2^s, \dots, a_{m+1}^s) \quad (s = 1, \dots, r),$$

where  $a_{m+1}^s = -a_s$ ,  $a_i^s = 1$  for  $i = j_s$ ,  $a_i^s = -1$  for  $i = i_s$ , and  $a_i^s = 0$  for the other  $i$ 's.

On the basis of Theorem 4 one may easily conclude that an optimal plan in this problem always exists. The plan is characterized by Theorem 3, which here reduces to the following.

**THEOREM 5.** For a plan  $\pi = (h_1, h_2, \dots, h_r)$  (satisfying conditions (1) and (2)) to be optimal it is necessary and sufficient that there exist numbers  $c_1, c_2, \dots, c_m$ , such that

$$(a) \quad c_{j_s} - c_{i_s} \leq a_s \quad (s = 1, \dots, r);$$

$$(b) \quad c_{j_s} - c_{i_s} = a_s, \quad \text{if } h_s \neq 0.$$

**NOTE 11.** The numbers  $c_i$  appearing in Theorem 5 are called potentials of the various points. The difference of potential shows by how much more expensive a unit of the given product is at one point than at another.

When planning transport in practice it is sometimes necessary to take into further account the limited carrying capacity of the various sections of line. This leads to a more general problem.

**PROBLEM G.** According to the conditions of Problem F, feasible plans are only those plans,  $\pi = (h_1, \dots, h_r)$ , for which the condition

$$(2') \quad h_s \leq q_s \quad (s = 1, \dots, r)$$

is observed (the numbers  $q_s$  characterize the carrying capacity of the various lines).

In the present case, the following proposition, which also follows from Theorem 2, characterizes the optimal plan.

**THEOREM 6.** For a plan  $\pi = (h_1, h_2, \dots, h_r)$  (satisfying conditions (1), (2) and (2')) to be optimal it is necessary and sufficient that numbers  $c_1, c_2, \dots, c_m, d_1, d_2, \dots, d_r$ , exist such that

$$(a) \quad c_{j_s} - c_{i_s} \leq a_s + d_s \quad (s = 1, \dots, r);$$

$$(b) \quad c_{j_s} - c_{i_s} = a_s + d_s, \quad \text{if } h_s > 0;$$

$$(c) \quad d_s \geq 0, \quad \text{while } d_s = 0, \quad \text{if } h_s < q_s.$$

**NOTE 12.** The numbers  $d_s$  are the rents (hire valuations) for the various sections of line, calculated per unit of load (for example, one wagon).

### *The Problem of a Production Complex*

We shall now suppose that there are several factories, for each one of which an optimal working plan has been compiled. In other words we shall consider the simultaneous solution of a number of

basic problems with certain common elements. The question naturally arises whether it is not possible to increase overall productivity through co-operation among the separate factories.

If the existing factories are situated at one point, or near each other (so that transport expenses may be ignored), the answer to this problem is given by

**THEOREM 7.** If it is impossible to establish common valuations for all kinds of factors of production (raw materials, intermediate and final products; immovable resources situated at different points are different) in such a way that the principle of profitability is satisfied at each factory, then the general productivity may be raised by means of changing the plans—by means of further co-operation between the various factories. On the other hand, if such valuations exist it is impossible to increase productivity through co-operation.

This theorem follows from Theorem 3 if the latter is applied to a problem in which the whole set of existing factories is considered as a single factory.

**NOTE 13.** An analogous theorem holds when the existing factories are separated appreciably one from another, when transport costs cannot be ignored. Here, it is true, the valuations of one or another product at different points may be different but their difference must not exceed the cost of transport of this product from one point to another (allowing for the hire valuations of the appropriate sections of the line) and must be equal to this quantity, if the goods are actually transported between two such points in the optimal plan.

### *Dynamic Problem*

By means of the basic production planning problem considered above the more general problem may be analysed, i.e. that of drawing up a production plan for a certain period of time divided into a series of intervals,  $t = 1, 2, \dots, T$  (problem of long-term planning).

One and the same product (or factor), produced (or used) in various periods of time, is treated here as a different element. Therefore the available technical methods are characterized now by means of the matrices

$$a^s = ||a_{it}^s|| \quad (i = 1, \dots, N; t = 1, \dots, T; s = 1, \dots, r),$$

whose elements show the amounts of the various products and factors produced in different intervals of time (negative elements denote



inputs). Included among the methods may be also those which refer to one period of time (methods which appeared in the compilation of the current plan). Technical progress in these methods may be taken into account by means of converting the inputs for their application at later periods and by stating the period from which the particular improved method is used, and so on. Clearly, this kind of initial data is of a prognostic nature and is inevitably quite approximate. Together with other factors, it is convenient to introduce, as special elements, some particular productive capacities. Among the latter are reproducible factors, and the possibility of their production is envisaged.

The concept of an optimal plan may be introduced in various ways; for example, with given resources for the first period (certain resources, say the natural ones, may be given for all periods) and with a given demand for final products (for each period of time) it is required to make up a plan in which balances are maintained, and the accumulation of final output of a specified composition (or of definite productive capacity) up to the end of the planning period is a maximum. However, for any natural definition of being optimal, a plan  $\pi = (h_1, h_2, \dots, h_r)$  is not optimal, if there exists a plan  $\pi' = (h'_1, h'_2, \dots, h'_r)$  in which all the elements are produced in greater quantities.

$$x_{it}^{\pi} = \sum_{s=1}^r a_{it}^s h_s < \sum_{s=1}^r a_{it}^s h'_s = x_{it}^{\pi'}$$

$$(i = 1, \dots, N; \quad t = 1, \dots, T).$$

This property turns out to be sufficient for the optimal plan  $\pi$  to be associated with a system of multipliers  $\{c_{it}\}$  ( $i = 1, \dots, N; t = 1, \dots, T$ ) (of valuations for all products and factors over the whole period) such that

$$(a) \quad c_{it} \geq 0 \quad (i = 1, \dots, N; \quad t = 1, \dots, T). \quad \text{and not all } c_{it} = 0;$$

$$(b) \quad \sum_{i,t} c_{it} a_{it}^s \leq 0 \quad (s = 1, \dots, r);$$

$$(c) \quad \sum_{i,t} c_{it} a_{it}^s = 0, \quad \text{if } h_s > 0.$$

The multipliers (valuations) may naturally be normalized. For instance if

$$c_{it} = \lambda_t c'_{it} \quad (i = 1, \dots, N; t = 1, \dots, T),$$

the valuations  $c'_{it}$  may be chosen to satisfy the conditions

$$c'_{i_1t} + c'_{i_2t} + \dots + c'_{i_{rt}t} = 1 \quad (t = 1, \dots, T),$$

or the valuations of a certain stated set of products in each interval of time are equal to 1. In carrying this out, the left-hand sides of the inequalities and equations in conditions (b) and (c) are replaced by the following

$$\sum_{t=1}^T \lambda_t \sum_{i=1}^r c'_{it} a_{it}^s \quad (s = 1, \dots, r),$$

i.e. in evaluating a technical method for production and expenditure belonging to different intervals of time, the valuations must be converted to a single interval by means of multipliers  $\lambda_t$ .

The ratio  $\lambda_t/\lambda_\tau$  is (for a given unit) a conversion coefficient for inputs in period  $t$  to period  $\tau$ . In particular the quantity

$$\left( \frac{\lambda_t}{\lambda_{t+1}} - 1 \right) = \frac{\lambda_t - \lambda_{t+1}}{\lambda_{t+1}}$$

gives the normal efficiency of capital investment in the transition from period  $t$  to the following one.

The quantities  $c_{it}$  characterize the dynamics of valuations; they are valuations of inputs and outputs which have been converted to a single instant of time. The quantities  $c'_{it}$  characterize the relative dynamics of valuations. Correspondingly, there are two methods for calculating the efficiency of certain capital investment (of a new method of production, calculated for a long period): namely, if inputs and outputs during the years of the capital investment are characterized by the matrix  $\| \bar{a}_{it} \|$ , then the question of the usefulness of its application is solved by determining whether the sum

$$\sum_{i,t} c_{it} \bar{a}_{it} = \sum_t \lambda_t \sum_i c'_{it} \bar{a}_{it}$$

is positive or not. The first expression is calculated directly according to the dynamics of valuations, and the second is calculated according to the relative dynamics followed by a conversion of inputs in each period to one particular period (cf. Conclusions 25 and 26).

The calculation is simplified if the relative valuations do not change with time. Then it is sufficient to know the valuations at the initial time  $c_{it}^i$  and the conversion coefficients  $\lambda_i$ ; to evaluate a process it is sufficient to apply the second of the expressions given above with  $c_{it}^i$  replaced by  $c_{it}^i$ .

### *Properties of Valuations. Variation of a Plan*

In the analysis of the basic production planning problem its geometrical meaning was elucidated. In particular, it was shown that o.d. valuations are given by the coefficients of the variables in the equation of the hyperplane  $H$ , which is a supporting hyperplane of the cone  $K$ , and which passes through the boundary point  $y^*$  of the intersection of the line  $y$  with this cone. Hence, first of all it is clear that o.d. valuations are completely realistic, that is, they are related to the situation (available technical methods, volumes of resources, assortment task) and change when the situation changes (*cf.* Conclusion 6). Indeed, during such changes, the cone of plans  $K$  and the assortment axis  $y$  change, and with them also the supporting hyperplane  $H$ .

However, if we exclude special cases, when the point  $y^*$  which corresponds to the optimal plan lies on the boundary of the cone  $K$  of dimensions less than  $N-1$ , then, for small changes in the assortment task and in the resources, the extreme point  $y^*$  remains on the same boundary; therefore o.d. valuations are not changed. For other small changes (in the methods) the cone  $K$  changes somewhat; this leads to small changes in the o.d. valuations.

Thus o.d. valuations possess a certain stability with respect to changes in the situation (*see* Conclusion 7).

By changing from the point  $y^* = (x_1^*, x_2^*, \dots, x_N^*)$  to the neighbouring point  $\bar{y}^* = (x_1^* + \Delta x_1, x_2^* + \Delta x_2, \dots, x_N^* + \Delta x_N)$  on the same supporting hyperplane, we arrive at an optimal plan corresponding to different resources and to a different assortment task. During this change, as has been noted, o.d. valuations  $c_1, c_2, \dots, c_N$ , do not alter: therefore

$$\sum_{i=1}^N c_i x_i^* = \sum_{i=1}^N c_i (x_i^* + \Delta x_i) = 0,$$

hence 
$$\sum_{i=1}^N c_i \Delta x_i = 0. \quad (11)$$

The last relation is called, naturally enough, the equation of variation of a plan; it specifies the condition for an equivalent substitution of some kinds of product and factors of production by others which must be satisfied in moving from a given optimum plan to a slightly altered optimal plan and which, generally speaking, is sufficient for the realization of the latter. In particular one unit of ingredient ( $i_1$ ) may be replaced by  $c_{11}/c_{12}$  units of ingredient ( $i_2$ ). When using other valuations (different from o.d. valuations), such a substitution is generally speaking impossible. Hence it is clear that relations, defined by o.d. valuations, between products and factors of different kinds are completely realistic (*see* Conclusion 8, and the more general Conclusion 13).

The above properties of o.d. valuations, and also the equation of variation, lead to many applications of these valuations in various problems of adjustments in plans and of obtaining separate partial solutions (applications of this kind have been described in detail in the main text of the book; *see*, for example, Conclusions 9, 10, 15, 16).

NOTE 14. We have pointed out that the solution to the problem of efficiency of some new method, characterized by the vector  $\bar{a} = (\bar{a}_1, \bar{a}_2, \dots, \bar{a}_N)$ , is determined according to whether the sum

$$\sum_{i=1}^N c_i \bar{a}_i$$

is positive or not. However, this refers only to the case when the method may be applied at any level. Methods of production commonly occur, which may be used only at a given level (indivisible methods or investments). In evaluating such a method, the necessary condition for its application

$$\sum_i c_i \bar{a}_i \geq 0$$

is retained; this condition, however, may not be sufficient, since inclusion of this method in the plan may require variations which are greater than the feasible ones. Hence, in order to solve this problem it may be necessary to reconsider the plan with the inclusion of the new method, and to compare the products and inputs of the plan thus obtained with the original plan.

### *Rent and Hire Valuation*

The analysis of the basic problem of industrial planning carried out above has shown that in the application of methods for optimal planning to concrete problems it turns out to be convenient to take into account other forms of output besides those usually considered in economic analysis. Among inputs may be included, for example, the use of more fertile ground, the use of production floor-space, the rent for a definite time of scarce equipment (apart from its wear), the rent of circulating resources, and so on. If these factors are available in limited amounts and are fully used, they receive positive valuations. Hence their omission from economic calculations often leads to incorrect solutions. Many examples of this kind have been introduced in the main text of the book.

Thus, in order to obtain correct solutions in an economic calculation, it is necessary to take into account rents (for the use of more favourable natural conditions) and hire valuations (for the rent of scarce equipment). Numerical values for these quantities are determined together with the other o.d. valuations.

### *Indicators which Characterize the Operation of Factories*

We shall consider the possibility of using o.d. valuations for constructing statistical-economic indicators which characterize the operation of factories.

In the models which have been studied it would have been possible to assume that within each production unit a certain group of processes are included which comprise part of the general plan. On each production unit certain factors and goods made by other production units are expended to a definite extent, and certain goods are produced by it; and the total planned valuation of production and use of a production element (rents and hire valuations being taken into account according to the o.d. valuations for the whole complex) must equal zero,

$$\sum_{i=1}^N c_i \bar{x}_i^{\Pi} = 0$$

( $\bar{x}_i^{\Pi}$  are the planned volumes of output and input on the production unit under consideration). However, by means of reducing inputs

and increasing outputs compared with the first draft of the plan, the sum formed from the actual data can, as a rule, be made to turn out positive. Its value may be taken as a fundamental index characterizing the value of the contribution of the production unit:

$$R = \sum_{i=1}^N c_i \bar{x}_i^{\phi},$$

where  $\bar{x}_i^{\phi}$  are the actual volumes of production and use. This index is similar to the usual index of profitability, but it differs in that production is calculated now according to the o.d. valuations, while among costs are included rent and the hire valuation. Hence attempts to improve this index should result in reduction of input, increase in the output of required products, more complete and intense use of equipment, and also the application of production methods which are most appropriate under existing conditions. The use of methods, however, which are inappropriate under existing conditions (in particular, ones which were rejected in drawing up the optimal plan), turns out to be disadvantageous and leads to a worsening of the index. Apart from the above index it is necessary, obviously, to take into account (when evaluating the operation of a factory as well as in economic calculations) that the planned task must conform with regard to its composition of products and inputs.

#### *Calculation of the Necessary Costs in Terms of Average Labour*

As we have seen, with an optimal plan definite o.d. valuations for the different types of product can be obtained (within the limits of the production system considered). The problem naturally arises as to whether they are in contradiction with the labour theory of value, according to which, under socialism as well, the value of products must be determined by the socially necessary expenditure of labour.

Analysis of this problem shows that the structure of o.d. valuations associated with an optimal plan is in complete agreement with the labour theory of value; furthermore, the methods for finding these valuations provide an approach to calculating the full social expenditure of labour.

It must be said that the problem of calculating the social expenditure of labour in a socialist society is far from simple, and in more complicated cases (such as constantly arise in modern production)

there is a lack of complete clarity as to what is to be understood by this expenditure and how one is to set about calculating it. Thus, in various processes which may be efficient for obtaining given products, the real (factual or planned) labour costs are different, but the socially necessary time must be the same. The production of various kinds of goods are inter-connected, their costs are mutually dependent, and it is necessary to observe the principle of allocation of costs.

It appears to us that under conditions of centrally planned socialist production (at least, in the case of state industry), in agreement with the Marxist theory of value the following premises may be taken as starting points for calculating social costs: (a) full expenditure of social labour on a given product has to be taken into account; (b) the manufacture of the given product must be considered to be specific for a given state of the forces of production; (c) the costs in the optimal plan must be taken into account, that is, the costs which are in fact *necessary* for society; (d) the calculation must be in terms of *average labour*, that is, corresponding to average social conditions.

Without raising the problem of calculating social costs in full detail we shall discuss it in terms of the mathematical model of production, which is considered in Problem D.

We shall assume that the plan is being made for a closed production system which manufactures its products independently (consideration of a group of systems always allows one to reduce matters to this case). Under these conditions the factors of production in the system will be labour, and factors which increase the productive power of labour (productive capacities, various types of equipment more favourable natural sources, and so on), and in this labour is the only source of value.

In an optimal plan let the input of direct labour be  $-x_1$  units,† let the costs of other factors be  $-x_2, -x_3, \dots, -x_m$  ( $x_1, x_2, \dots, x_m$  are negative), and let the quantities of the products being produced be  $x_{m+1}, x_{m+2}, \dots, x_{m+n}$ , according to their types.

A definite o.d. valuation  $c_{m+j}$  corresponds to each kind of product. Hence a general valuation for the products would be (in relative units)

$$\sum_{j=1}^n c_{m+j} x_{m+j}$$

† We are considering one type of labour (simple labour), and assume that other types (different in qualifications required, and in intensity) have been converted to this type.

and since the whole of this production resulted from  $-x_1$  units of labour (average for the given system),

$$\frac{-x_1}{\sum_{j=1}^n c_{m+j} x_{m+j}}$$

units of labour are expended on one relative unit of production. According to this, one unit of product of type  $(m+j)$  ( $j = 1, \dots, n$ ) evaluated as  $c_{m+j}$  relative units, requires an input of

$$\bar{c}_{m+j} = \frac{-x_1}{\sum_{j=1}^n c_{m+j} x_{m+j}} c_{m+j}$$

units of average labour (it is clear that the quantity  $\bar{c}_{m+j}$  is independent of the choice of the relative unit). It follows from this expression that the o.d. valuations  $c_{m+j}$  of production are proportional to the labour costs  $\bar{c}_{m+j}$ , constructed in the manner described.

We shall show that the labour costs, defined in this way, for a unit of output correspond in fact to those which are required for achieving this output under given conditions.

For this purpose we shall assume that the planned resources of the system have been increased by  $-\Delta x_1$  ( $\Delta x_1 < 0$ ) units of labour. In order that the conditions of labour remain unchanged by this it is necessary to assume that the factors of production determining these conditions receive proportional increases,

$$-\Delta x_2 = -\frac{x_2}{x_1} \Delta x_1, \dots, -\Delta x_m = -\frac{x_m}{x_1} \Delta x_1.$$

Directing these means towards an increase of product type  $(m+j)$ , we obtain an additional quantity  $\Delta x_{m+j}$  of the product, while for other types of product whose volume of output remains unchanged,  $\Delta x_{m+l} = 0$  ( $l \neq j$ ). As a result we arrive at an unchanged optimal plan.

The variational equation for the plan (see (11), p. 287) gives

$$c_1 \Delta x_1 + c_2 \frac{x_2}{x_1} \Delta x_1 + \dots + c_m \frac{x_m}{x_1} \Delta x_1 + c_{m+j} \Delta x_{m+j} = 0.$$



Hence the cost of producing one unit of the product is

$$\frac{-\Delta x_1}{\Delta x_{m+j}} = \frac{x_1}{\sum_{i=1}^m c_i x_i} c_{m+j} = \frac{-x_1}{\sum_{l=1}^n c_{m+l} x_{m+l}} c_{m+j} = \bar{c}_{m+j},$$

where we have taken into account that, in view of relation (c) (Theorem 3), for every method actually used and therefore also for the plan as a whole,

$$\sum_{i=1}^m c_i x_i + \sum_{l=1}^n c_{m+l} x_{m+l} = 0. \quad (12)$$

Therefore, for one unit of product of type  $(m+j)$  (in this variation we arrive once more at an optimal plan)  $\bar{c}_{m+j}$  units of labour are indeed required. This result is caused by the fact that the total change in the costs in the group, connected with the increase in output, is taken into account, allowance being made for the redistribution of the means. In this, the magnitudes of the costs are calculated on the basis of the methods used in the optimal plan. The validity of such a calculation is shown also in another approach, in which the social costs in labour of a certain product (in the present case, the labour costs in a group) are found according to the direct costs of labour, account being taken of the condition in which it is used.

We shall consider a specific method of production for obtaining a particular product in an optimal plan (it would be possible to take also a certain combination of methods which corresponds to the plan for a definite production unit), in which only the  $(m+j)$ th product is produced, other products being neither produced nor used. Since this method is in use in the optimal plan, we have for this method

$$c_1 x_1^s + \dots + c_m x_m^s + c_{m+j} x_{m+j}^s = 0. \quad (13)$$

Hence the direct labour costs for a unit product of type  $(m+j)$  in this method are equal to

$$\frac{-x_1^s}{x_{m+j}^s} = \frac{c_1 x_1^s}{\sum_{i=1}^m c_i x_i^s} c_{m+j}.$$

These are different from the values  $\bar{c}_{m+j}$  found above by a factor

$$k = \frac{c_1 x_1}{\sum_{i=1}^m c_i x_i} : \frac{c_1 x_1^*}{\sum_{i=1}^m c_i x_i^*}.$$

This factor, the conversion coefficient, characterizes the difference of the conditions for the given production unit from the average ones for the complex, as regards ensuring the supply of factors useful to labour. The coefficient  $k > 1$ , if the conditions are more favourable than average, and  $k < 1$ , if they are less favourable. In particular, if labour is used without the application of favourable factors which are available to a limited extent (it may be called unequipped labour), the coefficient which converts it to average is

$$k_H = \frac{c_1 x_1}{\sum_{i=1}^m c_i x_i} < 1.$$

Thus we see that the necessary costs expressed in average labour may be obtained from the direct labour costs multiplied by a conversion coefficient which allows one to convert them to average conditions. However, the last course which we have been considering, though it helps in an understanding of the essence of the problem, is not effective, since in practice it is difficult to construct these conversion coefficients.

However, it proves to be possible to find labour valuations for products by means of analysing the expenses incurred in their manufacture, if we take into account indirect labour costs as well as direct ones.

For this purpose valuations of favourable industrial factors must be obtained and expressed in terms of average labour. These will be

$$\bar{c}_i = \frac{x_i}{\sum_{r=1}^m c_r x_r} c_i \quad (i = 1, 2, \dots, m).$$

In particular, for  $i = 1$ , we obtain a valuation of one unit of unequipped labour, expressed in terms of average labour.

We shall now calculate the labour costs of one unit of a product of type  $(m+j)$  in process  $s$ , taking into account both direct labour costs and the indirect costs of favourable factors according to their valuations expressed in average labour. Using (12) and (13), we find

$$\begin{aligned} -\frac{\sum_{i=1}^m \bar{c}_i x_i^s}{x_{m+j}^s} &= -\frac{1}{x_{m+j}^s} \sum_{i=1}^m \frac{x_i}{\sum_{r=1}^m c_r x_r} c_i x_i^s = -\frac{\sum_{i=1}^m c_i x_i^s}{x_{m+j}^s} \cdot \frac{x_1}{\sum_{r=1}^m c_r x_r} \\ &= \frac{x_1}{\sum_{r=1}^m c_r x_r} c_{m+j} = \frac{-x_1}{\sum_{l=1}^m c_{m+l} x_{m+l}} c_{m+j} = \bar{c}_{m+j}. \end{aligned}$$

Hence, this calculation gives the previous value for labour costs incurred. This method of calculation may also be applied when other types of product are included in the costs, as long as these products are reckoned according to their labour valuations.

In defining labour costs within a group we started from their natural measurements. In more complicated conditions calculation of social cost can be realized apparently only in value form. However, the particular properties which have been demonstrated in the above analysis, namely the necessity to take into account indirect expenses and the conversion of conditions of labour to average ones, retain their significance.

### *The Significance of Mathematical Models and Their Application in Economic Analysis*

With the passage of time mathematical methods have been gaining more significance and a wider use. Whereas previously their basic field of application was in natural science and technology, nowadays they are finding appreciable application in other fields of science and human activity. Characteristic examples are the application of mathematical methods in philology (in connection with machine translation) and in military matters (operational research).

A highly important and natural field for the use of mathematical methods is provided by economics (analysis of economic planning), which by its very nature has a decidedly quantitative character.

Mathematical symbolism and methods occupy an important place in the economic researches of Marx, and in the economic and statistical works of V. I. Lenin which deal with the economic analysis of capitalism. These methods must receive special emphasis in the economic problems of a socialist society. The task of Marxist economic science is to review the social nature of capitalist society and to study its general laws and its tendencies of development and of decline. In socialist society economic science must serve as a tool for finding concrete solutions to problems of national economic development. Economic laws in a socialist society have an objective character but they are realized under the conditions of a planned economy to a large extent by means of deliberate solution. Therefore, the successful application of the laws in the interest of society depends on how completely and deeply we master them.† Hence it is clear that Marxist analysis of economic problems in socialist society and of the mechanism of the operation of its laws must be as accurate, detailed and specific as possible. One naturally expects that in this kind of analysis mathematics will be especially useful. In view of the complication and interdependence of economic problems in modern production one cannot expect that it will be possible to succeed in the quantitative analysis of these problems with the most simple mathematical means. Here undoubtedly the latest achievements of modern mathematics will be required.

Nevertheless, until recent times mathematical analysis was not only rarely used in economic problems, but it was even necessary to contend with definite objections to its use. Such objections cannot be accepted as justifiable.

The lack of appreciation of the possibility of applying quantitative mathematical methods in the analysis of economic phenomena because of their specific character is in our view a survival of notions about the unobjective character of the economic laws of socialism.

Equally unjustified is the prejudice against mathematical methods

† As an illustration of the various ways in which objective laws may be realized, we recall two classical problems of the calculus of variation: the problem of the catenary and the brachistochrone (the curve of steepest descent). A heavy thread sags in a catenary independently of whether the person holding its ends knows the solution to the relevant problem or not. How close a curve of descent is to a brachistochrone depends on the extent to which the constructor of the curve has mastered the laws of the calculus of variations.

because of their partial use by bourgeois economic schools. Clearly, the precedents of the incorrect use of mathematics for purposes different from ours cannot prevent Soviet scientists from using mathematical methods in economic problems in a way which is correct and of advantage in the building of communism.

Mathematical analysis is not applicable directly to problems of reality. Usually, by means of abstractions, a mathematical model of the phenomenon considered is constructed, and this model may be treated mathematically. Such a model naturally does not include all its aspects but only some of the more important ones chosen for particular consideration. Hence the solutions and conclusions obtained as a result of the analysis are applicable to the real problem only to a certain degree of approximation. Often the succeeding qualitative analysis assists one in knowing in which direction the model should be made more accurate so that it may better reflect the real problem.

At the same time, if a model exists its mathematical analysis may be used not only for obtaining certain quantitative data, but also to reveal new conformities with laws, to analyse causal relationships and dependencies, and to predict new phenomena (examples of this kind in natural science are provided by the discovery of Neptune, and the theoretical prediction of certain phenomena occurring in supersonic speeds and in atomic physics).

Of prime significance for the effectiveness and applicability of a particular model is the correctness of the initial premises used in its construction; it is also necessary that the important factors are in fact included and that the lesser ones are discarded. Thus, certain models of capitalist economies introduced by bourgeois economists turn out to be necessarily fallacious, since in their construction the authors have ignored the existence of unemployment and similar phenomena which are always inherent in their social structure. Naturally, the conclusions drawn from these premises do not merit the least credence.

Consequently, in applying mathematical analysis and in constructing mathematical models for the study of economic problems in a socialist society, it is necessary that the initial premises should be in agreement with the basic principles of the Marxist method of economic analysis, namely dialectical thought, the objective character of research, social analysis of the relations of production.

pre-eminence of production and recognition of labour as the sole source of value.

The fundamental criteria for estimating the significance and correctness of such research, as for the truth of any knowledge in general, must be the Leninist criterion of practice. In other words, the greatest importance for its evaluation must lie in the agreement of the results obtained with reality and in their ability to explain and influence the phenomena of our economic reality, so that they may help in the development of more effective measures and solutions.

Socialist society as a whole, and in its various departments, is by its nature capable of securing a more complete and rational use of productive resources for the better satisfaction of the needs of society. Therefore, for each sector of socialist production and for socialist society as a whole, an optimal plan has a concrete reality, and the consistency of such a plan with economic laws corresponds to the real economic conformities with the laws of socialist society (similar to the way in which mechanical motion is governed by the extremal variational principles of mechanics). From this it is clear that in the quantitative mathematical analysis of the planned economy in a socialist society the basic approach must be research into extremal mathematical problems.†

In the conditions of socialist society, a critical problem is that of raising the level of production. In the study of this problem and of the economics of socialist society it is known to be correct to separate the problem of production from the problem of distribution. Independent consideration of the problem of the optimum organization of production is permissible since in socialist economy without crises and with public socialist ownership of the means of production in its two forms goods cannot be produced according to the requirements of society and then remain unused. In view of this we must accept the models of production planning (the fundamental problem and the dynamic model), which we have considered, as fully justified, and the same applies to the corresponding problems of current and long-term planning in which the object is to obtain the maximum level of

† In capitalist society the existence of unemployment, crises and systematic under-utilization of production capacities shows that the use of the maximum principle for studying its economics as a whole is inadmissible. Hence the attempts on the part of the economists-apologists for capitalism (for example, Pareto) to study the economic laws of capitalism, starting from the mathematical conditions for a maximum, are faulty in their method.

production of the necessary composition from given resources or equivalently, to obtain the fast increase of output.† The results of analysing these schemes confirm that the requirement for a given plan of production to be optimal enables one to obtain rather substantive conclusions together with its important quantitative characteristics.‡

One must not be surprised at the circumstance that, besides the qualitative difference in principle in the laws of socialist and capitalistic society and in the meaning of the fundamental economic categories, manifest formal analogies are to be found in various quantitative indicators and relations—for example, in normal efficiency and normal profit, normal valuations and the price of production. V. I. Lenin drew attention to this possibility when he noted that “the unity of nature is displayed in the ‘strikingly analogous’ differential equations relating to various fields of phenomena”.§

As we have noticed, the models under study must find application both in problems of national economic planning and in more special problems of the various units of socialist production and of particular planning problems.

The analysis of these schemes leads also to a certain system of objectively determined valuations.||

Socialist economy is concerned with obtaining scientifically based magnitudes of costs for various types of products. It is essential to know these magnitudes for solving problems of labour distribution and of replacing one product, or certain inputs, by others. Such a manifestation of value relations in socialist society is fundamental.

† In capitalism the use of such models for the general analysis of economics is impossible. There the economics of a country as a whole cannot follow a single plan, let alone a maximal one. In a capitalist economy, not only are the interests of society continually subordinated to those of the capitalist corporations, but instead of true requirements they follow short-term conditions of demand which altogether misrepresent these requirements.

‡ In a figurative sense we may say that the problem of efficiency would be solved, if a supporting hyperplane to the cone of all economic plans were found.

§ V. I. Lenin: *Works*, vol. 14, p. 276 (Russian edition).

|| It must be emphasized that analysis of a model is a necessary stage in studying an object even though the model does not represent the object completely. For example, in construction engineering the calculation for a building does not include all the beams, tie-rods and girders; however, without mastering the methods for calculating these elements it is impossible to make correct calculations for the building as a whole.

The choice of one solution or another as a separate economic problem makes no basic change to the general plan but is related only to one of its variations. Hence o.d. valuations that correspond to conditions of production in an optimal plan and permit us, as we have seen, to make a correct comparison of the results of different variations of a plan, are well adapted for determining the economic effect of the choice of specific economic solutions. This is a result of the fact that o.d. valuations correspond to the magnitude of the costs of (average) labour, which is necessary for producing this output under the given conditions. The fact that in so doing it is necessary to take into account the inputs of factors which determine the conditions for the application and economy of labour and that these factors also have o.d. valuations (rent, hire valuations of equipment) is connected simply with the necessity to calculate labour costs correctly, with allowance for the national economic conditions of its use; it is connected with the need to obtain not only partial labour costs in any production unit, but full labour costs.

Such a complete calculation also ensures that the level of costs of a given product characterizes not only the costs of varying the volume of its output, but turns out as a rule to be universal for all (rational) methods used for producing it on all units and hence coincides with the global (average) level of costs.†

It is important to note the realism of the above statement of the optimal planning problem. It is determined by the fact that the system of o.d. valuations, which is constructed together with the optimal plan, provides a means for solving a number of problems which are necessary for the practical realization of a plan: for example, the possibility of changing the plan (while keeping it optimal) according to changes in the situation, the possibility of obtaining indicators for evaluating the work of the separate production units and for stimulating the carrying out of the optimal plan, and so on.

It should be emphasized, however, that the models of production planning which have been considered are only approximations to the real problem. To make these studies more accurate it would be

† Ignoring indirect expenditure leads to misrepresentation of the real cost relations which may be compared with that obtained in, say, problems of mechanics, if forces of reaction, inertia, and friction were excluded from consideration (or taken into account only qualitatively), and only "visible" active forces were retained



necessary to consider the assumption of linearity, which is not entirely justified, and to allow for the stochastic (problematical) nature of some initial data; finally, one should remember to allow for certain non-economic factors. However, the introduction of these considerations into the calculations cannot change the basic conclusions; hence it is of the greatest importance to follow those paths of the analysis of economic planning which the basic model provides.

In considering these models, of course, a number of important problems have been altogether ignored, for example, how to make adjustments to the composition of the final products in the sector of private demand on the basis of demand studies, problems of allocation, in particular the wage structure, and so on. All these problems require special study which does not come within the scope of this book; but it is to be assumed that here too mathematical methods and models would find their appropriate place.

The above area of problems as a whole requires a great deal of further research, which will probably introduce appreciable corrections to the propositions given in this book, and will lead to the treatment of many essential problems which have not been touched upon here.

However, there is no doubt that mathematical analysis will help towards a better understanding of the quantitative aspects of the economic laws which govern a socialist society and will produce a more complete discovery of the advantages of this highly perfected social structure. This will assist in a more complete realization of the possibilities of the socialist method of production in the national economy as a whole and in all its sectors.

## APPENDIX II

# NUMERICAL METHODS FOR THE SOLUTION OF PROBLEMS OF OPTIMAL PLANNING

THE characteristics of an optimal plan given above (Appendix I) lead to a series of effective methods for solving problems of optimal planning. The present appendix is devoted to a description of these methods.

In the methods explained earlier the multipliers corresponding to these characteristics provide the basic tool for finding an optimal plan. Hence all these methods may be considered as different special examples of the general method of solution multipliers described in reference [1].

As in the rest of this book, the arguments presented here are chiefly for persons who are not mathematical specialists. Hence we shall not dwell on the detailed mathematical basis for the methods proposed nor on a consideration of certain special cases which are comparatively rarely met in practice.

It should be noted that the general methods of classical analysis for solving these problems are not realizable in practice even in comparatively simple cases. For example, to solve Problem A (of the allocation of a programme) by means of these methods with  $m = 8$ ,  $n = 5$ , it would be necessary to solve about one hundred thousand sets of simultaneous linear algebraic equations with twelve unknowns, which is obviously unworkable even with the use of modern electronic computers.

### *The Analysis of an Existing Plan*

Directly connected with the problem of finding an optimal plan is the question of checking (analysing) an existing plan. The method of checking normally used in practice consists of an attempt to form

a new plan with a higher aggregate output. One could not succeed, as a rule, in investigating the great number of all possible alternatives, while a survey of only some of the alternatives can provide no certainty that the projected plan is optimal. Thus, in spite of the large amount of work involved, the above method of checking does not enable one to establish whether a chosen alternative is optimal; the quality of the check depends essentially on the qualifications of the worker and is therefore of a subjective character.

The tests obtained for an optimal plan (*see* Appendix I, Theorems 1-6) provide objective methods for checking an existing plan. To test whether a given plan is optimal it is sufficient to establish whether or not there exists a system of multipliers (valuations) which satisfies the relevant conditions.

EXAMPLE 1.† In Problem A (*see* Appendix I, p. 263) let  $m = 4$ ,  $n = 3$ ,  $k_1 = 5$ ,  $k_2 = 12$ ,  $k_3 = 10$  (we are concerned here, for example, with the distribution of three types of work among four production units); the productivities of the units  $a_{ij}$  ( $i = 1, 2, 3, 4$ ;  $j = 1, 2, 3$ ) are characterized by the matrix

$$\|a_{ij}\| = \begin{vmatrix} 40 & 250 & 250 \\ 20 & 0 & 500 \\ 80 & 200 & 220 \\ 0 & 120 & 180 \end{vmatrix}.$$

We shall consider the following feasible plan for the operation of the production unit:

$$\pi = \|h_{ij}\| = \begin{vmatrix} 0.436 & 0.564 & 0 \\ 0.565 & 0 & 0.435 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}.$$

According to this plan,  $x_1^\pi = 108.74$ ,  $x_2^\pi = 261$ ,  $x_3^\pi = 217.5$  units of the respective products are produced per unit time (for example,  $x_1^\pi = 40 \times 0.436 + 20 \times 0.565 + 80 \times 1 = 108.74$ ), i.e. per unit time

$$\mu(\pi) = \frac{x_1^\pi}{k_1} = \frac{x_2^\pi}{k_2} = \frac{x_3^\pi}{k_3} = 21.75$$

complete assortment sets are produced.

We attempt to find valuations for the products which satisfy the

† The data in this example correspond in substance to those of the problem given in Chapter I, Section 2.

conditions of Theorem 1. These valuations are determined only up to a factor of proportionality; hence we may take the valuation for one unit of the first product as  $c_1 = 1$ . Then the productivities of production units (1), (2) and (3), which are used in the manufacture of this product, are respectively  $d_1 = 1 \times 40 = 40$ ,  $d_2 = 1 \times 20 = 20$ ,  $d_3 = 1 \times 80 = 80$ . On unit (1), product type 2 is also made; hence the valuation for one unit of this product is  $c_2 = 40 \div 250 = 0.16$ . Further, we find the productivity of the production unit (4),  $d_4 = 0.16 \times 120 = 19.2$ , and the valuation of one unit of product type (3),  $c_3 = 20 \div 500 = 0.04$ .

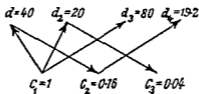


FIG. 10

The scheme for the successive determination of these valuations is shown in Fig. 10.

It is not difficult to check that the valuations obtained satisfy all the conditions of Theorem 1 (in particular, each machine is used for that kind of work for which its productivity is maximal).

Consequently these valuations are a system of o.d. valuations, and the plan under consideration is optimal.

$$\pi' = \left\| \begin{array}{ccc} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0.1925 & 0.8075 & 0 \\ 0 & 0.1622 & 0.8375 \end{array} \right\|.$$

We shall consider now another plan. For this plan we have

$$x_1^{\pi'} = 75.4; \quad x_2^{\pi'} = 181.0; \quad x_3^{\pi'} = 150.8.$$

$$\mu(\pi') = \frac{x_1^{\pi'}}{k_1} = \frac{x_2^{\pi'}}{k_2} = \frac{x_3^{\pi'}}{k_3} = 15.08.$$

In checking this plan, by a procedure analogous to the above, we find

$$c_1 = 1; \quad d_1 = 40; \quad d_2 = 20; \quad d_3 = 80; \\ c_4 = 80 \div 200 = 0.4; \quad d_4 = 0.4 \times 120 = 48; \quad c_3 = 48 \div 180 = 0.2667.$$

The valuations found do not satisfy Theorem 1; for example, the productivity of the production unit (2) for the manufacture of the third product is  $0.2667 \times 500 = 133.3 > 20 = d_2$ , and this possibility is not provided for in the plan (condition (b) is violated). Hence plan  $\pi'$  is not optimal; it may be improved by using the production unit (2) partly for manufacture of the third product.

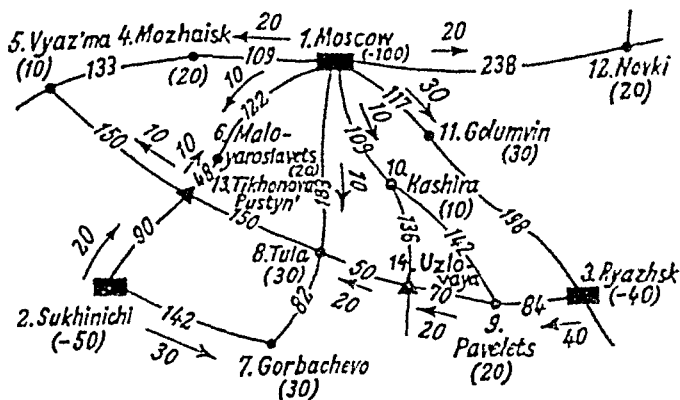


FIG. 11

EXAMPLE 2. We shall consider the transport problem (Problem F) under the specific conditions shown in Fig. 11.

Here there are  $m = 14$  points joined by a railway network, which consists of  $r = 19$  sections.† With the name of each point ( $i$ ) the level of its demand  $b_i$  is indicated in brackets. Points of production ( $b_i < 0$ ) are denoted by rectangles, points of demand ( $b_i > 0$ ) are indicated by circles, and intermediate points ( $b_i = 0$ ) by triangles. Against each section a number  $a_s$  is written, which denotes the cost of transporting one unit of goods along this section. For definiteness we shall assume that the numbers  $b_i$  express the level of demand in wagons per 24 hr, and the numbers  $a_s$  give the distance between the respective stations in kilometres; hence the costs are measured in wagon-kilometres.

Figure 11 also indicates a certain transport plan (the arrows and the figures under them indicate respectively the direction and

† More exactly, in the notation of Problem F, we should take  $r = 38$ , since, in the present case, goods may be transported in either one of two directions along each section of the system.

volumes of the flow of goods). According to this plan, products arrive at each station in the required quantities, so the plan is feasible. To establish the optimality of the plan we must seek potentials  $c_1, c_2, \dots, c_{14}$ , which satisfy the conditions of Theorem 5.

The potentials are determined only up to an additive constant; hence the potential for one of the stations may be chosen arbitrarily. We take, for example, Moscow's potential to be  $c_1 = 500$ . From Moscow goods are transported to Mozhaisk; consequently, the potential of this station is  $c_4 = 500 + 109 = 609$ . Analogously we find  $c_6, c_{13}, c_5, c_2$ , and so on;

$$\begin{aligned} c_6 &= 500 + 122 = 622; & c_{13} &= 622 - 48 = 574; & c_5 &= 574 + 150 = 724; \\ c_2 &= 574 - 90 = 484; & c_7 &= 484 + 142 = 626; & c_8 &= 500 + 183 = 683; \\ c_{14} &= 683 - 50 = 633; & c_9 &= 633 - 70 = 563; & c_3 &= 563 - 84 = 479; \\ c_{10} &= 500 + 109 = 609; & c_{11} &= 500 + 117 = 617; & c_{12} &= 500 + 238 = 738. \end{aligned}$$

The potentials obtained satisfy all the conditions of Theorem 5. Indeed, these potentials were constructed so that they fulfil conditions (b), and it is not difficult to check that they also satisfy conditions (a):

$$\begin{aligned} c_5 - c_4 &= 115 < 133; & c_8 - c_7 &= 57 < 82; & c_8 - c_{13} &= 109 < 150; \\ c_{14} - c_{10} &= 24 < 136; & c_{10} - c_9 &= 46 < 142; & c_{11} - c_3 &= 138 < 198. \end{aligned}$$

Hence the proposed plan is optimal.

In this example we managed to determine the potentials at all stations on the basis of only some of the conditions (b). It may happen in a particular transport plan that all the stations are divided into several groups which are not connected one with another by goods streams. Then it is impossible to determine the potential at all stations starting from conditions (b). However, using conditions (a) as well as conditions (b), one can obtain a set of inequalities for the required potentials. If this set is compatible the existing plan is optimal. If the set is incompatible the inequalities for one of the potentials are inconsistent; this means that the plan under consideration is not optimal; at the same time, a possible method for improving the existing plan emerges.

We shall make this clear by means of a concrete example. With the conditions of Example 2, let the given plan be that which is represented in Fig. 12. As is not difficult to see, this plan is feasible. We

shall seek potentials which correspond to it. Taking  $c_1 = 500$ , we find

$$c_4 = 609; \quad c_6 = 622; \quad c_{10} = 609; \quad c_{11} = 617; \quad c_{12} = 738.$$

The other points are not connected by flows of goods with the numbered stations. Hence condition (b) does not enable one to

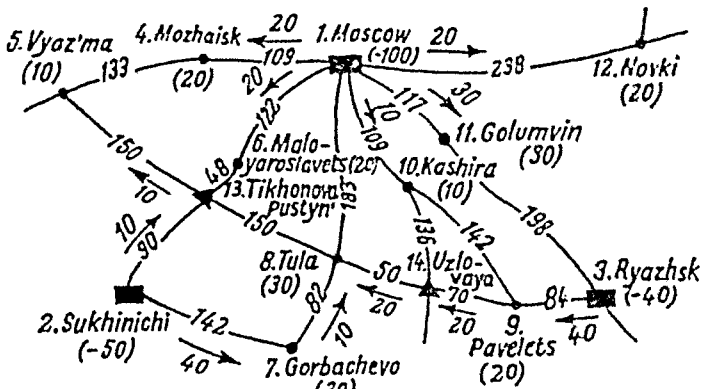


FIG. 12

determine their potentials. However, applying condition (a) to the Vyaz'ma-Mozhaisk section, we obtain

$$609 - 133 = 476 \leq c_5 \leq 609 + 133 = 742.$$

According to (b) and (a), we have for Tikhonova Pustyn'

$$\begin{aligned} 326 &= 476 - 150 \leq c_{13} \leq 742 - 150 = 592, \\ 574 &= 622 - 48 \leq c_{13} \leq 622 + 48 = 670, \end{aligned}$$

whence

$$574 \leq c_{13} \leq 592.$$

Further, according to (b), we find

$$484 \leq c_2 \leq 502; \quad 626 \leq c_7 \leq 644; \quad 708 \leq c_8 \leq 726.$$

Applying condition (a) now to the Moscow-Tula section, we obtain

$$317 \leq c_8 \leq 683.$$

For  $c_8$ , the potential at Tula, we obtain two inconsistent inequalities,

$$c_8 \geq 708, c_8 \leq 683;$$

this shows that the present plan is not optimal. It may be improved by introducing goods transport on the Moscow-Tula section with a corresponding reduction in transport along the Gorbachevo-Tula section (the former section was used for obtaining the lower limit for the potential at Tula, 708, and the latter section for obtaining the upper limit, 683). Adjustments of this plan are considered below.

We give now a more general algebraic description of the proposed method for checking a basic production planning problem (see Appendix I, Problem D).

To each feasible vector (plan)  $\pi = (h_1, h_2, \dots, h_r)$ , as we have seen (see (8), p. 277), there corresponds a point

$$y^0 + \lambda z = \sum_{s=1}^r h_s a^s + \sum_{i=1}^N h_{r+i} e^i, \quad (1)$$

where  $y^0 = (b_1, \dots, b_m, 0, \dots, 0)$ ,  $z = (0, \dots, 0, k_1, \dots, k_n)$ ,

$$a^s = (a_1^s, a_2^s, \dots, a_n^s)$$

are given vectors, which characterize respectively the limits of the first  $m$  elements, the required set of the last  $n$  elements, and the existing technical methods;

$$e^i = \overbrace{(0, \dots, 0)}^{i-1}, -1, 0, \dots, 0)$$

are the unit vectors along the respective co-ordinate axes,  $\lambda = \mu(\pi)$  is the aggregate output,  $h_s$  ( $s = 1, \dots, r$ ) are the components of the vector  $\pi$ , and the coefficients of the unit vectors  $h_{r+i}$  are the surpluses of the various elements, which are determined by formulae (9) (p. 277). Retaining on the right-hand side of (1) only non-zero terms, we find the relation

$$y^0 + \lambda z = \sum_{k=1}^u h_{s_k} a^{s_k} + \sum_{i=1}^v h_{r+i_i} e^{i_i} \\ (h_s > 0, s = s_1, \dots, s_u, r+i_1, \dots, r+i_v). \quad (2)$$

In Appendix I (see Theorem 3) it was shown that for a feasible



plan  $\pi$  to be optimal, it is necessary and sufficient that there exist multipliers (o.d. valuations)  $c_1, c_2, \dots, c_N$ , such that

$$(a) \quad c_i \geq 0 \quad (i = 1, \dots, N), \quad \max_{1 \leq j \leq n} c_{\pi+j} > 0;$$

$$(b) \quad \sum_{i=1}^N c_i a_i^s \leq 0 \quad (s = 1, \dots, r);$$

$$(c) \quad \sum_{i=1}^N c_i a_i^{s_k} = 0 \quad (k = 1, \dots, u);$$

$$(d) \quad c_{i_l} = 0 \quad (l = 1, \dots, v).$$

This enables one to formulate the following general rule for checking.

*Rule 1.* To check an existing feasible plan  $\pi = (h_1, h_2, \dots, h_r)$  one must consider the set of equations (c) and (d) with unknowns  $c_1, c_2, \dots, c_N$ .

(1) If this set has no solution which satisfies (a), plan  $\pi$  is not optimal; it may be improved without including other technical methods by changing the level of application of the methods used in it.

(2) Let the simultaneous equations under consideration have a unique solution  $\bar{c}_1, \bar{c}_2, \dots, \bar{c}_N$  (up to an arbitrary multiplicative constant) which satisfies conditions (a). We find then valuations for all the technical methods,

$$\sum_{i=1}^N \bar{c}_i a_i^s \quad (s = 1, \dots, r). \quad (3)$$

If none of these valuations is positive, conditions (b) are also fulfilled; hence plan  $\pi$  is optimal, and the numbers  $\bar{c}_i (i = 1, \dots, N)$  are a system of o.d. valuations. If this is not so the plan under review is not optimal, and it may be improved by adding a technical method for which the valuation is positive.

(3) If the equations (c-d) are indeterminate, i.e. if their general solution contains several arbitrary constants, then conditions (a) and (b) provide a set of inequalities for determining these constants. If this set of inequalities is compatible, the plan  $\pi$  is optimal; if not, then the plan is not optimal.

*Note.* In practical problems, case (3) occurs much less commonly than cases (1) and (2).

In Example 1 considered above valuations for checking the plan  $\pi$  were in fact determined from the equations

$$\begin{aligned} -d_1 + 40c_1 &= 0; & -d_2 + 20c_2 &= 0; & -d_3 + 80c_3 &= 0; \\ -d_1 + 250c_2 &= 0; & -d_2 + 500c_3 &= 0; & -d_4 + 120c_2 &= 0, \end{aligned}$$

which, as may easily be seen, correspond to conditions (c) and (d). Further, we checked that each element is used for the work for which it has maximum productivity, or that the inequalities

$$-d_i + a_{ij}c_j \leq 0 \quad (i = 1, 2, 3, 4; \quad j = 1, 2, 3)$$

are satisfied. In this way we established that all the quantities (3) are non-positive in the present case and, consequently, this plan is optimal.

#### *Construction of an Optimal Plan by Means of Successive Adjustments†*

In the previous paragraph it was shown that if a given plan is not optimal, then not only will this fact be established by the check but also a possible method of improving it will be discovered. More precisely, one establishes what improvement is possible by including in the plan a method which is not already used in it (the vector  $e^i$  may also act as such a method). Next, one determines the maximum level of applicability with which this method may be included in the plan. The inclusion of a new method leads, as a rule, to the exclusion of one of the methods used previously. As a result of such a correction we obtain a plan with a higher aggregate productivity. If this plan is still not optimal then a possible method for improving it will be discovered again by this check, and so on. After a finite number of such steps, we arrive at an optimal plan and a system of o.d. valuations. This is one of the effective methods for solving problems of optimal planning.

† The method of successive adjustments is given in references [3, 4] for the plan for solving a transport problem, and in [5] for a plan for the treatment of a complex raw material. This method has much in common with Dantzig's simplex-method, see [9] which was developed independently; however, o.d. valuations (solution multipliers) are not used in Dantzig's method, which makes it necessary at each stage to expand all the vectors  $a^s$  and  $e^i$  in terms of bases. The present method involves a smaller amount of computation.

We shall illustrate this method initially on the same numerical examples as above.

In Example 1 (p. 303) it was established in checking the plan that it was not optimal and that it could be improved by using production unit (2) partly for the manufacture of the third product.

We denote that part of the working time of unit (2) which is devoted in the new plan to the manufacture of the third product by  $\varepsilon$ . With the full utilization of this unit for manufacturing the third product one obtains a gain of  $133.3 - 20 = 113.3$  units, which corresponds to  $113.3 / (1 \times 5 + 0.4 \times 12 + 0.2667 \times 10) = 9.088$  assortment sets. We obtain thus a gain of  $9.088\varepsilon$  assortment sets.

On production unit (2) in time  $\varepsilon$ ,  $500\varepsilon$  units of the third product are produced, and only  $10 \times 9.088\varepsilon = 90.88\varepsilon$  units of it are needed. Hence a part of the working time of unit (4), equal to  $(500\varepsilon - 90.88\varepsilon) / 180 = 2.273\varepsilon$ , remains free. Using this time for the manufacture of the second product, we obtain  $120 \times 2.273\varepsilon = 272.8\varepsilon$  units of it instead of  $12 \times 9.088\varepsilon = 109.1\varepsilon$  which are needed. This enables unit (3) to be relieved by  $(272.8\varepsilon - 109.1\varepsilon) / 200 = 0.8185\varepsilon$ . Using this time for the manufacture of the first product, we obtain  $80 \times 0.8185\varepsilon = 65.48\varepsilon$  units of it. However, on production unit (2)  $20\varepsilon$  units less of this product are now obtained. Consequently, the gain is  $65.48\varepsilon - 20\varepsilon = 45.48\varepsilon$ , which corresponds to the required increase in the first product of  $5 \times 9.088\varepsilon = 45.44\varepsilon$  (some discrepancy is caused by rounding off).

Thus the new plan takes the form

$$\pi^r = \begin{vmatrix} 1 & 0 & 0 \\ 1 - \varepsilon & 0 & \varepsilon \\ 0.1925 + 0.8185\varepsilon & 0.8075 - 0.8185\varepsilon & 0 \\ 0 & 0.1622 + 2.273\varepsilon & 0.8378 - 2.273\varepsilon \end{vmatrix} \quad (4)$$

We are interested in choosing a maximal  $\varepsilon$ ; however, in doing this, all the numbers  $h_{ij}$  must remain non-negative. Hence we take  $\varepsilon = 0.8378 / 2.273 = 0.3686$ . Substituting this value in (4), we obtain a plan

$$\pi^r = \begin{vmatrix} 1 & 0 & 0 \\ 0.6314 & 0 & 0.3686 \\ 0.4942 & 0.5058 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

whose aggregate productivity is higher than that of  $\pi'$  by  $9.088 \varepsilon = 3.35$  assortment sets.

A check of plan  $\pi''$  shows that it also is not optimal. By improving it we arrive at plan  $\pi$  (p. 303) which, as has already been shown, is optimal.

In Example 2 we satisfied ourselves that the plan represented in Fig. 12 was not optimal. The contradiction, which became apparent on determining the potentials, showed that this plan may be improved by sending a certain number of wagons from Moscow to Tula. We join these stations by the unclosed circuit Moscow-Maloyaroslavets - Tikhonova Pustyn' - Sukhinichi - Gorbachevo-Tula, which consists of the sections used in determining the upper limit 683 for the potential at Tula. The smallest number of wagons travelling in this direction is 10 (on the section Gorbachevo-Tula). Hence we decrease the goods stream in this direction by 10 wagons; that is to say, we reduce the number of wagons over the arrows pointing in the stated direction by 10, we increase the number of wagons over the arrows pointing in the opposite direction by 10, and on the section where there was no goods stream at all (the Maloyaroslavets-Tikhonova Pustyn' section) we introduce a stream of 10 wagons in the direction from Tula to Moscow. So that the general balance is not violated (the plan remains feasible), we sent 10 wagons directly from Moscow to Tula. Thus, we obtain the plan represented in Fig. 11, which, as has been shown, is optimal.

In the above example of obtaining an optimal plan it was necessary to carry out only one adjustment to the existing plan. Generally speaking, several such adjustments have to be carried out.

We come now to the general algebraic description of the method. To simplify the explanation, we shall assume that for each feasible plan  $\pi = (h_1, h_2, \dots, h_r)$  (with aggregate productivity  $\mu(\pi) > 0$ ) as represented by the point  $y^0 + \mu(\pi)z$  in the form (2), the number of terms is†

$$u + v \geq N - 1. \quad (5)$$

† If this condition is violated in the process of constructing the optimal plan which we are considering, certain difficulties may occur. We have met such a situation in the second part of Example 2, which, however, did not interfere with our finding an optimal plan. Nevertheless, we cannot dwell here on a detailed study of this case. We note only that, generally speaking, condition (5) must be observed, but that its violation is connected with cases of degeneracy; hence, this condition can always be satisfied by infinitesimal changes in the initial data.

The process starts with a feasible plan  $\pi = (h_1, h_2, \dots, h_r)$  for which the following equation holds:

$$u + v = N - 1. \quad (6)$$

In this case, as will be shown below, the vectors

$$z, a^{s_1}, \dots, a^{s_u}, e^{i_1}, \dots, e^{i_v} \quad (7)$$

are a basis for the space considered, i.e. any vector may be uniquely expressed as a linear combination of these vectors. Hence it follows that the equations

$$\sum_{j=1}^n k_j c_{m+j} = 1, \quad \sum_{i=1}^N a_i^{s_k} c_i = 0 \quad (k = 1, \dots, u), \quad -c_{i_l} \quad (l = 1, \dots, v) = 0 \quad (8)$$

have a unique solution  $c_1, c_2, \dots, c_N$ .

If this solution satisfies the conditions

$$(a) \quad c_i \geq 0 \quad (i = 1, 2, \dots, N),$$

$$(b) \quad \sum_{i=1}^N c_i a_i^{s_0} \leq 0 \quad (s = 1, \dots, r).$$

then the plan  $\pi$  is an optimal plan, and the numbers  $c_1, c_2, \dots, c_N$  are a system of o.d. valuations.

If one of these conditions is violated, the plan  $\pi$  is not optimal, and aggregate productivity may be increased by including in the plan the vector

$$x = \begin{cases} e^{i_0}, & \text{if } c_{i_0} < 0, \\ a^{s_0}, & \text{if } \sum_{i=1}^N c_i a_i^{s_0} > 0. \end{cases} \quad (9)$$

To include this vector, we express  $x$  as a linear combination of the vectors (7),

$$x = f \cdot z + \sum_{k=1}^u g_{s_k} a^{s_k} + \sum_{l=1}^v g_{r+i_l} e^{i_l}, \quad (10)$$

where, as one may easily show by taking the scalar product of both

sides of this equation with the vector  $c = (c_1, c_2, \dots, c_N)$  and by using (8),

$$f = (c, x) = \begin{cases} -c_{i_0} > 0, & \text{if } x = e^{i_0}, \\ \sum_{i=1}^N c_i a_i^{s_i} > 0, & \text{if } x = a^{s_i}. \end{cases}$$

Using (2) and (10), we obtain the identity

$$y^0 + (\lambda + \varepsilon f)z = \sum_{k=1}^u (h_{s_k} - \varepsilon g_{s_k})a^{s_k} + \sum_{l=1}^v (h_{r+l_l} - \varepsilon g_{r+l_l})e^{l_l} + \varepsilon x. \quad (11)$$

The quantity  $\lambda + \varepsilon f$  characterizes the aggregate productivity of the respective plan. Consequently we attempt to make  $\varepsilon$  take its maximum value. However, in doing this, all the coefficients in (11) must remain non-negative; therefore we take

$$\varepsilon = \min_{g_s > 0} \frac{h_s}{g_s} \quad (s = s_1, \dots, s_u, r+i_1, \dots, r+i_v). \quad (12)$$

Thus we arrive at a representation in the form (2) for the point  $y^0 + \lambda'z$ , where  $\lambda' = \lambda + \varepsilon f > \lambda$  and the number of terms is  $u' + v' = N - 1$ ; the process may be continued. (If all the coefficients in (10)  $g_s \leq 0$ , then, as shown by (11), there are plans with aggregate productivity as high as we please and, consequently, an optimal plan does not exist. The same follows also from Theorem 4, Appendix I, since, with the assumption made, relation (10) indicates that condition  $\beta$  of this theorem is violated. In practical problems, as has already been noted, this situation does not occur.)

The process described cannot be continued indefinitely since there is only a finite number of points such as  $y^0 + \lambda z$  for which the number of terms in (2) satisfies (6), and in view of the monotonic nature of the process ( $\lambda' > \lambda$ ) we cannot arrive twice at the same point. Therefore, after a finite number of steps, we reach an optimal plan  $\pi = (h_1, h_2, \dots, h_r)$  and a system of o.d. valuations  $c_1, c_2, \dots, c_N$ , or else show that in the given problem an optimal plan does not exist (that there are plans with arbitrarily high aggregate productivities).

The most laborious portion of the process described is the solution of the equations (8) and the expansion of the vectors  $x$  in terms of the components (7). We shall explain how it is possible, by use of matrices,† to simplify these operations, beginning with the second stage of the process.

We denote by  $A$  the matrix composed of the coefficients of the unknowns in equations (8). We shall assume that the inverse matrix  $A^{-1}$  is known. Then, as may be easily seen, the elements in the first column of this matrix are a solution of the equations (8), and the coefficients in the expansion (10) are easily found:

$$(f, g_{s_1}, \dots, g_{s_w}, g_{r+i_1}, \dots, g_{r+i_v}) = x \cdot A^{-1}. \quad (13)$$

Calculation of the inverse matrix is, of course, in no way simpler than solving two systems of equations (determination of the coefficients in (10) also amounts to the solution of a system). However, the set of equations (8), obtained at each successive stage, differs from the previous set only in one equation; hence, in treating its matrix (starting from the second stage) the following rule may be used.

*Rule 2.* Let there be two square matrices,  $A_1 = \|x_{ij}\|$  and  $A_2 = \|x'_{ij}\|$ , of order  $N$ , which differ only in the elements of the  $v$ -th row,

$$x_{ij} = x'_{ij} \quad (i = 1, \dots, v-1, v+1, \dots, N; j = 1, \dots, N),$$

and let the inverse matrix  $A_1^{-1} = \|y_{ij}\|$  be given. Then the inverse matrix  $A_2^{-1} = \|y'_{ij}\|$  is found in the following way:

(1) The vector

$$(\alpha_1, \alpha_2, \dots, \alpha_N) = (x'_{v1}, x'_{v2}, \dots, x'_{vN}) \cdot A_1^{-1}$$

is found.

(2) The  $v$ -th column of the matrix  $A_1^{-1}$  is divided by  $\alpha_v$ , and then this transformed column, multiplied by the numbers  $\alpha_j$  ( $j = 1, \dots, v-1, v+1, \dots, N$ ), is subtracted from the respective columns of the

† As regards matrix notation and numerical operations on matrices, see, for example, V. N. Faddeyeva: *Numerical methods of linear algebra*, Gostekhizdat, 1950.

matrix  $A_1^{-1}$ . In other words, the elements of the inverse matrix  $A_2^{-1}$  are determined thus:

$$y'_{ij} = \begin{cases} \frac{1}{\alpha_v} \cdot y_{iv} & \text{for } j = v, \\ y_{ij} - \frac{\alpha_j}{\alpha_v} \cdot y_{iv} & \text{for } j \neq v. \end{cases}$$

Now the whole process may be described in the form of a certain routine.

*Rule 3.* Searching for an optimal plan and a system of o.d. valuations may be reduced to the following sequence of operations.

(1) Starting from some feasible plan  $\pi = (h_1, h_2, \dots, h_r)$ , for which condition (6) is satisfied, we form a matrix  $A$ , whose rows are the components of the vectors

$$z, a^{k*} \quad (k = 1, \dots, u), \quad e^{i_l} \quad (l = 1, \dots, v).$$

(2) We find the inverse matrix  $A^{-1}$  (using any of the methods usually recommended in algebra).†

(3) For the elements of the first column of matrix  $A^{-1}$ , which give the values  $c_1, c_2, \dots, c_N$ , we check conditions (a) and (b). If these conditions are satisfied, the elements are a system of o.d. valuations, and the plan  $\pi = (h_1, h_2, \dots, h_r)$  is optimal; the process is completed. If the conditions are not satisfied, we choose a vector  $x$  corresponding to (9), and we proceed to the next stage.

(4) We calculate the coefficients  $f, g_s$  ( $s = s_1, \dots, s_u, r + i_1, \dots, r + i_v$ ) and the quantity  $\varepsilon$  according to (13) and (12). The quantities found are used for determining the coefficients in the new relation (2),

$$\lambda' = \lambda + \varepsilon f, \quad h'_s = h_s - \varepsilon g_s \quad (s = s_1, \dots, s_u, r + i_1, \dots, r + i_v),$$

where  $\varepsilon$  is the coefficient of the new vector  $x$ .

If all the coefficients  $g_s \leq 0$ , an optimal plan does not exist (and the process has been completed).

(5) In the row of the matrix  $A$  which corresponds to the zero coefficient  $h'_s$  we write out the new elements—the components of the

† In cases where the form (2) is composed chiefly of the unit vectors, treatment of the initial matrix  $A$  is particularly simple.



vector  $x$ —and we treat the matrix thus obtained according to Rule 2. In this way the quantities  $x_j$  ( $j = 1, \dots, N$ ) coincide with the corresponding coefficients  $f$  and  $g_s$ , calculated in the previous sub-section.

(6) We proceed to sub-section (3).

It remains only to consider the question of forming an initial feasible plan.

Let us suppose first of all that we have at our disposal a certain feasible plan  $\pi = (h_1, h_2, \dots, h_r)$ , but that for this plan there is a strict inequality sign in (5). In this case, the vectors (7) are linearly dependent (their number exceeds the dimension of the space), and the following relation holds:

$$fz + \sum_{k=1}^u g_{s_k} a^{s_k} + \sum_{l=1}^r g_{r+l} e^{t_l} = 0; \quad (14)$$

without loss of generality we may suppose that  $f \geq 0$  and for  $f = 0$  at least one of the coefficients  $g_s > 0$ .

Using (2) and (14), we obtain the identity

$$y^0 + (\lambda + \epsilon f)z = \sum_{k=1}^u (h_{s_k} - \epsilon g_{s_k}) a^{s_k} + \sum_{l=1}^r (h_{r+l} - \epsilon g_{r+l}) e^{t_l}.$$

Substituting in it the quantity  $\epsilon$  calculated according to (12) we obtain the point  $y^0 + \lambda'z$  in form (2), where  $\lambda' = \lambda + \epsilon f \geq \lambda$ , and the number of added terms  $u' + v' < u + v$ . (If all the coefficients  $g_s \leq 0$ , then, as above, this indicates that in the particular case an optimal plan does not exist.)

After a finite number of such stages (not more than  $u + v - N + 1$ ) we obtain a formula in the form (2), for which condition (6) is satisfied and which consequently may be taken as an initial formula in the process just described (if, however, at a certain stage relation (14) is obtained, where all  $g_s \leq 0$ , then in this problem an optimal plan does not exist).

We may easily check the fact which we have used above, that when condition (6) is satisfied the vectors (7) form a basis for the given space. Indeed, if these vectors had turned out to be linearly dependent, then as we have seen, it would have been possible to obtain a formula in form (2) with a smaller number of added terms, a formula for which  $u + v < N - 1$ , which contradicts the assumption made.

We shall not dwell in detail on the compiling of an initial plan if no feasible plan is known at all. We point out only that in practical problems the finding of a particular feasible plan usually presents no difficulty whatever. For example, if there are positive reserves of the first  $m$  elements (the numbers  $b_i < 0, i = 1, \dots, m$ ) and in all the technical methods the last  $n$  elements are not fully expended ( $a_{m+j}^r \geq 0; s = 1, \dots, r; j = 1, \dots, n$ ), then we may take as an initial plan any one in which one or several technical methods are used and all the last  $n$  elements are produced in positive quantities. Moreover, it is desirable by taking advantage of the existing freedom in compiling an initial plan, to obtain immediately a feasible plan with the greatest possible aggregate productivity; this can decrease appreciably the number of steps necessary in the process. Some examples of this kind are given below. We shall consider now a numerical example.

EXAMPLE 3. Five elements are involved in production. The existing technical methods are characterized by the vectors

$$\begin{aligned} a^1 &= (-1; -5; 2; 12; 0); & a^6 &= (-8; -2; 1; 8; 1); \\ a^2 &= (-5; -4; 1; 0; 11); & a^7 &= (-8; -7; 3; 7; 7); \\ a^3 &= (-5; -9; 3; 5; 8); & a^8 &= (-4; -6; 2; 15; 8); \\ a^4 &= (-5; -5; 2; 5; 4); & a^9 &= (-3; -7; 4; 20; 0); \\ a^5 &= (-5; -9; 4; 1; 8); \end{aligned}$$

There are 18 and 24 units of resources in the first two ingredients; the last three elements must be obtained in the proportions 1:2:3, i.e.

$$y^0 = (-18; -24; 0; 0; 0); \quad z = (0; 0; 1; 2; 3).$$

As an initial plan we take one in which only one technical method (4) is used. The use of this method is limited by the first element; hence we take

$$h_4 = (-18):(-5) = 3.6; \quad \pi_1 = (0; 0; 0; 3.6; 0; 0; 0; 0; 0).$$

In this plan

$$\begin{aligned} x_1^{\pi_1} &= -18; & x_2^{\pi_1} &= -18; & x_3^{\pi_1} &= 7.2; \\ x_4^{\pi_1} &= 18; & x_5^{\pi_1} &= 14.4; & \mu(\pi_1) &= 4.8 \end{aligned}$$

and the corresponding formula in form (2) is

$$y^0 + 4.8z = 3.6a^4 + 6e^2 + 2.4e^3 + 8.4e^4$$

(the coefficients of this representation are given in the first row of Table I, pp. 320-1).

According to sub-sections (1) and (2) of Rule 3, we form the matrix  $A_1$  and find its inverse matrix  $A_1^{-1}$  (in the present case this is immediately achieved):

$$A_1 = \begin{vmatrix} 0 & 0 & 1 & 2 & 3 \\ -5 & -5 & 2 & 5 & 4 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{vmatrix} :$$

$$A_1^{-1} = \begin{vmatrix} 0.267 & -0.200 & 1 & -0.133 & -0.467 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0.333 & 0 & 0 & 0.333 & 0.667 \end{vmatrix} .$$

The elements of the first column of the matrix  $A_1^{-1}$ , as has been noted, are a solution of equations (8).

Following sub-section (3), we calculate the valuations for the technical methods (the valuations of the various elements,  $c_i$ , are the same as the elements of the first column of the matrix  $A_1^{-1}$ ):

$$\sum_{i=1}^5 c_i a_i^1 = -0.27; \quad \sum_{i=1}^5 c_i a_i^2 = 2.33; \quad \sum_{i=1}^5 c_i a_i^3 = 1.67;$$

$$\sum_{i=1}^5 c_i a_i^4 = 0; \quad \sum_{i=1}^5 c_i a_i^5 = 1.33; \quad \sum_{i=1}^5 c_i a_i^6 = -1.80;$$

$$\sum_{i=1}^5 c_i a_i^7 = 0.20; \quad \sum_{i=1}^5 c_i a_i^8 = 1.60; \quad \sum_{i=1}^5 c_i a_i^9 = -0.80$$

(for example,

$$\sum_{i=1}^5 c_i a_i^2 = 0.267 \times (-5) + 0 \times (-4) + 0 \times 1 + 0 \times 0 + 0.333 \times 11 = 2.33).$$

TABLE I

Approximations	Coefficient in expansion (2)									
	$\lambda$	$h_1$	$h_2$	$h_3$	$h_4$	$h_5$	$h_6$	$h_7$	$h_8$	$h_9$
1	4 800	—	—	—	3 600	—	—	—	—	—
2	6 480	—	0 720	—	2 880	—	—	—	—	—
3	6 781	—	0 753	—	2 679	0 167	—	—	—	—
4	9 977	—	1 937	—	0 051	1·052	—	—	—	0 932
5	9 990	—	1 951	—	—	1·064	—	—	—	0 946
6	10 000	—	2 000	—	—	1 000	—	—	—	1 000

We note technical method (2), for which the valuation is the largest.

According to sub-section (4) we find

$$(f, g_4, g_{9+2}, g_{9+3}, g_{9+4}) = a^2 \cdot A_1^{-1} = (2\cdot333; 1\cdot000; -1\cdot000; \\ 3\cdot333; 9\cdot667);$$

$$\varepsilon = \min\left(\frac{3\cdot6}{1}; \frac{2\cdot4}{3\ 333}; \frac{8\cdot4}{9\cdot667}\right) = 0\cdot720,$$

and also the coefficients  $\lambda$  and  $h_s$ , which are given in the second row of Table I (for example,  $\lambda = 4\ 800 + 0\cdot720 \times 2\cdot333 = 6\cdot480$ ). Thus we obtain a new representation in the form (2),

$$y^0 + 6\cdot48z = 2\cdot88a^4 + 6\ 72e^2 + 0\cdot72a^2 + 1\cdot44e^4,$$

and the corresponding matrix is

$$A_2 = \begin{vmatrix} 0 & 0 & 1 & 2 & 3 \\ -5 & -5 & 2 & 5 & 4 \\ 0 & -1 & 0 & 0 & 0 \\ -5 & -4 & 1 & 0 & 11 \\ 0 & 0 & 0 & -1 & 0 \end{vmatrix}.$$

By Rule 2 we find the matrix  $A_2^{-1}$ . For this purpose we divide the elements of the fourth column of the matrix  $A_1^{-1}$  by  $\alpha_4 = g_{9+3} = 3\cdot333$ , and we write out the fourth column of the matrix  $A_2^{-1}$  thus

(plans)					Valuations of the elements				
$h_{9-1}$	$h_{9-2}$	$h_{9+3}$	$h_{9+4}$	$h_{9+5}$	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$
—	6.000	2.400	8.400	—	0.267	0	0	0	0.333
—	6.720	—	1.440	—	0.360	0	0.700	0	0.100
—	6.084	—	—	—	0.377	0	0.093	0.207	0.163
—	—	—	—	—	0.146	0.525	0.923	-0.023	0.041
0.089	—	—	—	—	0	0.416	0.777	-0.010	0.081
—	—	—	1.000	—	0.111	0.333	0.667	0	0.111

obtained alongside the matrix  $A_1^{-1}$ ; then we subtract the transformed column, multiplied by the numbers

$$\alpha_1 = f = 2.333; \alpha_2 = g_4 = 1.000; \alpha_3 = g_{9+2} = -1.000; \\ \alpha_5 = g_{9+4} = 9.667$$

(these multipliers are written in a row above the new matrix  $A_2^{-1}$ ) from the 1st, 2nd, 3rd, and 5th columns respectively of the matrix  $A_1^{-1}$ , and obtain the remaining columns of the matrix  $A_2^{-1}$ . The calculation has the following written form:

4th column: 3.333

$$A_1^{-1} = \begin{vmatrix} 0.267 & -0.200 & 1 & -0.133 & -0.467 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0.333 & 0 & 0 & 0.333 & 0.667 \end{vmatrix} \begin{matrix} -0.040 \\ 0 \\ -0.300 \\ 0 \\ 0.100 \end{matrix}$$

$$\begin{matrix} 2.333 & 1.000 & -1.000 & 9.667 & \times \end{matrix}$$

$$A_2^{-1} = \begin{vmatrix} 0.360 & -0.160 & 0.960 & -0.040 & 0.080 \\ 0 & 0 & -1 & 0 & 0 \\ 0.700 & 0.300 & 0.300 & -0.300 & 2.900 \\ 0 & 0 & 0 & 0 & -1 \\ 0.100 & -0.100 & 0.100 & 0.100 & -0.300 \end{vmatrix}$$

The elements of the first column of the matrix obtained are as before a solution to equations (8) (they are shown in the second row of Table I).

Now we go over again to sub-section (3) of Rule 3 and, proceeding as before, we obtain the 3rd, 4th, 5th and 6th approximations (see Table I). For the last of these approximations the valuations of the technical methods give

$$\sum_{i=1}^5 c_i a_i^1 = -0.444; \quad \sum_{i=1}^5 c_i a_i^2 = 0;$$

$$\sum_{i=1}^5 c_i a_i^3 = -0.555;$$

$$\sum_{i=1}^5 c_i a_i^4 = -0.777;$$

$$\sum_{i=1}^5 c_i a_i^5 = 0; \quad \sum_{i=1}^5 c_i a_i^6 = -0.777;$$

$$\sum_{i=1}^5 c_i a_i^7 = -0.444;$$

$$\sum_{i=1}^5 c_i a_i^8 = -0.222; \quad \sum_{i=1}^5 c_i a_i^9 = 0.$$

Hence, the plan  $\pi = (0; 2; 0; 0.1; 0; 0; 0; 1)$  is optimal, and the numbers  $c_1 = 0.111$ ,  $c_2 = 0.333$ ,  $c_3 = 0.667$ ,  $c_4 = 0$ ,  $c_5 = 0.111$  are a system of o.d. valuations.

#### *The Method of Adjusting Multipliers (Valuations)†*

This method consists of the following. Given a system of valuations which has the property that for each technical method, the total valuation for the production of goods does not exceed the total valuation of costs, we determine some methods in which these valuations are the same (profitable methods). Using only these methods we shall construct a plan with maximum aggregate productivity. This plan determines the changed system of valuations. If with the new valuations the total valuation for the production of goods by each technical method is not higher than the valuation

† This method is systematically used also in reference [1] (see also [5, 7]). In very recent years a similar method has begun to be used in other countries (see the work of Dantzig, Ford, Fulkerson in the collected work [10]).

of expenditure the plan is optimal and the valuations obtained are a system of o.d. valuations. If not we start from two existing systems of valuations, and construct a new system (generally speaking, in this system the valuations of those elements which do not limit production turn out to be relatively lower). This new system of valuations makes it possible by using only profitable methods to obtain a plan with a higher aggregate productivity, and so on. After a finite number of stages we obtain an optimal plan and a system of o.d. valuations.

Thus, in the process of compiling a plan according to this method, a particular kind of competitive battle takes place between the different technical methods with price oscillations, and this reveals those methods the use of which in the given circumstances would be most advantageous. This battle proceeds here only in the process of calculation and hence bears obviously no relation to the large losses which inevitably accompany real competition in a capitalist society.

We proceed now to an algebraic description of this method. For simplicity we shall assume, as we did above, that for any feasible plan  $\pi$  the number of terms in the representation (2) satisfies the inequality (5) (p. 312).

The process starts from an arbitrary system of valuations  $d_1, d_2, \dots, d_n$ , which satisfies the following conditions.

(1) These valuations are non-negative, at least one of the products included in the assortment set having a positive valuation, i.e.

$$d_i \geq 0 \quad (i = 1, 2, \dots, N); \quad \max_{1 \leq j \leq n} d_{m+j} > 0.$$

(2) For each technical method the total valuation of the products produced does not exceed the total valuation of the factors utilized.

$$\sum_{i=1}^N d_i a_i^s \leq 0 \quad (s = 1, 2, \dots, r).$$

(3) There exists a feasible plan  $\pi$ , in which only profitable methods are used, i.e. methods (s) such that

$$\sum_{i=1}^N d_i a_i^s = 0.$$

We calculate the valuations of the products for each technical method,

$$D_s = \sum_{i=1}^N d_i a_i^s \quad (s = 1, 2, \dots, r) \quad (15)$$

and we note the set  $S$  of profitable methods ( $D_s = 0$ ).

We then construct a plan  $\pi$  with the largest aggregate productivity, in which only methods  $s \in S$  are used. For this purpose we use the method of successive adjustments explained above. In the present case this method is appreciably simplified owing to the fact that the set  $S$  contains as a rule considerably fewer elements than the initial set of methods. Thus we achieve a feasible plan  $\pi = (h_1, h_2, \dots, h_r)$  which corresponds to the representation (2), where

$$s_k \in S \quad (k = 1, \dots, u)$$

(only profitable methods are used), and we obtain a new system of valuations  $c_1, c_2, \dots, c_N$ , which satisfy the conditions

$$(a) \quad c_i \geq 0 \quad (i = 1, 2, \dots, N); \quad \sum_{j=1}^n c_{m+j} k_j = 1;$$

$$(b) \quad \sum_{i=1}^N c_i a_i^s \leq 0 \quad (s \in S);$$

$$(c) \quad \sum_{i=1}^N c_i a_i^{s_k} = 0 \quad (k = 1, 2, \dots, u);$$

$$(d) \quad c_{i_l} = 0 \quad (l = 1, 2, \dots, v).$$

If the general valuation of production and costs for the other methods ( $s \notin S$ ) is also non-positive, so

$$C_s = \sum_{i=1}^N c_i a_i^s \leq 0, \quad (16)$$

then the plan  $\pi$  is optimal, and the numbers  $c_1, c_2, \dots, c_N$  are a system of o.d. valuations (for the initial problem, where the use of technical methods  $s \notin S$  is also permitted).

If conditions (16) are violated, the valuations obtained are not a system of o.d. valuations, and plan  $\pi$  is not optimal. In this case we construct a new system of valuations  $d'_1, d'_2, \dots, d'_N$  such that at



least one of the methods for which (16) is violated is included in the set  $S'$  of profitable methods. For this we take

$$d'_i = d_i + \varepsilon(c_i - d_i) \quad (i = 1, 2, \dots, N), \quad (17)$$

where

$$\varepsilon = \min_{c_i > 0} \frac{D_i}{D_i - C_i}. \quad (18)$$

Starting from valuations (17), we repeat the operations which have been described. As a result we obtain a plan  $\pi'$  with a higher aggregate productivity,  $\mu(\pi') > \mu(\pi)$ . Hence the process described cannot be continued indefinitely, and we arrive consequently after a finite number of stages at an optimal plan and a system of o.d. valuations.

We note that in solving a small problem (with a limited number of technical methods) by means of successive adjustments of the plan, it is convenient to take as an initial plan the one obtained in the previous stage (starting from the second stage; each time it is necessary, as a rule, to make only one adjustment). In doing this it turns out to be possible to use the simplified method described above for computing the inverse matrix.

This method may be formulated by the following rule.

*Rule 4.* We start from the arbitrary valuations  $d_1, d_2, \dots, d_N$ , which satisfy conditions 1, 2 and 3.

(1) We calculate the valuations of production and costs for all technical methods according to formula (15), and we note the set  $S$  of profitable methods,

$$S = \{s | D_s = 0\}.$$

(2) By the method of successive adjustments of a plan we solve the auxiliary problem in which the whole set of methods consists of methods  $s \in S$ . We obtain an optimal plan  $\pi = (h_1, h_2, \dots, h_r)$  ( $h_s = 0$  for  $s \notin S$ ) for this subsidiary problem and the matrices  $A$  and  $A^{-1}$  which correspond to the formulation (2). In the first column of the latter matrix there are valuations  $c_1, c_2, \dots, c_N$ , which form a system of o.d. valuations under the conditions of the subsidiary problem.

(3) According to formula (16) we calculate valuations  $C_s$  for the remaining methods  $s \notin S$ . If none of these valuations is positive, plan  $\pi$  and the valuations  $c_1, c_2, \dots, c_N$  are a solution of the initial

TABLE II

Approximations	Valuations of the elements					Coefficients in				
	$d_1; c_1$	$d_2; c_2$	$d_3; c_3$	$d_4; c_4$	$d_5; c_5$	$\lambda$	$h_1$	$h_2$	$h_3$	$h_4$
1	5 0000 0	5 0000 0 3333	2 0000 1 0000	2 0000 0	2 0000 0	8 000	—	—	—	—
2	2-2727 0	2 4546 0 3721	1-4546 0 6512	0 9091 0	0 9091 0 1163	8 930	—	—	—	—
3	0 0987 0-2836	0-4626 0 1940	0 6861 0 5522	0 0395 0	0 1507 0-1492	9-761	—	1-881	—	—
4	0 2349 0-1111	0 2648 0 3333	0 5875 0-6667	0 0104 0	0 1496 0 1111	10 000	—	2-000	—	—

problem, and the process is completed. If not, we proceed to the following sub-section.

(4) We calculate new valuations  $d'_1, d'_2, \dots, d'_N$  according to formulae (17) and (18) and we proceed to sub-section (1).

*Note.* In sub-section (2), starting from the second stage, we take as initial plan the one which was obtained in the previous stage.

We shall not dwell here in detail on the methods for determining initial valuations  $d_1, d_2, \dots, d_N$ , which satisfy conditions 1, 2 and 3. We point out only that in practical problems these valuations are usually easy to find. For example, in Problems A, B and C (see Appendix I) the valuations for the products may be chosen arbitrarily, and the valuations of the industrial factors are determined so that they satisfy conditions 2.

We shall illustrate now the method described here by the same numerical Example 3, as above.

As initial valuations we take

$$d_1 = 5; d_2 = 5; d_3 = 2; d_4 = 2; d_5 = 2.$$

(These valuations are obtained in the following fashion: we suppose  $d_1 = d_2 = x$ ,  $d_3 = d_4 = d_5 = y$ , and find the maximum ratio  $x/y$  for which the valuations of production and of costs in all technical methods  $D_s \leq 0$ .)

According to Rule 4, sub-section (1), we find by means of formulae (15)

$$D_1 = -2; D_2 = -21; D_3 = -36; D_4 = -28; D_5 = -44; \\ D_6 = -30; D_7 = -41; D_8 = 0; D_9 = -2; S = \{8\}.$$

expansion (2) (plans)									
$h_5$	$h_6$	$h_7$	$h_8$	$h_9$	$h_{9+1}$	$h_{9+2}$	$h_{9+3}$	$h_{9+4}$	$h_{9+5}$
—	—	—	4.000	—	2.000	—	—	44.000	8.000
—	—	—	3.349	0.558	2.930	—	—	43.535	—
—	—	—	1.075	1.433	—	—	—	25.253	—
1.000	—	—	—	1.000	—	—	—	1.000	—

Using only the technical method (8), we obtain the greatest productivity when this method is used at the highest possible level. However, we are limited by the available resources; hence we take  $\pi_1 = (0; 0; 0; 0; 0; 0; 0; 0; 4; 0)$ . The coefficients  $\lambda$  and  $h_i$  in representation (2), and also the valuations of the elements  $c_i$  corresponding to this plan, are given in the first row of Table II.

Next, according to formulae (16) we find  $C_1 = 0.333$ ,  $C_2 = -0.333$ ,  $C_3 = 0$ ,  $C_4 = 0.333$ ,  $C_5 = 1.000$ ,  $C_6 = 0.333$ ,  $C_7 = 0.667$ ,  $C_8 = 0$ ,  $C_9 = 1.667$ . Since some of these numbers are positive, we proceed to sub-section (4). According to (17) and (18) we find

$$\varepsilon = \min \left\{ \frac{-2}{-2-0.333}; \frac{-28}{-28-0.333}; \frac{-44}{-44-1}; \frac{-30}{-30-0.333}; \frac{-41}{-41-0.667}; \frac{2-}{-2-1.667} \right\} = 0.5454,$$

$d_1 = 2.273$ ,  $d_2 = 2.455$ ,  $d_3 = 1.455$ ,  $d_4 = 0.909$ ,  $d_5 = 0.909$  (for example,  $d_1 = 5 + 0.5454 \times (0 - 5) = 2.273$ ). Then we begin again at sub-section (1).

As was done previously, we obtain the following approximations (see Table II). For the last of these we have

$$C_1 = -0.444; C_2 = 0; C_3 = -0.555; C_4 = -0.777; C_5 = 0; \\ C_6 = -0.777; C_7 = -0.444; C_8 = -0.222; C_9 = 0.$$

Hence plan  $\pi = (0; 2; 0; 0; 1; 0; 0; 0; 1)$  is optimal, and the numbers  $c_1 = 0.111$ ,  $c_2 = 0.333$ ,  $c_3 = 0.667$ ,  $c_4 = 0$ ,  $c_5 = 0.111$  are a system of o.d. valuations.

*The Method of Double Limits of O.D. Valuations†*

In spite of the fact that o.d. valuations are not usually known at first certain inequalities may be obtained for their possible values from a consideration of the various technical methods and also of very simple plans. Quite accurate limits bounding o.d. valuations may be obtained by increasing progressively the accuracy of the rough limits originally taken. For example, for any plan we know that the total valuation for the product must not exceed the general valuation of the elements expended, which provides a certain inequality involving the o.d. valuations. In particular, having taken a lower bound for the o.d. valuations of all elements produced except one, and an upper bound for the elements expended, we obtain a more accurate upper bound for the o.d. valuations of the isolated element. Inequalities in the opposite sense are constructed similarly. It often proves useful to find limits bounding the o.d. valuations of complete sets.

The greater the accuracy of the above method, the closer the approximations used (in the process of making the limits for o.d. valuations more accurate) are to the optimal plan. Generally speaking, when the system of o.d. valuations is unique, this method enables one to obtain limits for o.d. valuations as accurately as desired. For this it is sufficient to use plans in which the respective products or factor is over-produced or under-produced as compared with the required quota.

The method of double limits may be used for the complete solution of optimal planning problems. In particular, this method proves to be useful in solving problems of the allocation of a programme, of the treatment of a complex raw material and in transport problems.

Besides this, the above method may be used in combination with others. For example, without improving the accuracy of the upper and lower bounds until they coincide, after narrowing the gap only roughly we can change over to the method of correcting the multipliers, taking as our initial system of multipliers the average values of the limits found for the o.d. valuations. By doing this, both the levels of technical methods used and the required assortment quota

† This method is explained in the context of the problem of treating a compound raw material in reference [5] (Chapter I, Section 8).

become considerably more accurate, and therefore the number of necessary stages in obtaining an optimal plan is appreciably reduced.

Finally, the method of double valuations enables one to obtain limits for o.d. valuations even when certain initial data are not completely known, and one is given only the limits within which they lie.

### *Approximate Solution of Problems of Optimal Planning*

With the use of only approximate values for the o.d. valuations one can usually draw up immediately quite a satisfactory plan. In many cases this plan turns out to be so close to the optimal plan that its improvement is of no further practical interest. In any case, by taking it as an initial plan in the method of successive adjustments we appreciably reduce the number of necessary stages.

We shall illustrate the construction of such an approximate plan by means of a specific example of solving a problem of type A.

EXAMPLE 4.† Let there be 8 shelling machines on which it is required to treat 5 types of material in a given ratio (see Table III).

TABLE III. ASSORTMENT TASK

Type of material	I	II	III	IV	V
Fraction of the total quantity (%)	10	12	28	36	14

In Table IV the productivities of the machines for the different kinds of work are shown.

As approximate o.d. valuation for the amounts of labour involved in treating the various materials we may take numbers which are inversely proportional to the total productivities of the machines,

$$c_1 = 30.30; c_2 = 17.45; c_3 = 13.76; c_4 = 9.09; c_5 = 7.4$$

(for example,

$$c_1 = \frac{1000}{4.0+4.5+5.0+4.0+3.5+3.0+4.0+5.0} = \frac{1000}{33} = 30.30).$$

† The solution of this example by the method of adjusting the multipliers is given in reference [1].

The respective valuations for the productivities of the machines are given in Table V (the columns of this table are obtained from the columns of Table IV multiplied by the valuation of the corresponding

TABLE IV. PRODUCTIVITIES OF THE MACHINES

Number of machine	Type of material				
	I	II	III	IV	V
1	4.0	7.0	8.5	13.0	16.5
2	4.5	7.8	9.7	13.7	17.5
3	5.0	8.0	10.0	14.8	18.0
4	4.0	7.0	9.0	13.5	17.0
5	3.5	6.5	8.5	12.7	16.0
6	3.0	6.0	8.0	13.5	15.0
7	4.0	7.8	9.0	14.0	17.0
8	5.0	8.0	10.0	14.8	18.0

material; the maximum valuation, and the ones closest to them, are noted in each row of the table).

If we succeed in using every machine on the work for which its productivity valuation is highest, we obtain  $122+136+152+$

TABLE V. PRODUCTIVITIES OF THE MACHINES (RELATIVE UNITS)

Number of machine	Type of material				
	I	II	III	IV	V
1	121	122	117	118	122
2	136	136	133	125	130
3	152	140	138	135	133
4	121	122	124	123	126
5	106	113	117	115	119
6	91	105	110	123	111
7	121	122	124	127	126
8	152	140	138	135	133

$126+119+123+127+152 = 1057$  relative units of output. The assortment set of products has a valuation of  $30.3 \times 0.1 + 17.45 \times 0.12 + 13.76 \times 0.28 + 9.09 \times 0.36 + 7.41 \times 0.14 = 13.29$  relative units. Hence the number of assortment sets in the plan cannot exceed  $1057/13.29 = 79.5$ .

Using the machines as far as possible on the work for which they have greatest productivity, we try to accommodate the given number of sets, i.e. we shall allocate the task as in Table VI.

TABLE VI

Type of material	I	II	III	IV	V
Quota	7.95	9.54	22.26	28.62	11.13

(for example, for item I:  $79.5 \times 0.1 = 7.95$ ).

According to Table V, material of type I should be treated on machines Nos. 2, 3 and 8. However, their total productivity exceeds the task ( $4.5 + 5 + 5 > 7.95$ ). Hence some of these machines should be used also for other work. It is more expedient to use machines Nos. 3 and 8 on item I, since machine No. 2 may be effectively used also for treating material II. We shall load machine No. 3 fully; this provides 5.0 units of material I. In order to obtain 7.95 units of material I we have to use a further 0.59 units of working time of machine No. 8. The remaining time belonging to this machine—0.41 units—is devoted to treating material II since for this material the machine's productivity valuation is very nearly maximal. Thus we obtain  $8 \times 0.41 = 3.28$  units of material II. The remaining  $9.54 - 3.28 = 6.26$  units of this material are treated on machine No. 2, for which 0.80 unit of its working time is required. The remaining time of machine No. 2 is given over to treating material III. Continuing this process for allocating the task, we arrive at Plan I (Table VII).

TABLE VII. PLAN I (FRACTION OF THE WORKING TIME OF MACHINES SPENT ON VARIOUS KINDS OF WORK)

Number of machine	Type of material				
	I	II	III	IV	V
1	—	—	0.27	0.05	0.68
2	—	0.80	0.20	—	—
3	1	—	—	—	—
4	—	—	1	—	—
5	—	—	—	1	—
6	—	—	—	1	—
7	—	—	1	—	—
8	0.59	0.41	—	—	—

In this plan not enough of material IV is treated or 1.8 units less than was planned. The valuation for the deficiency in this material is  $9.09 \times 1.8 = 16.36$  relative units, which correspond to  $16.36/13.29 = 1.23$  assortment sets. Starting from a task of  $79.5 - 1.23 = 78.27$  assortment sets, as previously we compile Plan 2 (see Table VIII) which accords completely with the given task.

Allowing for the fact that the volume of production cannot exceed 79.5 assortment sets, we see that the method described for con-

TABLE VIII. PLAN 2

Number of machine	Type of material				
	I	II	III	IV	V
1	—	—	0.19	0.15	0.66
2	—	0.76	0.24	—	—
3	1	—	—	—	—
4	—	—	1	—	—
5	—	—	—	1	—
6	—	—	—	1	—
7	—	—	1	—	—
8	0.57	0.43	—	—	—

structing a first approximation in the present case gave an error which, however, is not greater than  $(1.23/78.4) \times 100 = 1.6$  per cent. However, if we find an ultimate optimal plan (for example, by successive adjustments of Plan 2), it turns out that the error in fact is only 0.7 per cent.

### *The Use of Physical Models*

As we have seen, the detailed properties of an optimal plan lead to quite effective computational methods for solving problems of optimal planning. These properties often provide a basis for the construction of models for the automatic solution of problems. A characteristic example of this kind is the model for solving the transport problem (Problem F).

For the sake of clarity we give here a description of a hydraulic model, though an electrical model based on the same principles may prove to be more useful (these models are mentioned in reference [4]).



Let there be  $m$  vessels. Into each vessel ( $i$ )  $-b_i$  units of liquid flow every second (in the case of vessels for which  $b_i > 0$  liquid is flowing out at the rate of  $b_i$  units per second). The vessels  $i_s$  and  $j_s$  ( $s = 1, \dots, r$ ) are connected in such a way that they communicate with each other when the difference of liquid levels in them is

$$H_{i_s} - H_{j_s} \geq a_s.$$

It may easily be verified that when the process has achieved equilibrium the quantity of liquid flowing every second from one vessel

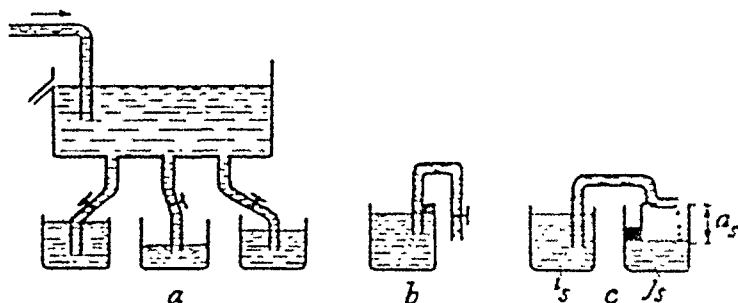


FIG. 13

( $i_s$ ) to another ( $j_s$ ) corresponds to the volume of goods transport along the section ( $s$ ) of a network under an optimal plan. Thus we may take the numbers

$$c_i = H - H_i \quad (i = 1, \dots, m)$$

as the potentials, where  $H_i$  is the liquid level in vessel ( $i$ ), and  $H$  is an arbitrary real number.

The scheme described may be realized in practice by the following connected vessels. The vessels, which correspond to points of production ( $b_i < 0$ ), are fed from a common reservoir (in which a constant head of liquid is maintained) by means of tubes equipped with graduated valves (see Fig. 13a). From vessels corresponding to points of demand ( $b_i > 0$ ), liquid is led off also through tubes fitted with graduated valves; to maintain a constant head, the outlet end of these tubes is attached to a float (see Fig. 13b). The connection between the vessels ( $i_s$ ) and ( $j_s$ ) is shown in Fig. 13c; owing to the fact that the outlet end of the siphon is secured at a fixed height above the float, a difference of levels not exceeding  $a_s$  is maintained.

*An Example of the Simultaneous Analysis of the Distribution of Production and Goods Streams*

In the transport problem  $F$  considered above we shall suppose that at points of production (we shall take the first  $m_1 < m$  points to be of this type) the volumes of production  $-b_i$  may be varied within certain limits,

$$0 \leq -b_i \leq l_i \quad (i = 1, \dots, m_1),$$

and that the total productive capacity exceeds the demand for the particular product †

$$\sum_{i=m_1+1}^m b_i < \sum_{i=1}^{m_1} l_i.$$

The prime cost of a unit of production  $d_i$  ( $i = 1, \dots, m_1$ ) at each production point is given (we shall take it that the cost  $d_i$  and the expenses  $a_i$  for transport of the product are expressed in the same units, say in roubles). It is required to plan the volumes of production  $-b_i$  ( $i = 1, \dots, m_1$ ) and to construct a transport plan such that the total price of the product, including the transport costs, should be a minimum.

An optimal plan in the present case is characterized by the presence of a system of potentials  $c_1, c_2, \dots, c_m$ , such that conditions (a) and (b) of Theorem 5 are satisfied, and furthermore, the potentials must be such that

(c) The potentials at points of production satisfy the relations

$$\begin{aligned} c_i &= d_i, & \text{if } 0 < -b_i < l_i, \\ c_i &\leq d_i, & \text{if } b_i = 0, \\ c_i &\geq d_i, & \text{if } -b_i = l_i. \end{aligned}$$

We shall illustrate this problem by a numerical example.

**EXAMPLE 5.** With the conditions of Example 2, let the volumes of production be varied at the points of production. The prime cost

† Such a situation occurs, for example, in providing certain kinds of raw material for the building industry.

of one unit of a product and the maximal outputs at various points of production are characterized by the following data:

$$d_1 = 500, d_2 = 450, d_3 = 550, l_1 = 120, l_2 = 75, l_3 = 80.$$

As above, the plan represented in Fig. 11 is here feasible. However, under the changed conditions it will not be optimal. Indeed, condition (c) is violated for the potentials corresponding to this plan (see p. 306):

$$c_2 = 484 > 450 = d_2.$$

This means that total costs may be reduced by changing the volumes of production —  $b_i$  ( $i = 1, 2, 3$ ).

In Fig. 14 another feasible plan is given for the allocation of production and of goods streams, in which the total costs for manu-

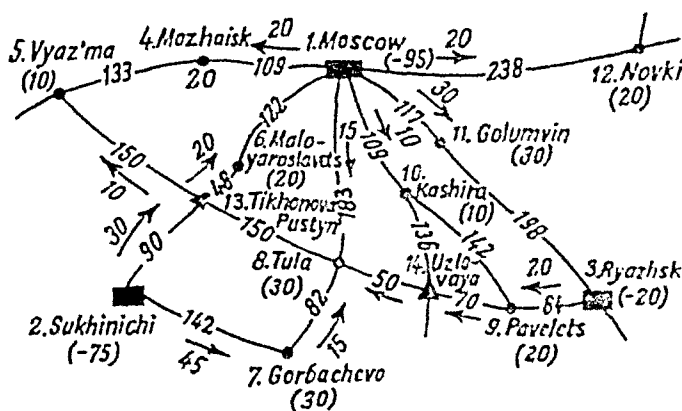


FIG. 14

facture and transport of the products are 1895 units lower than in the previous plan.

The potentials corresponding to this plan are

$$c_1 = 500, c_2 = 459, c_3 = 550, c_4 = 609, c_5 = 699, c_6 = 597, c_7 = 601, \\ c_8 = 683, c_9 = 634, c_{10} = 609, c_{11} = 617, c_{12} = 738, c_{13} = 549, c_{14} = 633,$$

which, as may be easily checked, satisfy conditions (a) and (b) of Theorem 5, and also conditions (c). Hence, this plan is optimal.

We note that it may be obtained from the plan given in Fig. 11 by means of successive adjustments to the latter. To obtain the optimal plan in the present case it is necessary to carry out three iterations which in principle are little different from those carried out in Example 2.

*Adjustment 1.* Ten wagons are removed from the Moscow-Maloyaroslavets section and 10 wagons are added to the Sukhinichi-Tikhonova Pustyn'-Maloyaroslavets section. At Moscow the volume of production is reduced by 10, and at Sukhinichi it is increased by 10.

*Adjustment 2.* Fifteen wagons are removed from the Ryazhsk-Pavelets-Uzlovaya-Tula section, and 15 wagons are added to the Sukhinichi-Gorbachevo-Tula section. At Ryazhsk the volume of production is decreased by 15, and at Sukhinichi it is increased by 15.

*Adjustment 3.* Five wagons are removed from the Ryazhsk-Pavelets-Uzlovaya-Tula section, and 5 wagons are added to the Moscow-Tula section. At Ryazhsk the volume of production is reduced by 5, and at Moscow it is increased by 5.

#### *An Example of a Calculation of a Dynamic Problem*

We shall consider a rather simplified problem of compiling a long-term plan.

EXAMPLE 6. For the preparation of a certain product it is necessary to use equipment (machines) and a labour force; there exist also a number of methods which involve technology to varying extents and require correspondingly various amounts of labour. (We shall assume, however, that only one type of machine is used for a long period of service.) The resultant production is used partly to satisfy demand, and partly for acquiring (or manufacturing) new machines. We are given the resources of labour per year, the number of machines available in the first year, and the quantity of production allotted to demand per year. It is required to construct a production plan in such a way that at the end of the given period the productive capacities are maximal:

- (1) We shall consider a four-year period.
- (2) In the first year 30 machines are available.
- (3) The labour resources and magnitudes of demand are given in Table IX.

(4) The various technical methods are given in Table X.

(5) Machines may be acquired to any amount required at a price of twenty units of production per machine. The acquisition and use of machines in the course of several years may also be described as one of the methods. These methods are given in Table XI.

TABLE IX. LABOUR RESOURCES AND DEMAND

Year	Labour resources (man-hours)	Demand (units of output)
I	100,000	1500
II	100,000	1600
III	100,000	1700
IV	100,000	1800

TABLE X. CURRENT PRODUCTION METHODS (ANNUAL)

Methods	Costs and products		
	Labour (man-hours)	Productive capacity (units)	Production (units)
1	-50,000	0	1000
2	-40,000	-20	1000
3	-30,000	-50	1000
4	-25,000	-70	1000

TABLE XI. PRODUCTION METHODS FOR A NUMBER OF YEARS

Methods	Costs and products							Machines at the end of the period
	Annual production				Annual productive capacity			
	I	II	III	IV	II	III	IV	
5. Machines acquired in year I	-20	—	—	—	1	1	1	1
6. Same in year II	—	-20	—	—	—	1	1	1
7. Same in year III	—	—	-20	—	—	—	1	1
8. Same in year IV	—	—	—	-20	—	—	—	1

In compiling an optimal long-term plan it is necessary to choose those methods which secure least costs per unit of production for the stated ratios of technical and labour resources. For this it is sufficient to reduce the present dynamic problem to the basic problem of production planning (*see* analysis of the dynamic problem in Appendix I, pp. 284-5) and to solve the latter problem by any of the methods described above.

TABLE XII. OPTIMAL LONG-TERM PLAN

Year	Production by the method				Expenditure on		Gross production	Acquisition of machines (use of methods 5-8)
	1	2	3	4	Labour	Productive capacity		
I	800	1500	—	—	100,000	30	2300	40
II	—	2071	571	—	100,000	70	2642	52
III	—	957	2057	—	100,000	122	3014	66
IV	—	—	2718	744	100,000	188	3462	83

In Table XII an optimal plan is given which corresponds to the specific conditions. In this plan the machines in stock by the end of the fourth year will number 271.

We shall not pause to consider the calculation of this plan, and we shall limit ourselves only to a verification of it being optimal. For this we construct o.d. valuations for a unit of labour and for a unit of output, and the hire valuations for the machines; these valuations correspond to each of the four years under consideration. We denote them respectively by  $T_1, T_2, T_3, T_4, P_1, P_2, P_3, P_4, M_1, M_2, M_3, M_4$  (these valuations are assumed to be converted to a common period of time; *see* p. 285). Let the valuation for a unit of work in the fourth year be  $T_4 = A$ . Starting from the fact that a unit of production is equal to the sum of the valuations of the expenditures, and that this sum is identical for all the methods used, we have

$$25,000T_4 + 70M_4 = 1000P_4,$$

$$30,000T_4 + 50M_4 = 1000P_4$$

or

$$T_4 = A, M_4 = 250.0A, P_4 = 42.5A.$$

In changing from the fourth year to the third, we must take into account the fact that a machine which is bought one year earlier

must have a higher valuation (generally, within the limits of each year, the valuation of a machine is equal to  $20P$ ). That is, in order to obtain the valuation for a machine in a particular year it is necessary to add its hire valuation for that year to its valuation in the following year. Hence, for the third year, the valuation of the machine equals

$$20P_4 + M_4 = 20 \times 42.5A + 250.0A = 1100A;$$

the valuation of production in the third year is then

$$P_3 = 1100A : 20 = 55A.$$

TABLE XIII. OBJECTIVELY DETERMINED VALUATIONS  
(CONVERTED TO THE FIRST YEAR)

Year	Valuation of production $P$	Valuation of labour $T$	Hire valuation $M$	$T:P$	Efficiency (%) (in terms of output units)
I	1.0000	0.02000	10.000	0.02000	36
II	0.7368	0.01579	5.263	0.02143	36
III	0.5429	0.01163	3.878	0.02144	32
IV	0.4100	0.00988	2.470	0.02353	—

By repeating this argument, we obtain all the remaining valuations for the third year, and then in turn those for the second and first years. Having chosen the proportionality factor  $A$  such that the valuation for one unit of production in the first year is equal to 1, we obtain a final table of valuations (Table XIII).

TABLE XIV. PRODUCTION AND COSTS FOR ONE-YEAR TECHNICAL METHODS (RELATIVE UNITS)

Year	Costs of method				Valuation of production
	1	2	3	4	
I	1000	1000	1100	1200	1000
II	790	737	737	763	737
III	582	543	543	562	543
IV	494	445	420	420	420

It is not difficult to verify (*see* Table XIV) that for all the one-year technical methods the valuation of production does not exceed

the valuation of expenditure, and that, for the methods used in the plan, these valuations are equal. This check may be made for the methods given in Table XI, from which it is clear that the valuations we have found are o.d. valuations and that the plan under consideration is optimal

In the last column of Table XIII the efficiency for each year (except the last) is shown. This norm is obtained by taking the production as the unit of measurement (for example, for the first year  $1.0000/0.7368 - 1 = 0.36$ , i.e. 36 per cent). We note that in other units of measurement the efficiency norm has other values. For example, if labour is taken as the unit of measurement, then for the first year the norm is  $0.02000/0.01579 - 1 = 0.27$ , i.e. 27 per cent.

We note the steady decrease of all valuations and the increase of the ratio of the labour valuation to the product valuations; these tendencies have a general character.

A distinguishing feature of dynamic problems is the existence of technical methods which last for several periods. The existence of dynamic valuations (Table XIII) allows one to produce a valuation of the efficiency of these methods even if they were not considered in compiling the original plan.

In the present example let there be a further, ninth, method which requires the following inputs for turning out 1000 units of production during the second year; 15,000 man-hours and 25 machines in the first year, and 5000 man-hours and 31 machines in the second year. In all cases we speak of the years after production by this method has begun.

We use the existing valuations for studying the efficiency of this method. In the first and second year the method is inappropriate since

$$15,000T_1 + 5000T_2 + 25M_1 + 31M_2 = 792.1 > 1000P_2 = 736.8;$$

in the second and third years it is also inappropriate, since

$$15,000T_2 + 5000T_3 + 25M_2 + 31M_3 = 546.8 > 1000P_3 = 542.9;$$

in the third and fourth years this method is advantageous, since

$$15,000T_3 + 5000T_4 + 25M_3 + 31M_4 = 397.4 < 1000P_4 = 420.0.$$

The use of this ninth method in the third and fourth years allows one to obtain a new plan (see Table XV) in which the technical



TABLE XV. OPTIMAL PLAN WITH THE USE OF A TWO-YEAR TECHNICAL METHOD

Year	Production by the method					Expenditure on		Gross production	Acquisition of machines (use of methods 5-8)
	1	2	3	4	9	Labour	Productive capacity		
I	800	1500	—	—	—	100,000	30	2300	40
II	—	2071	571	—	—	100,000	70	2642	52
III	—	957	1836	—	—	100,000	122	2793	55
IV	—	—	3264	—	442	100,000	177	3706	95

equipment reaches 272 machines by the end of the period. One may easily satisfy oneself that this plan is optimal by means of the new o.d. valuations given in Table XVI (in this the total valuation of expenses for a two-year method must be compared with the valuation of production in the year of its realization).

TABLE XVI. NEW O.D. VALUATIONS

Year	Valuation of production $P$	Valuation of labour $T$	Hiring valuation $M$
I	1.0000	0.02000	10.000
II	0.7368	0.01578	5.263
III	0.5429	0.01165	3.883
IV	0.4024	0.00872	2.816

### *The Use of Electronic Digital Machines*

As we have seen, if there are a few kinds of output and of factors of production, the construction of an optimal plan and its characteristics may easily be carried out by means of the computational methods which we have discussed without the use of auxiliary computational means, or at least with the use of only simple equipment such as a slide rule, a comptometer, or a desk calculating machine. However, when there are dozens of products, and more so in the case of hundreds and thousands of kinds, such calculations require modern computational methods as provided, for example, by electronic digital machines which can be suitably programmed. In this

case, the computational methods described in this appendix may be used with small modifications. The compilation of a programme (list of instructions) is carried out by the usual methods and involves no special difficulties. Trial calculations carried out on a Strela machine showed that with the use of electronic calculating machines problems with several dozens of different products may be solved in the course of a few minutes.

In this connection we must point out the importance of the combination of the use of electronic calculating machines with the improvement in the method of planned economic calculations resulting from the application of mathematical methods to these problems.

The use of electronic calculating machines, with retention of the usual methods in planned economic calculation, would lead to a certain speeding up of calculation, but will provide no essential improvement in the analysis, and will not enable one to remove the serious drawbacks from which it suffers. Furthermore, a machine cannot help in the supplementary qualitative considerations of the problem which, in many cases, enables one to correct deficiencies in the analysis. A systematic survey of all possible alternatives, or of a number of randomly taken alternatives, also cannot prove to be effective.

On the other hand, with a scientifically based method derived from a precisely stated economic problem and having an accurate and complete mathematical description, optimal plans may be solved directly by electronic machines; and this despite the considerable volume and difficulty of the necessary calculations.

## REFERENCES TO APPENDICES I and II

1. L. V. KANTOROVICH: *Mathematical methods for organizing and planning industry* (Mathematicheskiye metody organizatsiyi i planirovaniya proizvodstva). Leningrad State University Press (1939).
2. A. TOLSTOI: "Methods for eliminating irrationalities from transport planning", *Sotsialisticheskii Transport*, No. 9 (1939).
3. L. V. KANTOROVICH: "On the movement of masses", *Dokl. Akad. Nauk SSSR* 37, Nos. 7-8 (1942).
4. L. V. KANTOROVICH and M. K. GAVURIN: "The use of mathematical methods in analysing problems of goods transport", in the collection *Problems of Increasing Transport Efficiency* (Problemy povysheniya effektivnosti raboty transporta). Academy of Sciences Press (1949).
5. L. V. KANTOROVICH and V. A. ZALGALLER: *Calculation of rational cut-out of industrial materials* (Raschet ratsional'nogo raskroya promyshlennykh materialov). Lenizdat (1951).
6. V. A. ZALGALLER: *A new development in the composition of low saving consignments* (Novoye v sostavlenii postavov dlya raspilovki brevent) Central Scientific Research Laboratory "Sevzaples", Leningrad (1956).
7. G. SH. RUBINSTEIN: "A problem of a limiting point of intersection of a line with a polyhedron", *Dokl. Akad. Nauk SSSR* 100, 4 (1955). "Generalization of the problem of a limiting point", *Dokl. Akad. Nauk SSSR* 113, 5 (1957).
8. L. V. KANTOROVICH: "Methods for analysing certain extremal problems in production planning", *Dokl. Akad. Nauk SSSR* 115, 3 (1957).  
The basic results of this paper were reported at a scientific session at Leningrad State University, 12 May 1941.
9. T. C. KOOPMANS (Editor): *Activity analysis of production and allocation*. Wiley, New York (1951).
10. H. W. KUHN and A. W. TUCKER (Editors): "Linear inequalities and related systems", *Ann. Math. Studies*, 38. Princeton (1956).

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